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Research Article

How are students' reflective thinking for problem solving?

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Abstract

Mathematics students with a reflective thinking tend to have self-confidence in solving mathematics easily. In recent years, however, there have not been the results of the researches that classify students' reflective thinking for solving non-routine questions of Analytical Geometry questions, in particular. The present research paper reveals the new classification of the students' reflective thinking for solving the problems. These are solving Analytical Geometry which can cause the students' confusion to apply their reflective thinking. The classification of the study comprises four components. It employed a descriptive-qualitative approach. The amounts of the subjects were 21 out of 140 students who used their reflective thinking for solving questions. The data collection consisted of test, observation, and in-depth interview. The data validation employed a method triangulation technique. The results of the study show there are three classification of reflective thinking, namely: assumptive, virtual, and connective. The three classifications differ in the way they deal with confusion. Meanwhile, the three classifications have similarities. The students tended to use all of the reflective thinking components although there were a few indicators that were not realized maximally. These were making a plan before solving the problems, using an efficient way, and relating among concepts. The factor is students play an inactive role or low active in reflective thinking. Moreover, they think that the most important thing is answering the questions correctly and they do not understand the purposes of the problem. Therefore, the students need to get the treatments.

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Introduction

By thinking, human beings build and develop knowledge Halpem (2013) suggested that humans' thinking is used for knowledge development. However, each human has different thinking levels (Siswono, 2011; Zhang et al. 2019). According to King et al. (1997), the highest level is reflective thinking so it needs to examine reflective thinking continuously (Whalen & Paez, 2019). Reflective thinking helps problem solvers control their process and get the right strategy for the next step, better plan to solve new problems, and manage repetition (Hong & Choi, 2011).

According to Kholid et al. (2020), reflective thinking is seen as a very extremely active and rigorous activity by referring to knowledge. An individual with reflective thinking uses his or her two components, attitude, and knowledge for decision making. Attitude consists of wholeheartedness, directness, open-mindedness, responsibility, and readiness. Knowledge can be viewed an individual's ability to relate among concepts (Hill et al. 2008). Reflective thinking can change an action that is not based on any reason in a smart one. By using his or her reflective thinking, an individual has self-confidence in solving a problem easily (Kalelioglu, 2015). It is viewed as a process of the

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integration among thinking skill, experience, and insight (Pagano, 2009), so that an individual needs to apply his or her reflective thinking in learning process and problem solving.

In the present research paper, reflective thinking is defined as any confusion of problem solving so an individual must be able to analyse, evaluate, and motivate by himself to overcome a problem of it (Gurol, 2011). Referring to the research results, it can be concluded that reflective thinking plays a role in minimizing an individual's weakness when he has difficulty to find solution and draw a problem conclusion (Agustan & Siswono, 2017). He will be able to correct the solution he has found in order to draw a conclusion. Reflective thinking encourages an individual to think about a strategy and evaluate it for taking an appropriate in problem solving (Gencel & Saracaloğlu, 2018). In other words, the better an individual's reflective thinking skill is, the higher academic performance he achieves (Ghanizadeh, 2017; Hsieh & Chen, 2012). Similarly, Hong & Choi (2011) suggested that reflective thinking helps an individual control by himself during problem solving process. It encourages him to take a very extremely rigorous activity when he is solving a new problem that has been difficult to overcome. Thus, reflective thinking begins with individuals experiencing confusion and evaluation to find problem solving.

It is different from a real condition. Lectures and students with all level academic achievement have been low in reflective thinking (Yang et al. 2016). These lectures have focused on their students' answers. They have not thought about how the students take the process. If their answers are not right to answer keys, the lecturers tend to think about them as incorrect without understanding of how to answer it (Susandi & Widyawati, 2017) on one hand. On the other hand, the students tend to use their knowledge without evaluating and developing it (Kholid et al. 2020). Additionally, Chee & Mehrotra (2012) proposed that students have not used reflective thinking while it plays a role in problem solving (Agustan et al. 2017; Gurol, 2011). It plays a role in encouraging an individual to understand what he does and why he does (Ambrose & Ker, 2014).

Reflective thinking begins from an individual's confusion (Suharna et al. 2020). Therefore, a question or problem-solving type that can be used for understanding an individual reflective thinking is a question that can cause an individual's confusion. It may be a non-routine question (Hong & Choi, 2011). It is an unfamiliar question for students so it will cause an individual to have reflective thinking (Hidajat et al. 2019).

A non-routine question to explore the students' reflective thinking is related to an analytical geometry problem. For solving this question, furthermore, they need to understand it accurately (Aytekin et al. 2016), because analytic geometry is study how to solve geometry problems by employing algebra. Meanwhile, algebra and geometry as an alternative language indicates complete competence in problem-solving (Sholihah et al. 2019). It is an unfamiliar question for students so it can cause any confusion and encourage them to apply their knowledge and experience for solving it. Thus, it highly conforms to exploring students' reflective thinking.

Prior studies related to reflective thinking have been conducted. These are grouped into three main categories. First, the research focuses on developing instruments to measure reflective thinking skills. The instruments are: 1) the scale of reflective thinking (Basol & Gencel, 2013), 2) the reflective thinking questionnaire (Can & Yildirim, 2014), 3) the guidelines table for conducting reflective thinking (Ghanizadeh & Jahedizadeh, 2017), 4) the rubric for evaluating reflective thinking/REPORT (Pennington, 2011), and 5) mathematical problems in analytic geometry to measure individual reflective thinking (Zehavi & Mann, 2005). Next, the studies focus on the role of reflective thinking into problem solvers' performance. The conclusions are: reflective thinking has a role to minimize individual weaknesses when they encounter difficulties and misconceptions on solutions and problem conclusions (Agustan & Siswono, 2017), and encourages individuals to rethink and re-evaluate the strategies to make the right decisions (Gencel & Saracaloğlu, 2018). In other words, the better the individual's reflective thinking skills the better academic achievement (Ghanizadeh, 2017; Hsieh & Chen, 2012; Yang et al. 2016). Third category research discovers levels of reflective thinking (Hong & Choi, 2011), aspects of reflective thinking (Zehavi & Mann, 2005), and reflective thinking categories based on how problem solvers deal with confusion (Suharna, 2018). Based on the description, there is an opportunity to classify students' reflective thinking based on four aspects.

Problem of Study

The statement problem of the study is how the students' reflective thinking for solving a non-routine question of analytical geometry content by using four components of techniques, monitoring, insight, and conceptualization. It aims at classifying the students' reflective thinking based on those components for solving a non-routine question of analytical geometry content.

The research paper will address how the students' reflective thinking classification based on four components of techniques, monitoring, insight, and conceptualization for solving a non-routine question? Additionally, it will enrich the question types which can be meaningful to investigate reflective thinking based on the components. The results of the study will be employed as a theory for further research to increase students' reflective thinking through defragmenting or scaffolding techniques. It is a technique to rearranges the components and indicators of reflective thinking on individual cognitive. Moreover, the Indonesian National Research Master Plan (Rencana Induk Riset Nasional Indonesia/RIRN) document stated that in 2017-2045 research in Indonesia will pay attention to theme of education, social humanities, arts, and culture. Thus, the study of reflective thinking as a sub focus of educational research aims to produce highly qualified and competitive human resources. This indicates that reflective thinking research needs attention from the state as the decision maker and for education practitioners.

Literature Review

Reflective thinking is a study the researchers have investigated. It begins from an individual's confusion that has difficulty to overcome with the integration among knowledge, experience, and attitude. In overcoming it, there are there types of individuals, including reflective-clarification, reflective-connective, and reflective-productive (Suharna, 2018). In a reflective-clarification type, an individual overcomes any confusion by clarification. He investigates a problem solving repeatedly. In a reflective-connective type, an individual overcomes any confusion by relating among mathematics concepts; and in a reflective-productive type, an individual overcomes any confusion by other ways.

Additionally, there are an individual's reflective thinking indicators. A reflective thinker does something based on a plan (Dervent, 2015), develops his knowledge (Gencel & Saracaloğlu, 2018; Rieger et al. 2013), takes a good self-monitoring so it impacts on a positive achievement (Ghanizadeh, 2017; Hsieh & Chen, 2012; Pennington, 2011; Sivaci, 2017), gives a reasonable evidence for making in a problem decision (Satjatam et al., 2016), and rarely makes an error in problem solving (Ambrose & Ker, 2014). Those are associated with a reflective thinking component as suggested by Zehavi & Mann (2005).

The study adopted the reflective thinking components by Zehavi & Mann (2005). They are techniques, monitoring, insight, and conceptualization. Meanwhile, the indicators for each component developed by the author in preliminary research. Technique is a component how an individual uses some techniques for problem solving and applies strategies to solve a mathematics problem by using effective and efficient principles. Monitoring is an activity for monitoring solution to mathematics problems. It indicates that an individual rechecks solution to a mathematics problem whether it is correct or not. Insight is a component how an individual uses his intellectuality and emotion for problem solving. It is related to what an individual's extent employs two elements for problem solving when he fails. Thus, this component is the integration between knowledge and attitude. Conceptualization is a component how an individual employs his ability for developing concepts and understanding meanings and how an individual uses his ability for analysing knowledge and applying skills in solving a problem correctly. Reflective thinking components and indicators are reported in Table 1.

Table 1.
Reflective Thinking Components and Indicators

Components	Indicators	Codes
Techniques	1. Finding how to understand what the given question really means	T1
	2. Finding how to understand what the question is	T2
	3. Inferring the question's meaning	T3
Monitoring	1. Monitoring the steps of solution to mathematics questions	M1
	2. Monitoring whether the answers are correct or not	M2
	3. Using strategies for solving the questions	M3
Insight	1. Being ready to correct the wrong questions	I1
	2. Understanding how to prevent any difficulty	I2
Conceptualization	1. Thinking about other ways for solving the questions	C1
	2. Relating relevant concepts to solving the questions	C2

Method

Research Design

The study used a descriptive-qualitative approach that described all the facts, phenomena, and symptoms without manipulation (Sagala et al. 2019). It described the classification of the students' reflective thinking for solving non-routine questions based on techniques, monitoring, insight, and conceptualization components.

Participants

Analytical geometry is a course of Mathematics Education Department, Faculty of Teacher Training and Education, Universitas Muhammadiyah Surakarta, Central Java, Indonesia. It is structurally distributed at the students of Semester 2. In academic year of 2018/2019, there were 140 students consisting of 32 male and 108 female students who took this course. By using a purposive sampling method, the subjects of the study amounted to 21 people (Putranta & Jumadi, 2019) because they employed reflective thinking for problem solving. Additionally, they experienced any confusion while solving a problem, but they remained to overcome it very actively and rigorously. They were voluntarily ready to be a subject of the study.

Instruments

The instruments of the study comprised: 1) a test with non-routine question of analytical geometry content, 2) a field note sheet for understanding the students' reflective thinking for problem solving, and 3) a semi-structure interview guide for confirming the students' tendency to solve a problem. The three instruments had been validated by inviting the senior lecturers and professors in mathematics education and mathematics.

Data Collection Method

The data were collected by using the test, observation, and interview methods. The instruments were used for understanding the students' reflective thinking for problem solving. These had been conformed to techniques, monitoring, insight, and conceptualization components and indicators. The students were asked to solve the non-routine questions of analytical geometry content (see Figure 1) by applying a think-aloud method. It is a method by asking a subject to reveal what he is thinking actively and rigorously when solving problem. According to (Eccles & Arsal, 2017), a think-aloud method applies a person's memory and it can be used for understanding a higher order thinking process effectively.

Non-Routine Questions Of Analytic Geometry Content To Understand Reflective Thinking

Source: kapal-cargo.blogspot.com

A boat is at a coordinate point (a, b) with a as a positive odd number, $a < 7$ and a as a prime number. b is a positive even number divisible by 3 and 2, then $5 < b < 14$. a is multiplied by $b = 60$, and b can not be divided by a. Find:

a. Equation of the maximum range from the boat's radar.
b. What can the boat detect another boat that is at a coordinate point (50, 25)?

Figure 1.

The Non-routine Questions of Analytic Geometry Content

The researchers observed and recorded by using the video in a semi structure format. They recorded all of the subjects' activities when solving the non-routines questions based on an observation sheet for classifying the students' reflective thinking. Interviewing deeply with the subjects by using the video recorder was also used for gathering the data. The data were validated by applying a triangulation method for comparing the results of the subjects' tests (answer sheet and think-aloud transcript), observations, and interviews (Fielding, 2012). The triangulation results were analyzed to determine the students' reflective thinking tendency for solving the non-routine questions of analytical geometry content based on the indicators.

Data Analysis

Data were analyzed in two stages, data reduction and data presentation. In reduction stage, data were reduced and taken by referring to the objective of the research. In presentation stage, the data of the students' reflective thinking for solving the non-routine questions of analytical geometry based on the components of techniques, monitoring, insight, and conceptualization were presented.

Results

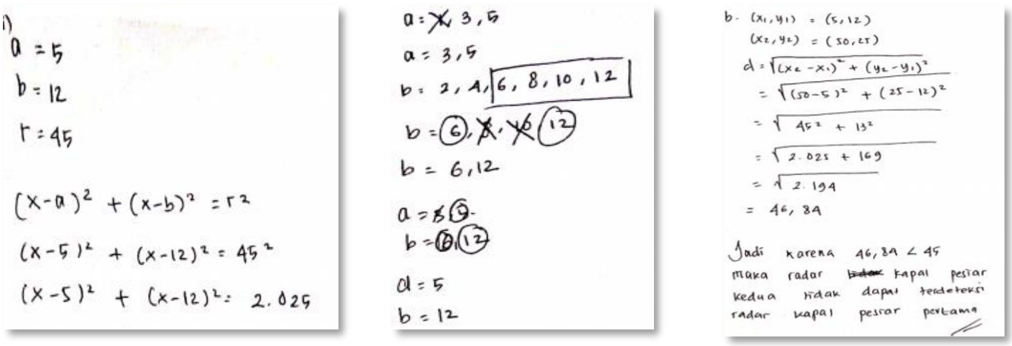
Three classifications of the students' reflective thinking for solving the non-routine questions of analytical geometry content are described below.

Classification I: Assumptive

Figure 2 reports the answer of the students' reflective thinking classification in type 1. There are nine students who understand how to gain information as needed (Indicator T1). They record all the probable values *a* and *b* (T2) by writing value $a = 1,3,5$ (positive odd numbers and less than 7, not inferring information that *a* is a prime number) and $b = 2,4,6,8,10,12$ (*b* is a positive even number with *b* namely $5 < b < 14$, not inferring information that *b* is a positive even number divisible by 3 and 2) (C2).

The students tend to solve a problem based on raw data. Then, they think the next prerequisite that *a* is a prime number so they often encircle values 3 and 5 as a probable value *a* (T3). Next, they think value *b* as an even number divisible by 3 and 2 so they encircle values 6 and 12 as a probable value *b*. For meeting the prerequisite of values *a* and *b* (*a* is multiplied by $b = 60$), they tend not to write what their ideas are. They only think abstractly and cognitively (I2) so it can be concluded that the value that meets the prerequisite is $a = 5$ and $b = 12$ (M3). They do not use a way to understand a problem as asked so they do not understand how to solve it efficiently. They ever experience any confusion how to write a mathematics model as intended. For overcoming this, they read repeatedly what is asked so they understand that the model as intended uses a circle equation at center (5, 12) and circle radius equals to 45.

After they write the model as intended, they recheck the stages of problem solving as written whether these stages are suitable to what they think about or not (M1). In sub-question "b", afterwards, they can relate the concepts of a two-point distance that is used for solving this problem. Based on the students' answer sheets, it seems they write the answer and reason clearly that the second boat can't be detected by the first one because the distance of the second boat is farer than the maximum range capacity of the first boat's radar. They ever think about again whether the answers are correct or not (M2). It can be seen at scratches on conclusion sheets. Nevertheless, they do not check them accurately. As written, the answer is "46.84 < 45" and it should be "46.84 > 45". In general, subjects in the Classification I tend to solve problems with trial and error methods. They employee assumptions and reasoning to get the most appropriate decision.



Translate Version

a)
$a = 5$
$b = 12$
$r = 45$
$(x-a)^2 + (y-b)^2 = r^2$
$(x-5)^2 + (y-12)^2 = 45^2$
$(x-5)^2 + (y-b)^2 = 2{,}025$

$a = 1,3,5$
$a = 3,5$
$b = 2,4,6,8,10,12$
$b = 6,8,10,12$
$b = 6, 12$
$a = 3,5$
$b = 6, 12$
$a = 5$
$b = 12$

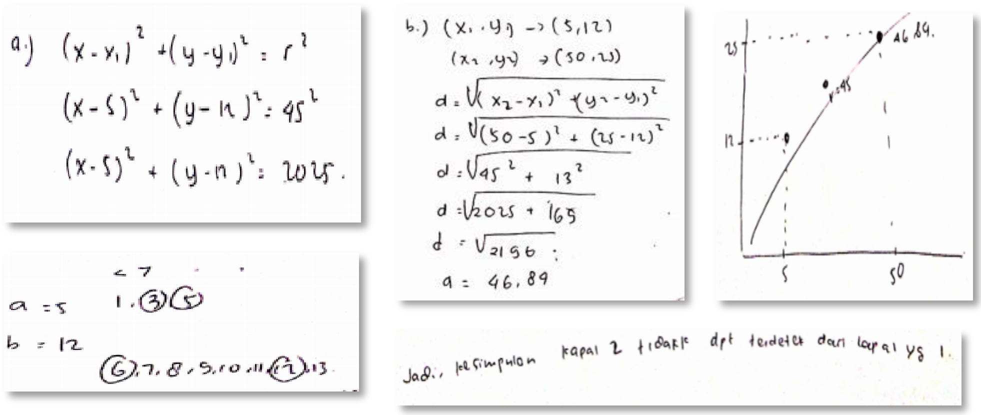
b)
$(x_1,y_1) = (5,12)$
$(x_2,y_2) = (50,25)$
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(50-5)^2 + (25-12)^2}$
$= \sqrt{45^2 + 13^2}$
$= \sqrt{2{,}025 + 169}$
$= \sqrt{2{,}194}$
$= 46.84$
To sum up, because of 46.84 < 45 so the boat's radar can't detect another boat.

Figure 2.
One of the Students' Nine Assignment Examples

Classification II: Virtual

The students in type 2 tend to infer more information of the problem (T3). They directly record value $a = 3, 5$ (*a* is a prime number, *a* is an odd number, and *a* is less than 7) and $b = 6,7,8,9,10,11,12$ (value *b* namely $5 < b < 14$, not inferring the information that b is a positive even number divisible by 3 and 2) (T1). They can be more skillful in inferring and using information than those in type 1 (T2). For answering values *a* and values *b* correctly (*a* is multiplied by *b* namely 60), furthermore, they conform by encircling the probable value *a* = 3, 5 and b = 6, 12. Then, they think about value *b* as an even number divisible by 2 and 3 so the encircle values 6 and 12 as the probable value *b*. For meeting the prerequisite of values *a* and *b* (*a* is multiplied by *b* namely 60), they tend not to write what their ideas are. They only think abstractly and cognitively (M3) so it can be concluded that the value which meets the prerequisite is *a* = 5 and *b* = 12. They do not use a way to understand the problem as asked so they can't be fluent in understand in an efficient way for solving the question. They ever experience any confusion how to write a mathematics model as intended, i.e. whether a mathematics model asked uses a circle equation or elliptic equation. They try to overcome the confusion (I1) by relating the meaning of the questions to circle and elliptic definitions (C2). After understanding the radar range is closely related to a circle definition, they write a circle equation at center (5, 12) and circle radius equals to 45. After the students write a mathematics model as asked, they do not recheck the stages of problem solving as written whether these are suitable to what they are thinking or not.

In sub-question "b", the students can relate the concepts of a two-point distance for representing the distance between two boats. After calculating, it is found the distance of two boats at 46.84 kilometers. In the stage, they experience any confusion to make a conclusion whether the first boat can detect the second one. They seem to think in a few second (about 30 seconds) how to overcome the problem. Based on their mind and thought, they describe the situation of the two boats at Cartesian coordinate (I2). It seems that they describe a point at a coordinate point (5, 12) and (50, 25) as a position of the first and second boats. They relate two points to a line representing that the distance of the two boats is 46.84 kilometers and they draw one point at the line of a 45-kilometer distance from the second boat. Then, they think that the maximum range point (as circle radius) takes place between the two boats so it can be concluded that the second boat can't be detected by the first one. They do not recheck and write the stages and final answers. In general, subjects in the Classification II need an illustration to overcome confusion. They believe that making virtual illustrations helps them in preventing the mistakes.



Translate Version

a) $(x-x_1)^2 + (y-y_1)^2 = r^2$

$(x-5)^2 + (y-12)^2 = 45^2$

a = 1,3,5

a = 5

b = 6,7,8,9,10,11,12,13

b =12

b) (x_1,y_1) □ (5,12)

(x_2,y_2) □ (50,25)

d = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

= $\sqrt{(50 - 5)^2 + (25 - 12)^2}$

= $\sqrt{45^2 + 13^2}$

= $\sqrt{2,025 + 169}$

= $\sqrt{2,194}$

=46.84

So, the conclusion is the second boat can't detect another boat

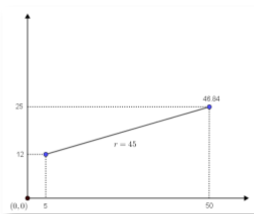
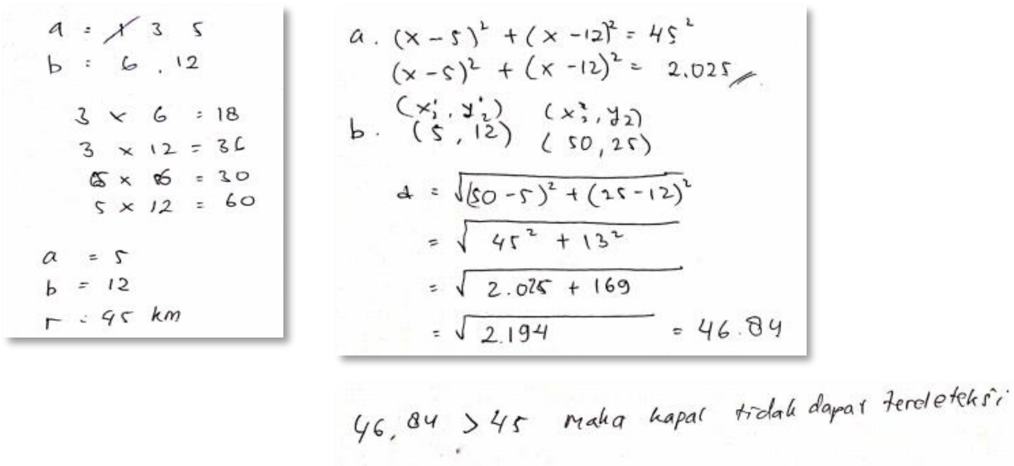


Figure 3.
One of the Students' Seven Assignment Examples

Classification III: Connective

Besides two types of answers, there are other types of answers by 5 students in this type (see **Figure 4**). The students in type 3 infer the information of the problem maximally (T3). They directly write value a = 1, 3, 5 (T1). In the stage, they begin to experience any confusion in value *a*. The confusion is overcome by reading the question repeatedly (I1). They perceive that *a* is also a prime number so they scratch "1" as the probable value *a* so *a* = 3, 5. For determining value *b*, they can directly answer correctly, namely *b* = 6, 12 (value *b* namely 5 < *b* < 14, *b* is a positive even number divisible by 3 and 2). The students in this type perceive how to understand the question more efficiently. However, they remain to read repeatedly the prerequisite of value *b* and conform to the answers. After being sure that the probable values *a* = 3, 5 and *b* = 6, 12, they read the questions repeatedly (T2) that *a* multiplied by *b* is 60. Afterwards, they do a cross-product multiplication by using a permutation process between values *a* and *b* so it is found *a* = 5 and *b* = 12. It indicates that they can relate the mathematics concepts appropriately, namely relating a cross-product concept to permutation process of an analytic geometry problem (C2). Before thinking the next stage, they read the questions repeatedly until they understand that the mathematics model asked uses a circle equation centered at (5, 12) with circle radius, namely a maximal range of the first boat at r = 45.

In sub-questions "b", the students relate a two-point concept as a distance of the second boat. After calculating, it is found the distance of the two boats is 46.84 kilometers. In the stage, they experience any confusion again whether the distance of the two boats is 46.84. The confusion comes into being when they say "why is the result not an integer number?" What do I calculate incorrectly? In a few seconds, they recheck the stages (M1) and answers (M2) in sub-questions "a". After being sure that the answers are correct, they make a conclusion that the second boat can't be detected because the distance of the two boats is farer than the maximum range of the first boat's radar. In general, subjects in classification III recall mathematical concepts that can be employed in solving a problem. The ability to connect between concepts is a significant characteristic of classification III.



Translate Version

a = 1, 3, 5
b = 6, 12
3 x 6 = 18
3 x 12 = 36
5 x 6 = 30
5 x 12 = 60
a = 5
b = 12
r = 45

a. $(x-5)^2 + (y-12)^2 = 45^2$
   $(x-5)^2 + (y-12)^2 = = 2{,}025$
b. (x_1, y_1) and (x_2, y_2)
   (5, 12) and (50, 25)
   $d = \sqrt{(50-5)^2 + (25-12)^2}$
   $= \sqrt{45^2 + 13^2}$
   $= \sqrt{2{,}025 + 169}$
   $= \sqrt{2{,}194}$
   =46.84
   46.84 > 45 so the first boat can't be detected.

Figure 4.
One of the Students' Five Assignment Examples

Discussion

Referring the results of the data analysis above, it can be stated that the students of Mathematics Education Department, Universitas Muhammadiyah Surakarta tend to use all of the reflective thinking components although there were a few indicators that were not realized maximally. It seems that they have not employed knowledge, experience, and attitude when overcoming the confusion in problem solving. Only twenty one out of 140 students apply reflective thinking. It is relevant to the research by (Chee & Mehrotra, 2012). It shows that many students play an inactive role or low active in reflective thinking. They think that the most important thing is answering the questions correctly. They do not think what the components must be applied when problem solving (Hidajat et al. 2019). It is consistent with the research by Handayani et al. (2020) that they do not understand the purposes of the problem. The students can't use experience and ideas for problem solving. Previously, Laurillard (2016) also concluded that students have not been able to involve knowledge and experience maximally in solving the problem.

In techniques component, the students tend to perceive how to understand what is asked in the questions by reading these repeatedly, on one hand. On the other hand, they can't infer some information. It is relevant to the research by Rohmah & Sutiarso (2018) and Sumitro et al. (2019). The students can't perceive the information of the questions appropriately. Suharna (2018) added the ability of students to understand what is known and asked means on this mathematical problem is classified at the understanding of the problem stage. This stage is characterized by the ability of individuals to understand what is known and asked means.

In monitoring component, the students tend to recheck the written stages and answers as whether these are correct or not. However, many students rechecked them inaccurately so it is found the wrong answers. In addition, it does not seem that they make a plan before solving a problem. It is relevant to the research by Purnomo et al. (2017). The students recheck the written answers, but they do not make a plan before solving a problem. The importance of monitoring in solving mathematical problems has been expressed by Ozsoy & Ataman (2009) and (Schneider & Artelt, 2010). Monitoring has a vital role in correcting individual mistakes. If monitoring is not carried out optimally, there is a possibility that individuals will fail in solving problems.

In insight component, the students are ready to correct the wrong answers. It is relevant to the research by Önder (2016). It is concluded that the students are ready to correct the wrong answers. In essence, the error is due to a subject's low interest (Pressley et al. 2003). It can be overcome by implementing problem solving in the class. In addition, they tend understand how to avoid any difficulty. It seems that they represent the position of the two boast and maximum range of radar at Cartesian coordinate. They need visualization to overcome confusions and construct understanding (Zayyadi et al. 2020). Although they do not understand the characteristics of visualization (Pradana et al. 2020; Purnomo et al. 2020), the visualization can be used as mathematical expression communication criteria (Sumaji et al. 2020). To avoid difficulties, students need intelligence called adversity quotient. This intelligence was first introduced by Stoltz (1997). Adversity quotient is an individual intelligence that can encourage individuals to correct failures or errors in solving mathematical problems. Research by Hidayat & Husnussalam (2019) shows that adversity quotient affects students' mathematical understanding of 57.3%, while other factors influence the rest.

In conceptualization component, it seems that the students can't relate another concept to an analytical geometry problem, namely a cross-product concept to permutation process. It is relevant to the researches by Mohamed & Johnny (2010b), Sa'dijah et al. (2020), and Sandie et al. (2019). The students have difficulty to relate experience to a concept of a new problem. When solving a non-routine question, the students tend to be able to answer a question in procedures by memorizing the formula and stages used for problem solving (Dündar & Yaman, 2015). It is relevant to the theory by (Suharna, 2018) the ability of individuals to connect between concepts is called rationalization. Rationalization includes the ability of individuals to accommodate concepts that have been learned to be applied to new concepts.

All three classifications show differences in reflective thinking tendencies. In the classification I, students tend to use initial assumptions and conjectures in problem solving. Assumptions are involved when students use the trial and error method. They assume that the trial and error method is a more effective and accurate method (Bardach & Patashnik, 2019). In the classification II, students prefer to illustrate the problem to a virtual figure. They believe that illustrating mathematical problems to a virtual figure can help them to solve problems correctly. The virtual figures help them to understand problems and decide appropriate solutions (Pradana et al. 2020). Students in the classification III have a good ability in relating among mathematical concepts. They are able to relate mathematical concepts that have been acquired (Miller & Halpern, 2014).

Conclusion

In this paper we have successfully presented that three classifications of reflective thinking have different characteristics. The Classification I show the subjects employee assumptions in solving problems. The trial and error method is the easiest method for them to find a solution. The Classification II show that the subject needs virtual illustrations to gain confidence in the answer. some subjects that depend on illustrations are problem solvers who fail in performing mathematical abstractions. Furthermore, in the Classification III, the subjects have a good ability to recall and relate mathematical concepts to solve problems.

The similarities of the three classifications are first, in problem solving process, the students tend to perceive how to understand what is seen and asked in the questions and they do not tend to apply an efficient principle, known as a technique component. Second, they recheck the written stages and answers whether these are correct or not. They do not make a plan before solving a problem. Thus, only a few indicators are applied in monitoring component. Third, they will be ready to correct if they make an error. They tend to perceive how to avoid any difficulty, called insight component. Fourth, most of the students do not relate another concept to an analytical geometry problem, termed as conceptualization component.

Recommendations

In general, the students have applied reflective thinking indicators, but these are not used maximally. Referring to the conclusion, it is recommended that lecturers give treatments to students for reflective thinking development. Giving the treatments can be learning model, worksheet instruments, and exercises or drills to stimulate students' thinking.

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