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Research Article

Level of combinatorial thinking in solving mathematical problems

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Abstract

Combinatorial thinking is an important reasoning process in building one's knowledge and experience. The purpose of this study is to describe the characteristics of the level of combinatorial thinking in solving mathematical problems. The subjects of the study were 40 students of Elementary Teacher Education Department (PGSD): 20 students of the second semester and the others of the sixth semester. The reason for choosing subjects from these two levels is to meet all levels of combinatorial thinking. All research subjects were given test questions about combinatorial problems. From 40 subjects, five students were selected to be interviewed as they had fulfilled all five levels of combinatorial thinking. The data validity was conducted by triangulation through recording interview results and comparing it with data from students' written test results to ensure the validity and reliability of this research. The results show that there are five levels of combinatorial thinking in solving mathematical problems: investigating "some cases', systematically checking cases, using the calculation order, systematically generating all cases, and changing the problem into another combinatorial problem. Level one is the identification of the possibility of students' understanding the questions incorrectly, or vice versa, already can answer the questions with systematic procedures, but the results are less precise. Level two is conducting systematic checking about students' understanding of the combination material. Besides, it also concerns about the ability to answer problems systematically using diagram trees. Level three is students are able to apply the calculation orders, which are addition and multiplication. Level four is systematically generating all cases about the ability to calculate possibilities without schematic, drawings, or diagrams. Level five is changing the problem into another combinatorial problem, it is the ability to calculate possibilities with complex problems Based on the research findings, it turns out there is another level of combinatorial thinking, which is using the calculation order and this is found between level two and level three. The researchers recommend further research to explore more on the application of calculation order.

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Introduction

Thinking and reasoning are needed for the present and the future (NRC, 1989). The statement is in line with (National Council of Teachers of Mathematics, 2000), that there are five standards in the process of learning mathematics: representations, reasoning, and proof, communication, connections, and problem-solving. As quoted by Setianingsih, et al. 2017, NCTM state that learning mathematics is perceived as a social effort in which mathematics class functions

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as communities where thinking, speaking, agreeing, and disagreeing are encouraged. Teachers provide students with strong mathematical problems to solve together and students are expected to justify and explain their solutions. Hufferd-Ackles, Fuson & Sherin (2004) stated that the research aims at expanding an individual's thinking and others'. Therefore, teachers and students are expected to master the five standard processes in learning activities, reasoning for instance. Procedural understanding focuses on experimenting facts and algorithms and conceptual understanding reflects students' ability to reason and understand mathematical concepts, operations, and relationships that will assist them in solving non-routine problems (Al-Mutawah et al. 2019).

Accordingly, for academic development, teachers must apply methods or strategies that are appropriate to elevate the level of students' potential development to their actual level (Kamau, 2016; Setianingsih et al. 2017). Learning mathematics is not only counting, but also requires skills in the reasoning logical thinking. Lay (2009) state that in the theory of mental development of Piaget through Test of Logical Thinking (TOLT), logical thinking includes: (1) controlling variables, (2) proportional reasoning, (3) probabilistic reasoning, (4) correlational reasoning, and (5) combinatorial thinking. NCTM states that combinatorial thinking is an essential element compared to other types of logical thinking and its existence is inseparable from Mathematics (English, 1991; Yuen, 2008)

Combinatorial thinking is one of the important reasoning processes in building one's knowledge and experience. The importance of combinatorial thinking will be more visible when students follow the learning process and work on problems. This is a basic skill that must be developed to build potential and critical thinking skills (Silwana, et al. 2021; Pamungkas & Khaerunnisa, 2020; Yuberti, et al. 2019) It will be more obvious when students participate in the learning process and complete several questions. As stated by Jazim et al. (2017), the principle of learning mathematics in schools is students must learn mathematics by understanding, actively building new knowledge from experience and knowledge. Effective mathematics learning requires students' understanding of what they know and what they need to learn, and then challenges and supports them to learn beyond. Cadenas, as quoted by Cuevas et al. (2018), mentions that to build new knowledge it is necessary for students to re-organize and expand their prior knowledge and teachers need to detect deficiencies, difficulties, and mistakes that prevent knowledge increasing significantly.

Research by Tsai & Chang (2008) reveals that combinatorial thinking triggers creativity, curiosity, and confidence of students in working on problems. Furthermore, combinatorial thinking is important because it is a basic thinking ability that must be continuously developed towards critical thinking abilities and skills. Still, many students encounter difficulty in combinatorial thinking (Batanero et al. 1997; English, 2005; Lockwood, 2012; Eizenberg & Zaslavsky, 2004)

Strategies to overcome combinatorial thinking difficulties have been applied by several researchers (Balera & Júnior, 2017; Hayashi & Ohsawa, 2013; Melusova & Vidermanova, 2015; Pizlo & Li, 2005). Hayashi & Ohsawa (2013) unravels that the game market innovator facilitates students to think combinatorially, which is to foster student creativity and innovation. The research by Pizlo & Li (2005) used puzzle 15 strategy to overcome combinatorial thinking difficulties. The weakness of the strategy is it requires a long time in solving problems. Melusova & Vidermanova (2015) examined the strategies used by students in working on combinatorial problems and associated it with students' answers. Additionally, some researchers used the latest software, T-Tupple Reallocation (TTR). TTR is a software program for computational problems and mathematical cases. The results indicate that there are better results in solving combinatorial cases, which means the use of TTR software has a better impact (Balera & Júnior, 2017).

Research by Lockwood (2013) found a combinatorial thinking model for students to analyze students' conceptual based on mathematical activities in solving counting problems (enumeration). The purpose of this model is to explain the relevant aspects of the students' counting process as well as to describe and explain aspects of the counting activities, with the final objective to describe the combinatorial way of thinking of students.

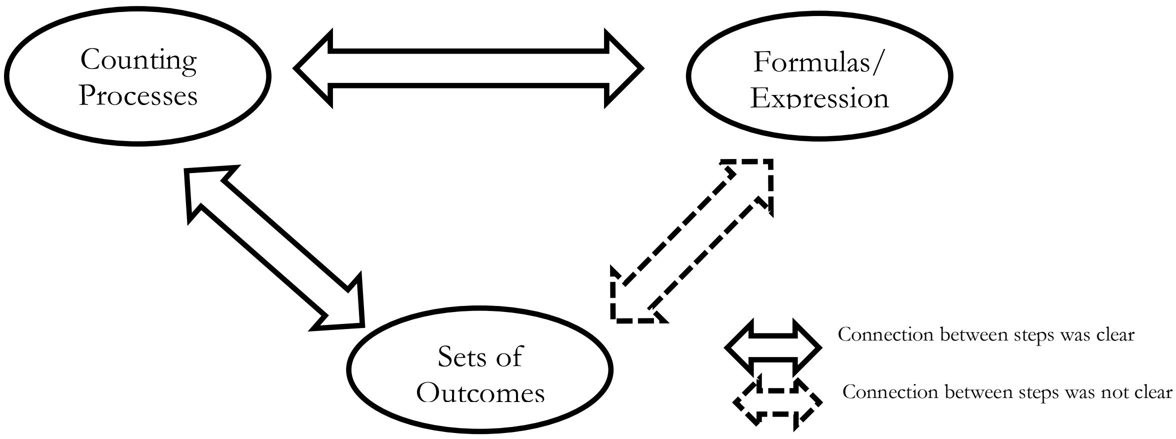


Figure 1.
Combinatorrial Thinking Model (Lockwood, 2013)

Rezaie & Gooya (2011) formulated four levels of combinatorial thinking: investigating "some cases", how am I sure that I have counted all the cases?, systematically generating all cases, and changing the problem into another combintorial problem. However, based on the results of preliminary studies conducted by researchers, there seems to be another level besides those four levels. Researchers found some students used addition and multiplication or calculation order in solving problems. The level is indicated to be between level two and level three. Therefore, it is necessary for further study about adding level of combinatorial thinking in solving mathematical problems. This is conducted to accommodate the stages of combinatorial thinking.

Problem of Study

What is the reconstruction of combinatorial thinking levels for mathematical problem solving? what are the descriptions of 1st, 2nd, 3rd, 4th, and 5th levels of combinatorial thinking?

Method

Research Design

This research is qualitative approach. The data of this study were obtained through interviews and tests (Cresswell, 2012). Researchers arranged meeting six times with students. The first meeting was a preliminary interview about combinatorial thinking and combinatorial material. As a result, students had obtained combinatorial material but had no knowledge about combinatorial thinking. The second to the sixth meeting were tests and interviews.

Participants

The subjects chosen in this study were 40 students of Elementary Teacher Education Department. The reason for choosing the subject is to obtain as much information as possible from various sources to formulate the level of combinatorial thinking, as well as expressing the characteristics of students' combinatorial thinking in solving mathematical problems. From 40 students five students were eligible for further interviews.

Table 1.
Demographic Structures of Participants

Selection by		f	%
Gender	Male	2	40
	Female	3	60
Grade	2nd semester	2	40
	6th semester	3	60

As stated in Table 1, the amount of the sample was 5 students of the 2nd and 6th semesters (two males and three females) with levels of different combinatorial thinking.

Procedure

Session 1, the researchers interviewed the subjects about combinatorial thinking and combinatorial materials. Based on the results of the interview, the subjects understood the combinatorial materials, but they did not understand what the combinatorial thinking really means. *Session 2,* the observation of the charasterictics of combinatorial thinking levels is realized by asking students to solve some mathematical problems. *Session 3,* the researchers interviewed the subjects. The type of interviews was a semi-structural so it does not need to make interview guidance. The interview was adjusted to the subjects' conditions.

Data Collection Tools

A test instrument related to the mathematical problem. The study reconstruct the indicators of combinatorial thinking (Rezaie & Gooya, 2011). The test instrument used in this study can be seen in Table 2

Table 2.

The Test Instrument of Mathematical Problem

Level	Questions
Investigating "some cases'	A restaurant serves two kinds of foods (chicken *satay* and fried rice) and two kinds of beverages (tea and orange). If there are two people and they order 1 kind of food and beverage, how many pairs of foods and beverages can be ordered?
How am I sure that I have counted all the cases?	A restaurant serves 2 kinds of foods (chicken *satay* and fried rice) and 2 kinds of beverages (tea and orange). If there are 2 people and they order one kind of food and beverage, how many pairs of foods and beverages can be ordered?
Using calculation order	A restaurant serves 6 kinds of foods (chicken *satay*, fried rice, baked chicken, Javanese traditional cooked rice, spicy fried eggs, and fried chicken) and 4 kinds of beverages (tea and orange). If there are 4 people and they order one kind of food and beverage, how many pairs of foods and beverages can be ordered?
Systematically generating all cases	A restaurant serves 14 kinds of foods (chicken *satay*, fried rice, baked chicken, Javanese traditional cooked rice, spicy fried eggs, fried chicken, fried eggs, chicken soup, meat soup, chicken curry, chicken soup, sweet meal soup, soup with meatballs, and goat *satay*) and 12 kinds of beverages (tea, orange, syrup, milk, coffee, ginger, lemon tea, water melon juice, orange juice, strawberry juice, *sour soup* juice, and guava juice). If there are 4 people and they order one kind of food and beverage, how many pairs of foods and beverages can be ordered?
Changing problems into another combinatorial problem	A computer's password comprises 8 characters in length. Each character can be alphabets or numeral, capital or small letters. How many passwords can be made?

The answers of 40 students were classified into five levels of combinatorial thinking according to students'understanding in solving mathematical. The distribution of the number of students' answers will be presented in Table 3.

Table 3.

Number of Students at Different Level of Combinatorial Thinking

Level of Combinatorial Thinking	Number of Students
Identifying possibilities	10 Students
Systematically checking all cases	12 Students
Using calculation order	8 Students
Systematically generalizing solutions	6 Students
Changing problem into another combinatorial problem	4 Students

The researchers interviewed the subjects. The type of interviews was a semi-structural so it does not need to make interview guidance.

The data validation used a method triangulation by the interview recordings and comparing its results with the paper-based test results to ensure the validity and reliability (Golafshani, 2003).

Data Analysis

The results of interviews and written tests were analyzed. The researcher listened to the first recording, taking into account the relationship between student results and related literature. During the second recording, the researcher listened in detail the results of the interview and the student's written test to determine each characteristic of the level of combinatorial thinking. After the characteristics of each combinatorial thinking were determined, the next step was to start processing data according to the level of combinatorial thinking. Determination of these characteristics is an important step in data analysis because it facilitates the interpretation of meaningful data. These characteristics emerge through a literature review, identifying each interview answer, and students' written tests that represents related concepts in the literature. Table 4 shows the coding scheme and the description of each characteristic of the level of combinatorial thinking.

Table 4.
Description Category of Combinatorial Thinking in Solving Mathematical Problems

Category	Description	Sub-category
Investigating "some cases'	The procedure is systematic Inaccurate in reading and understanding the problems There is the possibility of missing answers The results are more likely imprecise	Students understand the combination but inaccurate in reading and understanding the problems
How am I sure that I have counted all the cases?	The procedure is systematic whether in the form of pictures, schematics, and diagrams Through and meticulous in reading and understanding the problems Facilitate students to answer all possibilities	Students understand the combination, problems are precisely analyzed
Using calculation order	The procedure is systematic Using calculation order of addition and multiplication	Students can solve problems with the systematic procedure by using calculation order
Systematically generating all cases	The procedure is systematic with more complex cases that are not usually solved using pictures, schematics, and diagrams	Students can solve more complex problems than before without using pictures, schematics, and diagrams
Changing problems into another combinatorial problem	The procedure is systematic with more complex cases	Students can solve problems and change them into different forms and answer it correctly

Results and Discussion

1st Level : Investigating "Some Cases'

The following are the results of the study that the researchers will describe from the explanation of each answer that shows the level of combinatorial thinking of students in solving mathematical problems. The students at level one are investigating "some cases'. The students' answers can be seen in the following figure.

Question	Answer
A restaurant serves two kinds of foods (chicken *satay* and fried rice) and two kinds of beverages (tea and orange). If there are two people and they order 1 kind of food and beverage, how many pairs of foods and beverages can be ordered?	Foods → Chicken satay and fried rice Beverages → Tea and orange Person 1 → Chicken satay → tea → chicken satay + tea orange → chicken satay + orange Fried rice → Tea → Fried rice + Tea Orange → Fried rice + Orange Person 2 → Chicken satay → Tea → Chicken satay + Tea Orange → Chicken satay + Orange Fried rice → Tea → Fried rice + Tea Orange → Fried rice + Orange So, many pairs of foods and beverages that can be ordered by them amount to 8.

Figure 2.
P1-F1-18 Answer

Based on the answer, student had solved the problem systematically. The student answered by pairing one by one type of food and types of drinks that can be ordered. The result is incorrect because the student did not fully understand the question. This is shown in the following interview excerpt.

> *R: "Why is did you answer this way?" (Pointing the schema of P1-F1-18 answer sheet)*
> *P1-F1-18: "There are two foods and two drinks, so I wrote down the chicken satay as food and beverages that can be ordered with chicken satay are tea and orange juice. Food and beverage pairings will be chicken satay and tea and chicken satay and orange juice. And so on, ma'am (pointing to the results of the answer), and then there are two people coming in, the first person can order four pairs of food and beverage and so can the second person, so the answer is 8 pairs."*

Based on the results of the interview, *P1-F1-18* is included in the first level of combinatorial thinking and the student could solve the problems by using the scheme. From two pairs of foods, the student paired each food with two beverages. However, the student did not fully understand the problem so that the result was incorrect.

2nd level : How am I Sure that I have Counted All the Cases?

The second level is how am I sure that I have counted all the cases? At this level, students are able to solve problems by using pictures, schemes, and diagrams. Students can answer all possibilities precisely. Students create a diagram tree and pair each food with its beverage (see Figure 3).

Question	Answer
A restaurant serves 2 kinds of foods (chicken *satay* and fried rice) and 2 kinds of beverages (tea and orange). If there are 2 people and they order one kind of food and beverage, how many pairs of food and beverages can be ordered?	Chicken satay → Tea Orange Fried rice → Tea Orange There are 4 pairs foods and baverages that can be ordered by them.

Figure 3.
P2-M1-18 Answer

Based on the answer above, the student was able to answer with all the possibilities by pairing one by one the types of food and types of beverages that can be ordered. The student then orderly wrote down food and beverage pairs with 4 possibilities. It is presented in the following interview results.

R: "Why did you answer this way?" (pointing the schema of P2-M1-18 *answer sheet)*
P2-M1-18*: "There are two foods and two beverages. Then it questions how many possible pairs of food and beverages can be ordered. So, I wrote down the chicken satay as food and the beverage that can be ordered with chicken satay are tea and orange juice. The food and beverage pairs are chicken satay and tea, chicken satay, and orange juice. And so on, ma'am (pointing to the results of the answers), so there are 4 possibilities.*
R: "Oh, there are 4 possibilities? Then, there are 2 people coming in, what is the solution? "
P2-M1-18*: "The result is still 4 possibilities, ma'am. Because even 2 people coming in, there are only 4 food and beverage choices that can be ordered."*

Based on the results of the interview, P2-M1-18 is classified as the second level of combinatorial thinking, student solved problems using tree diagrams systematically and found the correct final answer. The student gave accurate reasons in solving the given problem.

3rd Level : Using Calculation Order

The third level is using calculation order. In this level, students are able to use more systematic procedures and use the principle of multiplication or addition. Students are able to solve problems using these rules. Students have understood the concept of combination. This is shown in the following answer sheet.

Question	Answer
A restaurant serves 6 kinds of foods (chicken *satay*, fried rice, baked chicken, Javanese traditional cooked rice, spicy fried eggs, and fried chicken) and 4 kinds of beverages (tea and orange). If there are 4 people and they order one kind of food and beverage, how many pairs of foods and beverages can be ordered?	The types of foods and 4 types of beverages. A pairs of foods and beverages that can be ordered by them. *Chicken satay → Tea → Orange → Milk → Syrup * Fried rice → Tea → Orange → Milk → Syrup * 6 x 4 = 24 So, many pairs of foods and beverages that can be ordered by them amount to 24.

Figure 4.
P3-F2-18 Answer

Based on the answers student P3-F2-18, the student could use a calculation order - multiplication. The student used multiplication when the word "and" is written. Student already understood that the multiplication rule is used if there is "and" conjunction. The addition rule is used if there is "or" conjunction. This can be seen in the following interview excerpt.

R: "Why did you answer this way?" (pointing P3-F2-18 *answer sheet)*
P3-F2-18*: "This is 'and' Ma'am, so I used multiplication (pointing the answer) and I used addition for 'or' "*
R: "Where did you get this concept from?"
P3-F2-18*: "From high school ma'am, this is the topic of calculation order"*
R: "So for all the questions with 'and' and 'or' can be multiplied and summed?"
P3-F2-18*: "For combination, yes, you can, ma'am"*
R: "Why is the answer 24?"
P3-F2-18*: "The problem stated 6 types of food and 4 types of beverages, while the question is the number of food and beverage pairs that can be ordered, so it can be multiplied as 6x4, so the result is 24"*
R: "The question wrote 4 people are coming, what is the solution?
P3-F2-18*: "The answer is still 6 x 4, ma'am. Because the question is how many food and beverage pairs can be ordered, so no matter how many people coming, there are only 24 food and beverages can be ordered."*

Based on the interview results, P3-F2-18 is categorized into the third level of combinatorial thinking, student solved problems using the calculation order. The student used multiplication. When researchers asked why students chose to use multiplication, the student could explain the reasons precisely.

4th Level : Systematically Generating All Cases

Level four is systematically generalizing the solution. At this level, students are able to use procedures systematically with more complex cases than previous which cannot use pictures, schematics, and diagrams or it takes a long time if it uses those methods. This can be seen from the following answer sheet.

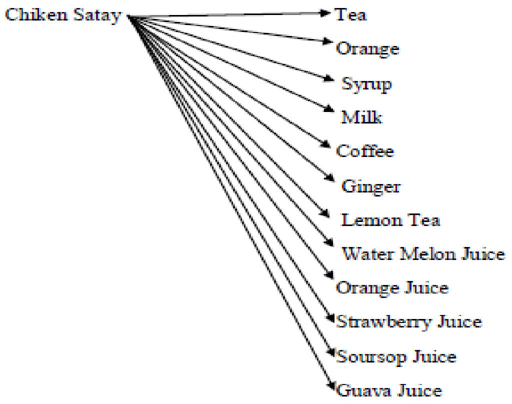
Question	Answer
A restaurant serves 14 kinds of foods (chicken *satay*, fried rice, baked chicken, Javanese traditional cooked rice, spicy fried eggs, fried chicken, fried eggs, chicken soup, meat soup, chicken curry, chicken soup, sweet meat soup, soup with meatballs, and goat *satay*) and 12 kinds of beverages (tea, orange, syrup, milk, coffee, ginger, lemon tea, Water melon juice, orange juice, strawberry juice, *sour soup* juice, and guava juice). If there are 4 people and they order one kind of food and beverage, how many pairs of foods and beverages can be ordered?	A pair of foods and beverages that can be ordered by them is 14 x 12 = 168. For example, 

Figure 5.
P4-M2-20 Answer

Based on the answers above, it can be seen that the student understood the question given. The Student solved the problem without using pictures, schematics, or diagrams. The student only gave one sample pair of food and 12 beverages. The student could explain the answer correctly as shown in the following interview excerpt.

> R: *"Please read the question first!"*
> P4-M2-20: *"(Student read the questions pointed by researchers)*
> R: *"What do you find from this problem?" (Pointing the question)*
> P4-M2-20: *"There are 14 types of food and 12 types of beverages, ma'am"*
> R: *"Then what is the question?"*
> P4-M2-20: *"How many food and drink pairs can be ordered if 4 people are coming".*
> R: *"Uhm, can you explain the solution?"*
> P4-M2-20: *"You only need to multiply it, ma'am. 14 x 12, the results are 168 pairs, 4 people coming are excluded just because no matter how many people coming, food and drink pairs that can be ordered are 168, ma'am."*

Based on the results of the interview, P4-M2-20 is included in level four of combinatorial thinking, student solved problems directly without using pictures, schemes or diagrams. The student used a calculation order, which is multiplication. The student could explain the answer correctly.

5th Level : Changing the Problem into Another Combinatorial Problem

Level five is changing the problem into another combinatorial problem. This level shows that students have a better understanding in solving combinatorial problems that are different from afore. Students can answer questions systematically. This can be seen from the following student answers.

Question	Answer
A computer's password comprises 8 characters in length. Each character can be alphabets or numeral, capital or small letters. How many passwords can be made?	6 characters = 36^6 = 2176782336 7 characters = 36^7 = 78364164096 8 characters = 36^8 = 2821109907456

Figure 6.
P5-F3-20 Answer

Based on the student answers, it can be seen that the student was able to count the number of letters and numbers that can be used as a combination of passwords with a length of six to eight characters. If the problem is changed, the student already knows how to solve the problem. This is shown from the following interview excerpt.

> *R: "Please explain the purpose of this problem?"*
> P5-F3-20*: "The purpose of this problem is how many passwords can be made of numbers and letters if they consist of four to eight characters if lowercase and uppercase letters are considered the same"*.
> *R: "Yes, so how?"*
> P5-F3-20*: "Because the password consists of letters and numbers. For the letters, lowercase and uppercase letters are the same, which are 26, from a to z; then for numbers of total ten are taken from 0 to 9. 26 added by 10 equals to 36. The password can be six to eight characters long so I made six columns where each column you can input the letters and numbers, which means 36 to the power of six and this is the result (showing the results of calculations on the answer sheet), for the password consisting of seven characters then 36 to the power of seven, this is the result (showing the calculation results on the answer sheet) for the eight characters then 38 to the power of eight."*

Based on the interview results, P5-F3-20 is included in level five of combinatorial thinking and the student solved combinatorial questions that are different from previous. The student could explain the process from the beginning to the result.

Discussion and Conclusion

The research by Rezaie & Gooya (2011) elaborates four levels of combinatorial thinking, investigating "some cases', how am I sure that I have counted all the cases?, systematically generating all cases, and changing the problem into another combinatorial problem. This article develops four levels of combinatorial thinking into five levels of combinatorial thinking by adding a level between level two and level three, which is using the calculation order.

Table 5.
Characteristics of Level of Combinatorial Thinking in Solving Mathematical Problems

Level of Combinatorial Thinking	General Description
Investigating "some cases'	Students do not fully understand the problem Students can solve problems with the systematic procedure but the results tend to be imprecise
How am I sure that I have counted all the cases?	Students understand combination materials Students can solve problems with systematic procedures using a tree diagram
Using calculation order	Students use calculation order, which is addition and multiplication Students understand multiplication and addition rules
Systematically generating all cases	Students can calculate the possibilities without using schematics, pictures or diagrams
Changing problems into another combinatorial problem	Students can solve other combinatorial problems Students can calculate possibilities with more complex problems

Based on Table 5, the results of the study show that there are five levels in combinatorial thinking, including investigating "some cases', how am I sure that I have counted all the cases?, using calculation order, systematically generating all cases, and changing the problem into another combinatorial problem. The student in the first level identified mathematical problem solving using systematic procedures, but the answers were incorrect because the student did not fully understand the problems. Therefore, the student encountered difficulty in solving the problems. Pramusinta et al. (2019); Suyono et al. (2019); Lockwood (2012) state that when students still encounter difficulty in combining combinatorial questions, the answers obtained tend to be incorrect.

The student on the second level solved problems with procedures systematically with tree diagrams. This is in line with the results of research by (Rosidin, et al. 2019) that students can learn easily some combinatorial ideas with the assistance of tree diagrams. Also, tree theory can be employed for probability theory, sorting letters at the post office, determining family trees, creating genetic trees, and connecting cities roads which have minimum length (Rosidin et al. 2019). Suyono et al. (2019) states that the use of tree diagrams is a basic procedure in solving combinatorial problems.

Student at level three was able to use more systematic procedures and used multiplication rules. In this case, the student began to use multiplication and knew the concept of multiplication well. This is consistent with the results (Pramusinta et al. 2019) research that after a formal operational period, teenagers should be able to find systematic combinatorial construction procedures. Students are included in this stage after passing the formal operational period. Research by Suyono et al. (2019) reveals that the use of calculation order in combinatorial procedure can also be referred to as numerical procedures. This finding is supported by Yuli et al. (2019) that the level of students' ability to solve mathematical problems is used as a consideration in preparing lesson plans.

Student at level four was able to calculate and solve more complex problems without using pictures, schematics, and diagrams. Student at this level was able to use systematic thinking to solve problems. This is in accordance with (Suyono et al. 2019) research that solving combinatorial problem will facilitate the process of enumeration, guessing, generalization, and systematic thinking.

The student at level five could see the concepts of mathematical problem solving. At this level, the student already saw a lot about the concepts of solving mathematical problems and the student began to solve various problems with combinatorial thinking skills. The student used five stages by rereading problems, checking calculations, checking plans, changing problems into other problems, and re-examining problems (Malloy & Jones, 1998; Suyono et al. 2019). The use of this stage is related to success in solving problems (Eizenberg & Zaslavsky, 2004; Pramusinta et al. 2019).

Based on the results of research and discussion, there are five levels of combinatorial thinking in mathematical problem solving. Level one is the identification of the possibility of students' understanding the questions incorrectly, or vice versa, already can answer the questions with systematic procedures but the results are less precise. Level two is conducting systematic checking about students' understanding of the combination material. Besides, it also concerns about the ability to answer problems systematically using diagram trees. Level three is students are able to apply the calculation orders, which are addition and multiplication. Level four is systematically generating all cases about the ability to calculate possibilities without schematic, drawings, or diagrams. Level five is changing the problem into another combinatorial problem, namely the ability to calculate possibilities with complex problems.

Recommendations

For Further Studies

Based on the research findings, it turns out there is another level of combinatorial thinking, which is using the calculation order and this is found between level two and level three. The researchers recommend further research to explore more on the application of calculation order. The researchers recommend further research to explore more on the application of calculation order. The research can explore that is explain about thinking style from combinatorial thinking. There are1)concrete sequential (CS); 2) concrete random (CR); 3) abstract random (AR); and 4) abstract sequential (AS)., Futrhermore, the researcher can explore in another level of student and different material.

For Applicants

The application of combinatorial thinking levels can be used to overcome learning difficulties. Next, the teacher can apply the level of combinatorial thinking with other questions.

Limitations of Study

The research just focus in students of Elementary Teacher Education Department (PGSD), and to find level of combinatorial thinking.

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