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| Research Article / Araştırma Makalesi |

Levels of Elementary Mathematics Teacher Candidates Determination Levels of Image Sets of Functions in R^2 and R^3

İlköğretim Matematik Öğretmeni Adaylarının R^2 ve R^3'teki Fonksiyonların Görüntü Kümelerini Belirleme Düzeyleri

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Keywords
two variable function
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Abstract

Purpose: This study aimed to reveal the relationship between elementary school mathematics teacher candidates' determination levels of image sets of functions in R^2 and R^3.

Design/Methodology/Approach: This study was conducted with 49 elementary mathematics teacher candidates and the correlation design from quantitative approaches was used. For the given purpose, the data were collected by 2D and 3D tests. The 2D test was used to reveal the students' level of determining the image sets of the functions in R^2 and the 3D test was used to reveal that in R^3. In the data collection process, the graphics of the questions in 2D and 3D tests drawn with the support of GeoGebra were presented to the students together with the tests. Correlation analysis was used to compare the levels of students in determining image sets of functions in R^2 and R^3.

Findings: According to findings, it was found that there was a high level, positive, and significant relationship between the students' levels of determining the image sets of the functions in R^2 and R^3. Another conclusion about the study was that the students were more successful in determining the image sets of functions in R^3 than in R^2. This is thought to be a result of the dynamic feature of the GeoGebra software.

Highlights: It was observed that the GeoGebra program was important in determining the image set of a function, especially in R^3. For this reason it is thought that using activities designed with the GeoGebra program in related lessons can be effective in teaching two-variable functions.

Öz

Çalışmanın amacı: Bu çalışmanın amacı ilköğretim matematik öğretmeni adaylarının R^2 ve R^3'teki fonksiyonların görüntü kümelerini belirleme düzeyleri arasındaki ilişkinin ortaya konmasıdır.

Materyal ve Yöntem: Nicel yaklaşımlardan korelasyon deseninin kullanıldığı bu çalışma 49 ilköğretim matematik öğretmen adayıyla yürütülmüştür. Çalışmanın amacı doğrultusunda veriler, 2D ve 3D testi ile toplanmıştır. 2D testi öğrencilerin R^2'deki, 3D testi ise öğrencilerin R^3'teki fonksiyonların görüntü kümelerini belirleyebilme düzeylerini ortaya koymak için kullanılmıştır. Veri toplama sürecinde 2D ve 3D testindeki soruların GeoGebra desteğiyle çizilmiş grafikleri, testlerle birlikte öğrencilere sunulmuştur. Öğrencilerin R^2 ve R^3'teki fonksiyonların görüntü kümelerini belirleme düzeylerini karşılaştırmak için korelasyon analizi kullanılmıştır.

Bulgular: Elde edilen bulgulara göre öğrencilerin R^2 ve R^3'teki fonksiyonların görüntü kümelerini belirleme düzeyleri arasında yüksek düzey, pozitif yönlü ve anlamlı bir ilişki olduğu bulunmuştur. Çalışmaya dair bir diğer sonuç ise öğrencilerin R^3'teki fonksiyonların görüntü kümelerini belirlemede R^2'den daha başarılı olduklarıdır. GeoGebra programının dinamik özelliğinin bu sonucu doğurduğu düşünülmektedir.

Önemli Vurgular: GeoGebra programının, özellikle R^3'teki bir fonksiyonun görüntü kümesini belirleme sürecinde etkili olduğu görülmüştür. Bu nedenle iki değişkenli fonksiyonların öğretiminde, GeoGebra programıyla tasarlanmış etkinliklerin ilgili derslerde kullanılmasının etkili olabileceği düşünülmektedir.

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INTRODUCTION

There are numerous studies conducted in the field of mathematics which show that the concept of function is one of the most basic concepts, and that those who want to learn advanced mathematics will not be able to learn mathematics without fully understanding this concept (Eisenberg, 1992; Harel & Dubinsky, 1992; Selden & Selden, 1992; DeMarois, 1996; Hollar & Norwood, 1999; Yerushalmy, 2000; Bell, 2001; Kalchman, 2001; LeVeque, 2003; Sajka, 2003; Fest, Hiob-Viertler & Hoffkamp, 2011). According to Kleiner (1989), the concept of function is one of the elements that distinguishes modern mathematics from classical mathematics. For this reason, many studies have been carried out and continue to be conducted on the teaching of the concept of function. A study conducted by DeMarois and Tall (1996), stated that the concept of function had been the focus of the studies conducted by mathematics educators in the last 10 years. Considering that DeMarois and Tall's study was conducted in 1996, it can be said that the concept of function had an important place in mathematics education until the 2000s. Just as function is at the core of analysis, it is also a prerequisite for technology, science, and advanced mathematics (Johari, 1998). Hence, a student who hopes to be successful in the analysis course must primarily grasp the concept of function very well (Harel & Dubinsky, 1992). Due to the importance of the concept of function in school mathematics (NCTM, 2000), not only students taking the analysis course but also teachers need to know the basic features to make sense of the function (Cooney, Beckmann, & Lloyd, 2010).

Although the place and importance of the concept of function in mathematics are indisputable, it is also a fact that there are some problems regarding the teaching and learning of this concept. Becker (1991) stated that only a few of the concepts that make up school mathematics include the concept of function, and those very few concepts of school mathematics are misunderstood or not fully understood as the concept of function. After the concept of function was considered to have an important place in school mathematics (Selden & Selden, 1992), it was aimed to develop students' processes of understanding the concept of function that would prepare the precondition for other concepts in the analysis, especially in high school mathematics curricula (NGA / CCSS0, 2010). Using different representations in teaching the concepts of analysis contributes to making meaning of these concepts (Berry & Nyman, 2003). On the contrary, students experience difficulties in making sense of the concept of function because of the different representations of the function, which is one of the concepts of analysis, and the difficulty of establishing relationships between these representations (Sierpinska, 1992).

The concept of function is one of the concepts that students have difficulty in learning and misunderstand. Güveli and Güveli (2002) pointed out that alternative methods should be used in teaching this concept and raised the question of how we could provide a better education. Elia and Spyrou (2006) stated that students' definitions of functions, students' daily life examples of functions, and different representations of functions should be used in order for students to understand the concept of function better. It is argued that one of the most important factors in learning the analysis course concepts, including the function, is visualization (Darmadi, 2011; Darmadi, 2015). Today, computer technologies are used for visualization. With the abandonment of the use of computers as effective calculation tools, now they are started to be seen as tools that allow the concretizing of abstract concepts in the electronic environment (Baki, 2008).

Software engineers and educators have tried to integrate traditional teaching methods in mathematics into technology and they have made it. (Baki, 2001). For this purpose, countless studies have been conducted on technology-assisted mathematics teaching. In addition, the main purpose of the studies suggesting the use of technology in teaching the subject of function concerns how to teach the concept of function to the students with the help of computer or graphic plotters in a conceptual way (Ayers, Davis, Dubinsky & Lewin, 1988; Wilson & Krapfl, 1994; O'Callaghan, 1998; Schwarz & Hershkowitz, 1999; Saidah, 2000; Mackie, 2002; Patterson & Norwood, 2004; Rider, 2004; Fest, Hiob-Viertler & Hoffkamp, 2011). Zukhrufurrohmah (2018), who used technology to reveal the graphical properties of second-order functions, mentioned the necessity of creating learning steps in this process. Also, Fest, Hiob-Viertler and Hoffkamp (2011) stated that interactive learning is important in teaching function since functions can be explained with more than one representation, but they emphasized that feedback should be planned well in this process. Mai and Meyer (2018) used a program that includes an evaluation system that provides feedback for drawings of functions. Using common information and communication technologies such as EBA and Vitamin, Urhan, Kuh, Günal and Arkün-Kocadere (2018) conducted function teaching with these applications. As a result of the study, it was seen that this application made an important contribution to the learning and teaching process of functions. In the study of Taylor (2013), who used GeoGebra in function teaching, it was stated that this teaching positively affected students because GeoGebra supported visualization. Again in the same study, contrary to most studies, it was argued that teaching multiple representations of functions together with GeoGebra-assisted teaching was not beneficial.

With all these and similar studies, the concept of function was tried to be visualized, and it was aimed that students learn the concept of function from a conceptual perspective. It was observed that the functions that were the subject of these studies were generally one-variable functions. In studies conducted with twovariable functions, it has been tried to reveal how students made sense of these functions by using APOS theory (Trigueros & Martínez-Planell, 2010; Şefik, 2017). In another study conducted with two-variable functions, the conceptual knowledge levels of mathematics teacher candidates about limits in one- variable and two-variable functions were compared (Biber & Argün, 2015). In that study, the authors stated that the teacher candidates used the same situations they used to find the limit of one-variable functions when they were finding the limit of two-variable functions, if they included generalizations for the extension. However, it was observed that the teacher candidates could not generalize the information they used in one-variable functions to two-variable functions in situations requiring generalization for restructuring.

Kabael (2011) revealed that the level of understanding the concept of function regarding one-variable functions is important in creating the concept of two-variable functions. Yerushalmy (1997) attributed the students' inability to generalize one-variable functions to multivariable functions to the presence of multiple representations in functions. Weber and Thompson (2014) created a conceptual analysis scheme for the situations of extending students' existing graphic images to the graphs of two-variable functions. Within this scheme, which is called the predictive learning roadmap, it has been revealed that covariational reasoning plays an important role in students' generalization processes. In addition to that, Trigueros and Martínez-Planell (2010) stated that there were many studies on the visualization of functions, but there were very few studies on the visualization of two-variable functions. In addition to the scarcity of related studies, Martinez-Planell and Trigueros-Gaismann (2009) stated that students had difficulty in graphical representations of two-variable functions. Their study revealed that the geometric visualization performed on two-variable functions supported conceptual understanding in students. Therefore, it can be said that the visualization of two-variable functions plays an essential role in conceptual learning. Martínez-Planell and Trigueros-Gaisman (2012), on the other hand, presented a schematic structure for two-variable functions for these functions to be comprehended. In this schematic structure, which is based on moving the structure from R2 to R3, there are also domains and image included. Therefore, it is considered significant to reveal the relationship between determining the image set of a function in R^2 and R^3. Based on these, this study aimed to determine the relationship between elementary mathematics teacher candidates' levels of determining image sets of functions in R^2 and R^3. Within the framework of this purpose, the research questions of the study are given below.

- What are the levels of elementary mathematics teaching program third-year students in terms of determining the image sets of the functions in R^2?
- What are the levels of elementary mathematics teaching program third-year students in terms of determining the image sets of the functions in R^3?
- What is the relationship between the levels of elementary mathematics teaching program third-year students in terms of determining the image sets of the functions in R^2 and R^3?

METHOD

The correlational research design, which is one of the quantitative approaches, was used in this study since the relationship between the levels of elementary mathematics teacher candidates in terms of determining the image sets of the functions in R^2 and R^3 was investigated. There is no guidance and intervention in correlational research. In addition, the correlational research method tries to find out to what extent some types of relationships exist (Büyüköztürk et al., 2011). In this study, where there was no experimental intervention; the correlational research method was adopted since the relationship between students' mathematical information which they use to determine the image sets of the functions in R^2, and the mathematical information which they use to determine the image sets of the functions in R^3 was examined. In the simple correlation in the correlational research method, the results of two variables taken from the same group were compared, and then the correlation coefficient is determined by using them. While doing this, different data collection tools can be used (McMillan & Schumacher, 2006). Considering in this context, the simple correlational method was used in the study, since two data collection tools were used, and the correlation coefficient was determined.

Sample

The study sample consisted of 49 third-year students studying in the elementary mathematics teaching program. They attended all courses that could contribute to the formation of the concept of image sets in functions. Therefore, it can be thought that they had sufficient knowledge about the concept of image set in functions. They took courses that support three-dimensional thinking skills such as Geometry, Abstract Mathematics, Linear Algebra I, Linear Algebra II, and Introduction to Algebra. Besides, the majority of the students were over the age of twenty. Considering these, it can be said that the students readiness in three-dimensional thinking skills was at a sufficient level. Codes such as EK, EA, DA consisting of the initials of the names and surnames of the students who voluntarily participated in the study were given and used in the study.

Data Collection Tools

To identify the students' levels of determining the image sets of the functions in R^2, the 2D test was used, whereas the 3D test was employed to reveal their levels of determining the image sets of the functions in R^3 (Annex 1 and Annex 2). Prepared by two specialists, these tests were applied to a different group of 40 people. Each of the 2D and 3D tests finalized with this pilot application consists of ten open-ended questions. The questions under the same question numbers in the prepared tests contain similar function structures. The students were asked to determine the image set of the given function in R^2 in the 2D test and the image set of the given function in R^3 in the 3D test.

Data Analysis

The answers given by the students to the 2D and 3D tests were categorized as "correct", "partially correct," and "incorrect". These categories were scored as 0-5-10, respectively. When the questions in the tests are examined, the determined image sets consist of intervals, real numbers and single point sets. In the questions involving functions whose image set is an interval, those who can determine neither the starting value nor the ending value of the interval get 0 points, those who can determine only one get five points, and those who can identify both get 10 points. In the functions whose image set is real numbers, those who can determine the image sets as real numbers get 10 points, and those who cannot determine that get 0 points. If a student determined the image set algebraically but showed it incorrectly on the graph, this student's answer deserved five points. Those who determined the single point forming the image set were scored as 10, those who determined the image set consisting of that single point received five, and those who could not determine the image set at all got 0 points.

Descriptive statistics were used while analyzing 2D and 3D tests data. The scores received from the two tests were subjected to correlational analysis since the students' levels of determining the image sets of functions in R^2 and R^3 were compared in this study. Criteria were created to observe the change in each question in the tests. The scores received from the first and second tests were used while creating these criteria. For each question, these criteria are 0-0, 0-5, 0-10, 5-0, 5-5, 5-10, 10-0, 10-5, 10-10, as the first one is the score received from the 2D test and the second one is from the 3D test. These criteria, created for each question in the two tests, were included in the categories named C1, C2, … C9, respectively. Percentage and frequency were used for the analysis. In the study, statistical analyses were performed using the SPSS/PC package program.

Process

The data obtained were collected in two stages. In the first stage, the questions in the 2D test given in Appendix 1 and in the second stage, the questions in the 3D test were directed to the students. In the pilot application conducted before the data collection process, it was observed that the 30-minute time given to the students for each test was not sufficient. The time given for each test was then increased to 50 minutes.

In the first part of the study, the 2D test was administered to the students. The students were asked to determine the image sets of functions. The graph of the function for each question was included in the test. In addition, the graphs of functions created with GeoGebra were projected on the board simultaneously while the students were thinking about the answers to the questions. Students saw the function graphs for each question both on the test and on the board. No guidance was provided to the students during the application. They then wrote down the image sets they identified in the section left blank to write their answers on the test. A section of the application is included in Figure 1 during the answering of the 2D test.

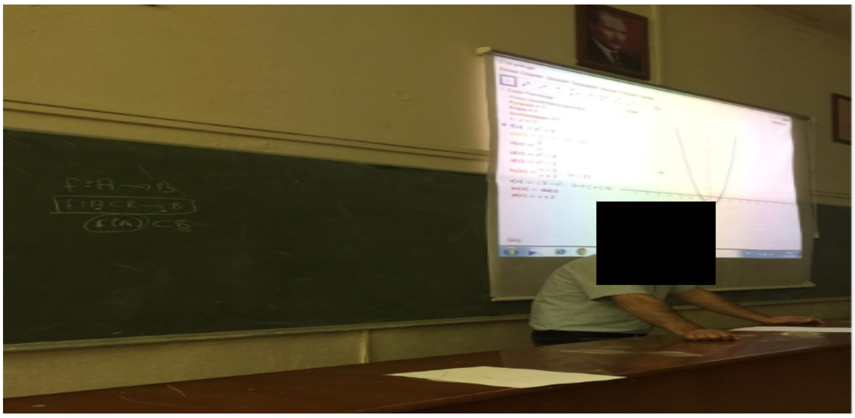

Figure 1. A section from the application moment of the 2D test

After the data for the 2D test was collected, the 3D test was distributed to the students. In this part, a similar process was followed. Unlike the first part, images of the function graphics in R^3drawn with the GeoGebra program from different angles were projected on the board. During the application, in which no guidance was provided, students saw a 3D graph of a function with this feature of the GeoGebra program. Students determined the image sets of functions. Afterwards, students were asked to write their answers to the place specified in the 3D test. An image of the moment the 3D test applied is presented in Figure 2.

Figure 2. A section from the application moment of the 3D test

FINDINGS

The scores obtained from the 2D and 3D tests were firstly subjected to descriptive statistics. The findings of the data analysis are presented in Table 1.

Table 1. Descriptive statistics regarding scores obtained from 2D and 3D tests

Tests	N	Minimum	Maximum	X	Sd	Level
2D Test	49	0	0	60.61	26.9	Partially Sufficient
3D Test	49	100	100	68.57	30.20	Sufficient

According to the findings obtained, some students received a minimum score of 0 and a maximum score of 100 in both tests. Since the tests were evaluated as "Insufficient", "Partially Sufficient" and "Sufficient", three groups were formed while forming a grouped frequency distribution. The range 0-33 was insufficient, 34-67 was partially adequate, and the range 68-100 was sufficient. In the total score evaluated out of 100, students' mean score on the 2D test was 60.61, while it was 68.57 on the 3D test. In this case, it is seen that the students were at a partially sufficient level in the 2D test and at a sufficient level in the 3D test. Therefore, it can be said that students were more successful at determining the image sets of the functions in R^3than those in $R^{2.}$

Correlation analysis was used to compare the students' levels of determining the image sets of the functions in R^2 and R^3. The findings of the correlation analysis are given in Table 2.

Table 2. Correlation analysis results for the relationship between students' 2D test scores and 3D test scores

		2D Test Score	3D Test Score
2D Test Score	r	1	.762**
	p		0.00
3D Test Score	r	.762**	1
	P	0.00	

(N:49) **. The correlation is significant at 0.01 level.

As shown in the table 2, there is a significant relationship between students' levels of determining the image sets of functions in R^2 and R^3. Büyüköztürk (2011) stated that the r-value obtained as a result of the correlation analysis being between 0.00 and 30 points to a low relationship, values between 30 and 70 indicate a moderate relationship, and values between 70 and 1.00 indicate a high-level relationship. Based on this information, when Table 2 was evaluated, it was found that there was a high level, positive, and significant relationship between the students' levels of determining the image sets of functions in R^2 and R^3 [r(49)=0.762; p<0.01]. Accordingly, students with high success in determining the image sets of the functions in R^2also had a high success in determining the image sets of the functions in R^3. Similarly, the students' low level of success when determining the image sets of the functions in R^2 was also low in R^3.

Functions with the same question numbers are similar. For example, the first question of the 2D test and the first question of the 3D test consist of similar functions. While students were asked to determine the image set of this function in R^2 in one question, they were asked to identify it in R^3 in the other. Based on the scores obtained from the two tests, a criterion were created to determine in which questions the students were able to transfer their mathematical knowledge from $R^2$to $R^{3.}$ For each question, these criteria were 0-0, 0-5, 0-10, 5-0, 5-5, 5-10, 10-0, 10-5, 10-10 as the first one was the score received from the 2D test and the second one was from the 3D test. These criteria were included in these criteria were included in the C1, C2, C3, C4, C5, C6, C7, C8 and C9 categories, respectively. The findings of the analysis are presented in Table 3.

Table 3. Frequency and percentage table regarding students' ability to transfer their mathematical information for each question

QUESTION	1	2	3	4	5	6	7	8	9	10
CATEGORY	frequency (%)	frequency (%)	frequency (%)	frequency (%)	frequency (%)	frequency (%)	frequency (%)	frequency (%)	frequency (%)	frequency (%)
C1	6 (2.9)	7 (3.43)	7 (3.43)	4 (1.96)	6 (2.94)	8 (3.92)	11 (5.39)	7 (3.43)	12 (5.88)	7 (3.43)
C2	2 (0.98)	1 (0.49)	0 (0)	3 (1.47)	3 (1.47)	0 (0)	2 (0.98)	0 (0)	1 (0.49)	1 (0.49)
C3	6 (2.94)	5 (2.45)	6 (2.94)	0 (0)	6 (2.94)	2 (0.98)	12 (5.88)	4 (1.96)	7 (3.43)	6 (2.94)
C4	2 (0.98)	3 (1.47)	0 (0)	2 (0.98)	4 (1.96)	1 (0.49)	0 (0)	6 (2.94)	6 (2.94)	0 (0)
C5	3 (1.47)	0 (0)	0 (0)	2 (0.98)	1 (0.49)	7 (3.43)	1 (0.49)	1 (0.49)	9 (4.41)	1 (0.49)
C6	9 (4.41)	2 (0.98)	1 (2.94)	2 (0.98)	1 (0.49)	4 (1.96)	8 (3.92)	6 (2.94)	3 (1.47)	7 (3.43)

QUESTION	1	2	3	4	5	6	7	8	9	10
CATEGORY	frequency (%)	frequency (%)	frequency (%)	frequency (%)	frequency (%)	frequency (%)	frequency (%)	frequency (%)	frequency (%)	frequency (%)
C7	1 (2.94)	1 (0.49)	5 (2.45)	6 (2.94)	2 (0.98)	4 (1.96)	2 (0.98)	0 (0)	2 (0.98)	2 (0.98)
C8	7 (3.43)	1 (0.49)	0 (0)	3 (1.47)	2 (0.98)	3 (1.47)	3 (1.47)	0 (0)	3 (1.47)	0 (0)
C9	13 (6.37)	29 (14.21)	30 (14.7)	27 (13.23)	14 (6.86)	20 (9.8)	10 (4.9)	25 (12.25)	6 (2.94)	25 (12.25)

As shown in the table 3, the students were mostly in the C1 and C9 categories. This situation supports the determined correlation. This situation can be interpreted as "the students who were successful in determining the image set in R^2 also had a high success in R^3, and that the students who were unsuccessful in R^2 also had low success regarding R^3."

When category C9 was evaluated, it was seen that the students were able to transfer their mathematical knowledge in question 3 the most. The third question in both tests was related to linear functions. It can be said that the students were successful in determining the image sets of the functions both in R^2 and R^3. The graphics of the linear functions being easier to comprehend might be the reason for this fact.

When the answers of the students in category C1 were examined, it was found that the students had the most difficulty in determining the image sets of the functions in the ninth question. While the function in the 2D test was $f(x) = \sqrt{(1 - x^2)}, -1 \leq x \leq 1$, the function in the 3D test was $f(x, y) = \sqrt{(1 - x^2 - y^2)}, -1 \leq x \leq 1, -1 \leq y \leq 1$. With the help of the GeoGebra program, the graph of the function in the 3D test was shown to the students from different angles. However, it was observed that the students who could not determine the image set of the function in the 2D test could not determine the image set of the function in the 3D test, either. The solutions of HZ to the ninth question are given in Figure 3 and Figure 4.

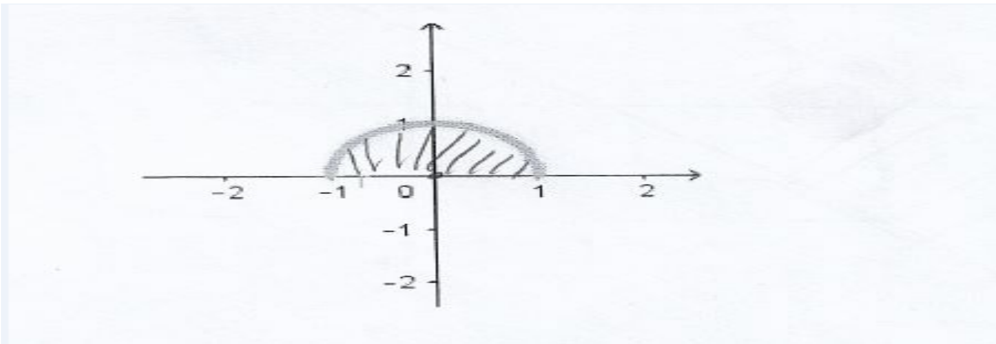

Figure 3. Solution of HZ to the ninth question in the 2D test

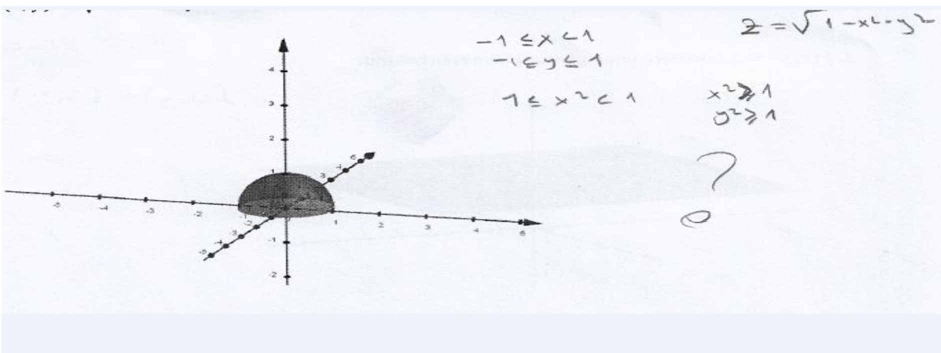

Figure 4. Solution of HZ to the ninth question in the 3D test

As shown in the figure 3, HZ indicated the image set as an area, similar to the study of Özkaya and İşleyen (2012). In Figure 4, similar to the other students who gave a wrong answer to this question, HZ tried to find the image set by algebraic operations without examining the function graph. Considering this situation, it can be said that the students did not use visualization, which is an important step in three dimensions. It can be considered that the students used the same processes that they used in determining the image set in R^2 and R^3 as well.

DISCUSSION

This study aimed to determine the relationship between the levels of elementary mathematics teacher candidates in identifying the image sets in R^2 and R^3. In this context, descriptive statistics were first employed. Statistical analysis revealed that while the students were partially sufficient in terms of determining the image sets of the functions in R^2, they were sufficient in determining those in R^3. In this case, it can be claimed that students were more successful in determining the image sets of the functions in R^3than in R^2. The underlying reason for this situation can be that the dynamic feature of GeoGebra program was used to display the function graphs in $R^3$from different angles. Considering the literature, it was emphasized that GeoGebra efficiently provides effective mathematics teaching (Hohenwarter, Preiner & Yi, 2007) and visualizing the concepts (Hohenwarter, Preiner, & Yi, 2007; Guncaga & Majherova, 2012). In addition, it was observed that students had difficulty making sense of geometric processes while performing algebraic processes for the concept of function (NCTM, 2000) included in the field of algebra learning (Berry & Nyman, 2003). The situation in question can explain why the students were not sufficient to determine the image sets of the functions both in R^2 and R^3. In the context of the answers given to the 2D test, it was determined that most of the students followed a correct process while determining the image sets. This result is incompatible with the study of Özkaya and İşleyen (2012). In their study, they determined that most of the first-year elementary mathematics teaching students had misconceptions while determining the domain and image sets of the functions in R^2. In this study, only one of the misconceptions identified was "Specifying the domain and image set as an area under or above the graphic." It is believed that the reason why the situations stated as misconceptions were so rare in this study is the GeoGebra application. In the context of the answers given to the 3D test, most of the students performed similar solutions to the solutions they found in the 2D test. This finding in the study is similar to the study of Martínez-Planell and Trigueros-Gaisman (2012). These authors put forth a structure for better interpretation of two-variable functions and mentioned the importance of transferring the structure in R^2 to R^3. Considering that visualization is important in two-variable functions (Trigueros & Martínez-Planell, 2010), it can be said that the GeoGebra program used in this study contributed to the students' success in determining the image sets of the functions included in the 3D test. In addition, it is understood that the students left the geometric representation aside because they focused on the algebraic representation. This is thought to be because, as Elia and Spyrou (2006) stated, the algebraic representation of the function is more understandable than its geometric representation. This situation shows that students could not think of the transition between the algebraic and geometric representation of functions although GeoGebra was used to support the visual. This result coincides with the study of Nagel (1994) who, revealed that transitions between different representations of functions could not be provided by technology.

CONCLUSION AND RECOMMENDATIONS

According to the data obtained, the students were more successful in determining the image sets of the functions in R^3 than in R^2 due to the dynamic feature of the GeoGebra program. It was observed that there was a high level of a positive and significant relationship between the students' levels of determining the image sets of the functions in R^2 and R^3. That is to say, a student who succeeded in determining the image set of the function in R^2 was also successful in R^3, and a student who failed to determine the image set of the function in R^2 also failed in R^3. It was observed that most of the students used the same approach they used when finding the image sets of the functions in R^2 and R^3.

This study determined that there was a parallelism between the students' levels of determining the image set of a function in R^2 and R^3. It was also observed that the GeoGebra program was important in determining the image set of a function, especially in R^3. Considering both the conclusions of the study and the effect of visualization in teaching the concept of function (Fest, Hiob, & Hoffkamp, 2011; Taylor, 2013; Mai & Meyer, 2018; Zukhrufurrohmah, 2018), future researchers may seek an answer to the question "What is the effect of a teaching designed with the GeoGebra program on making sense of two-variable functions?"

Ethics Committee Approval Information

The article data were collected in 2019, and there is no conflict of interest between the authors. It is declared that the practices were carried out in accordance with all ethical rules and publication ethics have been observed carefully. The authors received no financial support for the research, authorship and publication of this article.

Authors' Contribution

M.Ö. and T.İ. carried out misconceptions related to functions study. So M.Ö. and T.İ. conceived of the presented idea. They collected data together. All authors discussed the results and contributed to the final manuscript. The study was conducted and reported with equal collaboration of the researchers.

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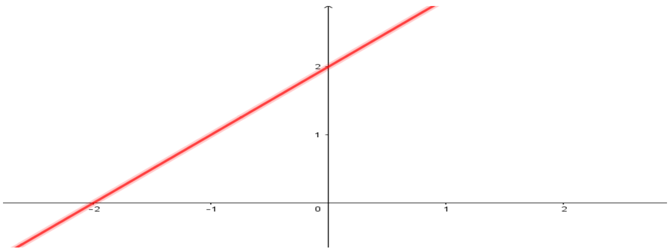
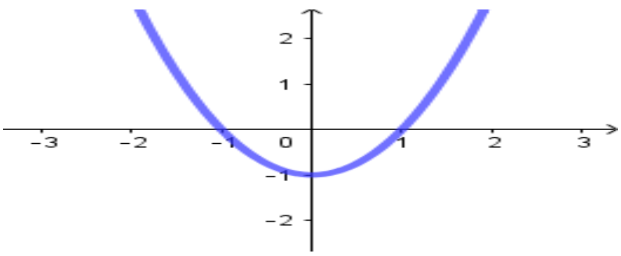
Appendix 1. 2D Test

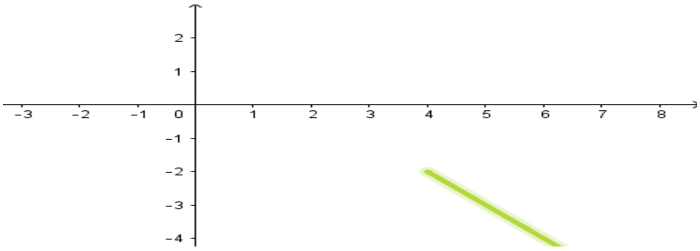
1. Find the image set of the function f(x)=x+2.



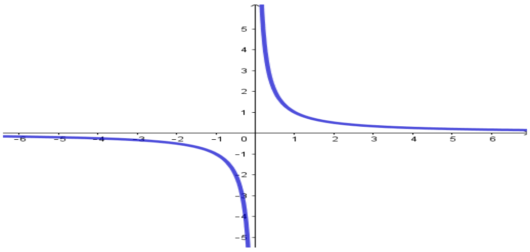
2.Find the image set of the function $f(x) = x^2$-1.



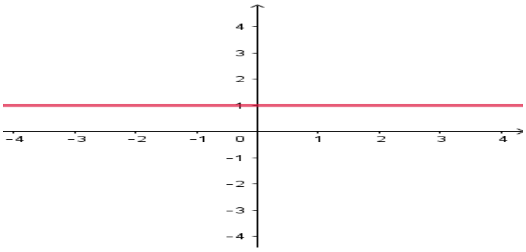
3. Find the image set of the function f(x)= 2-x, x≥4.



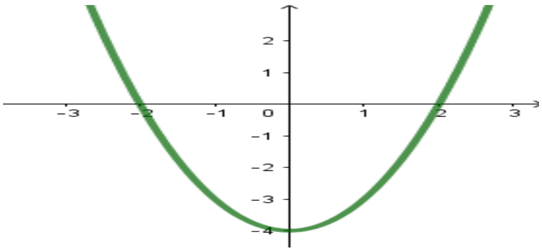
4. Find the image set of the function $f(x) = \frac{1}{x}$, x≠0.



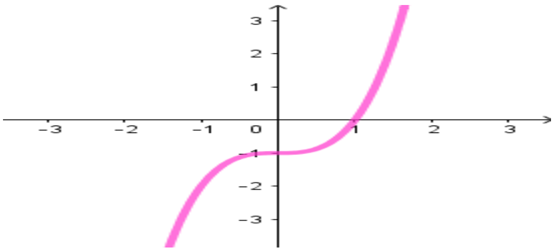
5. Find the image set of the function f(x) = 1.



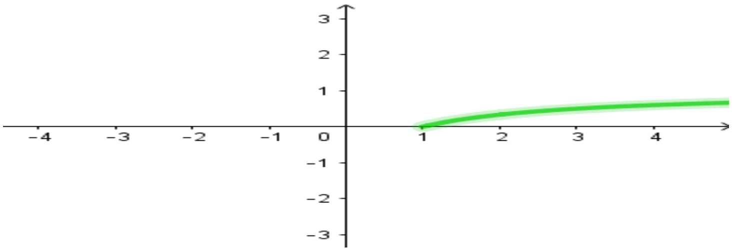
6. Find the image set of the function $f(x) = x^2$-4.



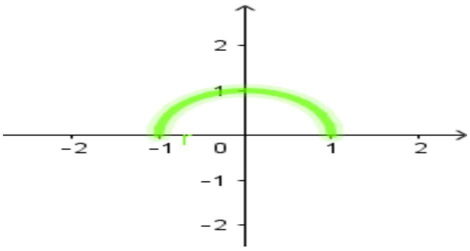
7. Find the image set of the function $f(x) = x^3 - 1$.



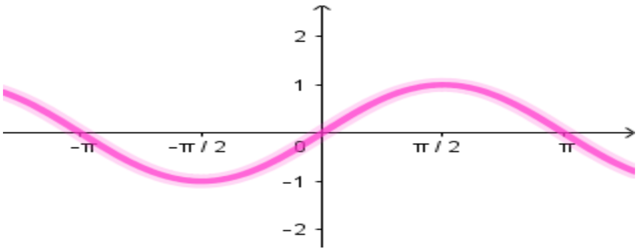
8. Find the image set of the function $f(x) = \frac{x-1}{x+1}$, x≥1.



9. Find the image set of the function f(x)= $\sqrt{1-x^2}$, -1≤ x ≤ 1.

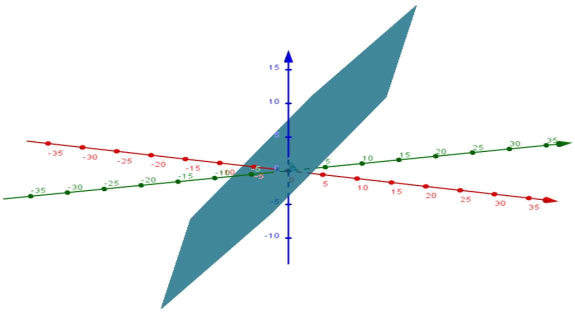


10. Find the image set of the function f(x)= sinx.

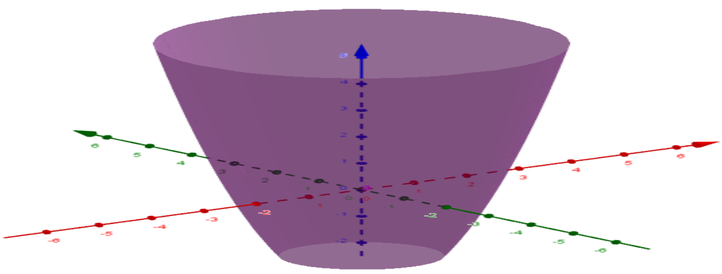


Appendix 2. 3D Test

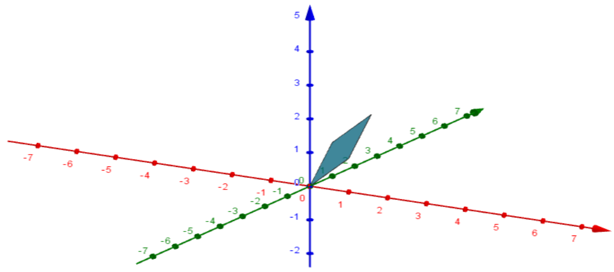
1. Find the image set of the function $f(x,y) = x + y + 2$.



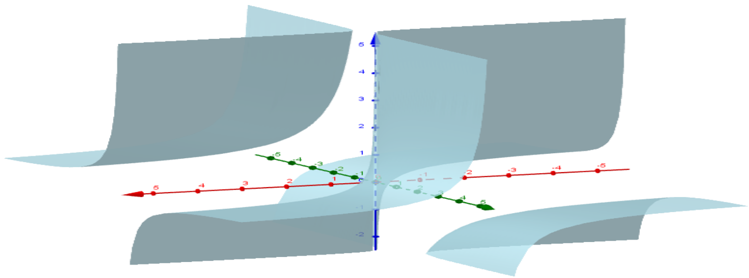
2. Find the image set of the function $f(x,y) = x^2 + y^2 - 4$.



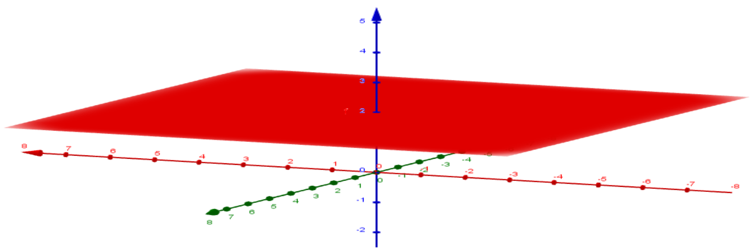
3. Find the image set of the function $f(x,y) = x + y\,, 0 \ \leq x \leq 1, 0 \leq y \leq 1.$



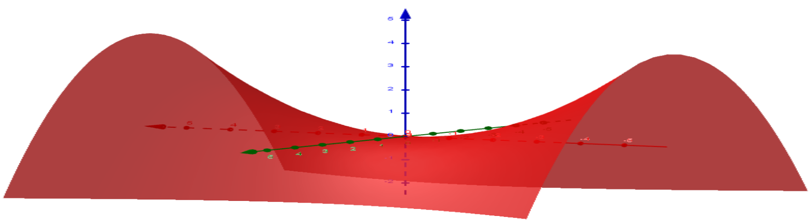
4. Find the image set of the function $f(x,y) = \frac{1}{x} - \frac{1}{y}\,, x \neq 0, y \neq 0.$



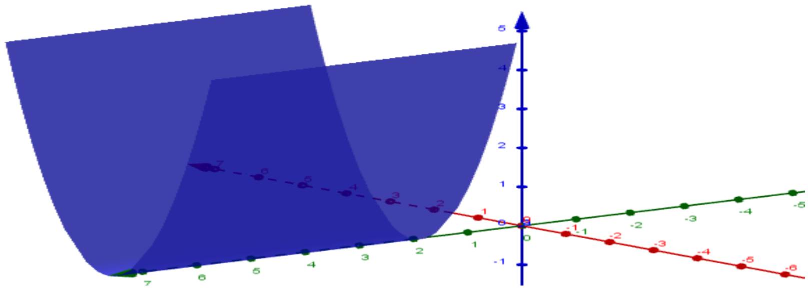
5. Find the image set of the function $f(x,y) = 2.$



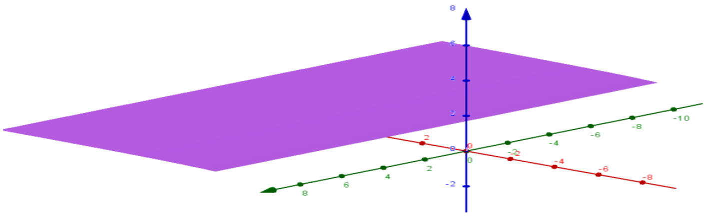
6. Find the image set of the function $f(x,y) = \frac{x^2}{9} - \frac{y^2}{4}.$



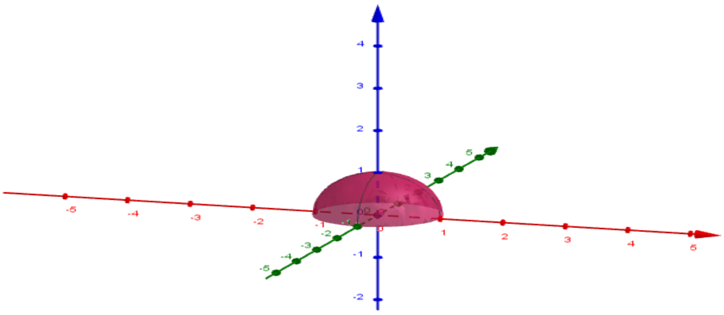
7. Find the image set of the function $f(x,y) = x^2, y \geq 2.$



8. Find the image set of the function $f(x,y) = \frac{x+1}{x+2}, x \geq 2.$



9. Find the image set of the function $f(x,y) = \sqrt{1 - x^2 - y^2}, -1 \leq x \leq 1, -1 \leq y \leq 1.$



10. Find the image set of the function $f(x,y) = sinx + siny.$