Turk. J. Math. Comput. Sci.
14(1)(2022) 1–7
© MatDer
DOI : 10.47000/tjmcs.796501

Independent Transversal Domination Number for Some Transformation Graphs G^{xyz} when $xyz = + - +$

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Received: 17-09-2020 • Accepted: 07-01-2022

Abstract. A dominating set of a graph G which intersects every independent set of a maximum cardinality in G is called an independent transversal dominating set. The minimum cardinality of an independent transversal dominating set is called the independent transversal domination number of G and is denoted by $\gamma_{it}(G)$. In this paper we investigate the independent transversal domination number for the transformation graph of the path graph P_n^{+-+}, the cycle graph C_n^{+-+}, the star graph $S_{1,n}^{+-+}$, the wheel graph $W_{1,n}^{+-+}$ and the complete graph K_n^{+-+}.

2010 AMS Classification: 05C69, 05C40, 68M10, 68R10

Keywords: Domination number, independent transversal domination number, independence number.

1. Introduction

In a communication network, the vulnerability measures the resistance of network to disruption of operation after the failure of certain stations or communication links. The stability of communication networks is of prime importance to network designers. If we think of the graph as modeling a communication network, many graph theoretical parameters have been used to describe the stability of communication networks including connectivity, toughness, integrity, binding number, domination, exponential domination, independent transversal domination. The independent transversal domination number is one of the measures of the graph vulnerability.

In this paper, we consider simple finite undirected graphs without loops and multiple edges. Let $G = (V(G), E(G))$ be a graph. For a vertex x of G, $N(x)$ denotes the set of all neighbors of x in G. The *distance* $d(u, v)$ between two vertices u and v in G is the length of a shortest path between them. The *diameter* of G, denoted by $diam(G)$ is the largest distance between two vertices in $V(G)$ [16]. The number of the neighbor vertices of the vertex v is called degree of v and denoted by $deg_G(v)$. The minimum and maximum degrees of a vertex of G are denoted by $\delta(G)$ and $\Delta(G)$. A vertex v is said to be pendant vertex if $deg_G(v) = 1$. A vertex u is called support if u is adjacent to a pendant vertex [11]. The eccentricity $e(u)$ of a vertex u in G is the distance from u to a vertex farthest from u. The minimum eccentricity of the vertices of the graph G is the *radius* of G denoted by $rad(G)$, while the *diameter* of G is the greatest eccentricity [7]. The *line graph* $L(G)$ of a graph G is that graph whose vertices can be put in one-to-one correspondence with the edges of G in such a way that two vertices of $L(G)$ are adjacent if and only if the corresponding edges of G are adjacent [7]. The *total graph* $T(G)$ has vertex set $V(G) \cup E(G)$, and two vertices of $T(G)$ are adjacent whenever they are neighbors in G [11]. It is easy to see that $T(G)$ always contains both G and $L(G)$ as induced subgraphs. The complement $\overline{G}$

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of a graph G is that graph with vertex set $V(G)$ such that two vertices are adjacent in $\overline{G}$ if and only if these vertices are not adjacent in G [7]. Let G be a graph and $S \subseteq V(G)$. We denote by $< S >$ the subgraph of G induced by S. For each vertex $u \in S$ and for each $v \in V(G) - S$, we define $\overline{d}(u, v) = \overline{d}(v, u)$ to be the length of a shortest path in $< V(G) - (S - u) >$ if such a path exists, and ∞ otherwise. Let $v \in V(G)$. The definition is

$$w_s(v) = \begin{cases} \sum_{u \in S} 1/2^{\overline{d}(u,v)-1} & , v \notin S \\ 2 & , v \in S. \end{cases}$$

We refer to $w_s(v)$ as the weight of S at v. If $\forall v \in V(G)$, we have $w_s(v) \geq 1$, then S is an exponential dominating set. The smallest cardinality of an exponential dominating set is the exponential domination number, $\gamma_e(G)$, and such a set is a minimum exponential dominating set, or $\gamma_e - set$ for short [2, 9].

A set S is said to be an *independent set* of G, if no pair of vertices of S are adjacent in G. The *independence number* of G, denoted by $\beta(G)$, is the cardinality of a maximum independent set of G. We denote by $\Omega(G)$ the set of all maximum independent sets of G. A vertex and an edge are said to *cover* each other if they are incident. A set of vertices which cover all the edges of a graph G is called a *vertex cover* for G, while a set of edges which covers all the vertices is an *edge cover*. The smallest number of vertices in any vertex cover for G is called its *vertex covering number* and is denoted by $\alpha(G)$ [11].

A *dominating set* S in a graph G is a set of vertices of G such that every vertex in $V(G) - S$ is adjacent to at least one vertex in S. The domination number of G, denoted by $\gamma(G)$, is the minimum cardinality of a dominating set of G [12, 13].

Given a graph G and a collection of subsets of its vertices, a subset of $V(G)$ is called a *transversal* of G if it intersects each subset of the collection. A dominating set of G which intersects every independent set of maximum cardinality in G is called an *independent transversal dominating set*. The minimum cardinality of an independent transversal dominating set is called the *independent transversal domination number* of G and is denoted by $\gamma_{it}(G)$. An independent transversal dominating set of cardinality $\gamma_{it}(G)$ is called a $\gamma_{it}(G) - set$. Thus, if D is an ITD-set of G, then D is a dominating set of G and $\beta(G) > \beta(G - D)$. The notion of independent transversal domination was first introduced by Hamid [6, 10]. This parameter is a new concept for graph theory. Independent transversal domination numbers for path graphs P_n, cycle graphs C_n, wheel graphs W_n, complete graphs K_n, bipartite graphs and tree graphs have been investigated in [10] and complexity of this parameter has been computed in [1].

In this paper, firstly known results are given. Then, we investigate the independent transversal domination number for some transformation graphs G^{+-+}. We displayed the relationship between the independent transversal domination number and the exponential domination number, the independence number β, the vertex covering number α as a corollary. Lastly, the conclusion section is presented.

2. Known Results

Theorem 2.1 ([10]). *If G is a complete multipartite graph having r maximum independent sets, then*

$$\gamma_{it}(G) = \begin{cases} 2 & , if \ r = 1 \\ r & , otherwise. \end{cases}$$

Theorem 2.2 ([10]). *For complete graph with order n and complete bipartite graph with order $m + n$, $\gamma_{it}(K_n) = n$ and $\gamma_{it}(K_{m,n}) = 2$, respectively.*

Theorem 2.3 ([10]). *For any path P_n of order n, we have*

$$\gamma_{it}(P_n) = \begin{cases} 2 & , if \ n = 2, 3 \\ 3 & , if \ n = 6 \\ \lceil \frac{n}{3} \rceil & , otherwise. \end{cases}$$

Theorem 2.4 ([10]). *For any cycle C_n of order n, we have*

$$\gamma_{it}(C_n) = \begin{cases} 3 & , if \ n = 3, 5 \\ \lceil \frac{n}{3} \rceil & , otherwise. \end{cases}$$

Theorem 2.5 ([10]). *If W_n is a wheel on n vertices, then*

$$\gamma_{it}(W_n) = \begin{cases} 2, & if\ n = 5 \\ 3, & if\ n \geq 7\ and\ is\ odd\ or\ n = 6 \\ 4, & otherwise. \end{cases}$$

Theorem 2.6 ([10]). *If G is a disconnected graph with compenents $G_1, G_2, ..., G_r$, then $\gamma_{it}(G) = min_{1\leq i\leq r}\{\gamma_{it}(G_i) + \sum_{j=1, j\neq i}^{r} \gamma(G_j)\}$.*

Theorem 2.7 ([10]). *If G has an isolated vertex, then $\gamma_{it}(G) = \gamma(G)$.*

Theorem 2.8 ([10]). *For any graph G, we have $1 \leq \gamma_{it}(G) \leq n$. Further $\gamma_{it}(G) = n$ if and only if $G = K_n$.*

Theorem 2.9 ([10]). *Let G be a graph on n vertices. Then $\gamma_{it}(G) = n - 1$ if and only if $G = P_3$.*

Theorem 2.10 ([10]). *Let G be a non-complete connected graph with $\beta(G) \geq \frac{n}{2}$. Then $\gamma_{it}(G) \leq \frac{n}{2}$.*

Theorem 2.11 ([10]). *If G is bipartite, then $\gamma_{it}(G) \leq \frac{n}{2}$.*

Theorem 2.12 ([10]). *Let a and b be two positive integers with $b \geq 2a - 1$. Then there exists a graph G on b vertices such that $\gamma_{it}(G) = a$.*

Theorem 2.13 ([10]). *If G is a non-complete connected graph on n vertices, then $\gamma_{it}(G) \leq \lceil \frac{n}{2} \rceil$.*

Theorem 2.14 ([10]). *For any graph G, we have $\gamma(G) \leq \gamma_{it}(G) \leq \gamma(G) + \delta(G)$.*

Corollary 2.15 ([10]). *If T is a tree, then $\gamma_{it}(T)$ is either $\gamma(T)$ or $\gamma(T) + 1$.*

Theorem 2.16 ([10]). *If G is a graph with $diam(G) = 2$, then $\gamma_{it}(G) \leq \delta(G) + 1$.*

Theorem 2.17 ([6]). *If G is a connected graph and u is a vertex of minimum degree in G, then*

$$\gamma_{it}(G) \leq \begin{cases} \delta(G) + 1 & if\ ecc_G(u) \leq 2 \\ \frac{n(G)}{2} + 1, & if\ ecc_G(u) \geq 3, \end{cases}$$

and these bounds are tight.

Theorem 2.18 ([6]). *If G is a graph with $\beta(G) \geq \frac{n(G)}{2}$, then $\gamma_{it}(G) \leq \gamma(G) + 1$, and this bound is tight.*

Theorem 2.19 ([9]). *For every graph G, $\gamma_e(G) \leq \gamma(G)$. Also, $\gamma_e(G) = 1$ if and only if $\gamma(G) = 1$.*

3. Independent Transversal Domination of a Graph

Definition 3.1 ([10]). A dominating set $S \subseteq V$ of a graph G is said to be an independent transversal dominating set if S intersects every maximum independent set of G. The minimum cardinality of an independent transversal dominating set of G is called the independent transversal domination number of G and is denoted by $\gamma_{it}(G)$. An independent transversal dominating set S of G with $|S| = \gamma_{it}(G)$ is called a $\gamma_{it} - set$.

The following figure shows the independent transversal domination number of a graph G.

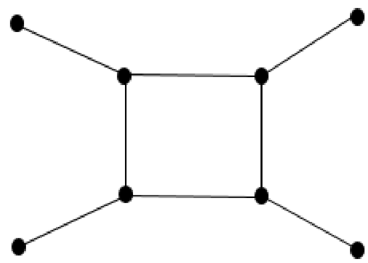

Figure 1. The graph G

In Fig. 1, $\beta(G) = 4, \gamma(G) = 4$. The maximum independent set of the graph consists of four pendant vertices or two pendant vertices and two support vertices on cycle having $deg(v) = 2$. The dominating set of the graph consists of four pendant vertices or four support vertices on cycle having $deg(v) = 2$. Let S be an independent transversal dominating set. If we pick the support vertices on cycle for S, then all vertices of the graph G are dominated. But the independence number of the graph doesn't decrease. $V - S$ contains any $\beta - set$. So, we must add also any pendant vertex to S. Hence, $\beta(G) > \beta(G - S)$ and we have $\gamma_{it}(G) = 5$ since $|S| = 5$.

Definition 3.2 ([15]). Let $G = (V(G), E(G))$ be a graph, and α, β be two elements of $V(G) \cup E(G)$. We define the associativity of α and β is $+$ if they are adjacent or incident, and $-$ otherwise. Let xyz be a 3-permutation of the set $\{+, -\}$. We say that α and β correspond to the first term x (resp. the second term y or the third term z) if both α and β are in $V(G)$ (resp. both α and β are in $E(G)$, or one of α and β is $V(G)$ and the other is in $E(G)$). The transformation graph $G^{x,y,z}$ of G is defined on the vertex set $V(G) \cup E(G)$. Two vertices α and β of $G^{x,y,z}$ are joined by an edge if and only if their associativity in G is consistent with the corresponding term of xyz.

Let x, y, z be three variables taking value $+$ or $-$. The transformation graph of G, G^{xyz} is a simple graph having as the vertex set $V(G) \cup E(G)$ and for $u, v \in V(G) \cup E(G)$, u and v are adjacent or incident in $G^{x,y,z}$ if and only if one of the following holds [3–5, 14, 15]:
(i) Let $u, v \in V(G)$. u and v are adjacent in G if $x = +$; u and v are not adjacent in G if $x = -$.
(ii) Let $u, v \in E(G)$. u and v are adjacent in G if $y = +$; u and v are not adjacent in G if $y = -$.
(iii) Let $u \in V(G)$ and $v \in E(G)$. u and v are incident in G if $z = +$; u and v are not incident in G if $z = -$.

Since there are eight distinct 3-permutations of $\{+, -\}$, we may obtain eight kinds of transformation graphs, in which G^{+++} is the total graph of G and G^{---} is its complement. Also, G^{--+}, G^{-+-} and G^{-++} are the complements of G^{++-}, G^{+-+} and G^{+--}, respectively.

In this paper, the independent transversal domination number of the transformation graphs for graphs paths, cycles, stars, completes and wheels have been computed when $xyz = + - +$.
The transformation graph P_6^{+-+} can be depicted as in the following figure:

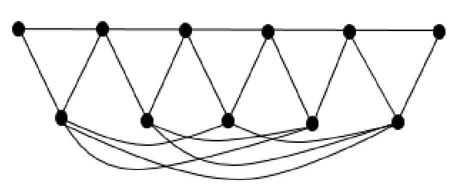

Figure 2. The graph P_6^{+-+}

Theorem 3.3 ([8]). *For any path P_n of order n, we have $\gamma(P_n) = \lceil \frac{n}{3} \rceil$.*

Corollary 3.4. *For any connected graph G, $\beta(G^{+-+}) = \lceil \frac{n}{2} \rceil + 1$.*

Theorem 3.5. *Let $G \cong P_n$ be any path graph of order $n > 10$. Then,*

$$\gamma_{it}(G^{+-+}) = \begin{cases} \lceil \frac{n-2}{3} \rceil + 2, & if\ n \equiv 2(mod\ 3) \\ \lceil \frac{n-2}{3} \rceil + 1, & otherwise. \end{cases}$$

Proof. The vertex set $V(G^{+-+}) = V(G) \cup V(\overline{L(G)})$ of the graph G^{+-+}, where $V(G) = \{v_1, v_2, ..., v_n\}$ and $V(\overline{L(G)}) = \{e_{12}, e_{23}, e_{34}, ..., e_{(n-1)n}\}$. Let D be a $\gamma - set$ of the graph G^{+-+}. Any of the vertices e_{12} and $e_{(n-1)n}$ must be in D, due to the maximum degree $\Delta(G^{+-+}) = deg(e_{12}) = deg(e_{(n-1)n}) = n-1$ of G^{+-+}. Suppose $\{e_{12}\} \subset D$. So, every vertices in $(V(\overline{L(G)}) - \{e_{23}\})$ and the vertices v_1, v_2 in $V(G)$ are dominated. Hence, we have the remaining graph $G \cong P_{n-2}$. There are three cases depending on n from Theorem 3.3.
Case 1. $n \equiv 0(\text{mod } 3)$
In this case, $D = \{e_{12}\} \cup \{v_{3i} : 1 \le i \le \lceil \frac{n-2}{3} \rceil\}$, where D is a $\gamma - set$ of the graph G^{+-+}. Therefore, $< V(G^{+-+}) - D >= (\frac{n}{3})K_2 \cup (\overline{L(G)} - \{e_{12}\})$. Every independent set in $V(G^{+-+}) - D$ contains at most $\frac{n}{3} + 2$ vertices due to $\beta(\overline{L(G)}) = 2$. Also, $\frac{n}{3} + 2 < \lceil \frac{n}{2} \rceil + 1 = \beta(G^{+-+})$. This means that $(V(G^{+-+})) - D$ doesn't contain any $\beta - set$ of G^{+-+}. This requires that $\beta - set$ of G^{+-+} contains at least one vertex of D. Hence, domination set D of the graph G^{+-+} is also an independent transversal domination set of the graph G^{+-+}. So, we have $\gamma_{it}(G^{+-+}) = \gamma(G^{+-+}) = |D| = \lceil \frac{n-2}{3} \rceil + 1$.
Case 2. $n \equiv 1(\text{mod } 3)$
$\gamma - set$ D of G^{+-+} is the same as in the Case 1. $D = \{e_{12}\} \cup \{v_{3i} : 1 \le i \le \lceil \frac{n-2}{3} \rceil\}$. We have $< V(G^{+-+}) - D >= (\frac{n-1}{3})K_2 \cup (\overline{L(G)} - \{e_{12}\}) \cup \{v_n\}$. Every independent set in $(V(G^{+-+}) - D)$ contains at most $\lceil \frac{n-1}{3} \rceil + 2 + 1 = \lceil \frac{n+8}{3} \rceil$ vertices. Obviously, $\lceil \frac{n+8}{3} \rceil < \lceil \frac{n}{2} \rceil + 1 = \beta(G^{+-+})$. This means that $(V(G^{+-+}) - D)$ doesn't contain any $\beta - set$ of G^{+-+}. This requires that $\beta - set$ of G^{+-+} contains at least one vertex of D. Hence, domination set D of the graph G^{+-+} is also an independent transversal domination set of the graph G^{+-+}. So, we have $\gamma_{it}(G^{+-+}) = \gamma(G^{+-+}) = |D| = \lceil \frac{n-2}{3} \rceil + 1$.
Case 3. $n \equiv 2(\text{mod } 3)$
$\gamma - set$ D of G^{+-+} is the same as in Case 1 and Case 2. So, $D = \{e_{12}\} \cup \{v_{3i} : 1 \le i \le \lceil \frac{n-2}{3} \rceil\} \cup \{v_n\}$. Therefore, $< V(G^{+-+}) - D >= (\frac{n}{3})K_2 \cup (\overline{L(G)} - \{e_{12}\})$. Also, every independent set in $(V(G^{+-+}) - D)$ contains at most $\frac{n+1}{3} + 2 = \frac{n+7}{3}$ vertices due to $\lceil \frac{n}{3} \rceil = \frac{n+1}{3}$ for $n \equiv 2(\text{mod } 3)$. It is easy to see that $\frac{n+7}{3} < \lceil \frac{n}{2} \rceil + 1 = \beta(G^{+-+})$. This means that $(V(G^{+-+})) - D$ doesn't contain any $\beta - set$ of G^{+-+}. This requires that $\beta - set$ of G^{+-+} contains at least one vertex of D. Hence, domination set D of the graph G^{+-+} is also an independent transversal domination set of the graph G^{+-+}. So, we have $\gamma_{it}(G^{+-+}) = \gamma(G^{+-+}) = |D| = \lceil \frac{n-2}{3} \rceil + 2$.
The proof is completed. □

Theorem 3.6. *Let $G \cong C_n$ be any cycle graph of order $n > 11$. Then,*

$$\gamma_{it}(G^{+-+}) = \lceil \frac{n}{3} \rceil + 1.$$

Proof. The vertex set $V(G^{+-+}) = V(G) \cup V(\overline{L(G)})$ of the graph G^{+-+}, where $V(G) = \{v_1, v_2, ..., v_n\}$ and $V(\overline{L(G)}) = \{e_{12}, e_{23}, e_{34}, ..., e_{(n-1)n}, e_{1n}\}$. Let D be a $\gamma - set$ of the graph G^{+-+}. If the vertices e_{23} and v_1 are added to D, then all vertices in $\overline{L(G)}$ and the vertices v_2, v_3, v_n in G are dominated. Hence, we have the remaining graph $G \cong P_{n-3}$. We know $\gamma(P_n) = \lceil \frac{n}{3} \rceil$ from the Theorem 3.3. So, $\gamma(G^{+-+}) = \lceil \frac{n-3}{3} \rceil + 2 = \lceil \frac{n}{3} \rceil + 1$. $(V(G^{+-+}) - D)$ doesn't contain any $\beta - set$ of G^{+-+}. Hence, domination set D of the graph G^{+-+} is also an independent transversal domination set of the graph G^{+-+}. So, we have

$$\gamma_{it}(C_n^{+-+}) = \lceil \frac{n}{3} \rceil + 1.$$

The proof is completed. □

Theorem 3.7. *Let $G \cong S_{1,n}$ be any star graph of order $n + 1$. Then, $\gamma_{it}(G^{+-+}) = 3$.*

Proof. The vertex set $V(G^{+-+}) = V(G) \cup V(\overline{L(G)})$ of the graph G^{+-+}, where $V(G) = \{c, v_1, v_2, ..., v_n\}$, where c is the center vertex and $V(\overline{L(G)}) = \{e_1, e_2, e_3, ..., e_{(n-1)}, e_n\}$. Let D be a $\gamma - set$ of the graph G^{+-+}. We know that the minimum vertex degree $\delta(S_{1,n}^{+-+}) = 2$, the diameter $diam(S_{1,n}^{+-+}) = 2$ and the domination number $\gamma(S_{1,n}^{+-+}) = 1$. We know $\gamma_{it}(S_{1,n}^{+-+}) \le \delta(S_{1,n}^{+-+}) + 1$ from the Theorem 2.16. So, we have $\gamma_{it}(S_{1,n}^{+-+}) \le 3$.

Assume that $D = \{c, v_i\}$ or $D = \{c, e_i\}$,where $1 \le i \le n$. In this case, $V - D$ contains a $\beta - set$ such that $\{v_1, v_2, ..., v_n\}$ or $\{e_1, e_2, ..., e_n\}$ or $\{v_x, e_y | x \ne y, 1 \le x, y \le n\}$ due to $\beta(S^{+-+}_{1,n}) = n$. Therefore, $\gamma_{it}(S^{+-+}_{1,n}) \ge 3$. So, we have $\gamma_{it}(S^{+-+}_{1,n}) = 3$.
The proof is completed. □

Theorem 3.8. *Let $W_{1,n}$ be any wheel graph of order $n + 1$. Then, $\gamma_{it}(W^{+-+}_{1,n}) = 3$.*

Proof. We can split the vertex set of $W^{+-+}_{1,n}$ into three sets $V(W^{+-+}_{1,n}) = V_1(W^{+-+}_{1,n}) \cup V_2(W^{+-+}_{1,n}) \cup V_3(W^{+-+}_{1,n})$ such as:
$V_1(W^{+-+}_{1,n}) = \{c, u_i | c$ is the center vertex of $W_{1,n}$ and $u_i \in V(W_{1,n}), 1 \le i \le n\}$ $V_2(W^{+-+}_{1,n}) = \{c_i|$ the vertices that are corresponding to the edges cu_i, c is the center vertex of $W_{1,n}$ and $u_i \in W_{1,n}, 1 \le i \le n\}$
$V_3(W^{+-+}_{1,n}) = \{v_{x,y}|\ y = x + 1$ or $x = n$ and $y = 1$ the vertices that are corresponding to the edges on the cycle of the wheel graph $\}$
The maximum independent set of the graph $W^{+-+}_{1,n}$ is the set $V_3(W^{+-+}_{1,n}) \cup c$ and $\beta(W^{+-+}_{1,n}) = n + 1$. Let $D = \{c \cup c_i \cup v_{x,y}\}$, where $i \ne x, y$ be a dominating set. Since $\beta(W^{+-+}_{1,n} - D) < \beta(W^{+-+}_{1,n})$, D is also an independent transversal dominating set of $W^{+-+}_{1,n}$ and $\gamma_{it}(W^{+-+}_{1,n}) = 3$. □

Theorem 3.9. *Let K_n be a complete graph of order n. Then, $\gamma_{it}(K^{+-+}_n) = 3$.*

Proof. We can split the vertex set of K^{+-+}_n into n sets such that:

$$\begin{aligned} V(K^{+-+}_n) = V(K_n) &= \{u_1, u_2, ..., u_n\} \cup V_1 = \{u_{12}, u_{13}, ..., u_{1n}\} \cup V_2 = \{u_{23}, u_{24}, ..., u_{2n}\} \cup V_3 \\ &= \{u_{34}, u_{35}, ..., u_{3n}\} \cup ... \cup V_{n-1} = \{u_{n-1,n}\}. \end{aligned}$$

The maximum independent set of the graph K^{+-+}_n is either V_1 or $\{V_2 \cup u_1\}$. Hence, $\beta(K^{+-+}_n) = n - 1$. Let S be an independent transversal dominating set of K^{+-+}_n. If we add u_1 and u_{12} to S, then the maximum independence number of K^{+-+}_n decreases and $(V(K^{+-+}_n) - S)$ contains no $\beta-$set. But, we need to add the vertex u_2 to S to dominate all vertices of K^{+-+}_n. Hence, we have $\gamma_{it}(K^{+-+}_n) = \gamma(K^{+-+}_n) = 3$.
The proof is completed. □

Theorem 3.10. *If G is a non-complete connected graph of order n and $\beta(G) = 2$, then $\beta(G) \le \gamma_{it}(G) \le \lceil \frac{n}{2} \rceil$*

Proof. If $\beta(G) = 2$ then $\gamma(G) \le 2$. Suppose $\{u, v\}$ be an independence set. If $\gamma(G) = 1$ then, there is a vertex w such that $d(u, w) = d(v, w) = 1$. Let $|D| = 1$ is a dominating set and S be an independent transversal dominating set of G. In this case, $\beta(G) = \beta(G - D)$. So, D is not an independent transversal dominating set and $S \ge 2$. If $\gamma(G) = 2$ then, $D = \{u, v\}$ is also a dominating set of G. Otherwise, there is a vertex z such that $d(z, u) > 1$ or $d(z, v) > 1$. So, we have an independence set $\{u, v, z\}$ and $\beta(G) = 3$, which is a contradiction. Since $\beta(G) \ge \beta(G - D)$, we have $|S| \ge 2$.
For the right side of inequality, if $\beta(G) = 2$ then, $\delta(G) \ge \frac{n-2}{2}$. We know $\gamma_{it}(G) \le \gamma(G) + \delta(G)$ from the Theorem 2.14. So, if $\beta(G) = 2$ then, we have $\gamma_{it}(G) \le 1 + \frac{n-2}{2} = \frac{n}{2}$.
The proof is completed. □

Theorem 3.11 ([11]). *For any nontrivial connected graph G of order n, $\alpha(G) + \beta(G) = n$.*

Corollary 3.12. *For $n \ge 4$ and $\beta(G) = 2$, we have $\beta(G) \le \gamma_{it}(G) \le \alpha(G)$.*

Proof. We know $\beta(G) + \alpha(G) = n$. If $\beta(G) = 2$ then, $\beta(G) \le \gamma_{it}(G)$ from the Theorem 3.10. and $\alpha(G) = n - 2$. Since $(n - 2) \ge \lceil \frac{n}{2} \rceil$ for $n \ge 4$, we have $\gamma_{it}(G) \le \alpha(G)$. □

Corollary 3.13. *Let G be any graph, then $\gamma_e(G) \le \gamma_{it}(G)$.*

Proof. We know $\gamma_e(G) \le \gamma(G)$ from the Theorem 2.19 and $\gamma(G) \le \gamma_{it}(G)$ from the Theorem 2.14. Hence, $\gamma_e(G) \le \gamma_{it}(G)$. □

4. Conclusion

In this paper, we have investigated the independent transversal domination number for the transformation graphs $P^{+-+}_n, C^{+-+}_n, S^{+-+}_{1,n}, W^{+-+}_{1,n}, K^{+-+}_n$. Calculation of the independent transversal domination number for simple graph types is important because if one can break a more complex network into smaller networks, then under some conditions the solution for the optimization problem on the smaller networks can be combined to a solution for the optimization problem on the larger network.

Conflicts of Interest

The author declares that there are no conflicts of interest regarding the publication of this article.

Authors Contribution Statement

The author has read and agreed to the published version of the manuscript.

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