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IDENTIFICATION OF KNIGHTS' RELATIONS FOR 5×5 KNIGHT GRAPH BY MODULARITY

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ABSTRACT

Modularity is a widely utilized technic to analyze graphs. The modularity divides the specified network into relational clusters. The clusters highlight the shared properties between the clustered nodes. In the present study, we analyze 5×5 knight graph by modularity to extract 5 Knight Covering Problem (5-KCP) solutions. Our investigation is completed for resolutions from 0.1 to 1.8. The maximum modularity score is 0.3 found for resolution 0.8. Moreover, resolution 0.1 is the best resolution to find more solutions of 5-KCP. Also, the analyses show resolution 1.0 is the best resolution to find the solutions of 5-KCP efficiently. Lastly, modularity extracts the solutions from 1 to 7 out of 172 solutions.

Keywords: *ht graph, modularity, 5-KCP*

5×5 AT ÇİZGESİ İÇİN MODÜLERLİK İLE AT İLİŞKİLERİ TANIMI

ÖZ

Modülerlik çok kullanılan çizge analiz tekniğidir. Modülerlik belirtilen ağı anlamlı ilişkili gruplara ayırır. Gruplar aynı gruptaki düğümler arasındaki ortak özelliklerin varlığını belirtir. Bu çalışmada 5 At Kaplama Problemi (5-AKP) çözüm arayışında 5×5 at çizgesi modülerlik ile analiz ediyoruz. Araştırmamız çözünürlüğün 0.1 ile 1.8 olduğu aralık için tamamlandı. Maksimum modülerlik puanı 0.8 çözünürlüğü için 0.3 bulundu. Buna ek olarak 0.1 5-AKP'nin bazı çözümlerini bulmak için ideal çözünürlüktür. Ayrıca, analizler 1.0 çözünürlüğünün 5-AKP'nin çözümleri bulmak için en optimize çözünürlük olduğunu gösterdi. Son olarak modülerlik 172 çözümden 1'den 7'ye kadar tanımlayabilmektedir.

Anahtar kelimeler: *At çizgesi, modülerlik, 5-AKP*

1. Introduction

The knights on chess are important pieces of complicated defenses and attacks because of their unintuitive moves. Thus, various problems are introduced based on the knights' movements on the chess-board-likes. For example, the knight's tour problem is one of the entertaining problems for mathematicians and computer scientists. There are many algorithms are developed to obtain solutions [1-5]. Additionally, it is extended to more complex problems without losing the heart of the problem [6-8]. The knight moves inspired to encrypt images [9-13]. Moreover, to place the knights on a chess-board-likes with the legal chess moves is the basis for the various versions of problems which are called the Knight Covering Problem (KCP) [14-21]. Since it is an NP-Hard problem, there is no known analytical solution. However, there are many algorithms are developed to identify KCP solutions such as the independent set [22, 23] and the Girvan-Newman clustering algorithm [24] of the responding knight graphs of KCP. In this study, the knight graph representation of 5-KCP (KCP on the 5×5 board) is introduced and investigated by the modularity method. 5-KCP is the problem to place certain number of knights on 5 by 5 chessboard-like, so every cell is either occupied or attacked. In Figure 1, 5-KCP solutions which are made of 5 knights are presented on the 5 by 5 boards. The solutions are shown in Figure 1.b and .e are rotationally symmetric solutions and Figure 1.c,d,f, and g are also rotationally symmetric solutions to each other. Thus, there are 3 unique solutions by 5 knights.

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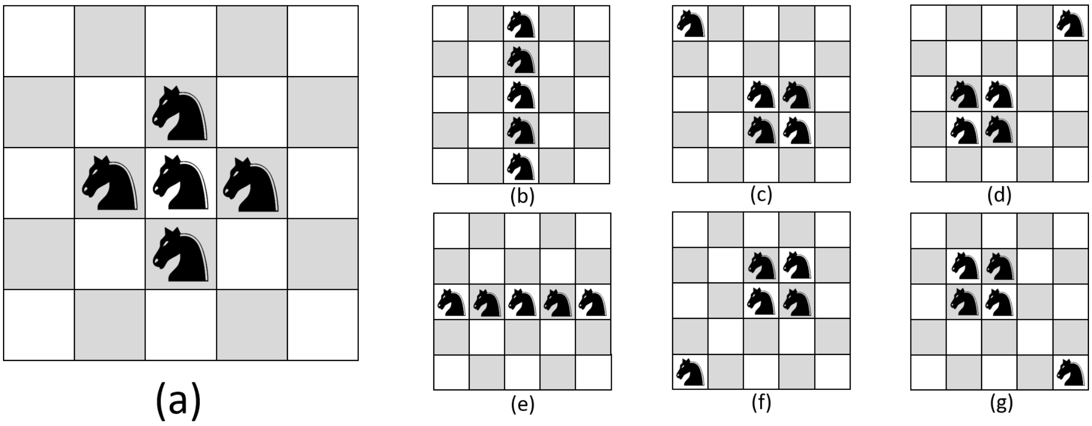
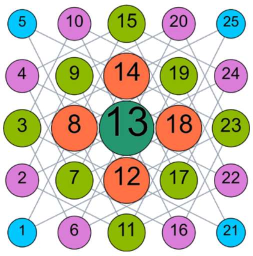


Figure 1. 5-KCP solution by utilizing 5 knights

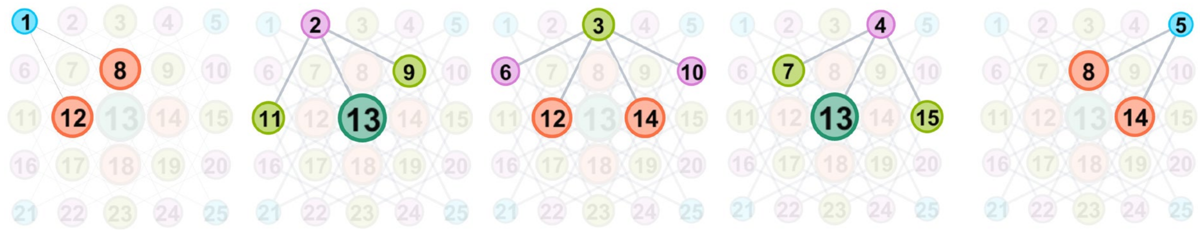
We intend to measure the effectiveness of the modularity algorithm to solve 5-KCP. Thus, the investigation of 5-KCP problem starts with conversion to the knight graph. **Figure** 2 shows the graph which nodes are colored and sized respect to the degrees of the nodes. Each node represents a cell on the board and is labeled by an index number. The cells on the corners (colored light blue) can attack 2 cells. The cells on the edges (colored purple) can attack 3 cells, and the cells (colored green) are next can attack 4 cells. The cells, colored orange, can cover 6 cells. Lastly, cell 13 (colored dark green) can attack 8 cells. The cells which are threatened are reachable by one move. Additional to the cells in which the knight attacks, an extra cell is occupied by the particular knight. In summary, the graph form of 5-KCP is composed of 25 nodes and 48 edges. The nodes have 2, 3, 4, 6, and 8 degrees. They are distributed from the entire graph by the portion respectively 16%, 32%, 32%, 16%, and 4%. Every knight and their relations on the board is explicitly shown in Figure 3.



Color code	Degree	Number of Nodes	Percentage in the graph (%)
●	2	4	16
●	3	8	32
●	4	8	32
●	6	4	16
●	8	1	4

Figure 2. 5-KCP graph is shown. The nodes are colored and sized proportional to the degrees of nodes (See Figure 3)

The graph analyses are important tools to extract information from networks of data such as computational networks [25-27], social networks [28-31], biological networks [32-34], word networks [35-37], infection networks [38, 39]. One of the analyses methods is clustering. Clustering divides the whole network into smaller clusters of the nodes. Each cluster presents a common property (or properties) of included nodes in the same cluster. The clustering algorithms are extensively utilized since it is relatively computationally efficient. There are numerous graph clustering algorithms which highlight the network properties. For example, the Girvan-Newman algorithm is based on edge betweenness, Highly Connected Clusters utilizes graph connectivity, k-means clustering divides the network by mean value, and Modularity generates modules (a.k.a. clusters) by means of the strength of division of a network. In this study, we investigated 5-KCP by modularity. The modularity algorithm is an adjustable algorithm with respect to the resolution. To increase resolutions divide the network to a greater number of clusters and to decrease the resolution is a lesser number of clusters. The appropriate resolution is found based on the intention with the help of the modularity score. Since our intention is to provide modularity application to 5-KCP, we changed the resolution from 0.1 to 1.8 for all meaningful resolutions.



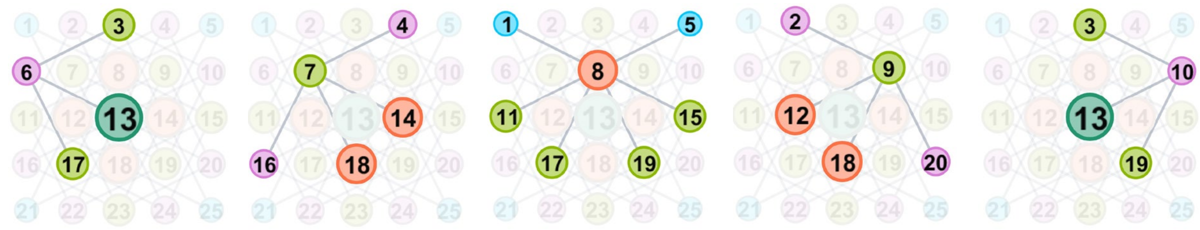
a) Cell 1 can cover cells 8 and 12

b) Cell 2 can cover cells 9, 11, and 13

c) Cell 3 can cover cells 6, 10, 12, and 14

d) Cell 4 can cover cells 7, 13, and 15

e) Cell 5 can cover cells 8 and 14



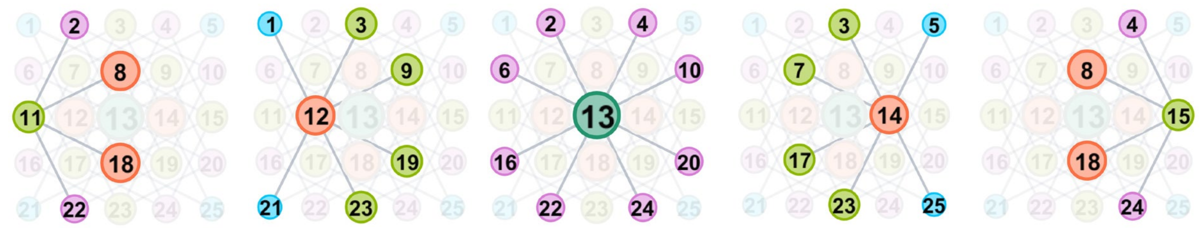
f) Cell 6 can cover cells 3, 13, and 17

g) Cell 7 can cover cells 4, 14, 16, and 18

h) Cell 8 can cover cells 1, 5, 11, 15, 17, and 19

i) Cell 9 can cover cells 2, 12, 18, and 20

j) Cell 10 can cover cells 3, 13, and 19



k) Cell 11 can cover cells 2, 8, 18, and 22

l) Cell 12 can cover cells 1, 3, 9, 19, 21, and 23

m) Cell 13 can cover cells 2, 4, 6, 10, 16, 20, 22, 24

n) Cell 14 can cover cells 3, 5, 7, 17, 23, and 25

o) Cell 15 can cover cells 4, 8, 18, and 24

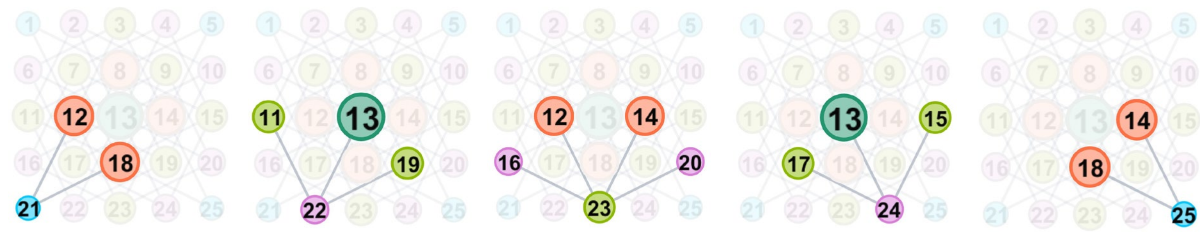
p) Cell 16 can cover cells 7, 13, and 23

q) Cell 17 can cover cells 6, 8, 14, and 24

r) Cell 18 can cover cells 7, 9, 11, 15, 21, and 25

s) Cell 19 can cover cells 8, 10, 12, and 22

t) Cell 20 can cover cells 9, 13, and 23



u) Cell 21 can cover cells 12 and 18

v) Cell 22 can cover cells 11, 13, and 19

w) Cell 23 can cover cells 12, 14, 16, and 20

x) Cell 24 can cover cells 13, 15, and 17

y) Cell 25 can cover cells 14 an 18

Figure 3. 5-KCP has 25 cells to place the knight. Each cell is represented by a node, and they are connected to the nodes which they can cover

Various algorithms are developed to solve N-KCP based on knight graphs [22]. In Figure 1, the 5-KCP solutions are presented which are found by the independent set algorithm. In the visualized solutions are limited by placing 5 knights to cover 5x5 board. The found solutions are grouped regarding the inherited rotationally symmetries. Likewise, we are benefited by graph forms in the search of 5-KCP solutions. Specifically, we use modularity to analyze the 5-KCP graph to find solutions. The considered analysis method divides the 5-KCP network into densely connected clusters. The clustered nodes present a stronger relationship between the knights. Thus, this highlights the knights which are less likely to be in the same solution. The details of the modularity method and the solution algorithm is introduced in the following sections.

2. Modularity

We used the graph modularity to identify the closely related knights for the 5-KCP. The modularity score is calculated by various formulas. The formula which we utilized is as follows [40]:

$$Q = \frac{1}{2\mathrm{m}} \sum_{i,j} \left(A_{ij} - \gamma \frac{k_i k_j}{2m} \right) \delta(c_i, c_j)$$

where δ-function is 1 if $c_i = c_j$; in other words, node i and j are in the same cluster. m stands for the number of edges in the graph. γ stand for resolution. k_i is the degree of node i and k_j is the degree of node j. A_{ij} represents the weights of the edge between nodes i and j. It is the same for all since the effect of all knights is equal.

Throughout our analysis, we used the Gephi [41-43]. The resolutions are limited from 0.1 to 1.8 which is defined specific to the 5-KCP graph. The analysis and implementation results will be given in the Results and Discussion section.

3. Results and Discussion

The investigated relational information of the 5-KCP graph by modularity score is extracted communities from 1 to 11 respect to modularity resolution.

The modularity identifies the strong relationships between nodes. However, the 5-KCP solutions are to place the knight which should have weak/no relations. Thus, the extracted clusters reveal the list of positions that are the least likely to be in the same solutions.

In **Figure 4**, modularity results on 5-KCP (for resolution = 0.1 – 1.8) graphs are presented. The resolution 1.8 is extracted 1 cluster in 5-KCP graph as shown in **Figure 4**.r. Thus, the generated solutions from the graphs show that there is no two nodes could be included simultaneously in a solution. Also, there is no solution by one knight hence 26 permutations which include no knight are not solutions for 5-KCP. In **Figure 4**.o, p, and q, 2 clusters are generated by modularity with the resolutions 1.5, 1.6, and 1.7. Similarly, 2 nodes (a.k.a. 2 knights) solutions do not exist for 5-KCP. Likewise, modularity application for resolutions = 1.1, 1.2, 1.3, 1.4, 1.5, and 1.6 cannot generate 5-KCP solutions. For the resolution = 1 in **Figure 4**.j, the 5-KCP graph is divided into 5 clusters. 2 solutions are found with the length of 5 are generated by the 7200 permutations. For resolution 0.8, 1 solution, length of 5, is found. The resolution 0.6 is found 3 solutions. While the resolution is 0.5, 2 solutions with length 5 is found. The resolution for 0.4 is lead to 3 solutions with length 5 and 1 solutions with length 9. The resolution 0.3 is found the solutions for length 5 and 9, and the number of solutions are 3 and 1 respectively. The resolution 0.2 results with 1 solution with length 9 and 3 solutions with length 5. The smallest resolution is 0.1 has the highest number of solutions with 7 solutions 3 of their length is 5 and the rest are with length 9. To sum up, the modularity mostly cover the length of the solutions 5 and then 9.

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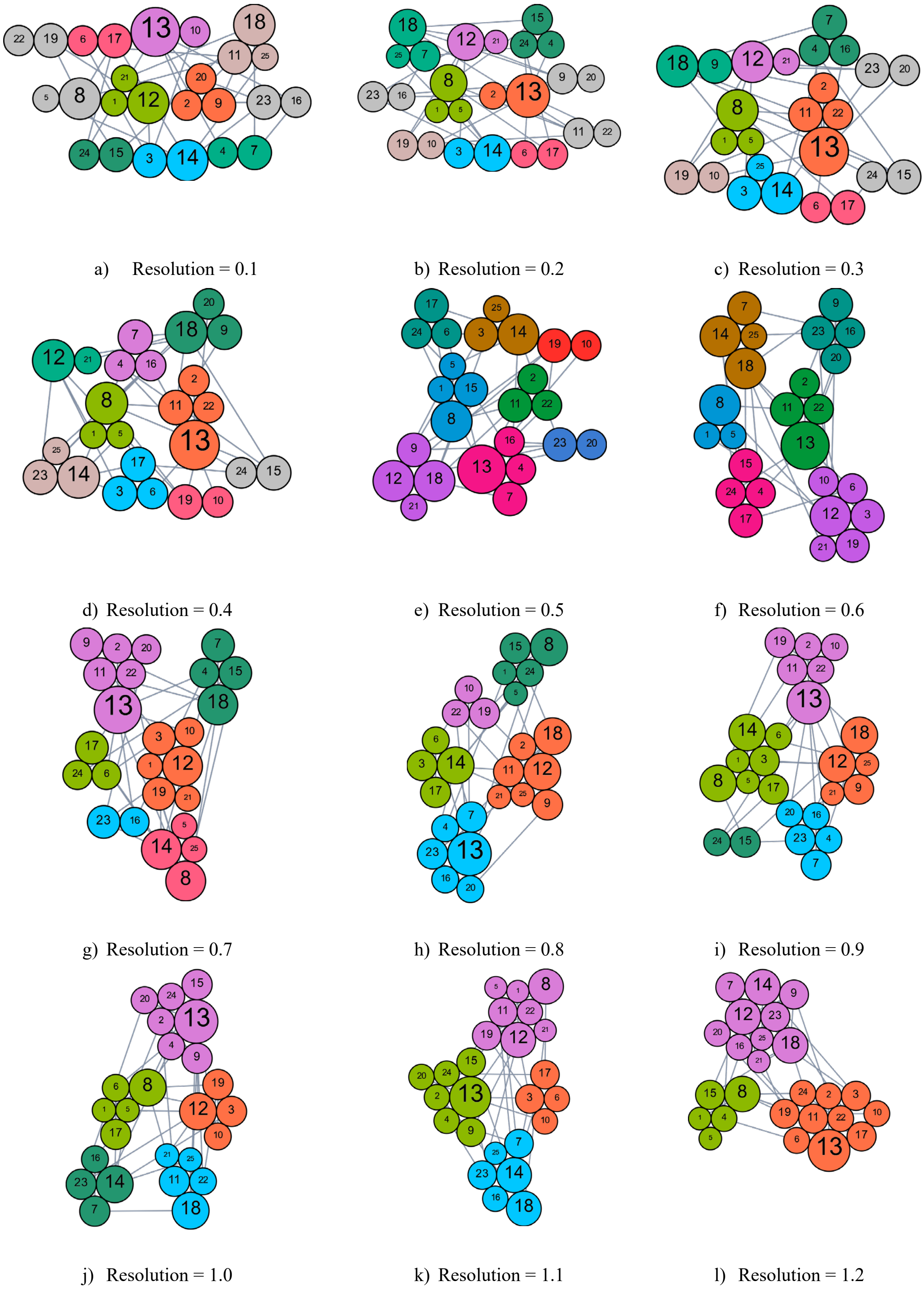


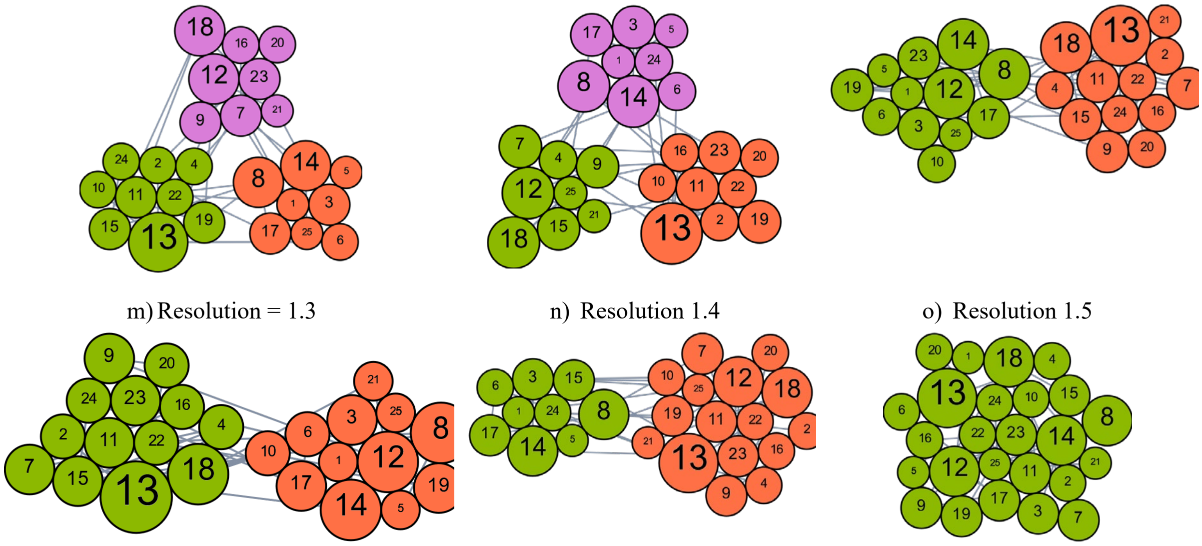
a) Resolution = 0.1 b) Resolution = 0.2 c) Resolution = 0.3

d) Resolution = 0.4 e) Resolution = 0.5 f) Resolution = 0.6

g) Resolution = 0.7 h) Resolution = 0.8 i) Resolution = 0.9

j) Resolution = 1.0 k) Resolution = 1.1 l) Resolution = 1.2

Figure 4. Modularity is applied to 5-KCP graphs for various resolutions from 0.1 to 1.8



m) Resolution = 1.3 n) Resolution 1.4 o) Resolution 1.5

p) Resolution 1.6 q) Resolution = 1.7 r) Resolution = 1.8

Figure 4. Modularity is applied to 5-KCP graphs for various resolutions from 0.1 to 1.8 (Continues)

The modularity is identified some solutions of 5-KCP. **Figure 5** presents the number of solutions vs the number of required knights. The found solutions are limited to length 5 and 9. While resolution 0.1 has the highest capability since it found 7 solutions, resolution 0.8 identifies the lowest number of solutions by 1 solution.

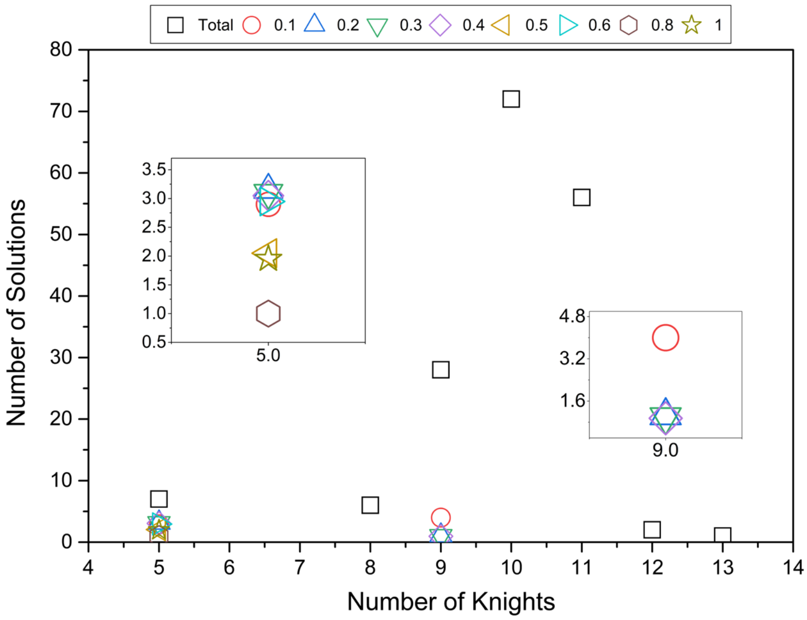


Figure 5. Number of the found solutions by modularity compared with total solutions

In **Table 1**, all the found solutions are summarized. The consistency of the found solutions for each resolution implies the robustness of the modularity algorithm. The solution includes 9 knights (1,2,6,14,15,16,19,20,21) is appeared in the modularity results of the resolutions namely 0.2, 0.3, and 0,4. The solutions by 5 knights (1,13,14,18,19 and 8,12,13,14,18) are found for the resolutions: 0.1, 0.2, 0.3, 0.4, and 0.6.

Table 1. The solutions are generated by modularity for various resolutions.

Resolution	Solution length is 5	Solution length is 9
0.1	1,13,14,18,19 8,12,13,14,18 8,9,13,14,21	1,2,3,4,5,17,18,19,23 1,2,3,4,5,17,18,22,23 1,2,3,4,5,18,19,23,24 1,2,3,4,5,18,22,23,24
0.2	1,13,14,18,19 8,12,13,14,18 8,9,13,14,21	1,2,6,14,15,16,19,20,21
0.3	1,13,14,18,19 8,12,13,14,18 8,9,13,14,21	1,2,6,14,15,16,19,20,21
0.4	1,13,14,18,19 8,9,13,14,21 8,12,13,14,18	1,2,6,14,15,16,19,20,21
0.5	1,13,14,18,19 8,12,13,14,18	
0.6	1,13,14,18,19 8,9,13,14,21 8,12,13,14,18	
0.8	1,13,14,18,19	
1	1,13,14,18,19 8,12,13,14,18	

The applied modularities for the resolutions between 0.1 to 1.8 clustered the 5-KCP graph. In **Figure** 6, the increasing resolutions divide the network into smaller numbers of clusters. The investigations show the increasing number of clusters more likely to find 5-KCP solutions (See **Figure** 5 and **Table 1**). The modularity score which identifies the quality of clustering has no explicit correlation with the number of found solutions for the specified resolutions as shown in **Figure** 6. The highest modularity score is 0.3 (for resolution 0.8). Thus, clustering for 0.8 presents the best sub-communities.

In **Figure** 7, the number of generated permutations is compared with the number of identified solutions for the resolution between 0.1 and 1.2. Although communities are identified, no solutions are obtained for the resolutions 1.2-1,8. The resolution 0.1 and 0.2 leads to 419904 permutations which identify 7 and 4 solutions, respectively. The resolutions 0.3 and 0.4 identify 4 solutions similar to resolution 0.2 with a smaller number of permutations. This is highlighted in computational efficiency (See **Figure** 8). The highest resolution which identifies 5-KCP solution is 1.0. 2 solutions are identified by 7200 permutations. The rest of the resolutions do generate no solutions regardless of generated permutations.

By comparison of the number of generated permutations and the number of found solutions, computational efficiency of the particular resolution is defined. The computational efficiency of a resolution is presented with respect to the resolution in **Figure** 8. The computational efficiency is formulated as:

$$Efficiency\ of\ the\ cluster = \frac{Number\ of\ found\ solutions * 100}{Number\ of\ permutations} \tag{1}$$

Resolution 0.1 introduces the best clustering which obtains relatively more solutions, 7 solutions. However, it is not computationally efficient (0.00167) because it is one of the resolutions which generate the highest number of permutations. The most computationally efficient resolution is 1.0 by 0.02778. It finds 2 solutions by 7200 permutations. Thus, resolution 1.0 extracts relatively more meaningful clusters by means of 5-KCP.

For the lower resolutions, generated permutations effect efficiency, so computational efficiency and the number of found solutions lose correlations. On the other hand, for the higher resolutions, the number of found solutions and computational efficiency are strongly correlated.

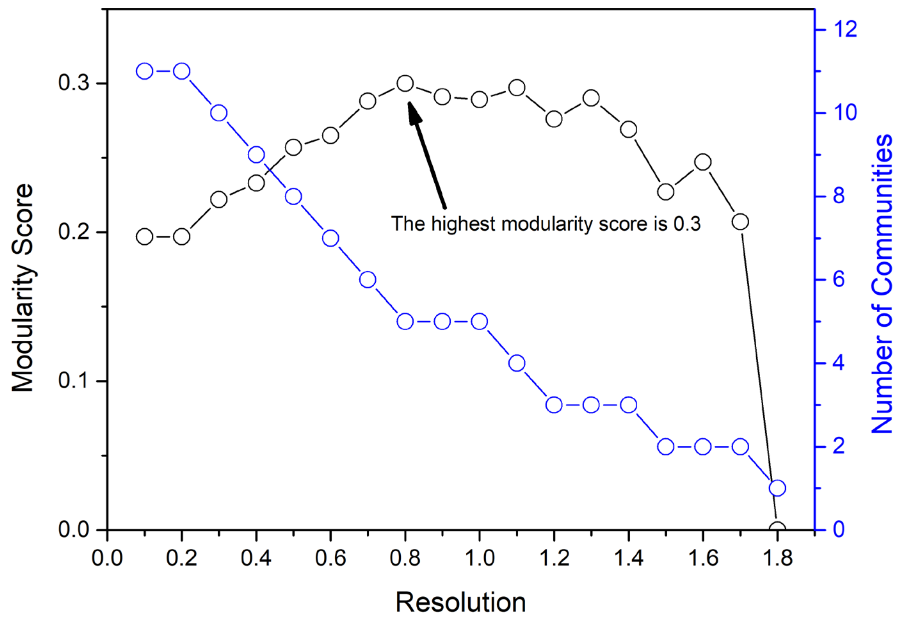


Figure 6. (Color online) An increase in the resolution causes to the lower number of communities for 5-KCP graph

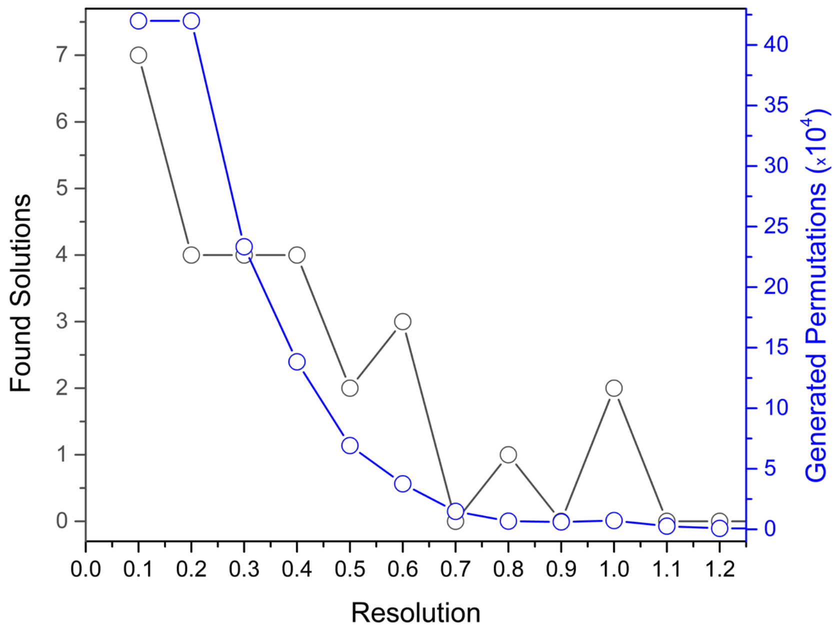


Figure 7. (Color online) Number of permutations have correlations with the number of found solutions.

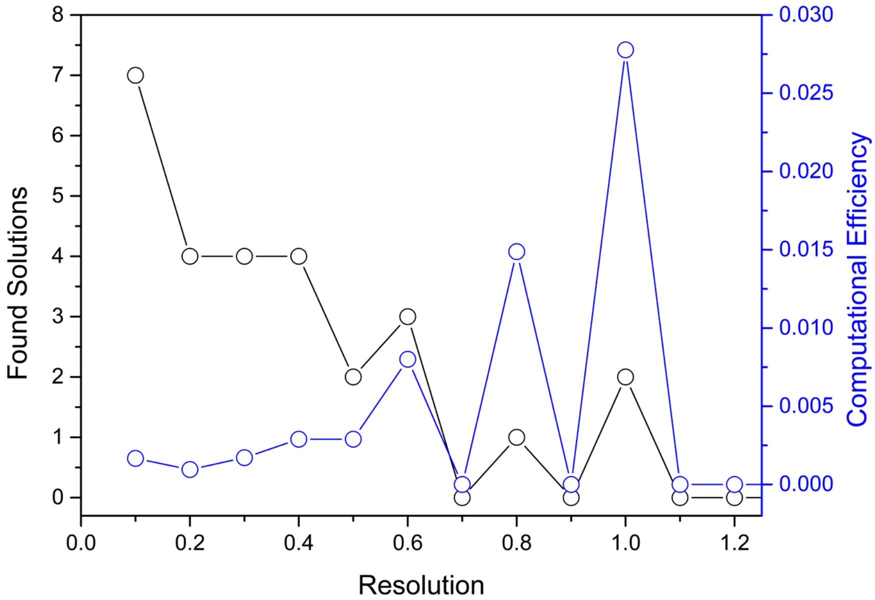


Figure 8. (Color online) Resolution 1.0 has the highest computational efficiency with respect to the other resolutions. On the other hand, the highest number of solutions, namely 7, are obtained by the resolution 0.1 which is considerable computationally less efficient than others.

4. Conclusion

In this study, we have applied the Modularity 5×5 knight graph. The analyses show resolution 1.0 is the computationally optimal resolution to find some solutions of 5-KCP. Moreover, the analysis shows resolution 0.1 is the best resolution to find all solutions of 5-KCP. The maximum modularity score is 0.3 found for resolution 0.8. Moreover, resolution 0.1 is the best resolution to find more solutions of 5-KCP. Also, the analyses show resolution 1.0 is the best resolution to find the solutions of 5-kcp efficiently. Lastly, modularity extracts the solutions from 1 to 7 out of 172 solutions.

Based on our analysis, the modularity is a promising method to solve N-KCP. Thus, in future studies, the analyses will be extended greater boards with modularity method.

Acknowledgment

The authors declare no conflict of interest.

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