Turkish Journal of Engineering & Environmental Sciences

http://journals.tubitak.gov.tr/engineering/

Research Article

Turkish J Eng Env Sci
(2014) 38: 184 – 196
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doi:10.3906/muh-1209-3

Thermal radiation and chemical reaction effects on magnetohydrodynamic free convection heat and mass transfer in a micropolar fluid

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Received: 08.09.2012 • **Accepted/Published Online:** 27.12.2014 • **Printed:** 30.06.2015

Abstract: This paper analyzes the flow and heat and mass transfer characteristics of the free convection on a vertical plate with uniform wall temperature and concentration in a micropolar fluid in the presence of a first-order chemical reaction and radiation. A uniform magnetic field B_0 is applied normal to the plate. The governing nonlinear partial differential equations are transformed into a system of coupled nonlinear ordinary differential equations using similarity transformations and then solved numerically using the Keller-box method. The numerical results are compared and found to be in good agreement with previously published results as special cases of the present investigation. The coefficient of skin-friction, wall couple stress, the rate of heat transfer in terms of Nusselt number, and the ratio of convective to diffusive mass transport in terms of Sherwood number at the plate are presented graphically for various values of coupling number, magnetic parameter, radiation parameter, chemical reaction parameter, Prandtl number, and Schmidt number.

Key words: Similarity solutions, free convection, micropolar fluid, magnetohydrodynamics, radiation, chemical reaction, heat and mass transfer

1. Introduction

Free convection flows are of great interest because of their various engineering, scientific, and industrial applications in heat and mass transfer. Free convection of heat and mass transfer occurs simultaneously in the fields of design of chemical processing equipment, formation and dispersion of fog, distributions of temperature, moisture over agricultural fields and groves of fruit trees, and damage of crops due to freezing and pollution of the environment. Extensive studies of free convection heat and mass transfer of a nonisothermal vertical surface under boundary layer approximation for Newtonian fluids have been undertaken by several authors. In recent years, several simple boundary layer flow problems have received new attention within the more general context of magnetohydrodynamic (MHD) studies. Several investigators have extended many of the available boundary layer solutions to include the effects of magnetic fields for those cases where the fluid is electrically conducting. The study of MHD flow for an electrically conducting fluid past a heated surface has important applications in many engineering problems such as plasma studies, petroleum industries, MHD power generators, cooling of nuclear reactors, the boundary layer control in aerodynamics, and crystal growth. In addition, there has been a renewed interest in studying MHD flow and heat transfer in porous media due to the effect of magnetic fields on flow control and on the performance of many systems using electrically conducting fluids. Alam et al. [1] studied the problem of free convection heat and mass transfer flow past an inclined semiinfinite heated surface of an electrically conducting viscous incompressible fluid with magnetic field and heat generation. Effect of

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heat and mass transfer on MHD free convection along a moving permeable vertical surface was analyzed by Abdelkhalek [2].

The study of non-Newtonian fluid flows has gained much attention from researchers because of its applications in biology, physiology, technology, and industry. In addition, the effects of heat and mass transfer in non-Newtonian fluid also have great importance in engineering applications: for instance, in the thermal design of industrial equipment dealing with molten plastics, polymeric liquids, foodstuffs, or slurries. Several investigators extended many of the available convection heat and mass transfer problems to include the non-Newtonian effects. The fluid model introduced by Eringen [3] exhibits some microscopic effects arising from the local structure and micromotion of the fluid elements. Furthermore, they can sustain couple stresses and include classical Newtonian fluid as a special case. The model of micropolar fluid represents fluids consisting of rigid, randomly oriented (or spherical) particles suspended in a viscous medium where the deformation of the particles is ignored. Micropolar fluids have been shown to accurately simulate the flow characteristics of polymeric additives, geomorphological sediments, colloidal suspensions, hematological suspensions, liquid crystals, lubricants, etc. The mathematical theory of equations of micropolar fluids and applications of these fluids in the theory of lubrication and in the theory of porous media were presented by Lukaszewicz [4]. The heat and mass transfer in micropolar fluids is also important in the context of chemical engineering, aerospace engineering, and industrial manufacturing processes.

The problem of free convection heat and mass transfer in the boundary layer flow along a vertical surface submerged in a micropolar fluid has been studied by a number of investigators. The boundary layer flow of a micropolar fluid over a semiinfinite plate was studied by Ahmadi [5]. Takhar and Soundelgekar [6] considered the heat transfer on a semiinfinite plate of micropolar fluid. Agarwal and Dhanapal [7] examined the flow and heat transfer in a micropolar fluid past a flat plate with suction and heat sources. Gorla et al. [8] studied the nonsimilar problem of the natural convection boundary layer flow of a micropolar fluid over a vertical plate with uniform heat flux boundary condition. Recently, Srinivasacharya and Ramreddy [9] analyzed the flow and heat and mass transfer characteristics of the free convection on a vertical plate with uniform and constant heat and mass fluxes in a doubly stratified micropolar fluid. El-Hakien et al. [10] studied the effects of the viscous and Joule heating on MHD free convection flows with variable plate temperatures in a micropolar fluid. El-Amin [11] considered MHD free convection and mass transfer flow in a micropolar fluid over a stationary vertical plate with constant suction.

Radiation effects on convective heat transfer and MHD flow problems have assumed increasing importance in electrical power generation, astrophysical flows, solar power technology, space vehicle reentry, and other industrial areas. Since the solution for convection and radiation equations is quite complicated, there are few studies about the simultaneous effect of convection and radiation for internal flows. Ghaly [12] discussed the effect of radiation on heat and mass transfer over a stretching sheet in the presence of a magnetic field. Raptis et al. [13] studied the effect of radiation on two-dimensional steady MHD optically thin gray gas flow along an infinite vertical plate taking into account the induced magnetic field. Seddek et al. [14] obtained an analytical solution for the effect of radiation on flow of a magnetomicropolar fluid past a continuously moving plate with suction and blowing.

Chemical reaction effects on heat and mass transfer are of considerable importance in hydrometallurgical industries and chemical technology. In many chemical engineering processes, a chemical reaction occurs between a foreign mass and the fluid. A chemical reaction can be described as either a heterogeneous or homogeneous process, which depends on whether it occurs at an interface or as a single-phase volume reaction. Furthermore, research on combined heat and mass transfer with chemical reaction and thermophoresis effect can help in food processing, cooling towers, chemically reactive vapor deposition boundary layers in optical materials processing, catalytic combustion boundary layers, chemical diffusion in disk electrode modeling, and carbon monoxide reactions in metallurgical mass transfer and kinetics. Several investigators have examined the effect of chemical reaction on the flow and heat and mass transfer past a vertical plate. Deka et al. [15] examined the effect of homogeneous first-order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Chamkha [16] analyzed the MHD flow of a uniformly stretched vertical permeable surface in the presence of heat generation/absorption and a chemical reaction. The problem of chemically reactive species of non-Newtonian fluid in a porous medium over a stretching sheet was investigated by Akyildiz et al. [17]. Magyari and Chamkha [18] obtained an analytical solution to study the combined effect of heat generation or absorption and first-order chemical reaction on micropolar fluid flows over a uniformly stretched permeable surface. Das [19] analyzed the effect of first-order chemical reaction and thermal radiation on hydromagnetic free convection heat and mass transfer flow of a micropolar fluid via a porous medium bounded by a semiinfinite porous plate with constant heat source in a rotating frame of reference. Bakr [20] considered the steady and unsteady MHD micropolar flow and mass transfers flow with constant heat source in a rotating frame of reference in the presence of chemical reaction of the first order, taking an oscillatory plate velocity and a constant suction velocity at the plate.

The aim of this investigation is to consider the effects of transverse magnetic field, radiation, and first-order chemical reaction on the free convection heat and mass transfer along a vertical plate with non-uniform wall temperature and concentration conditions embedded in a micropolar fluid. The governing system of partial differential equations is transformed into a system of nonlinear ordinary differential equations using similarity transformations. This system of nonlinear ordinary differential equations is solved numerically using the Keller-box method given by Cebeci and Bradshaw [21]. The effects of various parameters on the skin friction coefficient, wall couple stress, and heat and mass transfer rates are presented graphically.

2. Mathematical formulation

Considering a steady, laminar, incompressible, two-dimensional free convective heat and mass transfer along a semiinfinite vertical plate embedded in a free stream of electrically conducting micropolar fluid with temperature T_∞ and concentration C_∞, choose the coordinate system such that the x-axis is along the vertical plate and the y-axis is normal to the plate, as shown in Figure 1. The plate is maintained at non-uniform temperature $T_w(x)$ and concentration $C_w(x)$. These values are assumed to be greater than the ambient temperature T_∞ and concentration C_∞. A uniform magnetic field of magnitude B_0 is applied normal to the plate. The magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected in comparison with the applied magnetic field. The fluid is considered to be a gray medium absorbing emitting radiation, but nonscattering, and the Rosseland approximation [22] is used to describe the radiative heat flux in the energy equation. The radiative heat flux in the x-direction is considered negligible in comparison to the y-direction. The viscous dissipation and the Joule heating effects are assumed to be negligible in the energy equation. It is also assumed that there exists a homogeneous chemical reaction of the first order with rate constant R^* between the diffusing species and the fluid.

Using the Boussinesq and boundary layer approximations, the governing equations for the micropolar fluid are given by:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

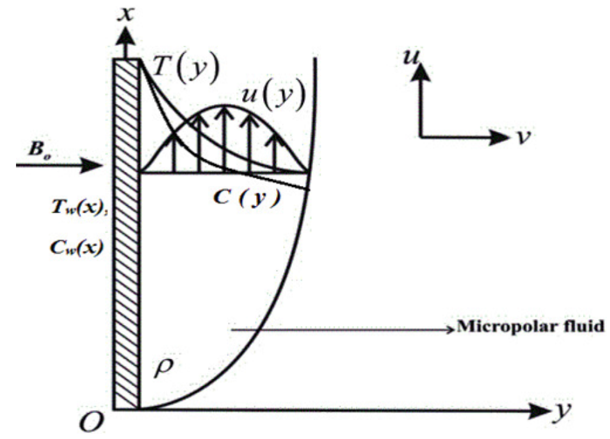

Figure 1. Physical model and coordinate systems.

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\left(\frac{\mu+\kappa}{\rho}\right)\frac{\partial^2 u}{\partial y^2}+\frac{\kappa}{\rho}\frac{\partial \omega}{\partial y}+g^*\left(\beta_T\left(T-T_\infty\right)+\beta_c\left(C-C_\infty\right)\right)-\frac{\sigma B_0^2}{\rho}u, \tag{2}$$

$$u\frac{\partial \omega}{\partial x}+v\frac{\partial \omega}{\partial y}=\left(\frac{\gamma}{\rho j}\right)\frac{\partial^2 \omega}{\partial y^2}-\frac{\kappa}{\rho j}\left(2\omega+\frac{\partial u}{\partial y}\right), \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y}, \tag{4}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-R^*\left(C-C_\infty\right), \tag{5}$$

where u and v are the components of velocity along the x- and y-directions, respectively; ω is the component of microrotation whose direction of rotation lies perpendicular the xy-plane; g^* is the gravitational acceleration; T is the temperature; C is the concentration; β_T is the coefficient of thermal expansion; β_C is the coefficient of solutal expansion; C_p is the specific heat capacity; B_0 is the coefficient of the magnetic field; μ is the dynamic coefficient of viscosity of the fluid; q_r is the radiative heat flux; ρ is the density; κ is the vortex viscosity; j is the microinertia density; γ is the spin-gradient viscosity; σ is the magnetic permeability of the fluid; α is the thermal diffusivity; and D is the molecular diffusivity.

The boundary conditions are

$$u=0; v=0; \omega=0; T=T_w(x); C=C_w(x) \text{ at } y=0, \tag{6a}$$

$$u\to 0; \omega\to 0; T\to T_\infty; C\to C_\infty \text{ as } y\to\infty, \tag{6b}$$

where the subscripts w and ∞ indicate the conditions at the wall and the outer edge of the boundary layer, respectively.

The radiative heat flux q_r is described by the Rosseland approximation [22] such that

$$q_r=-\frac{4\sigma^*}{3k_1}\frac{\partial T^4}{\partial y}, \tag{7}$$

where σ^* and k_1 are the Stefan–Boltzmann constant and the mean absorption coefficient, respectively. We assume that the differences of the temperature within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expansion in a Taylor series about T_∞ and neglecting higher-order terms; thus:

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4. \tag{8}$$

Using Eqs. (7) and (8) in the last term of Eq. (4), we obtain

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{16\sigma^* T_\infty^3}{3\rho C_p k_1}\frac{\partial^2 T}{\partial y^2}. \tag{9}$$

The continuity Eq. (1) is satisfied by introducing the stream function ψ such that

$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x}. \tag{10}$$

In order to explore the possibility of the existence of similarity, we assume

$$\psi = Ax^a f(\eta), \eta = Bx^b y, \omega = Ex^c g(\eta),$$

$$\frac{T - T_\infty}{T_w(x) - T_\infty} = \theta(\eta), T_w(x) - T_\infty = M_1 x^l,$$

$$\frac{C - C_\infty}{C_w(x) - C_\infty} = \varphi(\eta), C_w(x) - C_\infty = N_1 x^m, \tag{11}$$

where $A, B, E, M_1, N_1, a, b, c, l,$ and m are constants. Substituting Eqs. (10) and (11) into Eqs. (2), (3), (4), and (9), it is found that similarity exists only if $a = 1, b = 0, c = 1, l = m = 1$.

Hence, the appropriate similarity transformations are

$$\psi = Axf(\eta), \eta = By, \omega = Exg(\eta),$$

$$\frac{T - T_\infty}{T_w(x) - T_\infty} = \theta(\eta), T_w(x) - T_\infty = M_1 x,$$

$$\frac{C - C_\infty}{C_w(x) - C_\infty} = \varphi(\eta), C_w(x) - C_\infty = N_1 x. \tag{12}$$

Making use of the dimensional analysis, the constants A, B, E, M_1, and N_1 are, respectively, the dimensions of velocity, the reciprocal of length, the reciprocal of the product of length and time, the ratio of (temperature/length), and the ratio of (concentration/length).

Substituting Eq. (12) into Eqs. (2), (3), (5), and (9), we obtain

$$\left(\frac{1}{1-N}\right) f''' + \left(\frac{N}{1-N}\right) g' - (f')^2 + ff'' + \theta + L\varphi - f'M = 0, \tag{13}$$

$$\lambda g'' - \left(\frac{N}{1-N}\right) \vartheta \ (2g + f'') - f'g + fg' = 0, \tag{14}$$

$$\frac{1}{3\,\mathrm{Pr}}(3 + 4R)\theta'' + f\theta' - f'\theta = 0, \tag{15}$$

$$\frac{1}{Sc}\varphi'' - \delta\varphi + f\varphi' - f'\varphi = 0, \tag{16}$$

where $\Pr = \frac{\nu}{\alpha}$ is the Prandtl number; $Sc = \frac{\nu}{D}$ is the Schmidt number; $\lambda = \frac{\gamma}{j\rho\nu}$ is the spin-gradient viscosity; $N = \frac{\kappa}{\kappa+\mu}$, $(0 \leq N < 1)$ is the Coupling number; $L = \frac{\beta_c N_1}{\beta_T M_1}$ is the buoyancy parameter; $M = \frac{\sigma B_0^2}{\mu B^2}$ is the magnetic field parameter; $\vartheta = \frac{1}{jB^2}$ is the microinertia density; $R = \frac{4\sigma^* T_\infty^3}{kk_1}$ is the radiation parameter; and $\delta = \frac{R^*}{\nu B^2}$ is the chemical reaction parameter. The primes denote differentiation with respect to similarity variable η.

The boundary conditions of Eq. (6) in terms of f, g, θ, and φ become

$$f(0) = 0, f'(0) = 0, g(0) = 0, \theta(0) = 1, \varphi(0) = 1 \text{ at } \eta = 0, \tag{17a}$$

$$f'(\infty) \to 0, g(\infty) \to 0, \theta(\infty) \to 0, \varphi(\infty) \to 0 \text{ as } \eta \to \infty. \tag{17b}$$

3. Skin friction and wall couple stress

The wall shear stress and wall couple stress are

$$\tau_w = \left[(\mu+\kappa)\frac{\partial u}{\partial y} + \kappa\omega\right]_{y=0}, m_w = \gamma\left[\frac{\partial \omega}{\partial y}\right]_{y=0}. \tag{18}$$

The dimensionless wall shear stress $C_f = \frac{2\tau_w}{\rho A^2}$ and wall couple stress $M_w = \frac{Bm_w}{\rho A^2}$, where A is the characteristic velocity, are given by

$$C_f = \left(\frac{2}{1-N}\right) f''(0)\,\bar{x} \text{ and } M_w = \left(\frac{\lambda}{\vartheta}\right) g'(0)\,\bar{x}, \tag{19}$$

where $\bar{x} = Bx$.

4. Heat and mass transfer rates

The heat and mass transfers from the plate are respectively given by

$$q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} - \frac{4\sigma^*}{3k_1}\left(\frac{\partial T^4}{\partial y}\right)_{y=0} \text{ and } q_m = -D\left(\frac{\partial C}{\partial y}\right)_{y=0}. \tag{20}$$

The nondimensional rate of heat transfer, called the Nusselt number $Nu = \frac{q_w}{Bk(T_w - T_\infty)}$, and rate of mass transfer, called the Sherwood number $Sh = \frac{q_m}{BD(C_w - C_\infty)}$, are given by

$$Nu = -\theta'(0)\left(1 + \frac{4R}{3}\right) \text{ and } Sh = -\varphi'(0). \tag{21}$$

5. Results and discussion

The flow Eqs. (12) and (13), which are coupled, together with the energy and concentration Eqs. (14) and (15), constitute nonlinear nonhomogeneous differential equations for which closed-form solutions cannot be obtained. Hence, the governing Eqs. (12) through (15) have been solved numerically using the Keller-box implicit method [21]. This method has a second-order accuracy, is unconditionally stable, and is easy to be programmed, thus making it highly attractive for production use. A uniform grid was adopted, which is concentrated towards the wall. The calculations are repeated until some convergent criterion is satisfied and the calculations are stopped when $\delta f''(0) \leq 10^{-8}$, $\delta\theta'(0) \leq 10^{-8}$, and $\delta\varphi'(0) \leq 10^{-8}$. In the present study, the boundary conditions for η at ∞ are replaced by a sufficiently large value where the velocity, temperature, and concentration approach zero. In order to see the effects of step size ($\Delta\eta$), we ran the code for our model with three different step sizes of $\Delta\eta = 0.001$, $\Delta\eta = 0.01$, and $\Delta\eta = 0.05$, and in each case we found very good agreement between them for different profiles. After some trials we imposed a maximal value of η at ∞ of 6 and a grid size of $\Delta\eta$ as 0.01. In order to study the effects of the coupling number N, magnetic field parameter M, radiation parameter R, chemical reaction parameter δ, Prandtl number Pr, and Schmidt number Sc on the physical quantities of the flow, the remaining parameters are fixed as $L = 1$, $\lambda = 1$, and $\vartheta = 0.1$. The values of micropolar parameters λ and ϑ are chosen so as to satisfy the thermodynamic restrictions on the material parameters given by Eringen [3]

In the absence of coupling number N, magnetic parameter M, radiation parameter R, chemical reaction parameter δ, and buoyancy number L with $\lambda = \vartheta = 0$, $Pr = 1.0$, and $Sc = 0.24$, the results were compared with the exact values of Pop and Ingham [23] and Merkin [24], and it was found that they are in good agreement, as shown in Tables 1 and 2.

Table 1. Comparison of skin friction $f''(0)$ calculated by the present method and that of Pop and Ingham [23] and Merkin [24] for $N = M = R = \delta = L = 0$ and $Pr = 1.0$ and $Sc = 0.24$.

Pop and Ingham [23]	Merkin [24]	Present study
0.7395 (exact)	0.7395 (exact)	0.7395
0.9636 (series)	0.9636 (series)	

Table 2. Comparison of $\theta'(0)$ calculated by the present method and that of Pop and Ingham [23] and Merkin [24] for $N = M = R = \delta = L = 0$ and $Pr = 1.0$ and $Sc = 0.24$.

Pop and Ingham [23]	Merkin [24]	Present study
–0.5951 (exact)	–0.5951 (exact)	–0.5950
–0.5814 (series)	–0.5814 (series)	

The coupling number N characterizes the coupling of linear and rotational motion arising from the micromotion of the fluid molecules. Hence, N signifies the coupling between the Newtonian (μ) and rotational viscosities (κ) and hence $0 \leq N < 1$. With a large value of N, the effect of microstructure becomes significant, whereas with a small value of N the individuality of the substructure is much less pronounced. As $\kappa \to 0$, i.e. N $\to$ 0, the micropolarity is lost and the fluid behaves as a nonpolar fluid. Hence, N $\to$ 0 corresponds to viscous fluid.

Figures 2a and 2b depict the variation of heat and mass transfer rates (Nusselt number Nu and Sherwood number Sh) with coupling number N for different values of magnetic parameter M. It is observed that both the Nusselt number and Sherwood number decrease as the coupling number increases from 0 to 1. The coupling number N characterizes the coupling of linear and rotational motion arising from the micromotion of the fluid molecules. Hence, N signifies the coupling between the Newtonian and rotational viscosities. As $N \to 1$, the effect of microstructure becomes significant, whereas with a small value of N the individuality of the substructure is much less pronounced. As $\kappa \to 0$, i.e. $N \to 0$, the micropolarity is lost and the fluid behaves as a nonpolar fluid. Hence, $N \to 0$ corresponds to viscous fluid. It is noticed that the heat and mass transfer rates are higher in the case of viscous fluids. Therefore, the presence of microscopic effects arising from the local structure and micromotion of the fluid elements reduces the heat and mass transfer rates. It is seen that both the Nusselt number and Sherwood number are decreasing as the magnetic parameter is increasing. The presence of a magnetic field in an electrically conducting fluid introduces a force called the Lorentz force, similar to the drag force. This resulting force acts against the flow if the magnetic field is applied in the normal direction. This type of resistive force tends to slow down the flow. Accordingly, the rates of heat and mass transfer from the wall will be reduced. The effect of M on the rates of heat and mass transfer in the case of a micropolar fluid is less compared to that of viscous fluid.

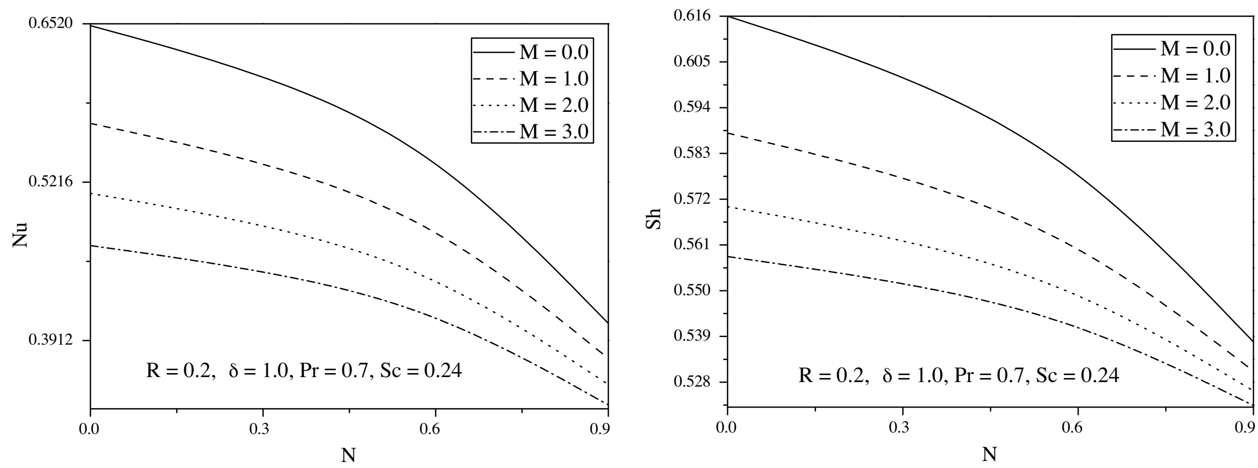

Figure 2. Effect of magnetic parameter M on (a) heat transfer rate and (b) mass transfer rate versus coupling number N.

The effect of radiation parameter on heat and mass transfer coefficient is displayed in Figures 3a and 3b. It is noticed from these figures that both the Nusselt number and Sherwood number increase with the increase in the radiation parameter. Higher values of radiation parameter R imply higher values of wall temperature. Consequently, the temperature gradient and hence the Nusselt number and Sherwood number increase. The effect of R on the heat and mass transfer rates in the case of a micropolar fluid is less compared to that of viscous fluid.

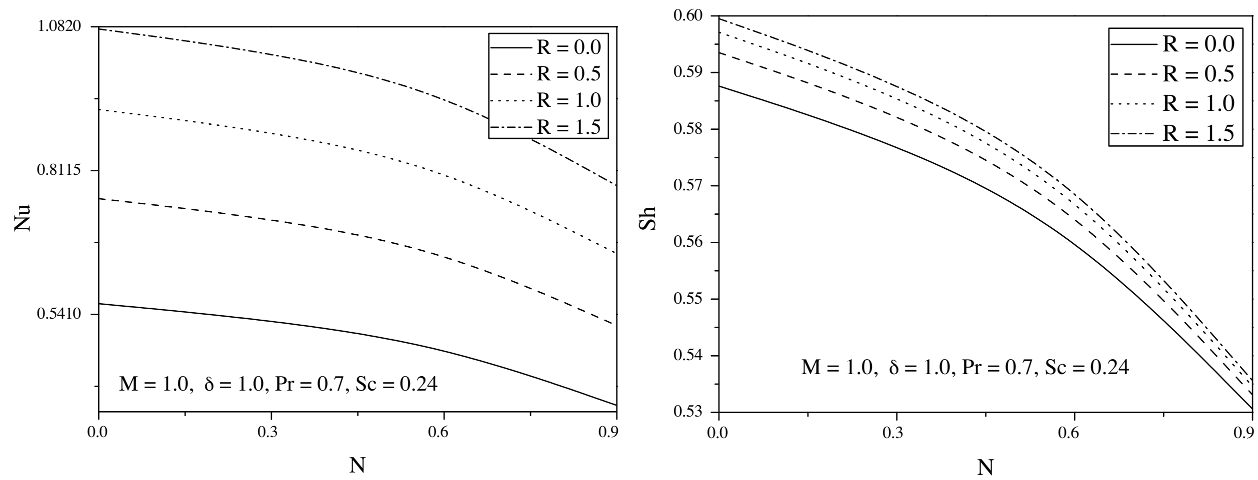

Figure 3. Effect of radiation parameter R on (a) heat transfer rate and (b) mass transfer rate versus coupling number N.

The variation of heat and mass transfer coefficients with coupling number for different values of chemical reaction parameter δ is depicted in Figures 4a and 4b. It is clear from these figures that an increase in the chemical reaction parameter δ leads to a decrease in the Nusselt number and an increase in the Sherwood number. Increase in the values of δ implies more interaction of species concentration with the momentum boundary layer and less interaction with the thermal boundary layer. Hence, the chemical reaction parameter has a more significant effect on Sherwood number than it does on Nusselt number.

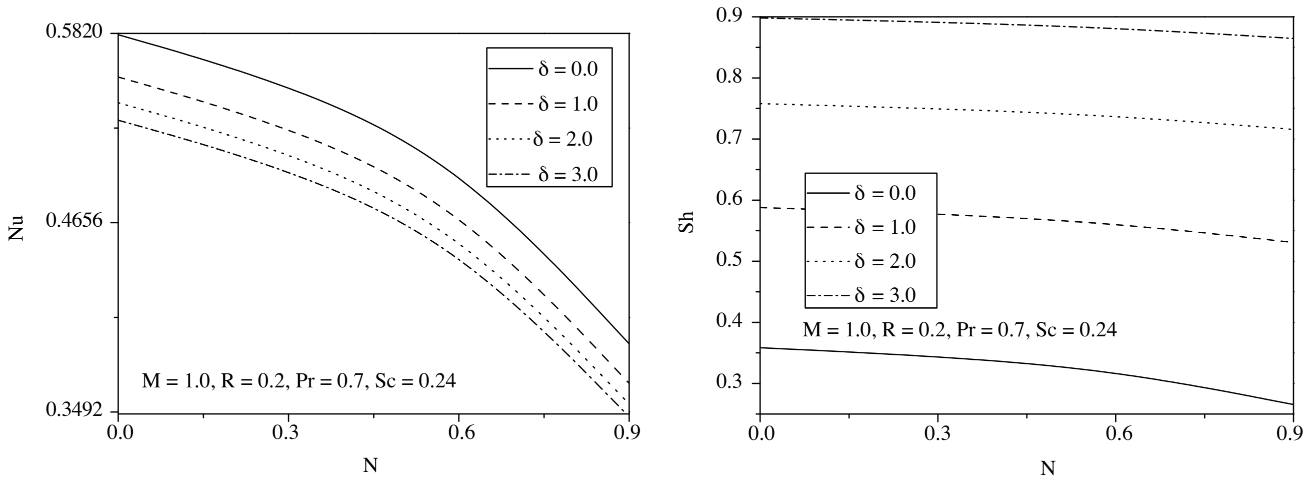

Figure 4. Effect of chemical reaction parameter δ on (a) heat transfer rate and (b) mass transfer rate versus coupling number N.

Figures 5a and 5b show the variation of heat and mass transfer coefficients with Prandtl number *Pr*. It is interesting to note that the Nusselt number is increasing but the Sherwood number is decreasing as the Prandtl number increases. The reason for this is that smaller values of *Pr* are equivalent to increasing the thermal conductivities, and therefore heat is able to diffuse away from the heated plate more rapidly than for higher values of *Pr*. Hence, in the case of smaller Prandtl numbers, as the boundary layer is thicker the rate of heat transfer is reduced.

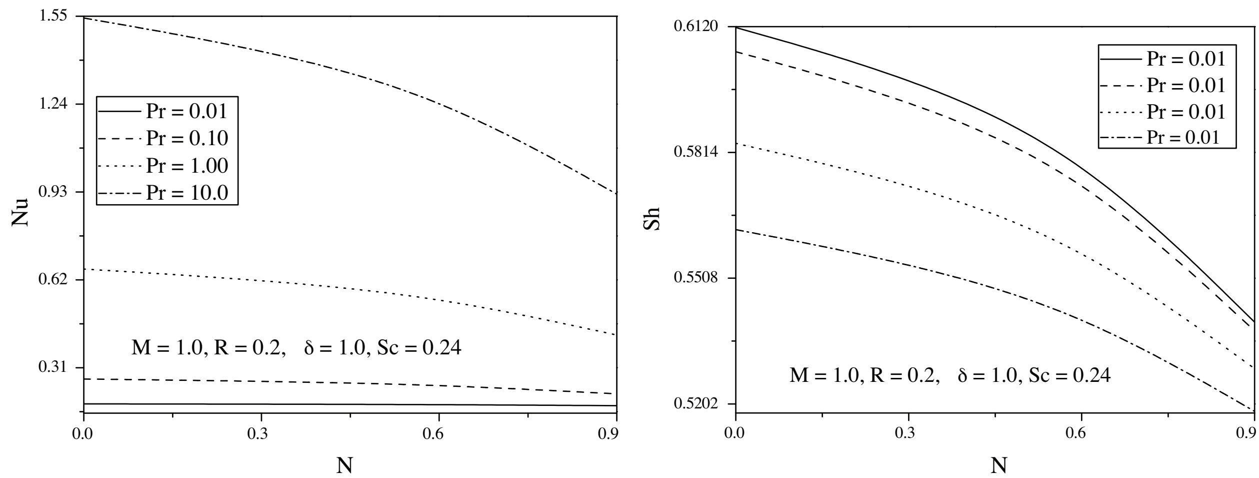

Figure 5. Effect of Prandtl number *Pr* on (a) heat transfer rate and (b) mass transfer rate versus coupling number N.

The effect of Schmidt number on the heat and mass transfer coefficients is plotted in Figures 6a and 6b. It is clear that the Nusselt number is decreasing while the Sherwood number is increasing with increasing values of *Sc*. The Schmidt number embodies the ratio of the momentum to the mass diffusivity. The Schmidt number therefore quantifies the relative effectiveness of momentum and mass transport by diffusion in the hydrodynamic (velocity) and concentration (species) boundary layers. Its effect on the species concentration has similarities to the Prandtl number's effect on the temperature: increase in the values of *Sc* cause the velocity and species concentration and the boundary layer thickness to decrease significantly.

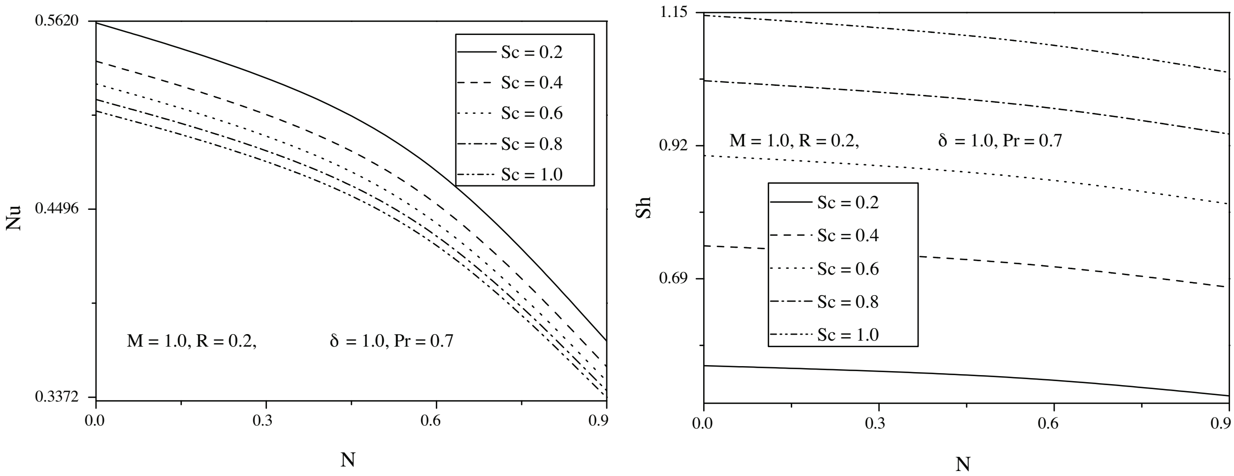

Figure 6. Effect of Schmidt number S_c on (a) heat transfer rate and (b) mass transfer rate versus coupling number N.

Table 3 shows the effects of coupling number N, Prandtl number *Pr*, Schmidt number *Sc*, radiation parameter R, chemical reaction parameter δ, and magnetic parameter M on skin friction C_f and dimensionless wall couple stress M_w. It is seen from this table that both the skin friction and the wall couple stress decrease with increasing coupling number N. For increasing values of N, the effect of the microstructure becomes significant and hence the wall couple stress decreases. The skin friction coefficient decreases and the wall couple stress increases with increasing Prandtl number and Schmidt number. The effect of the magnetic parameter is to decrease the skin friction coefficient and increase the wall couple stress. Furthermore, it is observed that the skin friction coefficient is increasing and wall couple stress is decreasing with increase in the value of radiation parameter. The increase in chemical reaction parameter decreases the skin friction coefficient and increases the wall couple stress.

6. Conclusions

In this paper, a boundary layer analysis for free convection heat and mass transfer in an electrically conducting micropolar fluid over a vertical plate with variable wall temperature and concentration conditions in the presence of a magnetic field, first-order chemical reaction, and radiation is considered. Using the similarity variables, the governing equations are transformed into a set of ordinary differential equations. The effects of different parameters on Nusselt and Sherwood numbers are presented through graphs. From the present study, we observe that:

- The Nusselt number, Sherwood number, and the skin friction coefficient decrease and the wall couple stress increases as magnetic parameter M increases.
- An increase in the value of radiation parameter Rimplies increase in the Nusselt number, Sherwood number, and skin friction coefficient and decrease in wall couple stress.

- An increase in the chemical reaction parameter implies decrease in the Nusselt number and skin friction coefficient and increase in the Sherwood number and wall couple stress.

Table 3. Effect of skin friction and wall couple stress for various values of N, Pr, Sc, M, R, and δ.

N	Pr	Sc	M	R	δ	$f''(0)$	$-g'(0)$
0.0	1.0	0.6	1.0	0.02	1.0	1.0108	0.0000
0.3	1.0	0.6	1.0	0.02	1.0	0.7976	0.0079
0.6	1.0	0.6	1.0	0.02	1.0	0.5446	0.0249
0.9	1.0	0.6	1.0	0.02	1.0	0.1947	0.0729
0.5	0.01	0.6	1.0	0.02	1.0	0.7564	0.0263
0.5	0.1	0.6	1.0	0.02	1.0	0.7318	0.0244
0.5	1.0	0.6	1.0	0.02	1.0	0.6352	0.0175
0.5	10	0.6	1.0	0.02	1.0	0.5071	0.0113
0.5	100	0.6	1.0	0.02	1.0	0.4156	0.0094
0.5	1.0	0.2	1.0	0.02	1.0	0.6954	0.0209
0.5	1.0	0.6	1.0	0.02	1.0	0.6352	0.0175
0.5	1.0	0.7	1.0	0.02	1.0	0.6264	0.0170
0.5	1.0	1.0	1.0	0.02	1.0	0.6061	0.0162
0.5	1.0	2.0	1.0	0.02	1.0	0.5676	0.0149
0.5	1.0	0.6	0.0	0.02	1.0	0.7487	0.0235
0.5	1.0	0.6	1.0	0.02	1.0	0.6352	0.0175
0.5	1.0	0.6	2.0	0.02	1.0	0.5651	0.0141
0.5	1.0	0.6	3.0	0.02	1.0	0.5155	0.0114
0.5	1.0	0.6	1.0	0.0	1.0	0.6337	0.0174
0.5	1.0	0.6	1.0	0.5	1.0	0.6617	0.0193
0.5	1.0	0.6	1.0	1.0	1.0	0.6787	0.0205
0.5	1.0	0.6	1.0	1.5	1.0	0.6903	0.0213
0.5	1.0	0.6	1.0	0.0	0.0	0.6960	0.0207
0.5	1.0	0.6	1.0	0.5	1.0	0.6352	0.0175
0.5	1.0	0.6	1.0	1.0	2.0	0.6043	0.0162
0.5	1.0	0.6	1.0	1.5	3.0	0.5840	0.0155

Acknowledgment

This work was supported by the Council of Scientific and Industrial Research, New Delhi, India (Project No. 25 (0189)/10/EMR-II).

Nomenclature

A	Characteristic velocity
B	Reciprocal of length
B_0	Magnetic field coefficient
C	Concentration
C_p	Specific heat at constant pressure
C_∞	Free stream concentration
C_f	Skin-friction coefficient
D	Mass diffusivity
E	Reciprocal of the product of length and time
f	Dimensionless stream function
g	Dimensionless microrotation
g^*	Acceleration due to gravity
j	Dimensional microinertia density
ϑ	Dimensionless microinertia density$(\frac{1}{jB^2})$
k_1	Mean absorption coefficient
L	Buoyancy parameter $\left(=\frac{\beta_c N_1}{\beta_T M_1}\right)$
m_w	Wall couple stress
M	Magnetic parameter $\left(=\frac{\sigma B_0^2}{\mu B^2}\right)$
M_1	The ratio (temperature/length)
M_w	Nondimensional couple stress on the wall
N	Coupling number $\left(=\frac{\kappa}{\mu+\kappa}\right)$
N_1	Ratio (concentration/length)

N_u Nusselt number
P_r Prandtl number $(=\nu/\alpha)$
q_r Radiative heat flux
R Thermal radiation parameter $\left(=\frac{4\sigma^* T_\infty^3}{kk_1}\right)$
R^* Rate of chemical reaction
q_w Heat flux
q_m Mass flux
S_c Schmidt number $(=\nu/D)$
S_h Sherwood number
T Temperature
T_∞ Free stream temperature
$u; v$ Velocity components in the x- and y-directions, respectively
x, y Cartesian coordinates along the plate and normal to it

Greek symbols

α Thermal diffusivity
β_T Coefficient of thermal expansion
β_c Coefficient of concentration expansion
γ Spin-gradient viscosity
δ Chemical reaction parameter $\left(=\frac{R*}{\nu B^2}\right)$
η Similarity variable
θ Dimensionless temperature
κ Vortex viscosity
λ Dimensionless spin-gradient viscosity $\left(=\frac{\gamma}{j\rho\nu}\right)$
μ Viscosity of the fluid
ρ Density of the fluid
σ Electrical conductivity
σ^* Stefan–Boltzmann constant
τ_w Wall shear stress
ϕ Dimensionless concentration
ψ Stream function
ω Microrotation component

Subscripts

w Condition at wall
∞ Condition at infinity

Superscripts

$'$ Differentiation with respect to η

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