



Orijinal Araştırma / Original Research

## INVESTIGATION OF BLAST FRAGMENTATION MODELS IN A SANDSTONE QUARRY

### PATLATMA KAYNAKLI PARÇALANMA MODELLERİNİN BİR KUMTAŞI OCAĞINDA İNCELENMESİ

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#### ABSTRACT

The existing blast fragmentation models were investigated in a sandstone quarry for bench blasting operations. It was tried to present different perspectives for evaluation of blast fragmentation. Size distribution curves were created for each blast. KuzRam model provided successful results. Kuznetsov-Cunningham-Ouchterlony (KCO) model was useful especially for the fine size predictions. Capability of the models for fine, mean and coarse size predictions were evaluated in detail. Uniformity coefficients proposed by the models were examined. Chung & Katsabanis model was practically tested and a modification implemented for the studied quarry. Model validation was performed using ten different error metrics. Applicability of symmetric errors and scaled errors was investigated for evaluation of blast fragmentation. KuzRam model predicted mean fragment size with an error lower than three centimeters. The results of the modified Chung & Katsabanis model is quite promise.

#### Anahtar Sözcükler:

Taş ocağı,  
Kaya patlatma,  
Parçalanma,  
Boyut dağılımı,  
Hata ölçütleri.

#### ÖZ

Parçalanma modelleri bir kumtaşı ocağında basamak patlatması operasyonları için incelenmiştir. Patlatma kaynaklı parçalanmanın değerlendirilmesi amacıyla farklı bakış açıları ortaya konmuştur. Her atım için boyut dağılım eğrileri çizilmiştir. KuzRam modelinin başarılı sonuçlar verdiği görülmüştür. KCO modeli özellikle ince boyutlu malzemenin tahmininde başarılıdır. Modellerin yetkinliği ince, orta ve iri boyutlu malzeme miktarının tahmini açısından detaylı bir şekilde incelenmiştir. Chung ve Katsabanis modeli pratik olarak uygulanmış ve ocak için bir modifikasyon önerilmiştir. Modellerin validasyonu için on farklı hata ölçütü kullanılmıştır. Simetrik hata ve ölçekli hata ölçütlerinin parçalanmanın değerlendirilmesi amacıyla kullanılabilirliği incelenmiştir. KuzRam modeli ortalama parça boyutunu üç santimetrenin altında bir hata ile tahmin edebilmektedir. Modifiye edilmiş Chung ve Katsabanis modeli de başarılı sonuçlar vermiştir.

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## INTRODUCTION

Rock blasting is the main rock excavation operation in open pit mines and quarries. Particle size distribution of muckpile influences all the downstream processes. Oversize blocks cannot be loaded and dumped into crusher bins. For some operations, fine materials are undesirable. Not only suitable particle size but also uniform size distribution is desired. Suitable size distribution increases crusher throughput. Blast fragmentation models are helpful tools for estimating size distribution of blast muckpile.

There are different blast fragmentation models appearing in the literature. Generally, the first target of a blast fragmentation model is to predict mean fragment size (50% passing size). Mean size is a central measure for prediction. Blast parameters are used to predict mean fragment size. Especially, main blast design parameters (burden, spacing, bench height, stemming height etc.) play important role in the determination of mean size. Some models also consider rock mass properties. In addition to mean size, sometimes, a particular characteristic size is used to define fragmentation. In second stage, a size distribution curve is constructed using a specific curve equation. Slope of the curve indicates uniformity of the distribution. Generally, models utilize blast design parameters to estimate uniformity coefficients.

Kuznetsov equation (1973) is one of the most used mean fragment size prediction formula. Larsson's formula (Larsson, 1974), SVEDEFO equation and Saroblast equation (Kou and Rustan, 1993) etc. are the samples of the other mean fragment size prediction formulas appearing in the blasting literature. KuzRam is the most well-known blast fragmentation model (Cunningham, 1983, 1987). KuzRam model relies on Kuznetsov mean size prediction equation and Rosin-Rammler size distribution curve (Rosin & Rammler, 1933; Weibull, 1951). Like the previous approaches, KuzRam model defines the 50% passing size as mean fragment size ( $x_{50}$ ). 63.2% passing size is defined as characteristics size ( $x_c$ ). The slope of the size distribution curve is characterized by the uniformity index. Uniformity index is calculated using blast parameters.

Several researchers have tried to modify KuzRam model to find solutions for the main drawbacks of

the model. Julius Kruttschnitt Mineral Research Centre (JKMRC) researchers investigated crushed zone around blastholes. They examined origin of the fine material. (Kanchibotla et al., 1999; Djordjevic, 1999). Chung and Katsabanis (C&K) Model (2000) is a modification of original KuzRam model. C&K Model calculates 80% passing size ( $x_{80}$ ) and defines the uniformity index using 80% passing size and mean fragment size ( $x_{50}$ ). Chung and Katsabanis (2000) also modified Kuznetsov's mean fragment size prediction equation. KCO model is relatively new. KCO model utilizes Kuznetsov equation to predict mean fragment size, however, it uses Swebrec function to construct size distribution curve. (Ouchterlony, 2005a). Ouchterlony claims that KCO model solves two main problems related to KuzRam model: poor prediction ability in the fines range and upper limit cut off block sizes.

KuzRam model has been applied by several researchers. However, KCO model generally been practiced by the model's creators. A comprehensive site application for C&K model has not been reported. The aim of this paper is the application of the aforementioned fragmentation models to a particular case. The models were used in a sandstone quarry. The results of the models were compared to each other. It was tried to present different perspectives for evaluation of blast fragmentation. Also, C&K model was modified considering the results obtained. The aim was to make the C&K model operative. Models were also compared by using different error metrics. In addition to mean and percent errors, applicability of symmetric errors and scaled errors was investigated for evaluation of blast fragmentation.

In the following pages, the site measurements performed in a quarry are explained. Blast fragmentation models are introduced and differences among models are discussed. The measured size distribution curves and the prediction of the fragmentation models are compared. Prediction capability of the models for mean size, coarse size and fine size fractions is examined in detail. In the section three, ten different error metrics are applied to calculate the forecasting errors. Finally, the discussion and conclusion parts give a brief summary and critique of the findings.

## 1. A SUMMARIZED EXPLANATION OF BLAST FRAGMENTATION MODELS

### 1.1. KuzRam Model

Kuznetsov equation (1973), which is the starting point of KuzRam Model, is the very first formula that makes it possible to calculate average fragment size using blast design parameters. Kuznetsov formula is presented as follow:

$$x_{50} = A \times \left(\frac{V_0}{Q_e}\right)^{0.8} \times Q_e^{0.167} \times \left(\frac{115}{E}\right)^{0.633} \quad (1)$$

where  $x_{50}$  is average fragment size (cm),  $A$  is the rock factor (7 for medium rock, 10 for hard highly fissured rock, and 13 for hard weakly fissured rocks),  $V_0$  is the rock volume broken per hole ( $m^3$ ),  $Q_e$  is the mass of explosive per hole (kg) and  $E$  is the relative weight strength of explosive to ANFO.

Rosin-Rammler function is used to construct particle size distribution of blasted material (Cunningham, 1983). The function is expressed in the form:

$$R = e^{-\left(\frac{x}{x_c}\right)^n} \quad (2)$$

where  $R$  is the proportion of the material retained on the screen (%),  $x_c$  is the characteristic size,  $x$  is the screen size (cm) and  $n$  is the uniformity index. The characteristic size is defined as 63.2% passing size. If the characteristic size ( $x_c$ ) is known, size distribution curve can be drawn:

$$x_c = \frac{x}{\sqrt[4]{0.693}} \quad (3)$$

Cunningham (1983, 1987) proposed a uniformity equation considering blast design parameters. Uniformity index ( $n$ ) is given by:

$$n = 1.1 \times \left[2.2 - 14 \times \left(\frac{B}{D}\right)\right] \times \left(\frac{1+S}{2}\right)^{0.5} \times \left(1 - \frac{W}{B}\right) \times \left(\frac{L}{H}\right) \quad (4)$$

In Eq. 4,  $D$  is the hole diameter (mm),  $B$  is the burden (m),  $S$  is the spacing between adjacent holes (m),  $W$  is the standard deviation of drilling accuracy,  $L$  is the hole length (m),  $H$  is the bench height (m).

Cunningham (1987) also suggested an equation to estimate the rock factor  $A$  more precisely. This suggestion was made based on the Lilly's Blastability Index (Lilly, 1986):

$$A = 0.06 \times (RMD+JPS+JPO+RDI+HF) \quad (5)$$

where  $RMD$  is the rock mass description,  $JPS$  is the joint plane spacing,  $JPO$  is the joint plane orientation,  $RDI$  is the rock density influence,  $HF$  is the hardness factor. Cunningham (2005) also made a review on rock factor and KuzRam approach 20 years after the creation of the model.

### 1.2. Chung & Katsabanis (C&K) Model and The Modification

Chung & Katsabanis (2000) suggested that delay time and distribution of explosive in rock mass should be considered when calculating the mean fragment size. They recommended different formulas which can be assumed as an extended KuzRam Model. Chung & Katsabanis cited Stagg and Rhol (1987) and concluded that delay time has a critical effect but after a certain point there is no a remarkable change in the fragmentation. Therefore, they removed the delay time parameter from the model. In C&K model, mean fragment ( $x_{50}$ ) size is given by:

$$x_{50} = A Q_e^{-1.193} B^{2.461} (S/B)^{1.254} H^{1.266} \quad (6)$$

Chung and Katsabanis (2000) also suggested new formulas for 80% passing size ( $x_{80}$ ), characteristic size ( $x_c$ ) and uniformity index ( $n$ ):

$$x_{80} = 3 A Q_e^{-1.073} B^{2.43} (S/B)^{1.013} H^{1.111} \quad (7)$$

$$x_c = e^{0.565 \ln x_{50} + 0.435 \ln x_{80}} \quad (8)$$

$$n = 0.842 (\ln x_{80} - \ln x_{50}) \quad (9)$$

As seen in Eq. 9, uniformity index is calculated using  $x_{50}$  and  $x_{80}$  passing size. A brief discussion about C&K Model was made by Ouchterlony (2003).

In this research, a modification was implemented based on the site measurements. The calculated uniformity index of C&K model is considered low. Original uniformity index equation (Eq. 9) is multiplied by "2". Modified uniformity index ( $n\phi$ ) is given by:

$$n' = 2 [0.842 / (\ln x_{80} - \ln x_{50})] \quad (10)$$

From this point forward, modified C&K Model is referred as "Mod. C&K". Original and modified

C&K models are also compared and discussed in the upcoming parts of the paper.

### 1.3. KCO Model

Ouchterlony (2005a) claimed that KuzRam model was not enough to define the fine and coarse size fractions. He suggested a new approach called KCO Model. In this model, mean fragment size is calculated by Kuznetsov equation. Swebrec function is used to determine size distribution curve. Swebrec function is presented as:

$$R_m = 1/[1 + (\ln \frac{x_{max}}{x} / \ln \frac{x_{max}}{x_{50}})^b] \quad (11)$$

where  $x_{max}$  is the maximum fragment size (cm),  $x$  is the screen size and  $b$  is the curve undulation parameters. Curve undulation parameter is expressed by:

$$b = \left[ 2 \ln 2 \ln \left( \frac{x_{max}}{x_{50}} \right) \right] n \quad (12)$$

Detailed structure of KCO model was presented by two research papers published in 2005. (Ouchterlony, 2005 a, b). The Swebrec function used in KCO model was also further analyzed by Sanchidrián et al. (2014). WipFrag image analysis software also contains KCO model in its Blastcast Module (Wipfrag, 2016).

## 2. SITE MEASUREMENTS

Site measurements were conducted in a sandstone quarry in Istanbul Kemerburgaz region as a part of a graduate study performed in Istanbul Technical University Mining Engineering Department (Akyıldız, 2015). The yearly production capacity of the quarry is 6 000 000 tons. The rock structure of the region is referred to as Trakya formation. Carboniferous-aged Trakya formation is commonly exposed in the European side of Istanbul (Tugrul and Undul, 2006). The formation mainly consists of sandstone, shale, siltstone and mudstone sediments. Sandstone is the most common rock type in this formation and limestone and conglomerate interbeds or lenses are found between layers. Trakya formation is of marine origin and very intensely folded, fractured, and weathered (Tugrul and Undul, 2006).

The excavated material in Akdaglar quarry is dark

grey, green or brown, thick-bedded sandstone. Sandstone contains 60% silicium. The physical and mechanical properties of the rock are given in Table 1. The blasts were measured in the eastern part of the quarry. The rock mass properties were meticulously examined. Tectonic deformations are very common in the studied quarry benches. Joints, folding and laminations are easily observed. The apparent in-situ block size of the benches was analyzed by WipJoint joint analysis software. WipJoint allows users to quantify bench characteristics such as joint orientation, spacing and in-situ block size. Mean fracture spacing in the blast faces is 1.15 m with a standard deviation of 0.69 m. The joint aperture is between 0.1 and 1 mm. The blastability of rock mass was examined based on suggestion of Kuznetsov (1973) and Cunningham (1987). The rock factor (A) was calculated as between 7 and 7.5 for the studied benches. The fractured sandstone can be evaluated as medium rock considering blastability. Also, the examined blasting operations in the quarry have indicated that the rock shows an average resistance to blasting.

Table 1. Physical and mechanical properties of the rock

Density (g/cm <sup>3</sup> )	2.70
The Mohs hardness	5-6
Average porosity (%)	1.03
Compressive Strength (MPa)	77.5
Tensile Strength (MPa)	5.60
Young modulus (GPa)	16.9

Totally 18 blast measurements were considered in this study. The diameter of the drill hole is 89 mm in the quarry. The main explosive used is ANFO. Primer explosive is emulsion type. Millisecond electrical and non-electrical detonators are used to initiate the blast holes. All the measured design parameters are presented in Table 2. The size distribution of the measured muckpiles is determined by WipFrag image analysis software. A sample image taken from blast muckpile is presented in Figure 1. In each blast, several images were captured from different location of blast muckpile. The images were analyzed separately and results were merged to construct final size distribution curves of the blast muck piles.

Table 2. Measured blast design parameters

Blast No	B (m)	S (m)	H (m)	T (m)	Hc	L (m)	Qe (kg/hole)	Pf (kg/m <sup>3</sup> )
1	2.2	2.5	8.0	3.0	5.5	8.5	29.1	0.66
2	1.8	2.0	8.0	3.0	5.5	8.5	29.1	1.01
3	2.5	2.8	7.5	3.0	5.0	8.0	26.4	0.50
4	1.8	2.3	6.5	3.0	4.0	7.0	21.1	0.79
5	2.5	2.5	6.0	2.5	4.0	6.5	21.1	0.56
6	2.6	3.0	10.0	3.0	7.5	10.5	39.6	0.51
7	2.0	2.5	6.5	2.5	4.5	7.0	23.8	0.73
8	2.5	2.8	6.5	2.5	4.5	7.0	23.8	0.52
9	2.0	2.5	7.0	2.5	5.0	7.5	26.4	0.76
10	2.5	3.0	10.0	4.0	6.5	10.5	34.4	0.46
11	2.0	2.5	8.0	2.5	6.0	8.5	31.7	0.79
12	2.0	2.5	7.0	2.5	5.0	7.5	26.4	0.76
13	2.0	3.0	8.0	3.0	5.5	8.5	29.1	0.61
14	2.0	2.5	7.0	3.0	4.5	7.5	23.8	0.68
15	2.0	2.5	8.0	2.5	6.0	8.5	31.7	0.79
16	2.5	2.5	7.0	3.0	4.5	7.5	23.8	0.54
17	2.2	2.8	6.0	2.5	4.5	7.0	23.8	0.64
18	2.5	3.0	7.0	3.0	4.5	7.5	23.8	0.45

B: Burden, S: Spacing between holes, H: Bench height, T: Stemming, Hc: Charge Height, L: Hole length, Qe: Explosive charge per hole, Pf: Specific charge

The results of the image analysis were compared to the blast fragmentation models.

The same approach was employed by several researchers. Formerly, Gheibie et al. (2009) compared the results of image analysis and KuzRam model. Hudaverdi et al. (2012) evaluated the mean fragment size values obtained by image analysis and the Kuznetsov equation.

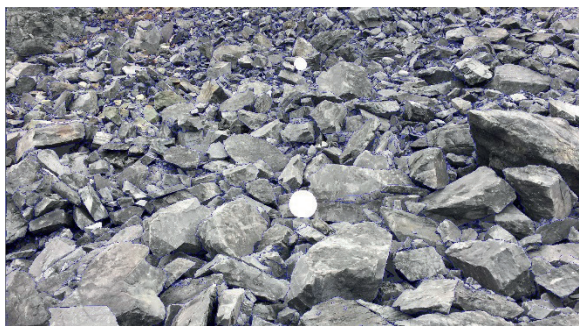


Figure 1. A sample image taken from blast muckpile

### 3. EVALUATION OF THE FRAGMENTATION MODELS

#### 3.1. Size Distribution Curves

Figure 2 shows particle size distribution curves for selected six sample blasts. The size distribution curves of the KuzRam, KCO, C&K and Mod. C&K models were drawn for each blast. Mostly the shapes of KuzRam and WipFrag curves are similar. The predictions of C&K model are excessive in the fine and coarse end of the curves due to its low uniformity index value. Modified C&K model (Mod. C&K) created more suitable curves for our case. Generally, the amount of the fine material predicted by KCO model is higher than the KuzRam predictions and WipFrag measurements. The results of the models are more similar for coarse size fractions (around + 20 cm for our case). The amount of the coarse material calculated by KCO model is relatively lower than those of the other models. Uniformity index is the key parameter to understand the shape of the curves.

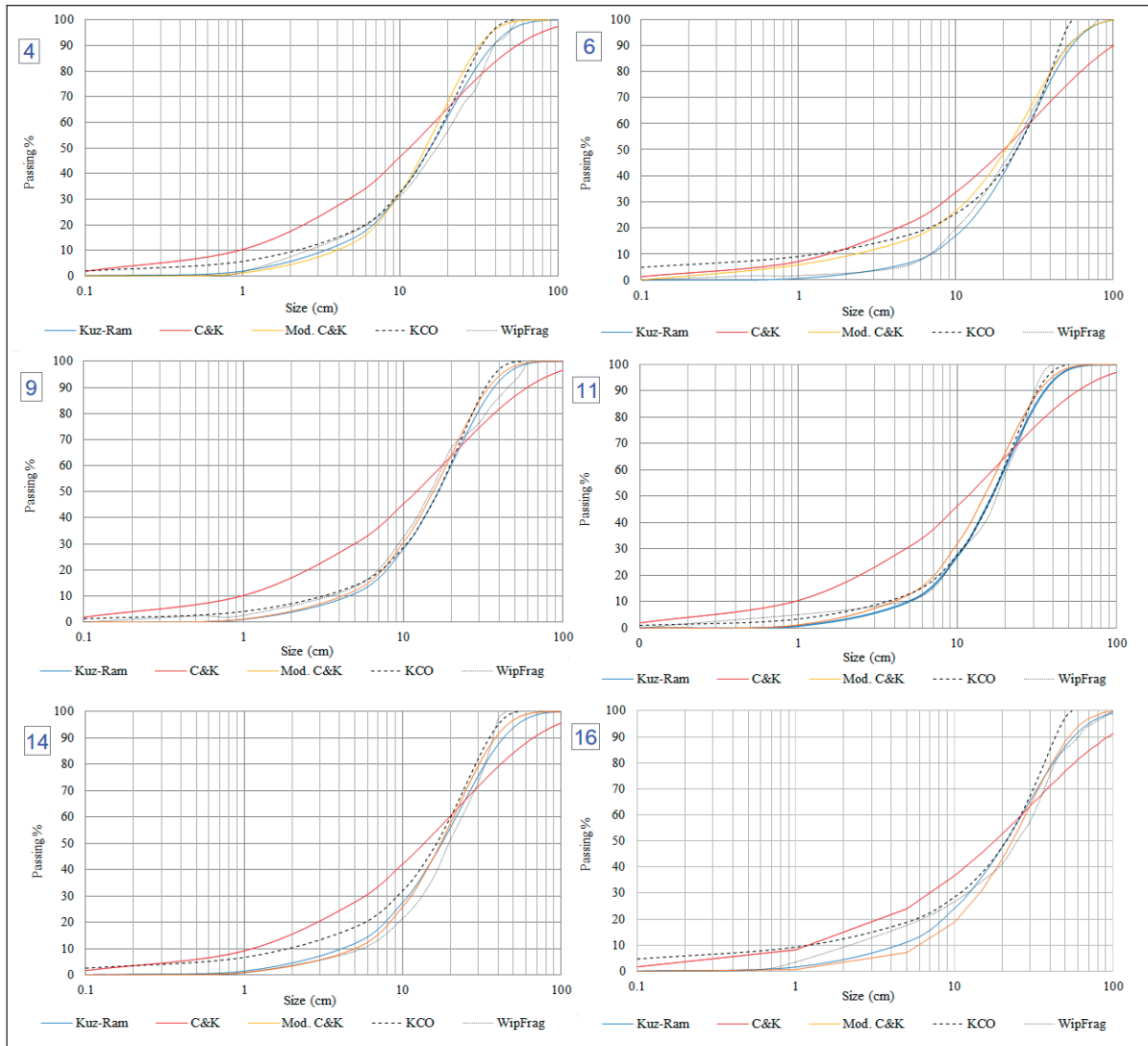


Figure 2. Size distribution curves belonging to the measured blasts

### 3.2. Fine, Mean and Coarse Size Predictions

The calculated and measured uniformity coefficients are presented in Figure 3. Uniformity coefficients determine the shape of particle size distribution curves. Mean uniformity index value ( $n$ ) for WipFrag measurements is 1.50 with a standard deviation of 0.153. Uniformity index of KuzRam tends to vary less than WipFrag uniformity index values. The calculated mean uniformity index for KuzRam model is 1.43 with a standard deviation of 0.11. The uniformity index of C&K model is almost constant. KCO model names the uniformity coefficient as curve undulation parameter ( $b$ ). Variation in curve undulation parameter is relatively high. Curve undulation parameters ( $b$ ) range between 1.08

and 2.93. Note that the uniformity index of Mod. C&K model is twice higher than that of original C&K model.

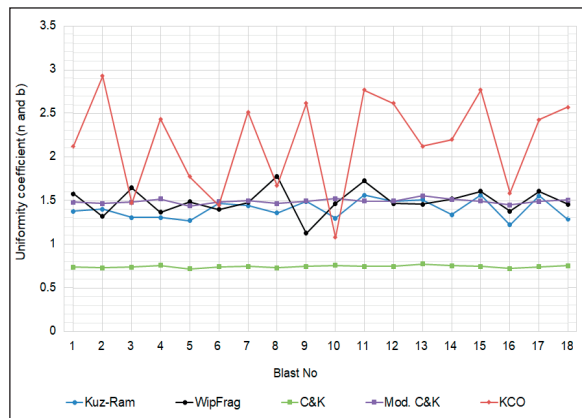


Figure 3. Comparison of the uniformity coefficients

The measured  $x_{50}$  values using WipFrag and the predicted  $x_{50}$  values by the models are presented in Figure 4. Mean size, also called median size by some researchers, is a central measure for blast fragmentation. Generally,  $x_{50}$  values are close to each other.  $x_{50}$  prediction of C&K model is smaller than that of KuzRam and WipFrag. The  $x_{50}$  values of Mod. C&K model are very close to those of the other models. Mod. C&K model apparently made better predictions than the original one. Particularly for the blasts 6, 9, 11 and 15, KuzRam predictions and WipFrag measurements are very close. It should be reminded that  $x_{50}$  predictions of C&K model is performed by using a slightly modified Kuznetsov Equation. Both KuzRam and KCO models use same Kuznetsov equation to calculate mean fragment size. Therefore, the predictions of KuzRam model in Figure 4 also represents KCO model.

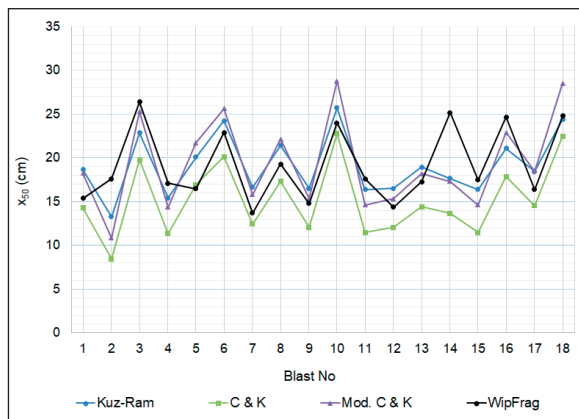


Figure 4. Predicted  $x_{50}$  values for the blast fragmentation models and WipFrag

Characteristics size (63.2% passing size,  $x_c$ ) is a fundamental parameter in KuzRam model. It is also used in C&K model. Figure 5 shows the calculated and measured  $x_c$  values. The calculated  $x_c$  values are more similar than the  $x_{50}$  values given in Figure 4. Particularly for the Blasts 1, 9, 11 and 12, calculated  $x_c$  values by the models are very close to each other. Mean  $x_c$  measured by WipFrag is 24.03 cm with a standard deviation 5.50 cm. The highest  $x_c$  was calculated for Mod. C&K model. The average  $x_c$  for Mod. C&K is 26.06 cm with a standard deviation of 7.33 cm.

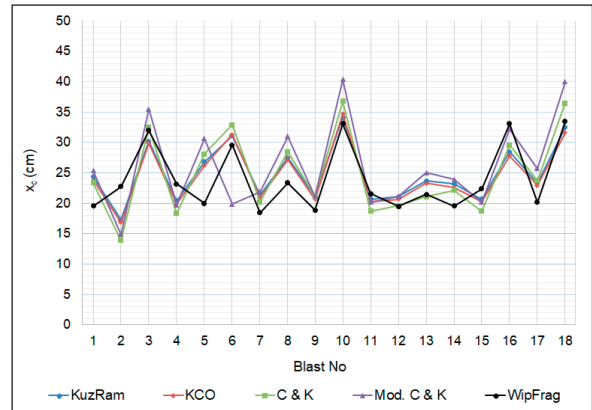


Figure 5. Predicted characteristic size ( $x_c$ ) values for the examined blasts

Figure 6 was created to examine the 20% passing size predictions of the fragmentation models. Largest differences among models occur in 20% passing size ( $x_{20}$ ).  $x_{20}$  is the indicator of the amount of the fine material. The calculated 20% passing size values for KCO model are smaller than those of KuzRam model and WipFrag measurements. In other words, KCO model estimates the amount of the fine material higher than that of KuzRam model and WipFrag. It is obvious that, generally the 20% passing size calculation of C&K model is excessive. Modified C&K Model gives more accurate results (Figure 6). For very fine size, the differences among KCO and other models are more visible. For example, for Blast 8, the amount of -1 cm material is measured as 1.70% using WipFrag software. However, it is 9.05% for KCO model. -1 cm fraction is measured as 1.92% for Blast 4. It is calculated as 5.72% by KCO model.

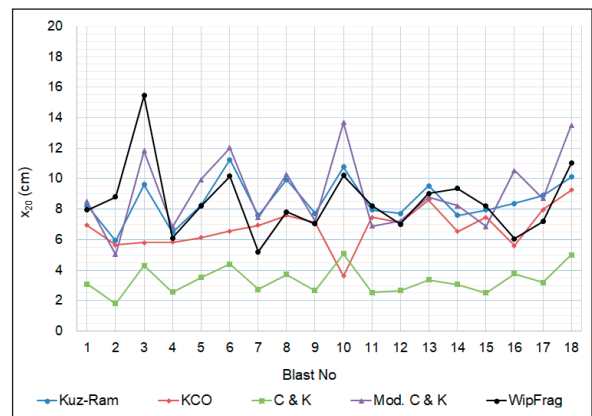


Figure 6. Comparison of the 20% passing size for each blast

80% passing size ( $x_{80}$ ) may be reviewed to examine the coarse end of the size distribution curves (Figure 7). 80% passing size is widely used for evaluation of coarse fractions in blasting literature. It is named as 80% feeding size in the Third Theory of Comminution developed by Bond (1952). Bond communication model estimates energy requirements to reduce fragments from an 80% feed size to an 80% product size. The measured and predicted  $x_{80}$  values are very close each other especially for Blasts 9, 11, 13 and 14 (Figure 7). Average  $x_{80}$  is 33.48 cm with a standard deviation of 9.48 cm for WipFrag measurements. The lowest mean  $x_{80}$  value is 31.96 cm with a standard deviation of 6.58 cm for KCO model. As seen in Figure 7, the coarse size prediction of original C&K model is excessive.

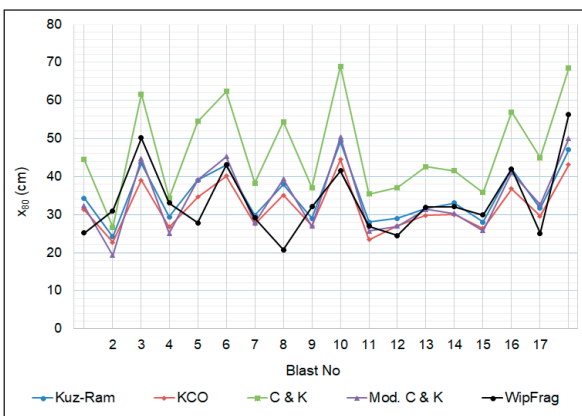


Figure 7. Comparison of the 80% passing size for each blast

### 3.3. Model Validation Using Error Metrics

Model validation is as important as model development stage. Generally, researchers consider a limited number of error measures for model validation. Actually, each error metric has some drawbacks. Ten different error measures were applied to determine prediction capability of the fragmentation models. It was aimed to perform a comprehensive model validation. All the used error measures were presented in Table 3. Absolute errors are largely used by engineers. They are easy to calculate. However, these errors are scaled dependent and they cannot be used to compare different series (Shcherbakov et al, 2013). Percentage errors show the error

magnitude as a percentage and they are very popular among non-expert users. Percentage errors are non-symmetrical and largely affected by extreme values. Symmetric errors proposed to solve asymmetry problem of percentage errors (Armstrong and Collopy, 1992). Scaled errors are relatively new error measures (Table 3). They were developed to solve scale dependency problem and to create a symmetrical error metrics (Hyndman and Koehler, 2006). Table 4 shows calculated error values for  $x_{20}$ ,  $x_{50}$  and  $x_{80}$  predictions. The predictions of fragmentation models were compared to WipFrag measurements. The mean absolute errors of KuzRam and Mod. C&K model are lower than 2 cm for 20% passing size ( $x_{20}$ ). Especially MAPE value of KuzRam model is very low, 16.15%. MAE, sMAPE and MASE values of Mod. C&K model are better than those of KCO model. However, according to median error values, KCO model performed better than Mod. C&K model for 20% passing size (Table 4). For  $x_{50}$  predictions, the calculated MAPE values of KuzRam, KCO and Mod. C&K models are lower than 20 percent. The calculated MAE and MASE values for KuzRam and KCO models are 2.56 cm and 0.28, respectively. It should be noted that KuzRam and KCO models both use Kuznetsov equation to predict mean fragment size. Mod. C&K model clearly performed better than original C&K model except median absolute error.

For 80% passing size predictions, KuzRam has lowest MAE and MdAE values. However, RMSE and RMSPE values of KCO model are slightly lower than those of KuzRam model (Table 4). Mod. C&K model performed better than KCO model according to only two median error values. MdAPE and sMdAPE values of Mod. C&K model are 12.47 and 13.31, respectively.

## DISCUSSION AND CONCLUSIONS

KuzRam is the most applied model in the blasting literature. Its main deficiency is the inadequate estimation of the fine size fractions. According to some researchers, KuzRam neglects fine material. It well predicts coarse fraction of size distribution curve. However, KuzRam has been proved itself by many industrial and experimental applications through the years. Even in recent



Table 3. Applied error measures

Class	Error Type	Formula
Absolute Error	Mean Absolute Error	$MAE = \frac{1}{n} \sum_{i=1}^n  m_i - p_i $
	Median Absolute Error	$MdAE = \text{median}_{i=1,n}  m_i - p_i $
	Root Mean Square Error	$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (m_i - p_i)^2}$
Percentage Error	Mean Absolute Percentage Error	$MAPE = \frac{100}{n} \sum_{i=1}^n \frac{ m_i - p_i }{m_i}$
	Median Absolute Percentage Error	$MdAPE = 100 \text{median}_{i=1,n} \frac{ m_i - p_i }{m_i}$
	Root Mean Square Percentage Error	$RMSPE = 100 \sqrt{\frac{1}{n} \sum_{i=1}^n \left(\frac{m_i - p_i}{m_i}\right)^2}$
Symmetric error	Symmetric Mean Absolute Percentage Error	$sMAPE = \frac{200}{n} \sum_{i=1}^n \frac{ m_i - p_i }{m_i + p_i}$
	Symmetric Median Absolute Percentage Error	$sMdAPE = 200 \text{median}_{i=1,n} \frac{ m_i - p_i }{m_i + p_i}$
Scaled Error	Mean Absolute Scaled Error	$MASE = \frac{1}{n} \sum_{i=1}^n \frac{ m_i - p_i }{\frac{1}{n-1} \sum_{i=2}^n  m_i - m_{i-1} }$
	Root Mean Square Scaled Error	$RMSSE = \sqrt{\frac{1}{n} \sum_{i=1}^n \left(\frac{ m_i - p_i }{\frac{1}{n-1} \sum_{i=2}^n  m_i - m_{i-1} }\right)^2}$

$m_i$  = Measured fragment size (cm),  $p_i$  = Predicted fragment size (cm),  $n$  = Case number  
 $\overline{m_i}$  = Mean measured fragment size

Table 4. Calculated error values for  $x_{20}$ ,  $x_{50}$  and  $x_{80}$  predictions

	Absolute error			Percentage error			Symmetric error		Scaled error		
	MAE	MdAE	RMSE	MAPE	MdAPE	RMSPE	sMAPE	sMdAPE	MASE	RMSSE	
$x_{20}$	KuzRam	1.37	0.81	1.94	16.15	9.88	21.39	15.87	9.41	0.15	0.21
	C&K	5.17	4.99	5.51	59.80	59.59	60.56	86.63	84.92	0.56	0.60
	Mod. C&K	1.85	1.62	2.25	22.40	19.67	28.34	20.90	18.40	0.20	0.25
	KCO	2.01	0.90	3.17	20.39	11.80	27.95	24.93	11.93	0.22	0.35
$x_{50}$	KuzRam	2.56	2.09	3.01	13.58	12.09	15.42	13.67	11.40	0.28	0.33
	C&K	4.05	2.77	5.05	20.65	16.48	24.97	24.50	17.96	0.44	0.55
	Mod. C&K	3.05	2.79	3.62	15.89	15.33	18.50	16.27	14.24	0.33	0.40
	KCO	2.56	2.09	3.01	13.58	12.09	15.42	13.67	11.40	0.28	0.33
$x_{80}$	KuzRam	5.06	4.09	6.82	17.37	12.17	26.32	15.32	12.97	0.55	0.74
	C&K	13.98	11.84	16.39	46.36	32.19	59.46	34.63	27.73	1.53	1.79
	Mod. C&K	5.81	5.26	7.42	19.86	12.47	28.66	18.26	13.31	0.63	0.81
	KCO	5.69	4.76	6.81	18.01	14.45	22.99	17.51	15.49	0.62	0.74

years, several researchers have practiced the KuzRam model or adapted–modified it for particular cases (Gheibie et al., 2009; Badroddin et al., 2013; Tosun et al., 2014). In general terms, KuzRam model and WipFrag produced similar results for our case study.

KCO model uses a new function to construct size distribution curve. It also introduces an upper limit to the fragment size ( $x_{max}$ ). KCO model does not require any complex parameters. Ouchterlony (2005a) claims that KCO model increase predictive capability for the fines range. For our cases, amount of the fine material calculated by KCO is higher than that of KuzRam and WipFrag as expected (Figure 6). Except the fine fraction of the size distribution curves, the results of WipFrag and KCO are similar. KCO model should be considered especially if fine size fractions are important for mining process.

C&K model is an extension of KuzRam. This approach was created using model-scale blasts. A limited site application has been reported. Application of C&K model to the studied quarry was unsuccessful because of low uniformity index values. Main difference between C&K and KuzRam model is the calculation of the uniformity index. Formulation of C&K uniformity index is simpler than that of KuzRam model. However, for our case, main drawback of C&K model was also uniformity coefficient. It was provided too low uniformity values. C&K model was modified by multiplying the uniformity index by 2. For future applications, it is expected that the uniformity index of Mod. C&K model ( $n\phi$ ) will be 2-2.5 times higher than that of original C&K model. It is believed that Mod. C&K model may be helpful as an alternative for KuzRam model. However, it should be practiced in various mines to increase its reliability.

Ten different error metrics were used to evaluate prediction capability of the models. Blast researchers tend to use mean errors for validation. The calculated median errors are lower than mean errors. Mean and median errors should be considered together to make a decision. Symmetric errors and scaled errors are found to be very useful for evaluation of blast fragmentation. The interpretation of symmetric errors are slightly difficult in comparison to percentage errors. The

range of scaled errors is quite narrow. For example, the calculated MAE values for KuzRam and Mod. C&K models are 1.37 and 1.85, respectively for  $x_{20}$  prediction. However, the scaled error values are 0.21 and 0.25.

Each blast fragmentation model explained in this paper considers different parameters to construct size distribution curve. Blast fragmentation is a very complex process. Researchers should be skeptical and use different models simultaneously. In this paper, a quarry application of the fragmentation models was discussed. Generally, quarries use small diameter blasthole. Therefore, burden, spacing between holes, and charge weight per hole are low. Also, quarries works with low benches. Interestingly, Kuznetsov equation, which is the fundamental parameter of the most fragmentation models, was also created in a quarry. In the future, attempts should be made to apply the fragmentation models to open pit mines, which operate using large diameter drill hole. That helps to understand the efficiency of the models for high benches with large burden and spacing values.

The capability of the models should be tested for specific or extreme rock conditions (very soft or hard-competent rocks). The researchers should not avoid modifying or adapting the models for their mines if needed. Models may be calibrated based on the experience of site workers. All the models assume that ideal explosive detonation occurs in the site. Performance of the explosive should be monitored especially for blended blasting agents like ANFO. In a wet blasthole, the performance of dry blasting agents may vary. Prediction accuracy of the models will be insufficient if there is no an ideal detonation in blast site.

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