Turkish J. Eng. Env. Sci.
34 (2010) , 201 – 213.

doi:10.3906/muh-1004-11

Investigation of undertow in reflective beaches using a GMDH-type neural network

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Received 05.04.2010

Abstract

Undertow is considered to be one of the dominant mechanisms in the erosion of beaches. In this paper, the distribution of undertow velocity is represented based on experimental evidence. A new polynomial model is suggested to calculate undertow velocity, based on experimental data from the Coastal Laboratory of Kagoshima University in Japan involving regular waves approaching natural and reflective beaches. This study addressed the question of whether GMDH-type neural networks could be used to estimate the undertow velocity based on specified variables. Results indicate that GMDH-type neural networks, in validation with the data obtained from the irregular wave experiments performed at the Hydraulic Laboratory of Imperial College (London, UK), provide an effective means of efficiently ($R^2 = 78\%$) recognizing the patterns in data and accurately predicting a performance index based on investigated inputs.

Key Words: GMDH, Erosion, Surf zone, Undertow, Reflective beach, Regular wave, Irregular wave

Introduction

Traditionally, undertow was thought to be to be a major control on shoreface erosion of a beach (Okayasu et al., 1990; Svendsen, 1984). In order to predict the sediment transport in the surf zone, it is necessary to estimate the cross-shore distribution of the undertow velocity. Although there exist advanced models, which predict the undertow velocity for natural beaches, surprisingly there have been only a limited number of works on estimating the undertow velocity in the case of reflective beaches, where partially standing waves are present (Neshaei, 1997; Neshaei et al., 2009a). Such beaches can be observed in front of reflective seawalls and natural steep slopes, particularly during storm conditions. The vertical and horizontal distributions of undertow velocity in front of a partially reflective seawall in a series of random wave experiments were measured by different researchers (Holmes and Neshaei, 1996; Mehrdad and Neshaei, 2004). Their investigation revealed that the magnitude of the undertow velocity is reduced in the presence of partially standing waves, which is in agreement with the work of Rakha and Kamphuis (1997) and Sato and Mitsunobu (1991) indicating a reduction in undertow velocity by reflected waves. In this paper, GMDH-type neural network modeling is applied to different sets of data to rectify the distribution of undertow velocity in the surf zone. The group method of data handling (GMDH) was first developed by Ivakhnenko (1971) as a multivariate analysis method for complex system modeling and identification, which can be used to model complex systems without having specific knowledge of the systems. The main idea of GMDH is to build an analytical function in a feed-forward network based on a quadratic node transfer function whose coefficients are obtained using the regression technique (Farlow, 1984). The results of the present work can be used for cross-shore sediment transport and beach evolution models where reflective conditions exist.

Theoretical Background

Okayasu et al. (1990) presented a model to describe the energy transfer under breaking waves by taking the energy of organized large vortexes and turbulence into account. The model estimates the dissipation rate and distribution of energy, and subsequently the cross-shore 2D distribution of the undertow velocity, as follows:

$$U = \frac{a'_\tau}{a_\nu}\left(z' - \frac{d_t}{2}\right) + \frac{a_\nu b'_\tau - a'_\tau \nu}{a_\nu^2}\left(1 + \log\frac{a_\nu z' + \nu}{a_\nu d_t + \nu} - \frac{\nu}{a_\nu d_t}\log\frac{a_\nu d_t + \nu}{\nu}\right) + U_m, \tag{1}$$

where:
U = undertow velocity at elevation z/ above the bed,
U_m = mean undertow velocity below the trough level,
v = kinematic viscosity of water,
d_t = water depth at wave trough,
and other parameters are coefficients related to the eddy viscosity of the wave.

Svendsen (1984) showed that undertow is driven by the local difference between radiation stress and the set-up pressure gradient, which only balance each other in average over the depth. He applied 2 aspects of the problem of incorporating the cross-shore circulation into mathematical-numerical nearshore models that also predict wave heights and set-up. One aspect is related to the proper choice of boundary conditions for the undertow velocity, and the other to the determination of the mean bottom shear stress, $\overline{T}_b$. Both methods also yield results that very accurately reproduce measured undertow velocity profiles and hence confirm the basic ideas. Finally, it was found that both methods can be used as parts of comprehensive nearshore models. The model of Svendsen (1984) reads as:

$$\frac{U - U_b}{\sqrt{gh}} = 0.5Ay^2 + \frac{2}{2r+1}\left(\frac{U_m}{\sqrt{gh}} - \frac{1}{6}A - \frac{U_s - U'_s\zeta_0}{\sqrt{gh}}\right) y\zeta_0 < \zeta < d_{tr}, \tag{2}$$

where:
U = undertow velocity,
ν_t = breaker-generated eddy viscosity of water,
U_m = mean undertow velocity below the trough level ($U_m = \frac{1}{d_{tr}}\int_0^{d_{tr}} U(\zeta)d\zeta$),
U_b = bottom velocity, and
A, r = coefficients related to the eddy viscosity and water depth.

Musumeci et al. (2005) developed a dynamic model of the wave propagation within the surf zone through a weakly dispersive, fully nonlinear Boussinesq-type model to predict the undertow velocity profile.

In this model, the velocity field was influenced by the effects of vorticity due to breaking, and the vorticity transport equation was solved analytically. Lin and Liu (2002) presented a one-dimensional model for undertow velocity and longshore current. The model predicted time- and depth-averaged undertow and longshore current velocities with longshore uniformity in depth, waves, and currents. It was calibrated with the field data obtained over longshore bars at Hazaki Oceanographical Research Station. Rakha and Kamphuis (1997) presented 2 formulations for the calculation of depth-integrated currents. The first formulation includes the additional mixing induced by the interaction between the undertow and the longshore current (UV). The second formulation assumes a higher eddy viscosity to replace this additional (UV) term.

Apart from these theoretical methodologies, the GMDH algorithm is a self-organizing approach by which gradually complicated models are generated based on the evaluation of their performances on a set of multi-input, single output data pairs (xi, yi) (i = 1, 2,..., M), explained in detail by Atashkari et al. (2007) and Nariman-zadeh et al. (2003, 2005).

Experiments

Hydraulic Laboratory of Imperial College, London, UK

To determine the velocity field inside the surf zone, laboratory experiments were performed in a large 2D wave tank at the Imperial College Hydraulics Laboratory. Figure 1 shows the experimental set-up. The overall

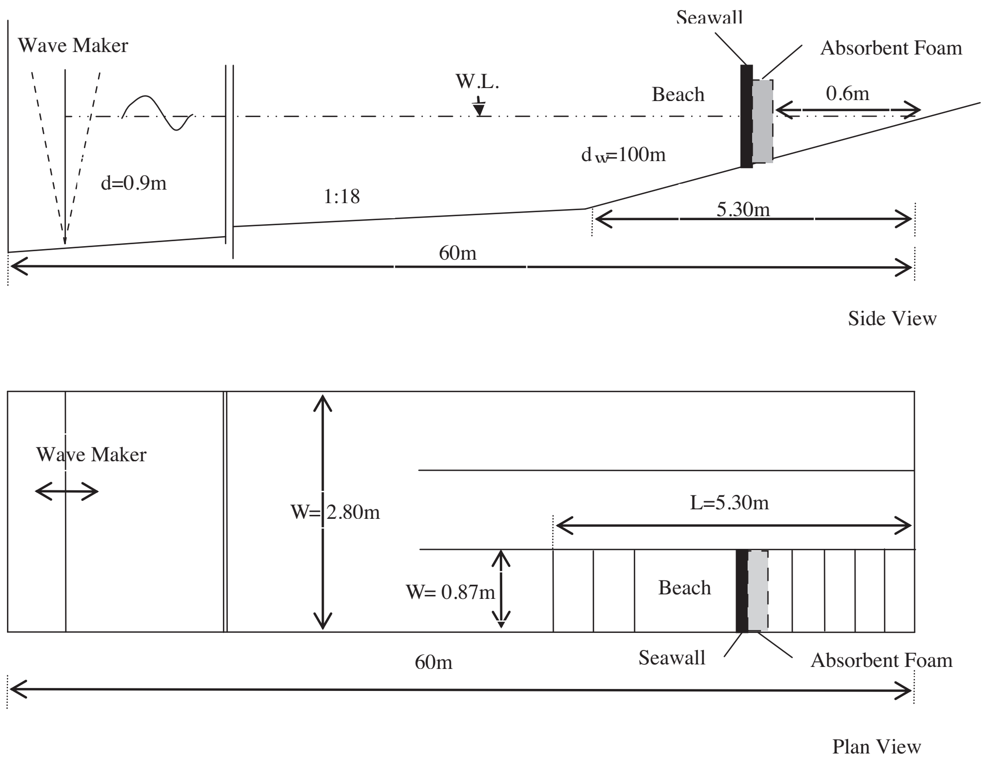

Figure 1. Illustration of experimental set-up; an absorbent foam was located behind a porous seawall, made of steel rods, to absorb the energy of incident waves.

dimensions of the tank were 2.80 m in width (which was divided into 3 sections), 1.5 m in depth, and 60 m in length. The width of the channel used in the experiments was 0.87 m. The water depth was 0.9 m for all experiments. A beach profile according to an equilibrium profile equation was built at the end of the tank. Previous studies have measured the undertow velocity under regular or random waves on a plane sloping beach. This study was different in that it measured the undertow velocity under random waves on a profile-type beach.

Using a typical JONSWAP spectrum, irregular waves were generated at one end of the tank generator, controlled by an electrohydraulic system that accepted an input voltage from a personal computer, and the water particle velocities were measured in the surf zone using a one-component fiber-optic laser Doppler anemometer (LDA). Table 1 summarizes the test conditions for this experiment (for more detail, see Holmes et al. (1996)). Four horizontal locations were chosen in the surf zone. At these locations, measurements were made at several points between the bottom and the still water level. The locations for all measured points are given in Table 2.

Table 1. Characteristics of random waves used in the experiments.

H_s (m)	T_z (s)	L_0 (m)	S_0
0.080	1.0	1.56	0.051
0.100	1.5	3.51	0.028
0.120	1.5	3.51	0.035

H_s = Significant wave height, T_z = Zero-crossing period L_0 = Deep water wave length, S_0 = Deep water wave steepness

Table 2. Specifications of measured points in random wave experiments.

Position	Distance from shoreline (m)	Water depth (m)	Elevation above the bed (mm)
1	0.6	0.097	5-25
2	0.9	0.1192	5-50
3	1.2	0.1438	5-55
4	1.8	0.187	5-65

In order to consider the effect of reflective structures on beach hydrodynamics, the experiment was repeated in front of a partially reflective seawall located in the surf zone. The main objective of this experiment was a quantitative comparison of near-bed velocities in the 2 cases, i.e. with and without the reflective structure (Holmes and Neshaei, 1996). For this purpose, a permeable seawall was built at the end of the beach. The exact distance of the seawall from the shoreline was 0.6 m, resulting in a water depth of 0.1 m in front of the structure. A sampling rate of 25 Hz with a record length of 6 min, similar to the first experiment, was selected to provide the suitable conditions in order to compare the velocity field in 2 different experiments. Figure 2 contrasts the vertical distribution of undertow on an open beach with that found in front of the reflective structure. It is clear that the presence of a permeable seawall with a reflection coefficient of about 30% caused a reduction in the magnitude of the undertow velocity in the surf zone.

Coastal Laboratory of Kagoshima University, Japan

In order to consider the effect of reflective beaches on the distribution of the mean flow, a series of experiments was performed in the Coastal Engineering Laboratory of Kagoshima University, Japan, and the magnitude and distribution of undertow velocity were obtained for different cases of reflective beaches (Hoque et al., 2001).

Figure 3 shows the experimental set-up. Monochromatic waves were generated and water particle velocities were measured in the surf zone using an electromagnetic current meter.

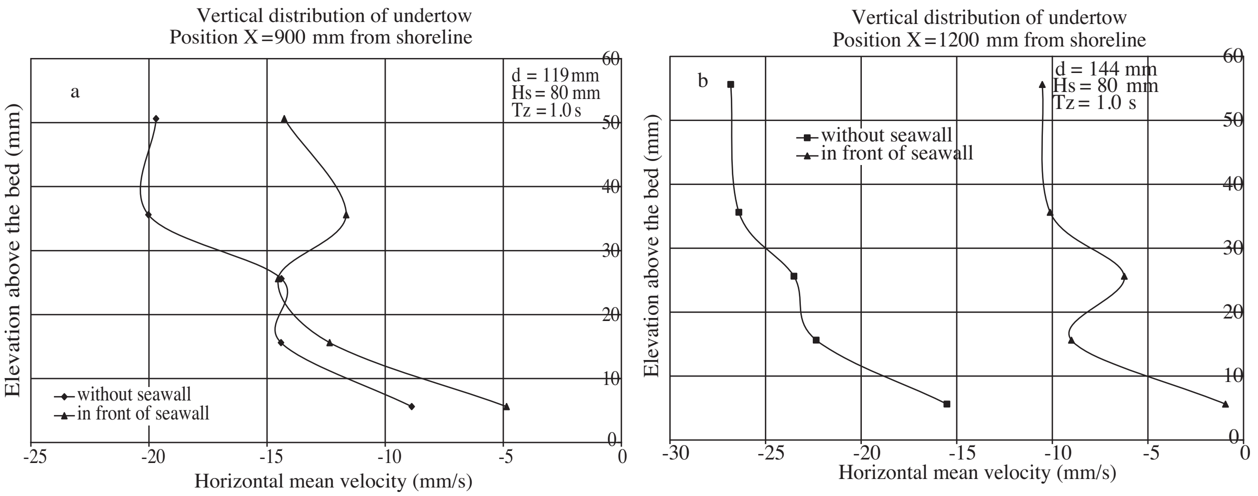

Figure 2. Comparison of vertical distribution of undertow in the surf zone between a natural beach and a beach fronting the seawall, a) 900 mm from the shoreline and b) 1200 mm from the shoreline (d = water depth).

The tank was 1.00 m wide, 1.2 m deep, and 26 m long. A plane beach profile with a constant slope of 1:20 was built at the end of the tank. Waves were generated at one end of the tank by a wave generator controlled by an electrohydraulic system, and the water particle velocities were measured in the surf zone using a 2-component electromagnetic current meter. Different horizontal locations were chosen in the surf zone, where the measurements were made at several points between the bottom and still water level. The locations for all measured points are given in Table 3. The measured points were in the middle of the flume (0.50 m from the inside face of the side-wall). Additionally, a resistance-type wave gauge measured the water surface elevation (synoptic with the velocity measurements) at each location. A deepwater wave gauge was mounted further offshore to measure the deep water incident wave spectrum. Data was acquired and analyzed using a personal computer with a sampling rate of 20 Hz for each channel. The recording length was 3.5 min, allowing approximately 200 waves into account. A solid reflective wall was placed at different locations across the surf zone, and the velocity measurements were repeated in front of the structure. Table 4 summarizes the different wave conditions (wave heights and wave periods) that were used in the experiments. The reflection coefficient of the beach was measured using the moving probe method to detect the envelope of partially standing waves formed in front of the seawall. The velocities were measured for 3 cases of seawall location, i.e. without seawall and with seawall located in the surf zone at water depths of 50 and 100 mm in front of the wall, respectively. The results were compared with those obtained from natural beaches with no reflection and existing theoretical models. The main objective was to undertake a quantitative comparison of the undertow velocity in 2 cases, with and without reflective conditions.

Figure 4 shows examples of comparison between the estimated (linear and nonlinear) and measured undertow velocities for different locations across the surf zone using different regular wave conditions. Calculated undertows were based on the model introduced by Neshaei et al. (2009b).

Table 3. Specifications of measured points for regular wave experiments.

Position	Distance from shoreline (m)	Water depth (m)	Elevations above the bed (mm)
1	1.00	0.0500	5-45
2	1.25	0.0625	5-55
3	1.50	0.0750	5-65
4	1.75	0.0875	5-75
5	2.00	0.1000	5-85
6	2.25	0.1125	5-95
7	2.50	0.1250	5-115
8	2.75	0.1375	5-125
9	3.00	0.1500	5-135
10	3.25	0.1625	5-155
11	3.50	0.1750	5-165
12	3.75	0.1875	5-175
13	4.00	0.2000	5-185
14	4.25	0.2125	5-195
15	4.50	0.2250	5-205
16	4.75	0.2375	5-225
17	5.00	0.2500	5-235

Table 4. Characteristics of monochromatic waves used in the experiments.

Deep water wave height (m)	Wave period (s)	Deep water wave length (m)	Deep water wave steepness
0.100	2.0	6.24	0.016
0.125	1.5	3.51	0.036
0.150	1.0	1.56	0.096

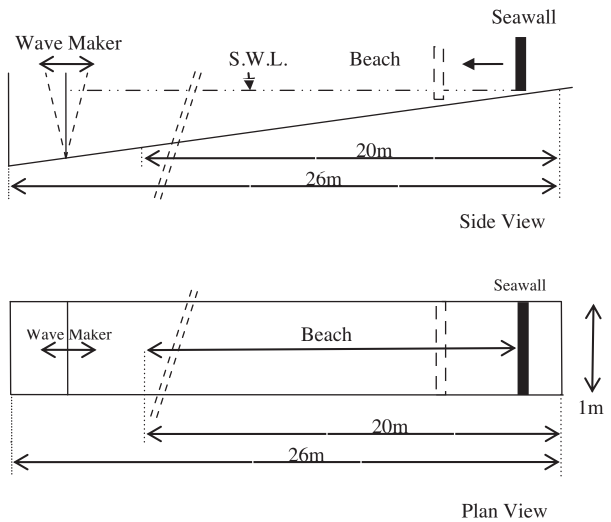

Figure 3. Illustration of the experimental set-up for regular wave experiments.

The work presented by Neshaei et al. (2009b) was a new modification of the originally based model developed by Okayasu et al. (1990). The main conceptual innovation of the model was taking the nonlinearity of waves, mass drift, and wave-current interaction into account in estimation of undertow velocity in the surf zone.

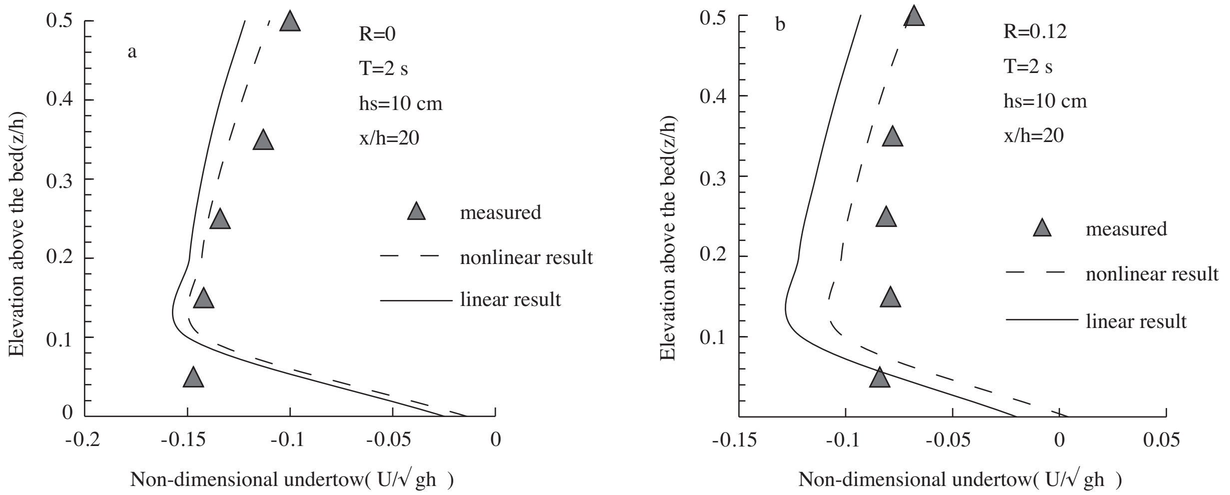

Figure 4. Comparison between calculated and measured undertow velocities for different reflection coefficients of the beach (R); a) R = 0, b) R = 0.12.

The presence of partially standing waves due to reflective conditions in the surf zone results in a reduction in the magnitude of undertow and changes its distribution across the surf zone. The level of reflectivity of the beach is an important parameter to control the magnitude and distribution of the undertow velocity. The results obtained from experiments and theoretical investigations showed that as the reflection coefficient of a beach increased, the magnitude of undertow decreased, which can affect the offshore sediment transport rate in the surf zone. This reduction was more pronounced for the inner surf zone points. The results obtained from regular wave experiments and the presented model were consistent and clearly support the conceptual elements of the proposed model to predict the undertow for reflective beaches.

In summary, comparison of the results by those obtained from the experiments clearly indicate that by taking into account the nonlinearity and wave-current interaction, the predictions of undertow in the surf zone were remarkably improved. Finally, it can be concluded that taking the experimental data into account for GMDH modeling can be supported by the basic conceptual idea reported by Neshaei et al. (2009b).

Principle of modeling Using GMDH-type neural Network By means of the GMDH algorithm, a model can be represented as a set of neurons in which different pairs in each layer are connected through a quadratic polynomial and, thus, produce new neurons in the next layer. The formal definition of the identification problem is to find a function $\hat{f}$ that can be approximately used instead of the actual one, f, in order to predict output $\hat{y}$ for a given input vector $X = (x_1, x_2, x_3, \ldots, x_n)$ as close as possible to its actual output y. Therefore, given M observations of multi-input, single output data pairs:

$$y_i = f(x_{i1}, x_{i2}, x_{i3}, \ldots, x_{in}) \quad i = 1, 2, 3, \ldots, M \tag{3}$$

It is now possible to train a GMDH-type neural network to predict the output values for any given input vector $X = x_{i1}, x_{i2}, x_{i3}, \ldots, x_{in}$, that is:

$$\widehat{y}_i = \widehat{f}\,(x_{i1}, x_{i2}, x_{i3}, \ldots, x_{in}) \quad i = 1, 2, 3, \ldots, M \tag{4}$$

The problem now is to determine a GMDH-type neural network such that the square of the differences between the actual output and the predicted one is minimized, that is:

$$\sum_{i=1}^{M} (\widehat{f}\,(x_{i1}, x_{i2}, x_{i3}, \ldots, x_{in}) - y_i)^2 \to \min \tag{5}$$

The general connection between the inputs and the output variables can be expressed by a complicated discrete form of the Volterra functional series in the form of:

$$y = a_0 + \sum_{i=1}^{n} a_i x_i + \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} x_i x_j + \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} a_{ijk} x_i x_j x_k + \cdots \tag{6}$$

This is known as the Kolmogorov-Gabor polynomial. This full form of mathematical description can be represented by a system of partial quadratic polynomials consisting of only 2 variables (neurons) in the form of:

$$\widehat{y} = G(x_i, x_j) = a_0 + a_1 x_i + a_2 x_j + a_3 x_i^2 + a_4 x_j^2 + a_5 x_i x_j \tag{7}$$

In this way, such a partial quadratic description is recursively used in a network of connected neurons to build the general mathematical relation of the inputs and output variables given in Eq. (6). The coefficients a_i in Eq. (7) are calculated using regression techniques, so that the difference between the actual output, y, and the calculated one, $\widehat{y}$, for each pair of x_i, y_i as input variables is minimized. Indeed, it can be seen that a tree of polynomials is constructed using the quadratic form given in Eq. (7), whose coefficients are obtained in a least squares sense. In this way, the coefficients of each quadratic function G_i are obtained to optimally fit the output in the whole set of input-output data pairs, that is:

$$E = \frac{\sum\limits_{i=1}^{M} (y_i - G_i())^2}{M} \to \min \tag{8}$$

In the basic form of the GMDH algorithm, all possibilities of 2 independent variables out of the total n input variables are taken in order to construct the regression polynomial in the form of Eq. (7) that best fits the dependent observations $(y_i, i = 1, 2, \ldots, M)$ in a least squares sense. Consequently, $\binom{n}{2} = \frac{n(n-1)}{2}$ neurons will be built up in the first hidden layer of the feed-forward network from the observations $\{(y_i, x_{ip}, x_{iq}); (i = 1, 2, \ldots, M)\}$ for different $p, q \in \{1, 2, \ldots, n\}$. In other words, it is now possible to construct M data triples $\{(y_i, x_{ip}, x_{iq}); (i = 1, 2, \ldots, M)\}$ from observations using such $p, q \in \{1, 2, \ldots, n\}$ in the form:

$$\begin{bmatrix} x_{1p} & x_{1p} & y_1 \\ x_{2p} & x_{2p} & y_2 \\ x_{Mp} & x_{Mp} & y_M \end{bmatrix}$$

Using the quadratic subexpression in the form of Eq. (7) for each row of M data triples, the following matrix equation can be readily obtained as

$$Aa = Y, \tag{9}$$

where a is the vector of unknown coefficients of the quadratic polynomial in Eq. (7):

$$a = \{a_0, a_1, a_2, a_3, a_4, a_5\}, \tag{10}$$

and

$$Y = \{y_1, y_2, y_3, \ldots, y_M\}^T \tag{11}$$

is the vector of the output's value from observation. It can be readily seen that:

$$A = \begin{bmatrix} 1 & x_{1p} & x_{1q} & x_{1p}x_{1q} & x_{1p}^2 & x_{1q}^2 \\ 1 & x_{2p} & x_{2q} & x_{2p}x_{2q} & x_{2p}^2 & x_{2q}^2 \\ 1 & x_{Mp} & x_{Mq} & x_{Mp}x_{Mq} & x_{Mp}^2 & x_{Mq}^2 \end{bmatrix} \tag{12}$$

The least squares technique from multiple regression analysis leads to the solution of the normal equations in the form of:

$$a = (A^T A)^{-1} A^T Y, \tag{13}$$

which determines the vector of the best coefficients of the quadratic Eq. (7) for the whole set of M data triples. It should be noted that this procedure is repeated for each neuron of the next hidden layer according to the connectivity topology of the network. However, such a solution directly from normal equations is rather susceptible to round-off errors and, more importantly, to the singularity of these equations (Nariman-Zadeh et al., 2005).

Modeling undertow velocity using a GMDH-type neural network In order to demonstrate the prediction ability of the evolved GMDH-type neural networks, experimental data were divided into 2 different sets, training and testing sets. The training set, which consisted of 178 input-output data pairs, was used for training the neural network model. The testing set, which consisted of 78 unforeseen input-output data samples during the training process, was merely used for testing to show the prediction ability of such evolved GMDH-type neural network models during the training process. Two hidden layers were considered for each model. To genetically design such neural networks, a population of 100 individuals with a crossover probability of 0.9, mutation probability of 0.1, and 300 generations was used. To illustrate neural network model-predicted performance in comparison with actual data, 100 data lines were selected (input-output) from the training set. The good behavior of such a GMDH-type neural network model is depicted in Figure 5. The corresponding polynomial representation of such a model for undertow velocity is as follows:

$$y_1 = 0.004 + 0.03x_1 - 0.004x_4 - 0.002x_1^2 - 0.033x_4^2 - 0.001x_1x_4 \tag{14}$$

$$y_2 = -0.22 + 0.07x_1 - 0.03x_2 - 0.004x_1^2 - 0.0002x_2^2 + 0.002x_1x_2 \tag{15}$$

$$y_3 = 0.005 - 0.014x_2 + 0.07x_5 + 0.0006x_2^2 + 0.141x_5^2 - 0.01x_2x_5 \tag{16}$$

$$y_4 = 0.04 + 1.48y_1 + 0.59y_2 + 7.78y_1^2 - 2.37y_2^2 + 1.39y_2 \tag{17}$$

$$y_5 = -0.041 - 1.115y_3 - 0.027x_3 - 24.49y_3^2 + 0.793x_3^2 - 2.53y_3x_3 \tag{18}$$

$$U = -0.02 + 0.282y_4 - 0.71y_5 + 5.02y_4^2 - 3.307y_5^2 - 16.63y_4y_5, \tag{19}$$

where $x_1,\ x_2,\ x_3, x_4 and x_5$ stand for H_s, d, R, T, and z/h, respectively.

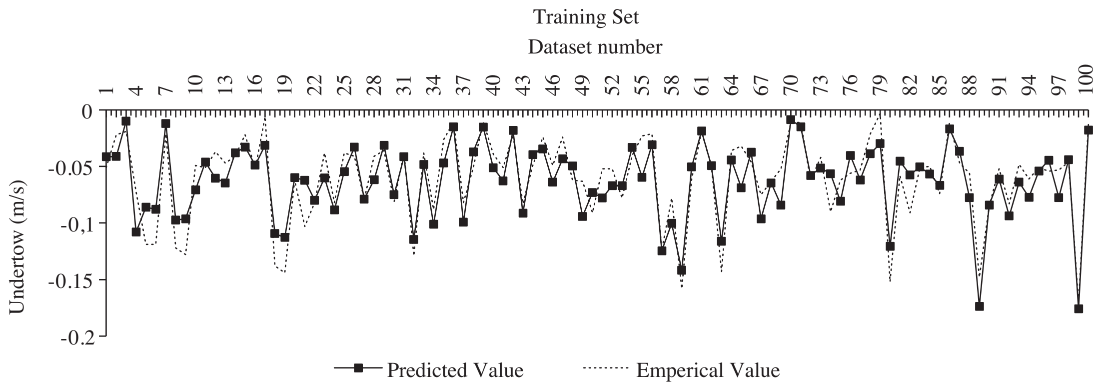

Figure 5. Neural network model predicted performance in comparison with actual data for the training set (100 input-output data).

The structure of the evolved 2-hidden-layer GMDH-type neural networks is shown in Figure 6, corresponding to the genome representations of adabbecc for undertow velocity in which a , b, c, d, and e stand for H_s, d, R, T, and z/h, respectively.

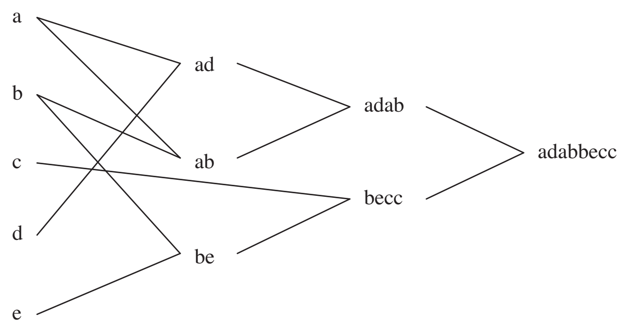

Figure 6. Evolved structure of generalized GMDH neural network for undertow velocity.

Some statistical measures are given in Table 5 to determine the accuracy of model. These statistical values are based on R^2 as an absolute fraction of variance, RMSE as root-mean squared error, MSE as mean squared error, and MAD as mean absolute deviation, which are defined as follows:

$$R^2 = 1 - \left[\frac{\sum_{i=0}^{M} \left(y_{i(Model)} - y_{i(Actual)}\right)^2}{\sum_{i=1}^{M} \left(y_{i(Actual)}\right)^2} \right] \tag{20}$$

$$RMSE = \left[\frac{\sum_{i=0}^{M} \left(y_{i(Model)} - y_{i(Actual)}\right)^2}{M} \right]^{1/2} \tag{21}$$

$$MSE = \frac{\sum_{i=0}^{M} \left(y_{i(Model)} - y_{i(Actual)}\right)^2}{M} \tag{22}$$

$$MAD = \frac{\sum_{i=1}^{M} \left|y_{i(Model)} - y_{i(Actual)}\right|}{M} \tag{23}$$

The obtained polynomial model was tested for the 100 unforeseen data during the training process, which accordingly demonstrated the prediction ability of the model. Figure 7 shows the comparison of such behavior with the actual values.

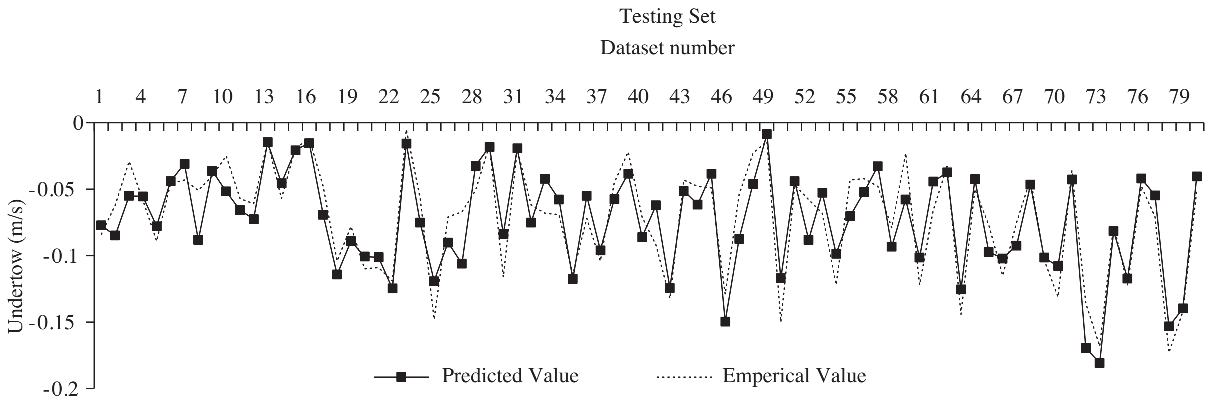

Figure 7. Neural network model predicted performance in comparison with actual data for the testing set (78 input-output data).

Validation of the proposed model The model was calibrated by a set of data obtained from the tests performed at the coastal laboratory of Kagoshima University in Japan (regular waves experiments) and validated by another set of data for random waves performed at the hydraulic laboratory of Imperial College, London, UK. The predictions were good enough to show the power of the GMDH method as a strong tool to evaluate such phenomenon. Figure 8 presents this method of comparison. In this figure, the predicted values for undertow

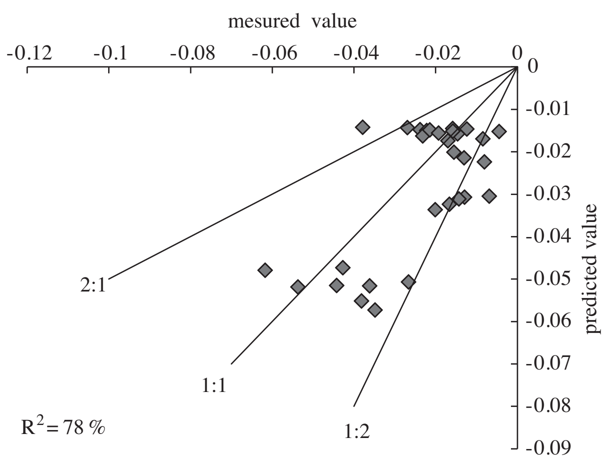

Figure 8. Validation of the present model.

velocity are compared to the random wave experimental data. As can be seen, most measured points lay within the upper and lower bands, and, particularly, most data are scattered around the middle line, indicating a good agreement.

From the engineering point of view, the result obtained from the present model can be used to estimate undertow velocity for reflective beaches with a reasonable level of accuracy. Using Eq. (19), it would be possible to predict the undertow velocity at different locations, particularly inside the surf zone, and consequently predict the amount of cross-shore sediment transport and the resulting beach profile evolution.

Conclusions

We attempted to deploy a powerful system identification technique to develop the undertow correlation over wave properties. Since the theoretical methods for calculating the undertow velocity for reflective beaches are highly complicated and indeterministic, applying soft computing methods such as GMDH-type neural networks could simplify the problem and estimate the distribution of undertow velocity with a reasonable level of accuracy. The evolved GMDH-type neural networks were successfully used to obtain a model for the prediction of undertow velocity. The result of the numerical simulation clearly indicates the power of the GMDH method to estimate the magnitude and distribution of undertow velocity at different points inside the surf zone. The R^2 value obtained was 78%, which shows a good correlation. The model can predict the measured data with a reasonable level of accuracy and can be further improved by adding more data. For future research, an investigation of the stability of the model concerning the error of measuring wave properties using genetic algorithm and the optimization process is proposed.

Nomenclatures

d	water depth
dt	water depth at wave trough
g	acceleration of gravity
H	wave height
H_b	wave height at the breaker point
H_s	significant wave height
L_0	deep water wave length
R	reflection coefficient of the beach
S_0	deep water wave steepness
T_z	zero-crossing period
U	undertow at elevation z′ from the bed
Ub	bottom velocity
Um	mean undertow below trough level
x/H	horizontal coordinate in cross-shore direction
z/h	relative depth
$\tan\beta$	bottom slope
γ	ratio of wave height to water depth
v	kinematic viscosity of water
v_t	breaker-generated eddy viscosity of water

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