Süleyman Demirel Üniversitesi Fen Edebiyat Fakültesi Fen Dergisi
Süleyman Demirel University Faculty of Arts and Sciences Journal of Science
2020, 15(2): 244-252
DOI: 10.29233/sdufeffd.785875
Atıf için / For Citation: U. Özdem, “Magnetic moment of Z_c(3900) as molecular state”, *Süleyman Demirel Üniversitesi Fen Edebiyat Fakültesi Fen Dergisi*, 15(2), 244-252, 2020.

Magnetic Moment of Z_c(3900) as Molecular State

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(Alınış / Received: 27.08.2020, Kabul / Accepted: 09.11.2020, Yayımlanma / Published: 29.11.2020)

Abstract: Employing light-cone QCD sum rule, we have acquired the magnetic moment of the Z_c(3900) resonance by considering it as the molecular form of $D\bar{D}^*$ and $D^*\bar{D}$ resonance with quantum numbers $J^{PC} = 1^{+-}$. The magnetic moment contains crucial knowledge about the internal organization of particles and their geometric configuration. Measurement of the magnetic moment of the Z_c(3900) resonance in future experiments can be quite useful comprehension the inner organization of exotic resonances. A comparison of our numerical values on the magnetic moment with those estimated by the other theoretical models existing in literature is carried out.

***Anahtar kelimeler**:* Işık konisi KRD toplam kuralları, Manyetik moment, Egzotik durumlar, Z_c(3900)

Z_c(3900) Parçacığının Molekül Durumundaki Manyetik Momenti

Özet: Z_c(3900) parçacığının $D\bar{D}^*$ ve $D^*\bar{D}$ molekül yapısında ve $J^{PC} = 1^{+-}$ kuantum sayılarına sahip olduğu varsayılarak manyetik momenti Işık konisi KRD toplam kuralları kullanılarak elde edilmiştir. Manyetik moment parçacıkların geometrik konfigürasyonu ve içyapısı hakkında önemli bilgiler içermektedir. Gelecekteki deneylerde Z_c(3900) parçacığının manyetik momentinin ölçülmesi, egzotik parçacıkların iç organizasyonunu anlamak için oldukça yararlı olabilir. Bu çalışmada elde edilen sonuçlar ile literatürde var olan farklı modeller kullanılarak elde edilen sonuçların karşılaştırılması yapılmıştır.

***Key words**:* Light-cone QCD sum rules, Magnetic moment, Exotic states, Z_c(3900)

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1. Introduction

Since 2003, many exotic states have been discovered experimentally, such as XYZ exotic states. The X, Y, Z are representing the axial-vector, vector and charged exotic states, respectively. Up to now, there are eight members in the family of the electrically charged exotic resonances: $Z_c(3900)$, $Z_c(4020)$, $Z_c(4200)$, $Z_c(4430)$, $Z_1(4050)$, $Z_2(4250)$, $Z_b(10610)$, and $Z_b(10650)$ discovered in decays into final resonances including a pair of heavy and light quarks [1–10]. Because of their electric charge they cannot be classified as conventional $\bar{Q}Q$ resonances, they have to be exotic resonances with minimal quark contents$\bar{Q}Q\bar{u}d$. As tetraquark resonances with quark contents, $\bar{c}c\bar{u}d/\bar{b}b\bar{u}$ these charged exotic resonances were generally investigated as hadronic molecules and diquark-antidiquark pictures. Recent experimental and theoretical progress about the exotic resonances can be found in Refs. [11–22]. The investigation of spectroscopic properties of the exotic states ensures us with helpful knowledge about the dynamics of quarks and gluons at the hadronic scale. Many theoretical approaches accurately estimate the spectroscopic parameters of the exotic states; however, the internal organization of this state is yet unknown. To put it another way, the spectroscopic parameters alone cannot separate the inner substructure of the exotic resonances. We need other tools to understand internal structure of these resonances. Investigations of the electromagnetic features of the exotic resonances can help us obtain important information about their charge radius and geometric shapes, the spatial distribution of electrical charge and magnetization inside them and finally their inner structure.

Magnetic moment (MM) of hadrons is one of the most significant properties in study of their electromagnetic structure, and it can ensure important knowledge about the dynamics of the QCD at low energy region. In this study, we consider the $Z_c(3900)$ to be the molecular picture with quantum numbers $J^{PC} = 1^{+-}$ and investigate its MM with the help of the LCSR (Hereafter, we will denote $Z_c(3900)$ resonance as Z_c). The LCSR is a powerful nonperturbative approach [23–25], and it has been widely and successfully used to investigate the various features of the conventional and exotic hadrons such as, electromagnetic multipole moments, form factors, strong decays, etc. [22, 26, 27]. This approach is on the basis of the operator product expansion near the light-cone and expansion is performed over the twists of the operators and the properties of the hadrons under examination are expressed associated with the properties of the vacuum and the distribution amplitudes of the particles. In the literature, there are a few studies where the MM of the Zc state is studied [28-30]. In Ref. [28], the magnetic and quadrupole moments of the $Z_c(3900)$ state have been obtained in the diquark-antidiquark picture in the framework of LCSR. In Ref. [29], the magnetic moment of the Z_c state have been extracted in the diquark-antidiquark configuration within QCD sum rules (QCDSR) in the external weak electromagnetic field. In Ref. [30], they obtained the magnetic moment of the Z_c state in the molecular picture by means of the QCDSR in the external weak electromagnetic field.

The paper is organized as follows: In Sec.2, we acquire the analytical calculations of the LCSR for the Z_c resonance. In Sec.3, we perform the numerical computations for the MM of the Z_c resonance and comprehensive discussions.

2. Material and Method

To acquire the MM of the Z_c resonance within the scope of the LCSR, we begin from the following correlator

$$\Pi_{\mu\nu}(p,q) = i\int d^4x\, e^{ipx} \langle 0|T\{j_\mu^{Z_c}(x) j_\nu^{Z_c\dagger}(0)|0\rangle_\gamma, \tag{1}$$

where J_μ is the interpolating current of the Z_c resonance with quantum numbers $J^{PC} = 1^{+-}$. The Z_c state is stay near the $D\bar{D}^*$ threshold, thus, it is one way to explain it as $D\bar{D}^*$ molecular scenario. In this study, we assume that Z_c state can be considered as the hadronic molecules composed of meson pairs. Based on this assumption, the molecular form of the interpolating current of the Z_c state is given as

$$j_\mu^{Z_c}(x) = \frac{1}{\sqrt{2}}\left\{[\bar{d}_a(x)i\gamma_5 c_a(x)][\bar{c}_b(x)\gamma_\mu u_b(x)] + [\bar{d}_a(x)\gamma_\mu c_a(x)][\bar{c}_b(x)i\gamma_5 u_b(x)]\right\}. \tag{2}$$

To evaluate the MM of the Z_c resonance in LCSR, we need to acquire the correlator in two different pictures. First, the related correlator is obtained in connection with the quark-gluon parameters, known as QCD representation. Second, the correlator is acquired with respect to the hadron features such as electromagnetic multipole moments, known as phenomenological representation. The QCD sum rules for the corresponding physical observables are acquired by equating these results of the representations of the correlator via the quark-hadron duality ansatz.

Inserting the complete set of states with the same quantum numbers as those of Z_c resonance, the phenomenological representation of the correlator is obtained as

$$\Pi_{\mu\nu}^{Ph}(p,q) = \frac{\langle 0|j_\mu^{Z_c}|Z_c(p)\rangle}{p^2 - m_{Z_c}^2} \langle Z_c(p)|Z_c(p+q)\rangle_\gamma \frac{\langle Z_c(p+q)|j_\nu^{Z_c\dagger}|0\rangle}{(p+q)^2 - m_{Z_c}^2} + ...$$

(3)

The matrix elements $\langle 0|j_\mu^{Z_c}|Z_c(p)\rangle$ and $\langle Z_c(p)|Z_c(p+q)\rangle_\gamma$ in Equation (3) are defined as follows:

$$\langle 0|j_\mu^{Z_c}|Z_c(p)\rangle = \lambda_{Z_c}\varepsilon_\mu^\theta, \tag{4}$$

$$\begin{aligned}\langle Z_c(p,\varepsilon^\theta)|Z_c(p+q,\varepsilon^\delta)\rangle_\gamma = -\varepsilon^\tau(\varepsilon^\theta)^\alpha(\varepsilon^\delta)^\beta\ &[G_1(Q^2)(2p+q)_\tau g_{\alpha\beta} \\ &+ G_2(Q^2)(g_{\tau\beta}q_\alpha - g_{\tau\alpha}q_\beta) \\ &- \frac{1}{2m_{Z_c}^2}G_3(Q^2)(2p+q)_\tau q_\alpha q_\beta],\end{aligned} \tag{5}$$

Where $G_1(Q^2)$, $G_2(Q^2)$ and $G_3(Q^2)$ are the electromagnetic form factors and λ_{Zc} residue of the Z_c resonance. The magnetic form factor ($F_M(Q^2)$) can be defined with respect to $G_2(Q^2)$ form factor as follows

$$F_M(Q^2) = G_2(Q^2). \tag{6}$$

At $Q^2 = 0$, the $F_M(Q^2 = 0)$ form factor is connected to the MM μ as

$$eF_M(Q^2) = 2m_{Z_c}\mu. \tag{7}$$

Inserting the matrix elements in Equations (4) and (5) into the correlator in Equation (3), we acquire the correlator with respect to the hadronic features as

$$\Pi^{Ph}_{\mu\nu}(p,q)=\frac{\lambda^2_{Z_c}}{[p^2-m^2_{Z_c}][(p+q)^2-m^2_{Z_c}]}[(q_\mu\varepsilon_\nu-q_\nu\varepsilon_\mu)+\frac{1}{2m^2_{Z_c}}\,p.\varepsilon(p_\mu q_\nu-p_\nu q_\mu)]$$
$$+\ \textit{other independent structures.}$$
(8)

For the QCD representation, one calculates the correlator, Equation (1), employing the interpolating current presented in Equation (2) clearly. To carry out the computations, first the possible contractions between the light and heavy quark fields are performed with the help of Wick's theorem. Consequently, we acquire

$$\begin{aligned}\Pi^{QCD}_{\mu\nu}(p,q)=-\frac{1}{2}\int d^4x e^{ipx}\langle 0|\{\,&Tr[\gamma_5 S_c^{aa'}(x)\gamma_5 S_d^{a'a}(-x)]Tr[\gamma_\mu S_c^{bb'}(x)\gamma_\nu S_d^{b'b}(-x)]\\ &+Tr[\gamma_5 S_c^{aa'}(x)\gamma_\nu S_d^{a'a}(-x)]Tr[\gamma_\mu S_u^{bb'}(x)\gamma_5 S_c^{b'b}(-x)]\\ &+Tr[\gamma_\mu S_c^{aa'}(x)\gamma_5 S_d^{a'a}(-x)]Tr[\gamma_5 S_u^{bb'}(x)\gamma_\nu S_c^{b'b}(-x)]\\ &+Tr[\gamma_\mu S_c^{aa'}(x)\gamma_\nu S_d^{a'a}(-x)]Tr[\gamma_5 S_u^{bb'}(x)\gamma_5 S_c^{b'b}(-x)]\}|0\rangle_\gamma\,,\end{aligned} \tag{9}$$

where $S_q(x)$ and $S_c(x)$ are light and c-quark propagators, respectively. In our computations for these propagators in x-space, we employed the following expressions

$$S_q(x)=i\frac{\not x}{2\pi^2x^4}-\frac{\bar{q}q}{12}\left(1+\frac{m_0^2x^2}{16}\right)+i\frac{g_s}{32\pi^2x^2}G^{\mu\nu}(x)[\not x\sigma_{\mu\nu}+\sigma_{\mu\nu}\not x], \tag{10}$$

$$\begin{aligned}S_c(x)=\frac{m_c^2}{4\pi^2}&\left[\frac{K_1(m_c\sqrt{-x^2})}{\sqrt{-x^2}}+\not x\frac{K_2(m_c\sqrt{-x^2})}{\left(\sqrt{-x^2}\right)^2}\right]-\frac{g_s m_c}{16\pi^2}\int_0^1 d\nu\, G^{\mu\nu}(\nu x)[(\not x\sigma_{\mu\nu}\\ &+\sigma_{\mu\nu}\not x)\frac{K_1(m_c\sqrt{-x^2})}{\sqrt{-x^2}}+2\sigma_{\mu\nu}K_0(m_c\sqrt{-x^2})],\end{aligned} \tag{11}$$

where K_i are modified Bessel functions of second kind, $G^{\mu\nu}(\nu x)$ is gluon field strength tensor.

The correlator in QCD representation includes two different contributions: perturbative and nonperturbative. Practically, perturbative contributions, the photon interacts with one of the quarks, can be computed by the replacing the one of the light or c-quark propagators by

$$S^{free}\rightarrow\int d^4z\, S^{free}(x-z)\not A(z)S^{free}(z), \tag{12}$$

Where

$$S_q^{free}(x)=i\frac{\not x}{2\pi^2x^4}-\frac{\bar{q}q}{12}\left(1+\frac{m_0^2x^2}{16}\right), \tag{13}$$

$$S_c^{free}(x) = \frac{m_c^2}{4\pi^2}\left[\frac{K_1(m_c\sqrt{-x^2})}{\sqrt{-x^2}} + \chi\frac{K_2(m_c\sqrt{-x^2})}{\left(\sqrt{-x^2}\right)^2}\right], \tag{14}$$

The left behind three propagators are the free ones. Nonperturbative contributions, the photon is radiated at long distances, can be computed by replacing one of the light quark propagators by

$$S_{\alpha\beta}^{ab} \to -\frac{1}{4}(\bar{q}^a\Gamma_i q^b)(\Gamma_i)_{\alpha\beta},$$

(15)

where Γ_i are full set of the Dirac matrices and left behind propagators are considered as full quark propagators. When Equation (15) is employed in computation of the nonperturbative contributions, we observe that matrix elements of the form $\langle\gamma(q)|\bar{q}(x)\Gamma_i q(0)|0\rangle$ and $\langle\gamma(q)|\bar{q}(x)\Gamma_i G_{\mu\nu}q(0)|0\rangle$ are required. These matrix elements are characterized associated with the photon distribution amplitudes (DAs) in Ref. [31]. We should be noted that the photon DAs expressions were calculated up to twist-4 accuracy. Therefore, the accuracy of our QCD side calculations are up to twist-4. The LCSR for the MM of Z_c resonance can be acquired by matching invariant amplitudes of a structure in $\Pi_{\mu\nu}^{QCD}(p)$ and $\Pi_{\mu\nu}^{Ph}(p)$. For this purpose, we choose the $\varepsilon_\nu q_\mu$ structure for MM. Then equating these two representations of the correlator via dispersion relation and carrying out double Borel transformations on the $-p^2$ and $-(p + q)^2$ to eliminate the contributions of higher resonances and continuum (the details of Borel transformations can be found in Refs. [32,33]). As a result, we get the requested LCSR for the MM as

$$\mu_{Z_c} = \frac{e^{m_{Z_c}^2/M^2}}{\lambda_{Z_c}^2}\Pi_{\mu\nu}^{QCD}.$$

(16)

The $\Pi_{\mu\nu}^{QCD}$ function is very lengthy and not illuminating, therefore the explicit expression of this function are not presented here.

3. Results

In this part of paper, numerical analysis of the magnetic moment of Z_c resonance will be acquired. In numerical calculations, we use $m_{Z_c} = 3899.0 \pm 3.6 \pm 4.9$ MeV, $m_c = 1.27 \mp 0.02$ GeV, $f_{3\gamma} = -0.0039$ GeV^2 [31], $\lambda_{Z_c} = m_{Z_c} f_{Z_c}$ with $f_{Z_c} = (0.69 \pm 0.21)\times 10^{-2}$ GeV^5 [34], $\langle g_s^2 G^2\rangle = 0.88$ GeV^4 [19], $\langle\bar{u}u\rangle = \langle\bar{d}d\rangle = (-0.24 \pm 0.01)^3$ GeV^3 [35], $m_0^2 = 0.8 \pm 0.1$ GeV^2 [35], and $\chi = 2.85 \pm 0.5$ GeV^{-2} [36]. The main input parameters of the LCSR are the DAs of the corresponding particle. The photon DAs and the parameters entering in their expressions are given in Ref. [31].

Furthermore above mentioned input parameters, the LCSR contain two extra arbitrary parameters: Borel mass parameter M^2 and the continuum threshold s_0. The physical observables, in our case MM, should be independent on these parameters. The working intervals of M^2 for the ground resonance is decided from the standard prescription; namely, both continuum contributions and power corrections in the sum rules must be suppressed in this interval. Numerical computations lead to the conclusion that both conditions are fulfilled in the range $4.0\ \mathrm{GeV}^2 \leq M^2 \leq 6.0\ \mathrm{GeV}^2$. The take into account of the pole dominance and operator product expansion convergence give rise to the following working interval for the s_0: $18.0\ \mathrm{GeV}^2 \leq s_0 \leq 20.0\ \mathrm{GeV}^2$. In Figure 1, we plot the MM of Z_c resonance as functions of M^2 and s_0. The results show good stability against the deviations of the arbitrary parameters as expected. Finally, we can extract the MM

$$\mu_{Z_c} = 0.60 \pm 0.25\ \mu_N.$$

The error in the result is because of the ambiguities of the input parameters and that of the arbitrary parameters. We should mention that the major source of ambiguities is because of the deviations of the results in connection with s_0. As we mentioned previously, there are a few studies in the literature where the electromagnetic properties of the Z_c states have been studied in the different quark configurations. The comparison of these results is presented in Table 1. When the obtained values are compared with those in the literature, it is seen that our results are very close to those obtained in Ref. [28]. In addition, it is seen that our results are consistent with the results obtained in Ref. [29] within the errors. However, there is a large discrepancy between results obtained using the molecular scenario in Ref. [30] and other results.

Table 1. Different results from theoretical approaches, which are LCSR [28], QCDSR [29,30] and our model (in unit of μ_N).

[28]	[29] Model-I	[29] Model-II	[30]	This Work
0.67 ± 0.32	$0.35^{+0.24}_{-0.19}$	$0.47^{+0.27}_{-0.22}$	$0.19^{+0.04}_{-0.01}$	0.60 ± 0.25

4. Consclusion and Comment

We have acquired the magnetic moment of the $Z_c(3900)$ resonance by considering it as the molecular form of $D\overline{D}^*$ and $D^*\overline{D}$ resonance with quantum numbers $J^{PC} = 1^{+-}$ in the framework of the light-cone QCD sum rule. The magnetic moment contains crucial knowledge about the internal organization of particles and their geometric configuration. While it is seen that the results obtained using the diquark-diquark scenario are consistent with each other within the errors, the results obtained in the molecule picture are quite different from each other. The results extracted using different approaches lead to different predictions, which can be used to distinguish these approximations. The seemingly measurable value of the magnetic moment we acquire show that this magnetic moment can assist a quite helpful tool for obtaining knowledge about the properties of the exotic resonances. But, the direct measurement of the of the magnetic moment of the $Z_c(3900)$ resonance is unlikely in the short run. Thus, any indirect estimation of the magnetic moment of the $Z_c(3900)$ resonance could be substantially functional.

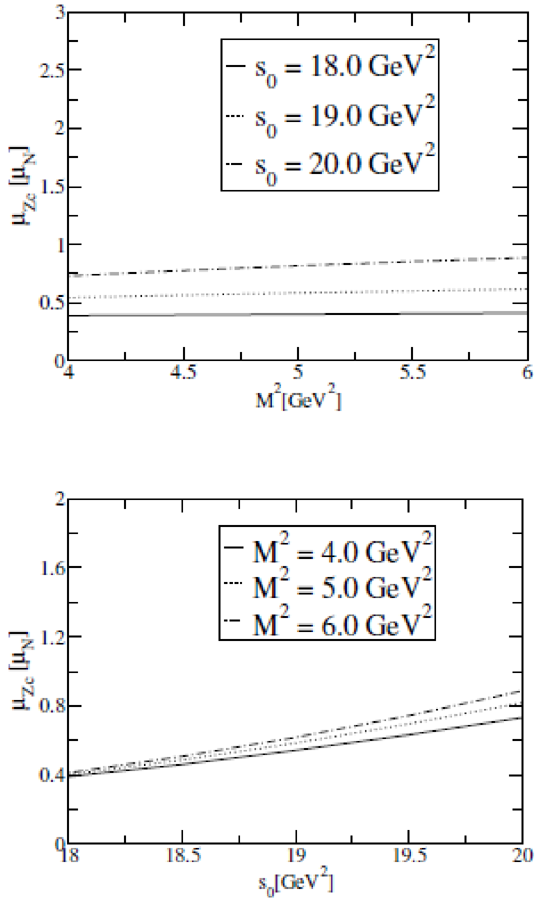

Figure 1. Variations of the MM μ_{Z_c} with M^2 and s_0.

Author Statement

Author Ulaş ÖZDEM: Conceptualization, Methodology, Investigation, Original Draft Writing, Review and Editing, Resource/Material/Instrument Supply.

Acknowledgement

As the author of this study, I declare that I do not have any support and thank you statement.

Conflict of Interest

As the author of this study, I declare that I do not have any conflict of interest statement.

Ethics Committee Approval and Informed Consent

As the author of this study, I declare that I do not have any ethics committee approval and/or informed consent statement.

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