SAKARYA ÜNİVERSİTESİ

FEN BİLİMLERİ ENSTİTÜSÜ DERGİSİ

Sakarya University Journal of Science
SAUJS

e-ISSN 2147-835X | Period Bimonthly | Founded: 1997 | Publisher Sakarya University |
http://www.saujs.sakarya.edu.tr/en/

Title: Solution of Test Problems with Grey Wolf Optimization Algorithm and Comparison with Particle Swarm Optimization

Authors: Alper KÖYBAŞI, İrfan YAZICI

Recieved: 2020-09-02 08:45:09

Accepted: 2020-09-15 18:34:31

Article Type: Research Article

Volume: 24
Issue: 6
Month: December
Year: 2020
Pages: 1252-1264

How to cite
Alper KÖYBAŞI, İrfan YAZICI; (2020), Solution of Test Problems with Grey Wolf Optimization Algorithm and Comparison with Particle Swarm Optimization. Sakarya University Journal of Science, 24(6), 1252-1264, DOI: https://doi.org/10.16984/saufenbilder.788681
Access link
http://www.saujs.sakarya.edu.tr/en/pub/issue/57766/788681

Sakarya University Journal of Science 24(6), 1252-1264, 2020

SAKARYA ÜNİVERSİTESİ

Solution of Test Problems with Grey Wolf Optimization Algorithm and Comparison with Particle Swarm Optimization

Alper KÖYBAŞI*[1], İrfan YAZICI[2]

Abstract

In this study, Grey Wolf Optimization (GWO), which is a new method with swarm intelligence is compared with another metaheuristic optimization method, Particle Swarm Optimization (PSO), using optimization benchmark functions. Simulation studies on test functions are presented as a table by obtaining mean, standard deviation, best and worst values. In addition, the effects of population and iteration number change on the GWO algorithm are presented in separate tables. The GWO algorithm has establish a good balance between exploration and exploitation. Simulation studies have shown that GWO has better convergence performance and optimization accuracy.

Keywords: Grey Wolf Optimization, Metaheuristic Optimization, Particle Swarm Optimization

1. INTRODUCTION

The process of finding the smallest or largest values under a given constraint that gives a purpose function that changes depending on various variables mathematically is defined as an optimization problem [1]. Optimization is used in a wide range of fields such as electronics, computers, economics, transportation, production. In the design of heuristic and metaheuristic algorithms, inspired by biological systems or the behaviour of physical events in nature [2]. For instance, Ant Colony Optimization (ACO), is based on the talent of ants to find the shortest way from the anthill to the food source [3], Whale Optimization Algorithm (WOA), imitating the hunting behaviour of whales [4]. Grey Wolf Optimization (GWO), which has been developed by imitating the hunting and social behaviour of grey wolves, has been one of the most studied metaheuristic methods in recent years. The reasons why population-based metaheuristic optimization methods such as GWO, Particle Swarm Optimization (PSO), Bat Algorithm (BA), ACO, WOA have become so popular can be shown; simplicity, flexibility, non-derivative system, and avoidance of local optimal values [5]. The purpose of these methods is to find

* Corresponding Author: alper.koybasi@gmail.com
[1] Sakarya University, Institute of Science, Electrical and Electronics Engineering
ORCID: https://orcid.org/0000-0003-4210-7757
[2] Sakarya University, Department of Electrical and Electronics Engineering
E-Mail: iyazici@sakarya.edu.tr ORCID: https://orcid.org/0000-0003-3603-7051

the best solution quality and better convergence performance [6].

In this study, GWO was compared with the PSO algorithm by using 23 test functions in the literature. Experimental solutions are presented as a table by obtaining mean, standard deviation, best and worst values. Experimental results have shown that GWO has better convergence performance and optimization accuracy.

2. GREY WOLF OPTIMIZATION (GWO)

GWO is a population-based metaheuristic optimization method created by Mirjalili et al., [5] by considering the hunting and social behaviour of grey wolves. Grey wolves live in flocks and which are at the top of the food chain. There are 4 types of grey wolves in the GWO method in terms of social hierarchy: alpha (α), beta (β), delta (δ) and omega (ω). It has a strict social hierarchy that decrease from top to bottom as shown in Figure 1.

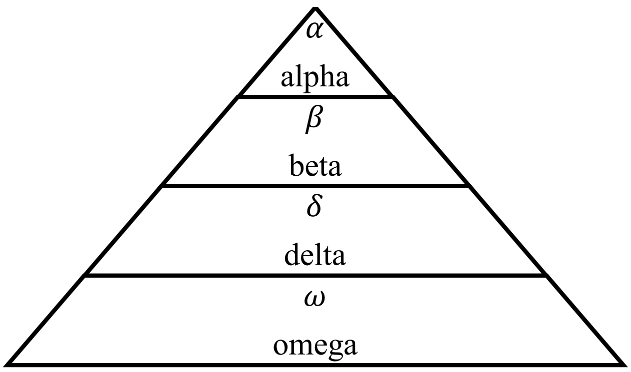

Figure 1 Grey wolf hierarchy (dominance decreases from top to bottom.)

Alpha is the group leader in GWO and responsible for taking decisions on topics such as hunting. Alpha's decisions are obeyed by the other wolves. Beta wolves help alpha in decision making. Delta wolves obeys alpha and beta wolves, and which is dominate omega. Omega wolves take the last place in the grey wolf hierarchy. Hunting in GWO takes place in 3 main steps. Tracking, encircling and attack towards the prey.

2.1. Social Hierarchy

In the GWO the social hierarchy and hunting behaviour of grey wolves are mathematically modelled. Alpha is considered the best candidate solution. Optimization is directed by alpha, beta and delta, respectively. These wolves are followed by omega.

2.2. Encircling Prey

Grey wolves surround their prey during hunting. The following equations are used for the mathematical model of the siege [5]:

$$\vec{D} = \left|\vec{C} \cdot \vec{X}_p(k) - \vec{X}(k)\right| \quad (1)$$

$$\vec{X}(k+1) = \vec{X}_p(k) - \vec{A} \cdot \vec{D} \quad (2)$$

where k indicates current iteration, $\vec{A}$, $\vec{C}$ and $\vec{D}$ are coefficient vectors, $\vec{X}_p$ is the position vector of prey, $\vec{X}$ points the position vector of grey wolves. The coefficients $\vec{A}$, $\vec{C}$ and $\vec{a}$ are calculated as follows:

$$\vec{A} = 2\vec{a} \cdot \vec{r}_1 - \vec{a} \quad (3)$$

$$\vec{C} = 2 \cdot \vec{r}_2 \quad (4)$$

$$\vec{a} = 2 - k * \left(\frac{2}{k_{max}}\right) \quad (5)$$

$\vec{A}$ and $\vec{C}$ are the coefficients, to equilibrium the exploration and the exploitation [7]. Value of $\vec{a}$ are updated from 2 to 0 as given (**5**), $\vec{r}_1$ and $\vec{r}_2$, can be randomly selected in the range [0-1]. Grey wolves can update their position around the prey according to (**1**) and (**2**). The $\vec{C}$ vector, can be also considered as the effect of impediments in nature in the hunting process.

2.3. Hunting

Hunting is done by being directed by alpha. Beta and delta can also join hunting. The best three solutions obtained are recorded and it is ensured that the positions of other wolfs (including omega) are updated regarding the position of the best search agents. The following formulas are recommended in this respect [5].

$$\vec{D}_\alpha = |\vec{C}_1\vec{X}_\alpha - \vec{X}|, \qquad \vec{D}_\beta = |\vec{C}_2\vec{X}_\beta - \vec{X}|,$$

$$\vec{D}_\delta = |\vec{C}_3\vec{X}_\delta - \vec{X}| \tag{6}$$

$$\vec{X}_1 = \vec{X}_a - \vec{A}_1(\vec{D}_a), \qquad \vec{X}_2 = \vec{X}_\beta - \vec{A}_2(\vec{D}_\beta),$$

$$\vec{X}_3 = \vec{X}_\delta - \vec{A}_3(\vec{D}_\delta) \tag{7}$$

$$\vec{X}(k+1) = \frac{\vec{X}_1+\vec{X}_2+\vec{X}_3}{3} \tag{8}$$

The GWO search process starts by creating a random population of grey wolf. During the iterations, alpha, beta and delta update the distance from the hunt by predicting the possible location of the prey. Value of $\vec{a}$ is updated as given (**5**), to emphasise exploration and exploitation. As shown in Figure 2, grey wolves move away from prey when $\vec{A}$>1, and approach prey when $\vec{A}$<1. GWO's equilibrium between exploration and exploitation it is carried out with parameters $\vec{A}, \vec{C}$ ve $\vec{a}$.

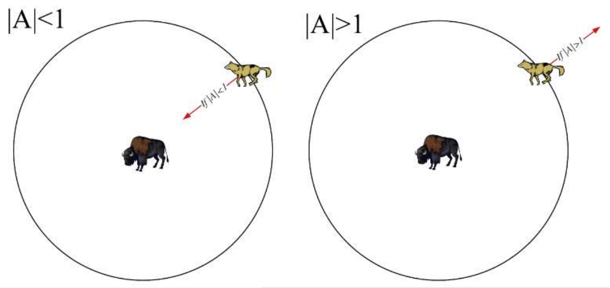

Figure 2 Attacking prey and searching for prey

GWO algorithm flow chart is as shown in Figure 3.

```
Initialize all the parameters
Calculation of the fitness of search agent
X_a = the finest solution
X_β = the second finest solution
X_δ = the third finest solution
 while (k < k_max)
     for each grey wolf
         Update the position of the current wolf by
          equation (8)
     end for
     Update parameters α, A, and C
     Calculate the fitness of all grey wolves
     Update X_a, X_β , and X_δ
      k = k+1
 end while
return X_a
```

Figure 3 GWO Pseudo Code

3. TEST STUDIES

3.1. Test Benchmark Functions

In this study, GWO algorithm has been compared with another metaheuristic optimization method, standard PSO algorithm. The PSO algorithm was proposed by Eberhart and Kennedy in 1995. PSO algorithm has been developed inspired by the behaviour of flocks of birds and fish [8]. Various studies on PSO such as Clubs-Based PSO [9], The Modified Power Mutation PSO [10] are continuing.

Optimization benchmark functions used in similar studies were used [11]. The $f_1 - f_7$ single-mode test functions shown in Table 1 have only one global optimum and no local optimum.

Table 1
Unimodal benchmark functions

Function	f_{min}	Range	Dimensions
$f_1(x) = \sum_{i=1}^{n} x_i^2$	0	[-100,100]	30
$f_2(x) = \sum_{i=1}^{n} \lvert x_i \rvert + \prod_{i=1}^{n} \lvert x_i \rvert$	0	[-10,10]	30
$f_3(x) = \sum_{i=1}^{n} \left(\sum_{j-1}^{i} x_j \right)^2$	0	[-100,100]	30
$f_4(x) = max_i\{\lvert x_i \rvert, 1 \leq i \leq n\}$	0	[-100,100]	30
$f_5(x) = \sum_{i=1}^{n-1} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$	0	[-30,30]	30
$f_6(x) = \sum_{i=1}^{n} ([x_i + 0.5])^2$	0	[-100,100]	30
$f_7(x) = \sum_{i=1}^{n} ix_i^4 + random[0,1]$	0	[-1.28,1.28]	30

The $f_8 - f_{13}$ multimodal test functions shown in Table 2 have multiple optima, making them more demanding than unimodal functions. Only one of the optimum points is global optimum and the others are local optimum [12].

Table 2
Multimodal benchmark functions

Function	f_{min}	Range	Dimensions
$f_8(x) = \sum_{i=1}^{n} -x_i \sin\left(\sqrt{\lvert x_i \rvert}\right)$	-418.9829× 5	[-500,500]	30
$f_9(x) = \sum_{i=1}^{n} [x_i^2 - 10\cos(2\pi x_i) + 10]$	0	[-5.12,5.12]	30
$f_{10}(x) = -20\, exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}\right) - exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)\right) + 20 + e$	0	[-32,32]	30
$f_{11}(x) = \frac{1}{4000}\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n}\cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	0	[-600,600]	30
$f_{12}(x) = \frac{\pi}{n}\left\{10\sin(\pi y_1) + \sum_{i=1}^{n-1}(y_i - 1)^2[1 + 10\sin^2(\pi y_{i+1})] + (y_n - 1)^2\right\} + \sum_{i=1}^{n} u(x_i, 10, 100, 4)$ $y_i = 1 + \frac{x_i + 1}{4}$ $u(x_i, a, k, m) = \begin{cases} k(x_i - a)^m & x_i > a \\ 0 & -a < x_i < a \\ k(-x_i - a)^m & x_i < -a \end{cases}$	0	[-50,50]	30
$f_{13}(x) = 0.1\left\{sin^2(3\pi x_1) + \sum_{i=1}^{n}(x_i - 1)^2[1 + sin^2(3\pi x_i + 1)] + (x_n - 1)^2[1 + sin^2(2\pi x_n)]\right\} + \sum_{i=1}^{n} u(x_i, 5, 100, 4)$	0	[-50,50]	30

The only difference of the $f_{14} - f_{23}$ fixed-size multimodal test functions shown in Table 3 from the multimodal functions is that they contain a small number of local minimums due to their low size [13]. If the exploration of an algorithm is poorly designed, it will not be able to effectively scan at a wide angle, causing the algorithm to get stuck at the local optimum. Therefore, multimodal functions with containing many local optima are shown as the most difficult problem classes for many algorithms [14].

Table 3
Fixed-dimensions multimodal benchmark functions

Function	f_{min}	Range	Dimensions
$f_{14}(x) = (\frac{1}{500} + \sum_{j=1}^{25} \frac{1}{j + \sum_{i=1}^{2}(x_i - a_{ij})^6})^{-1}$	1	[-65,65]	2
$f_{15}(x) = \sum_{i=1}^{11} [\ a_i - \frac{x_1(b_i^2 + b_i x_2)}{b_i^2 + b_i x_3 + x_4}]^2$	0.00030	[-5,5]	4
$f_{16}(x) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1x_2 - 4x_2^2 + 4x_2^4$	-1.0316	[-5,5]	2
$f_{17}(x) = (x_2 - \frac{5.1}{4\pi^2}x_1^2 + \frac{5}{\pi}x_1 - 6)^2 + 10(1 - \frac{1}{8\pi})\cos x_1 + 10$	0.398	[-5,5]	2
$f_{18}(x) = [1 + (x_1 + x_2 + 1)^2(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2)] \times [30 + (2x_1 - 3x_2)^2 \times (18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2)]$	3	[-2,2]	2
$f_{19}(x) = -\sum_{i=1}^{4} c_i exp(-\sum_{j=1}^{3} a_{ij}(x_j - p_{ij})^2)$	-3.86	[1,3]	3
$f_{20}(x) = -\sum_{i=1}^{4} c_i exp(-\sum_{j=1}^{6} a_{ij}(x_j - p_{ij})^2)$	−3.32	[0,1]	6
$f_{21}(x) = -\sum_{i=1}^{5} [(X - a_i)(X - a_i)^T + c_i]^{-1}$	−10.1532	[0,10]	4
$f_{22}(x) = -\sum_{i=1}^{7} [(X - a_i)(X - a_i)^T + c_i]^{-1}$	-10.4028	[0,10]	4
$f_{23}(x) = -\sum_{i=1}^{10} [(X - a_i)(X - a_i)^T + c_i]^{-1}$	-10.5363	[0,10]	4

3.2. Comparison of Test Results of GWO and PSO

The GWO and PSO pseudocodes are coded in MATLAB R2017A and implemented on Nvidia GeForce GTX1650, 16 GB Memory, i7 9750H Processor and 256 GB SSD. In all tests, the same parameter settings were used in both algorithms, with a population number of 30 and a maximum number of iterations of 500. All benchmark functions were run 30 times and presented as a table by obtaining mean, standard deviation, best values, worst values, and computation time. The algorithm with better average solution in each function is solved in bold font.

The $f_1 - f_7$ Functions are unimodal test functions used only to examine the convergence rates of optimization algorithms that have global optimum solution. As shown in Table 4, GWO outperformed 6 of these 7 $(f_1, f_2, f_3, f_4, f_5, f_7)$ functions. Performance curves of unimodal functions are shown in Figure 4 through Figure 10.

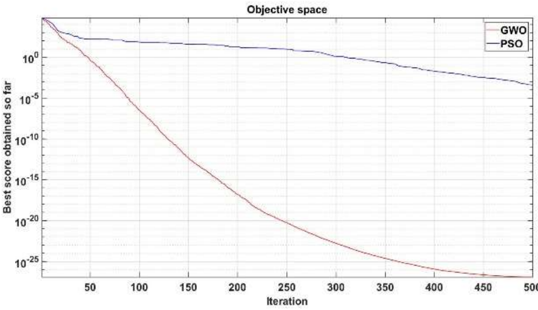

Figure 4 F1 Function convergence curve

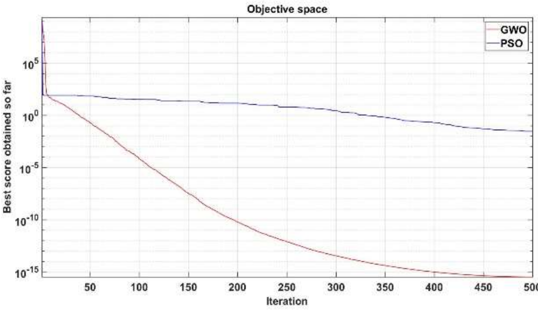

Figure 5 F2 Function convergence curve

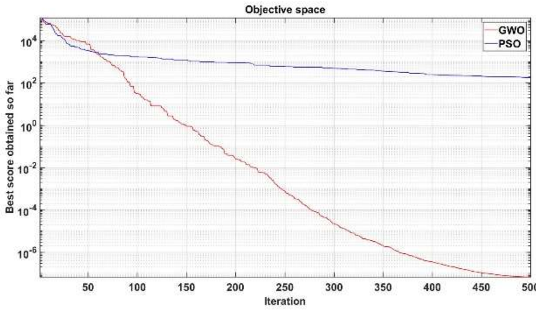

Figure 6 F3 Function convergence curve

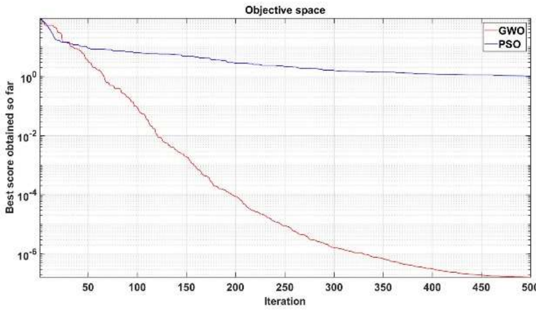

Figure 7 F4 Function convergence curve

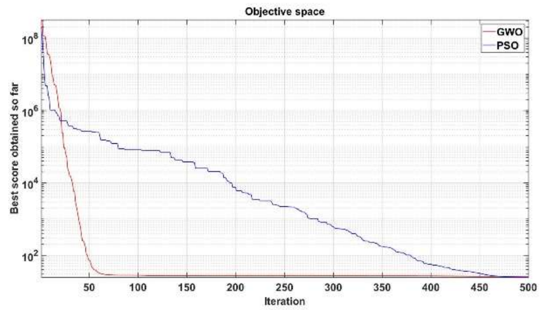

Figure 8 F5 Function convergence curve

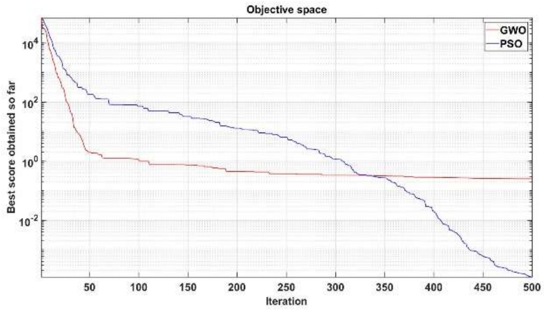

Figure 9 F6 Function convergence curve

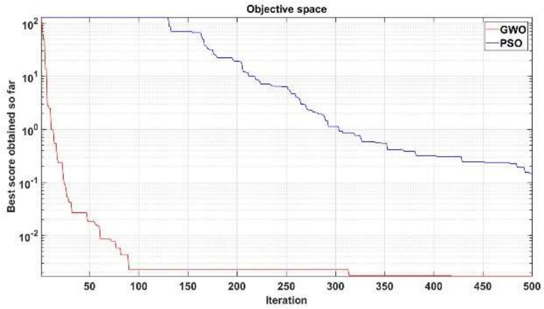

Figure 10 F7 Function convergence curve

As shown in Table 5, GWO outperformed 3 of these 6 multimodal functions (f_9, f_{10}, f_{11}) containing many local minimums. The performance curves of the multimodal functions are shown in Figure 11 to Figure 16.

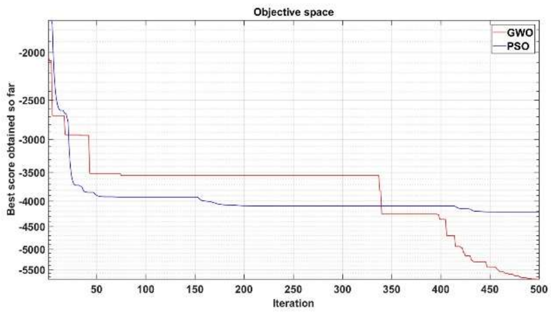

Figure 11 F8 Function convergence curve

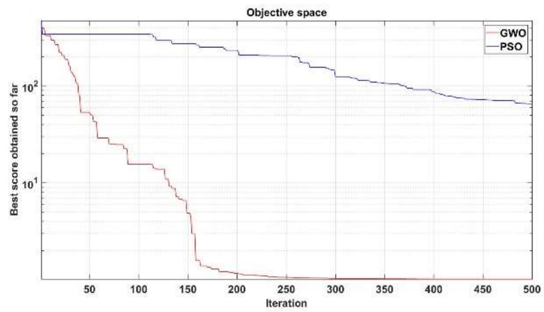

Figure 12 F9 Function convergence curve

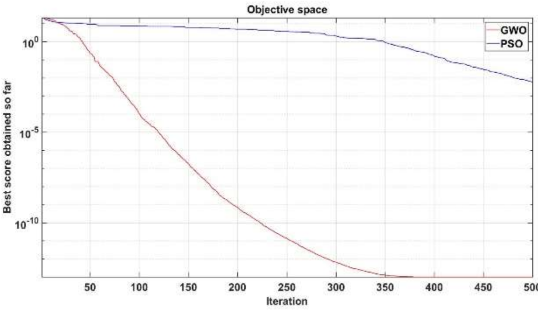

Figure 13 F10 Function convergence curve

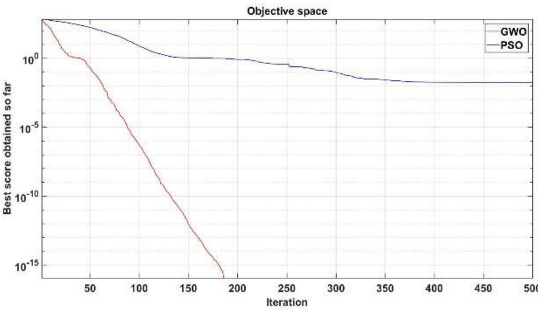

Figure 14 F11 Function convergence curve

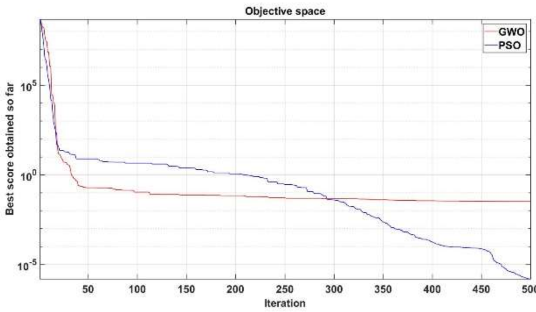

Figure 15 F12 Function convergence curve

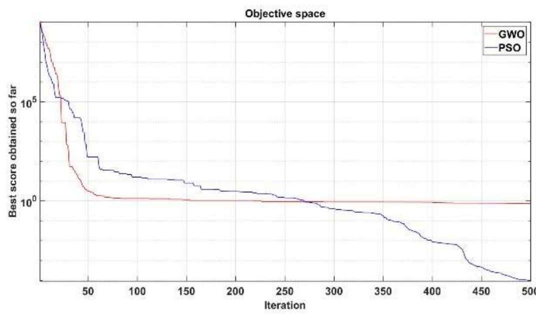

Figure 16 F13 Function convergence curve

GWO showed better results in 6 of 10 functions ($f_{17}, f_{19}, f_{20}, f_{21}, f_{22}, f_{23}$) that contain fewer local minimum and low dimensions compared to multimodal functions. Both algorithms showed good results in f_{16} functions. The results are shown in Table 6. Performance curves of fixed-size multimodal functions are shown between Figure 17 with Figure 26.

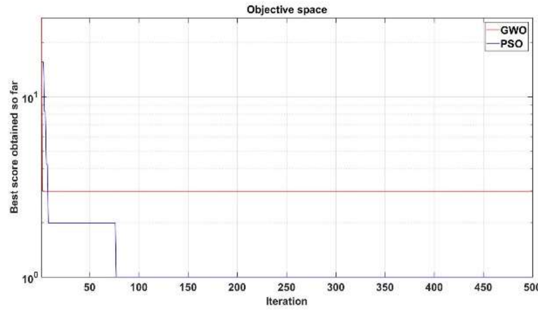

Figure 17 F14 Function convergence curve

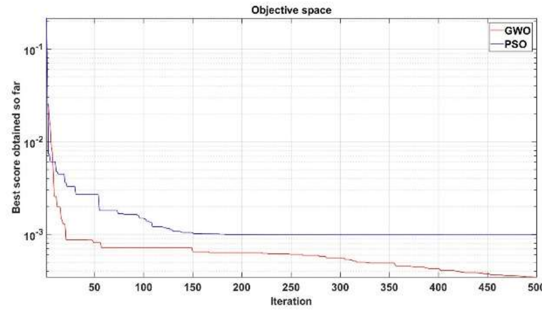

Figure 18 F15 Function convergence curve

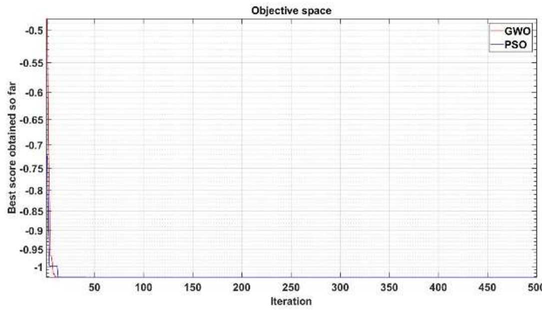

Figure 19 F16 Function convergence curve

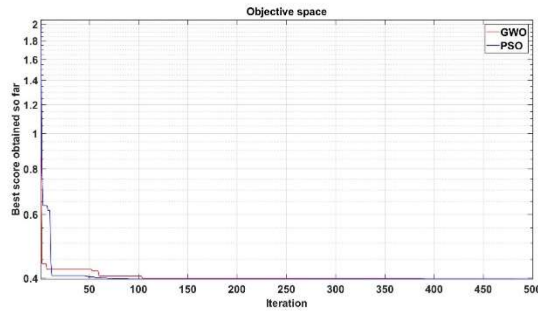

Figure 20 F17 Function convergence curve

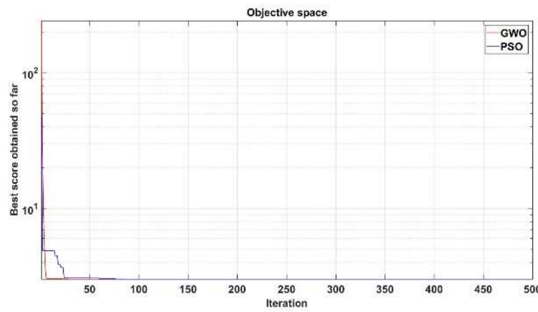

Figure 21 F18 Function convergence curve

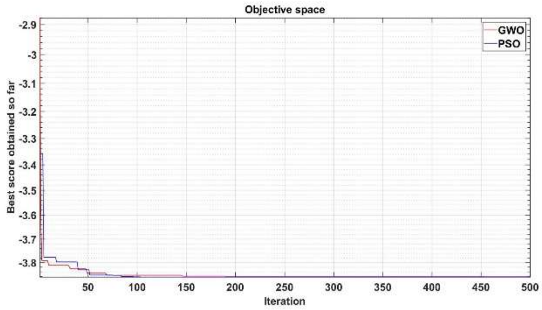

Figure 22 F19 Function convergence curve

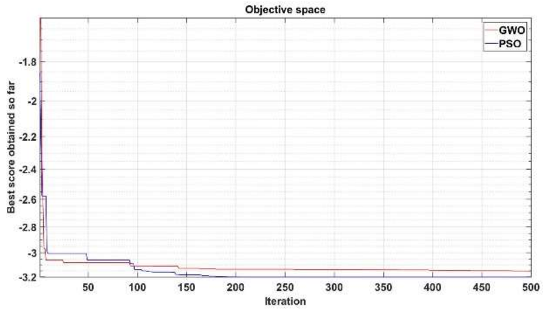

Figure 23 F20 Function convergence curve

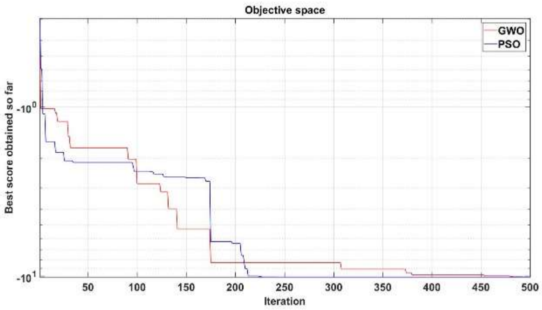

Figure 24 F21 Function convergence curve

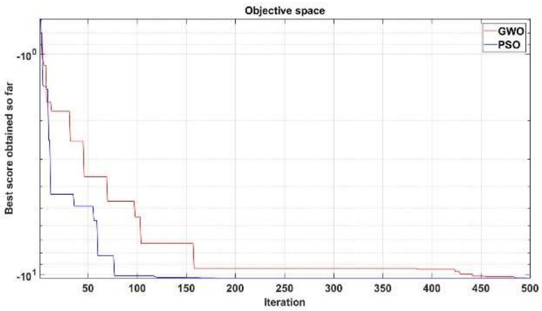

Figure 25 F22 Function convergence curve

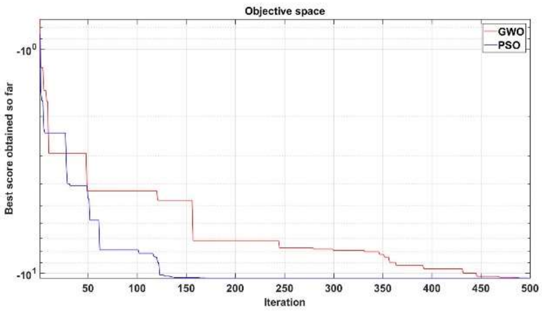

Figure 26 F23 Function convergence curve

3.3. The Effect of Change of Population Number and Iteration Number on GWO Algorithm.

In this part, the effects of the number of populations and iteration number on the GWO algorithm are examined. In the tests, the population number was applied as 15 and 30. The maximum number of iterations has been applied separately as 100 and 500. All benchmark functions were run 30 times and presented as a table by obtaining mean, standard deviation, best and worst values.

Increasing the number of populations and iteration had a positive effect on all single-mode test functions. The importance of the number of iterations was observed sharply in (f_3, f_4, f_5) functions. The results are shown in Table 7.

As shown in Table 8, in 5 of 6 multimodal functions, $(f_8, f_9, f_{10}, f_{12}, f_{13})$ high population number positively affected. The importance of the number of iterations was observed sharply in (f_9, f_{10}, f_{13}) functions.

As shown in Table 9, in 6 of 10 $(f_{14}, f_{15}, f_{18}, f_{20}, f_{21}, f_{23})$ fixed sized multimodal functions, high population and iteration number positively affected. The F_{16} function showed good results in both population and iteration numbers.

4. CONCLUSION

GWO is a metaheuristic optimization method developed inspired by the hunting and social behaviour of grey wolves. In this study, GWO was compared with PSO algorithm using 23 optimization test functions. Comparison results and performance curves are presented. GWO's exploration and exploitation performance has been observed to be better. In addition, increasing the number of populations and iterations in GWO has better convergence performance and optimization accuracy.

Funding

The authors received no financial support for research and publication of this article.

Acknowledgements

The authors thank the reviewers for their positive contribution to the article.

The Declaration of Conflict of Interest/ Common Interest

No conflict of interest or common interest has been declared by the authors.

Authors' Contribution

A.K: Literature research, simulation study, literary and technical editing.

İ.Y: Coordinating the studies related to article, directing A.K.

The Declaration of Ethics Committee Approval

The authors declare that this document does not require an ethics committee approval or any special permission.

The Declaration of Research and Publication Ethics

The authors of the paper declare that they comply with the scientific, ethical and quotation rules of SAUJS in all processes of the paper and that they do not make any falsification on the data collected. In addition, they declare that Sakarya University Journal of Science and its editorial board have no responsibility for any ethical violations that may be encountered, and that this study has not been evaluated in any academic publication environment other than Sakarya University Journal of Science.

REFERENCES

[1] P. Erdoğmuş and E. Yalçın, "Parçacık Sürü Optimizasyonu ile Kısıtsız Optimizasyon Test Problemlerinin Çözümü," Journal of Advanced Technology Sciences, vol. 4, no. 1, pp. 14–22, 2015.

[2] X.S. Yang, A New Metaheuristic Bat-Inspired Algorithm, in: Nature Inspired Cooperative Strategies for Optimization (NISCO 2010) (Eds. J. R. Gonzalez et al.), Studies in Computational Intelligence, Springer Berlin, 284, pp. 65-74, 2010.

[3] C. Blum, "Ant colony optimization: Introduction and recent trends," Physics of Life Reviews, vol. 2, pp. 353-373, 2005.

[4] S. Mirjalili and A. Lewis, "The Whale Optimization Algorithm," Advances in Engineering Software, vol. 95, pp. 51–67, 2016.

[5] S. Mirjalili, S. M. Mirjalili, and A. Lewis, "Grey Wolf Optimizer," Advances in Engineering Software, vol. 69, pp. 46–61, 2014.

[6] N. Singh and S. B. Singh, "Hybrid Algorithm of Particle Swarm Optimization and Grey Wolf Optimizer for Improving Convergence Performance," Journal of Applied Mathematics, vol. 2017, pp. 1-15, 2017.

[7] S. Cherukuri and S. Rayapudi, "Enhanced Grey Wolf Optimizer Based MPPT Algorithm of PV System Under Partial Shaded Condition," International Journal of Renewable Energy Development, vol. 6, no. 3, pp. 203-212, 2017.

[8] J. Kennedy and R. Eberhart, "Particle Swarm Optimization," Proceedings of ICNN'95 -International Conference on Neural Networks, Perth, WA, Australia, vol. 4, pp. 1942-1948, 1995.

[9] W. Elshamy, H. M. Emara and A. Bahgat, "Clubs-based Particle Swarm Optimization," IEEE Swarm Intelligence Symposium, Honolulu, HI, pp. 289-296, 2007.

[10] P. Chauhan, K. Deep, and M. Pant, "Power Mutation Embedded Modified PSO for Global Optimization Problems," Lecture Notes in Computer Science, vol. 6466, pp. 139-146, 2010.

[11] M. Molga and C. Smutnicki, "Test functions for optimization needs," Available:https://www.robertmarks.org/Classes/ENGR5358/Papers/functions.pdf.

[12] B. Alızada, "Sürü Tabanlı Karınca Aslanı ve Balina Optimizasyonu Algoritmalarının Fiziki Tabanlı Algoritmalarla Hibritleştirilmesi," Erciyes Üniversitesi / Fen Bilimleri Enstitüsü, Kayseri, 2019.

[13] G. Demir and E. Tanyıldızı, "Optimizasyon Problemlerinin Çözümünde Sinüs Kosinüs Algoritması (SKA)'nın Kullanılması," Fırat University Journal of Science and Engineering., vol.29, no. 1, pp. 225-236, 2017.

[14] M. Jamil and X.S Yang, "A Literature Survey of Benchmark Functions for Global Optimization Problems," Int. Journal of Mathematical Modelling and Numerical Optimisation, vol. 4, no. 2, pp. 150-194, 2013.

APPENDİX

Table 4
GWO-PSO performance comparison with unimodal benchmark functions

Function	Method	Best	Mean	Worst	Std	Computation Time
f_1	GWO PSO	2.8155e-29 1.8645e-05	**1.2770e-27** 1.8152e-04	1.0236e-26 0.0015	2.1109e-27 3.4239e-04	0.1171 0.0722
f_2	GWO PSO	1.8533e-17 0.0048	**9.5313e-17** 0.0320	3.6638e-16 0.1727	7.9800e-17 0.0392	0.1291 0.0778
f_3	GWO PSO	6.3728e-08 19.8364	**1.1175e-05** 73.6746	5.4032e-05 157.0681	1.6984e-05 30.9306	0.3805 0.5378
f_4	GWO PSO	3.4488e-08 0.6978	**1.0720e-06** 1.1559	5.3905e-06 1.7896	1.2244e-06 0.2664	0.1287 0.0699
f_5	GWO PSO	25.4990 17.0043	**26.9089** 98.0932	28.5607 498.7949	0.7535 104.7435	0.1473 0.1379
f_6	GWO PSO	6.5542e-05 9.0093e-06	0.7089 **1.3498e-04**	1.4975 5.1873e-04	0.3452 1.2357e-04	0.1388 0.0803
f_7	GWO PSO	5.1824e-04 0.0734	**0.0023** 0.1768	0.0067 0.3456	0.0014 0.0600	0.1860 0.2059

Table 5
GWO-PSO performance comparison with multimodal benchmark functions

Function	Method	Best	Mean	Worst	Std	Computation Time
f_8	GWO PSO	-3.1897e+03 -3.0109e+03	-6.0773e+03 **-4.9471e+03**	-7.3092e+03 -7.0724e+03	971.4041 1.2770e+03	0.2227 0.0994
f_9	GWO PSO	0 29.9706	**2.6344** 58.3851	15.4156 96.7560	3.9016 16.4032	0.2092 0.1409
f_{10}	GWO PSO	6.8390e-14 0.0022	**9.9179e-14** 0.0560	1.3234e-13 1.1558	1.5979e-14 0.2082	0.1334 0.0953
f_{11}	GWO PSO	0 2.1291e-06	**0.0035** 0.0078	0.0280 0.0320	0.0078 0.0089	0.1502 0.1046
f_{12}	GWO PSO	0.0193 3.9338e-08	0.0444 **0.0138**	0.0820 0.2074	0.0150 0.0450	0.2923 0.2555
f_{13}	GWO PSO	0.3080 5.0410e-06	0.5907 **0.0056**	1.3103 0.0210	0.1985 0.0067	0.2915 0.2577

Table 6
GWO-PSO performance comparison with fixed size multimodal functions

Function	Method	Best	Mean	Worst	Std	Computation Time
f_{14}	GWO PSO	0.9980 0.9980	5.1762 **2.8430**	12.6705 7.8740	4.2581 1.9284	0.5141 0.5071
f_{15}	GWO PSO	3.0750e-04 6.6311e-04	0.0031 **8.8895e-04**	0.0204 0.0012	0.0069 1.3007e-04	0.0846 0.0792
f_{16}	GWO PSO	-1.0316 -1.0316	**-1.0316** **-1.0316**	-1.0316 -1.0316	6.7752e-16 6.7752e-16	0.0683 0.0698
f_{17}	GWO PSO	0.39790 0.39789	**0.397891** 0.397890	0.397891 0.39789	3.4575e-06 1.6938e-16	0.1126 0.0623
f_{18}	GWO PSO	3 3	3.00003 **3**	3.0002 3	5.4667e-05 0	0.0783 0.0989

f_{19}	GWO	-3.8604	**-3.8611**	-3.8628	0.0026	0.1162
	PSO	-3.8628	-3.8628	-3.8628	3.6114e-15	0.1081
f_{20}	GWO	-3.3220	**-3.2879**	-3.1375	0.0586	0.2001
	PSO	-3.3220	-3.2784	-3.2031	0.0583	0.1107
f_{21}	GWO	-10.1530	**-8.9193**	-2.6303	2.5581	0.1679
	PSO	-10.1532	-6.0438	-2.6305	2.9052	0.1542
f_{22}	GWO	-10.4026	**-10.0480**	-5.0876	1.3434	0.1943
	PSO	-10.4029	-9.1608	-3.7243	2.5445	0.1887
f_{23}	GWO	-10.5362	**-10.3545**	-5.1284	0.9871	0.2526
	PSO	-10.5364	-9.2591	-2,8066	2.6405	0.2422

Table 7
The effects of changing the number of populations and iterations, on unimodal benchmark functions

Function	Method	Population Number	Iteration Number	Best	Mean	Worst	Std	Computation Time
f_1	GWO	30	500	2.8155e-29	**1.2770e-27**	1.0236e-26	2.1109e-27	0.1171
	GWO	15	500	3.3304e-21	6.2407e-20	2.4847e-19	6.5138e-20	0.0766
	GWO	30	100	1.8645e-05	0.0147	0.0326	0.0083	0.0750
	GWO	15	100	0.17745	0.6804	2.3128	0.4345	0.0570
f_2	GWO	30	500	1.8533e-17	**9.5313e-17**	3.6638e-16	7.9800e-17	0.1291
	GWO	15	500	6.4614e-13	2.3781e-12	6.1267e-12	1.6860e-12	0.0796
	GWO	30	100	0.0122	0.0230	0.0359	0.0069	0.0619
	GWO	15	100	0.1001	0.1950	0.3660	0.0646	0.0510
f_3	GWO	30	500	6.3728e-08	**1.1175e-05**	5.4032e-05	1.6984e-05	0.3805
	GWO	15	500	5.2722e-05	0.0102	0.0549	0.0152	0.2202
	GWO	30	100	52.7035	382.4511	2.4758e+03	459.5907	0.1864
	GWO	15	100	197.3679	1.2317e+03	4.6694e+03	946.8118	0.1578
f_4	GWO	30	500	3.4488e-08	**1.0720e-06**	5.3905e-06	1.2244e-06	0.1287
	GWO	15	500	5.9714e-06	8.9815e-05	4.7227e-04	9.5299e-05	0.0803
	GWO	30	100	0.5224	1.4898	2.9883	0.6714	0.0636
	GWO	15	100	0.9107	4.2749	8.0857	1.6376	0.0525
f_5	GWO	30	500	25.4990	**26.9089**	28.5607	0.7535	0.1473
	GWO	15	500	26.2671	27.7108	28.7986	0.7353	0.0828
	GWO	30	100	28.7672	30.9848	42.8574	3.0148	0.0699
	GWO	15	100	35.5374	88.1632	287.0783	51.4240	0.0542
f_6	GWO	30	500	6.5542e-05	**0.7089**	1.4975	0.3452	0.1388
	GWO	15	500	0.3679	1.4966	2.5180	0.6316	0.1208
	GWO	30	100	1.7964	2.9401	4.1530	0.6346	0.1107
	GWO	15	100	2.8815	4.8694	7.6166	1.0635	0.0973
f_7	GWO	30	500	5.1824e-04	**0.0023**	0.0067	0.0014	0.1860
	GWO	15	500	14.126e-04	0.0045	0.0100	0.0022	0.1098
	GWO	30	100	0.0081	0.0196	0.0399	0.0085	0.0819
	GWO	15	100	0.0065	0.0399	0.0734	0.0182	0.0786

Table 8
The effects of changing the number of populations and iterations, on multimodal benchmark functions.

Function	Method	Population Number	Iteration Number	Best	Mean	Worst	Std	Computation Time
f_8	GWO	30	500	-3.1897e+03	**-6.0773e+03**	-7.3092e+03	971.4041	0.2227
	GWO	15	500	-3.7541e+03	-5.7933e+03	-7.3390e+03	769.0020	0.1492
	GWO	30	100	-2.7952e+03	-5.5985e+03	-6.8392e+03	946.5625	0.0797
	GWO	15	100	-2.2474e+03	-4.6750e+03	-7.1282e+03	1.3882e+03	0.0781
f_9	GWO	30	500	0	**2.6344**	15.4156	3.9016	0.2092
	GWO	15	500	5.6843e-14	3.9220	15.5209	4.0141	0.0885
	GWO	30	100	10.0693	41.9163	229.1786	39.5007	0.0791
	GWO	15	100	23.3764	45.9260	81.3144	14.6026	0.0762
f_{10}	GWO	30	500	6.8390e-14	**9.9179e-14**	1.3234e-13	1.5979e-14	0.1334
	GWO	15	500	8.6127e-12	4.3345e-11	1.1429e-10	2.8252e-11	0.0955
	GWO	30	100	0.0127	0.0282	0.0792	0.0140	0.0852
	GWO	15	100	0.1232	0.3316	0.8121	0.1600	0.0659
f_{11}	GWO	30	500	0	0.0035	0.0280	0.0078	0.1502
	GWO	15	500	0	**0.0032**	0.0244	0.0074	0.0959
	GWO	30	100	0.0033	0.0919	0.1950	0.0615	0.0886
	GWO	15	100	0.3125	0.6201	0.9780	0.1637	0.0846
f_{12}	GWO	30	500	0.0193	**0.0444**	0.0820	0.0150	0.2923
	GWO	15	500	0.0370	0.0946	0.1763	0.0404	0.2821
	GWO	30	100	0.1037	0.4369	1.1312	0.2598	0.1264
	GWO	15	100	0.2496	0.9956	3.8609	0.6986	0.1099

f_{13}	GWO	30	500	0.3080	**0.5907**	1.3103	0.1985	0.2915
	GWO	15	500	0.3873	1.1492	1.5388	0.2549	0.2781
	GWO	30	100	1.4057	2.2815	3.2969	0.4376	0.2002
	GWO	15	100	2.3542	3.6059	6.6733	1.0975	0.1592

Table 9
The effects of changing the number of populations on fixed sized multimodal benchmark functions

Function	Method	Population Number	Iteration Number	Best	Mean	Worst	Std	Computation Time
f_{14}	GWO	30	500	0.9980	**5.1762**	12.6705	4.2581	0.5141
	GWO	15	500	1.9920	7.6726	12.6705	4.5098	0.4820
	GWO	30	100	0.9980	5.3432	12.6705	4.0264	0.2799
	GWO	15	100	1.1046	8.3693	17.3744	4.6689	0.1831
f_{15}	GWO	30	500	3.0750e-04	**0.0031**	0.0204	0.0069	0.0846
	GWO	15	500	3.0750e-04	0.0032	0.0204	0.0068	0.0712
	GWO	30	100	3.1176e-04	0.0049	0.0204	0.0080	0.0814
	GWO	15	100	3.8972e-04	0.0065	0.0204	0.0087	0.0811
f_{16}	GWO	30	500	-1.0316	**-1.0316**	-1.0316	6.7752e-16	0.0683
	GWO	15	500	-1.0316	**-1.0316**	-1.0316	6.7752e-16	0.0684
	GWO	30	100	-1.0316	**-1.0316**	-1.0316	6.7752e-16	0.0984
	GWO	15	100	-1.0316	**-1.0316**	-1.0316	6.7752e-16	0.0930
f_{17}	GWO	30	500	0.39790	0.3978913	0.39789	3.4575e-06	0.1126
	GWO	15	500	0.39791	**0.3979230**	0.39881	1.6762e-04	0.1094
	GWO	30	100	0.398	0.3981783	0.40253	9.0173e-04	0.0992
	GWO	15	100	0.398	0.3984403	0.40744	17.75e-04	0.0622
f_{18}	GWO	30	500	3	**3.00003**	3.0002	5.4667e-05	0.0783
	GWO	15	500	3	5.7001	84.0001	14.7885	0.0751
	GWO	30	100	3	3.0009	3.0028	8.6061e-04	0.1064
	GWO	15	100	3.0001	11.103	84.0036	24.7150	0.0545
f_{19}	GWO	30	500	-3.8604	-3.8611	-3.8628	0.0026	0.1162
	GWO	15	500	-3.8628	-3.8613	-3.8549	0.0027	0.0759
	GWO	30	100	-3.8601	-3.8610	-3.8553	0.0019	0.0664
	GWO	15	100	-3.8597	**-3.8604**	-3.8515	0.0030	0.0608
f_{20}	GWO	30	500	-3.3220	**-3.2879**	-3.1375	0.0586	0.2001
	GWO	15	500	-3.3220	-3.2525	-2.8404	0.1062	0.1180
	GWO	30	100	-3.3202	-3.2436	-2.8558	0.1037	0.0708
	GWO	15	100	-3.3197	-3.2319	-2.8403	0.1309	0.0651
f_{21}	GWO	30	500	-10.1530	**-8.9193**	-2.6303	2.5581	0.1679
	GWO	15	500	-10.1530	-8.2287	-2.6302	3.0591	0.1488
	GWO	30	100	-10.1499	-8.3714	-2.6244	3.2080	0.0856
	GWO	15	100	-10.1412	-9.1689	-2.6595	2.1374	0.0660
f_{22}	GWO	30	500	-10.4026	-10.0480	-5.0876	1.3434	0.1943
	GWO	15	500	-10.4028	**-10.3998**	-10.3964	0.0016	0.1787
	GWO	30	100	-10.3949	-9.70229	-2.76460	2.0055	0.0956
	GWO	15	100	-10.3852	-9.0812	-1.8358	2.8628	0.0684
f_{23}	GWO	30	500	-10.5362	**-10.3545**	-5.1284	0.9871	0.2526
	GWO	15	500	-10.5361	-10.3101	-3.8352	1.2229	0.2205
	GWO	30	100	-10.5311	-9.73273	-2.4172	2.3299	0.1123
	GWO	15	100	-10.5131	-8.84514	-2.4209	3.0127	0.1073