



Least Square Support Vector Machines Laguerre Hammerstein Model Identification and Non-linear Model Predictive Controller Design for pH Neutralization Process

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Highlights

- This paper outlines an alternative Hammerstein model Identification of a pH neutralization process.
- Model identification is done with Laguerre Least Square Support Vector Machines (LLSSVM).
- LLSSVM Hammerstein model is used to implement an efficient Nonlinear Model Predictive Controller.

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Abstract

The ability to describe the nonlinear process dynamics is an essential feature of the Hammerstein model that paved more research and application studies in system identification and control. Using the Hammerstein model, this study shows an alternative approach to identify and control the highly nonlinear pH neutralization process. This Hammerstein model called Laguerre Least Square Support Vector Machines (LLSSVM) models the static nonlinearity with LSSVM and the linear part with Laguerre filter. The identified LLSSVM Hammerstein model performance evaluation with Mean Squared Error (MSE) and Variance Accounted For (VAF) is better than the Linear Laguerre model. We apply the identified LLSSVM Hammerstein model to implement a Nonlinear Model Predictive Controller (NMPC) to control the pH neutralization process. Then evaluated NMPC performance in terms of Integral Squared Error (ISE), Integral Absolute Error (IAE), and Total Variation (TV) and Control Effort (CE) parameters to verify its effectiveness in set-point tracking and disturbance rejection problems. The comparison of the NMPC with the Linear Laguerre Model-based Predictive Controller (LMPC) shows better performance of the NMPC than the LMPC. Results show that the LLSSVM Hammerstein model replicates the pH neutralization process well than the Linear Laguerre model. Also, the identified LLSSVM Hammerstein model provides an efficient NMPC than the LMPC for the pH neutralization process.

1. INTRODUCTION

Major research areas in chemical, pharmaceutical, and other process industries include model identification and model-based control of nonlinear systems like pH neutralization, CSTR, and the distillation column [1]. Linear models can only approximate a process at one operating point. In contrast, nonlinear models can provide process dynamics in a better way [2]. A pH neutralization process has inherent non-linearity and is sensitive to disturbances as fluid process variation from alkaline to acidic and vice versa. This pH parameter is crucial in the food industry, chemical industry, and wastewater treatment. It showed the usefulness of a mathematical model in designing a predictive controller for nonlinear systems [3].

Several researchers have tried to get an identified model of nonlinear processes for the controller implementation [4-6]. Many processes are nonlinear in the chemical, food, and pharmaceutical industries and have multiple operating regions over their operating range. For nonlinear processes, linear MPC (LMPC) does not give the required control performance. Also, the computational burden is more in using the LMPC for nonlinear process control. These are a few reasons many researchers started focusing on

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NMPC design in many nonlinear dynamic process applications [7-8]. Block-oriented model identification with the Hammerstein model structure is a popular parameter estimation approach [9]. Its simple structure has fixed Eigen-values, flexible nature, and requires fewer tuning parameters [10-14]. The Hammerstein model structure can approximate nonlinear dynamics with high accuracy and replicate the physical system [15-19]. These are the few findings that motivated us to use the Hammerstein model to identify the pH neutralization model in this study. Hammerstein's model structure was implemented with the Recursive least squares (RLS) algorithm using an infinite impulse response (IIR) filter, least squares regression models, Auto-Regressive with eXogenous model input (ARX), and Auto-Regressive Moving Average with eXogenous model input (ARMAX) like polynomial models [20-22]. These identification methods give highly complex models, as they require many parameters.

The researchers perform model identification using machine learning with Support Vector Machine (SVM) and Least Square-Support Vector Machine (LSSVM). The SVM solves the quadratic programming methods but is time-consuming and complex. In contrast, the LSSVM resolves the quadratic programming problem, minimizing tracking errors with lesser complexity, time and also requires fewer tuning parameters [23-24].

To overcome the above problems, we apply an alternative approach with LLSSVM (Least Square Support Vector Machine- Laguerre) Hammerstein model [25] for modeling the pH neutralization system in this work. We model the static nonlinearity with LSSVM with the RBF (Radial Basis Function) Kernel [26]. In contrast, the Laguerre filter represents the linear part. This research study includes a nonlinear benchmark pH process [27] as the actual process model to check the fitness of the identified model. We evaluate the identified model in terms of parameters MSE (Mean Squared Error) and VAF (Variance Accounted For), which outperform the conventional Linear Laguerre model. Lesser is the MSE value; better is the quality of the estimated model. For validating the identified model, the VAF value must be higher. This VAF value shows the variance between identified and actual model outputs. The identified LLSSVM Hammerstein model is used to implement NMPC for controlling the pH neutralization process. The result analysis shows that in the set-point tracking and disturbance rejection, the designed NMPC performs better in ISE, IAE, TV, and CE parameters than in the LMPC.

2. LLSSVM HAMMERSTEIN MODEL IDENTIFICATION ALGORITHM

Identification of LLSSVM Hammerstein Models using LSSVM and Laguerre Filter structure is shown in Figure 1. This model identification uses the measurable input-output signals only. The LLSSVM Hammerstein model for the pH Neutralization process under study, shown in Figure 1, comprises the RBF Kernel based on LSSVM and Laguerre (L) Filter. LSSVM models the static nonlinearity for the pH neutralization process, whereas the Laguerre filter represents the linear part.



Figure 1. The LLSSVM Hammerstein Model structure for the pH Neutralization system

We use the input vector $x_i \in R_m$ and output vector $y_i \in R$ from the actual benchmark pH neutralization model as training and validation data for identifying the LLSSVM Hammerstein model. In SVM terminology, a nonlinear function $\Phi(x_i)$ maps input space (R_m) from lower to higher dimensional feature space. This mapping helps to identify the static nonlinearity of the Hammerstein model, as shown in Equation (1)

$$y_i(x_i) = w^T \Phi(x_i) + b \quad (1)$$

The LS-SVM parameters to be estimated are weighing factor- w and bias- b and are used to minimize training errors. The LS-SVM standard regression algorithm for identifying the optimized Hammerstein model structure uses Equation (2)

$$\text{Min}_{w, b_{bias}, e} J_p(w, b_{bias}, e) = \frac{1}{2} w^T w + \frac{\gamma}{2} \sum_{k=1}^N \{e(k)\}^2. \quad (2)$$

Subject to satisfying the equality conditions given below. Here, γ is a hyper-parameter used to tune the amount of regularization against the sum squared error. And e_i (deviation of the estimated output from the actual output) are the errors whose value decides the least-squares data fitting.

Equation (3) represents the LLSSVM Hammerstein model structure identification [25] approach:

$$y_i(x_i) = w^T \Phi(x_i) + b + e_i. \quad (3)$$

Equation (4) gives the optimization problem solution with the Lagrange function [26] as:

$$L(w, b, e, \alpha) = J(w, e) - \alpha_i (w^T \Phi(x_i) + b + e_i - y_i). \quad (4)$$

In Equation (4), $\alpha = \alpha_i$ and $\forall i \in [1, N]$ are the Lagrange multipliers set.

Solving the optimality conditions [28, 29] by taking partial differentiation of Equation (4) with respect to w , b , e_i , and α_i as described in Equations (5) to (8)

$$\frac{\partial L}{\partial w} = 0 \rightarrow w = \sum_{i=1}^n \alpha_i \Phi(x_i) \quad (5)$$

$$\frac{\partial L}{\partial b} = 0 \rightarrow \sum_{i=1}^n \alpha_i = 0 \quad (6)$$

$$\frac{\partial L}{\partial e_i} = 0 \rightarrow \alpha_i = \gamma e_i \quad (7)$$

$$\frac{\partial L}{\partial \alpha_i} = 0 \rightarrow w^T \Phi(x_i) + b + e_i - y_i = 0; \quad \forall i = 1, N. \quad (8)$$

Defining,

$$Y = [y_1, y_2, \dots, y_N]^T,$$

$$1 = [1, 1, \dots, 1]^T,$$

$$\alpha = [\alpha_1, \alpha_2, \dots, \alpha_N]^T.$$

The RBF kernel is:

$$\Omega_{il} = \Phi(x_i) \Phi(x_l) = K(x_i, x_l). \quad (9)$$

The RBF kernel helps to retrieve nonlinear dynamics associated with the plant [30].

Removal of the terms w and e makes the optimization or minimization problem simplified with linear equations as:

$$\begin{bmatrix} 0 & 1^T \\ 1 & \Omega + \frac{I}{\gamma} \end{bmatrix} \begin{bmatrix} b_{bias} \\ \alpha \end{bmatrix} = \begin{bmatrix} 0 \\ y \end{bmatrix}. \quad (10)$$

For invertible matrix $\begin{bmatrix} 0 & 1^T \\ 1 & \Omega + \frac{I}{\gamma} \end{bmatrix}$, ensure that $\gamma > 0$.

Here γ is the regularization constant to minimize training errors, model complexity, and data fitting. Then the solution for band α is given as:

$$\begin{bmatrix} b \\ \alpha \end{bmatrix} = \begin{bmatrix} 0 & 1^T \\ 1 & \Omega + \frac{I}{\gamma} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ Y \end{bmatrix}. \quad (11)$$

From the obtained band α , the output equation becomes:

$$y(x) = \sum_{i=1}^n \alpha_i K(x, x_i) + b. \quad (12)$$

In Equation (12), $K(x, x_i)$ is the RBF kernel function shown below

$$K(x, x_i) = \exp\left(-\frac{\|x - x_i\|^2}{2\delta^2}\right) \quad (13)$$

where δ represents RBF kernel bandwidth and the RBF kernel computes the variance $\|x - x_i\|$. The RBF kernel gives the best results than other kernel functions in terms of smoothness and also quicker calibration.

The Laguerre filters $L_i(z)$ implement the linear dynamics of the LS-SVM Hammerstein structure as represented in Equation (14) with output terms. The computational and dimensional complexity reduces with the lesser no. of Laguerre filters 'N'

$$\hat{y}(z) = \left(\sum_{i=1}^N c_i L_i(z)\right)u(z) \quad (14)$$

where

$$L_i(z) = \sqrt{(1 - \alpha^2)T} \frac{(1 - \alpha z)^{i-1}}{(z - \alpha)^i}. \quad (15)$$

The time scale parameters $p > 0$ and $p = T$ (time constant) achieve a faster convergence to satisfy controllability and observability conditions.

The Laguerre filters output representing the linear part of the system is:

$$\hat{y}(k) = c^T L(k) \quad (16)$$

where $c^T = [c_1, c_2, \dots, c_N]$ are Laguerre filter coefficients.

The approximation error or tracking error of sample k :

$$e_k = y(k) - \hat{y}(L(k)). \quad (17)$$

Equation (17) gives a cost function term used as a minimization term in the optimization problem.

3. LLSSVM HAMMERSTEIN MODEL IDENTIFICATION FOR THE pH NEUTRALIZATION PROCESS

This study includes a pH neutralization process as a benchmark system for identification and control, as shown in Figure 2. This pH neutralization process comprises three inlet streams, a q_1 -acid stream containing Nitric acid- HNO_3 , q_2 -buffer stream containing a weak base Sodium bicarbonate- NaHCO_3 , and q_3 -base stream of Sodium Hydroxide- NaOH with traces of Sodium bicarbonate- NaHCO_3 . In this process, the measured variables are tank liquid level (h) and pH. The acid stream is a disturbance variable, and the unmeasured disturbances are buffer and acid stream flows. Regulating the manipulated variable base flow helps to control the output variable pH.

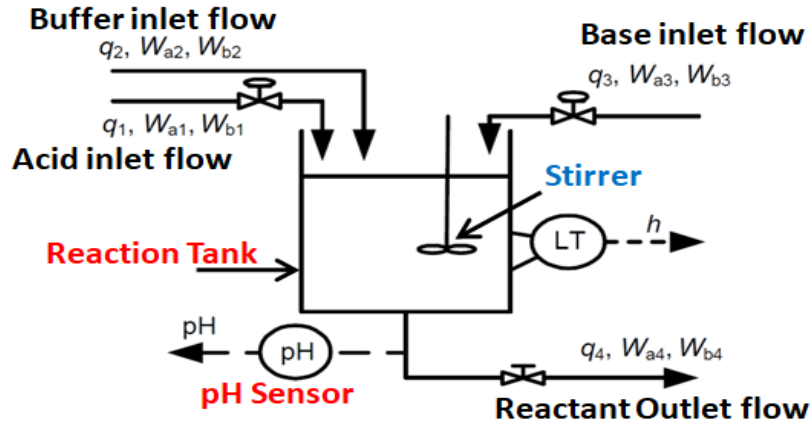


Figure 2. The pH neutralization process under study (Benson & Seaborg)

Table 1 shows the various parameters and their nominal values for the benchmark pH neutralization process.

Table 1. pH neutralization process parameters

Process Parameters	Nominal Value	Process Parameters	Nominal Value
Area of the tank, A	207 (cm ²)	Acid stream W _{b1}	0 (mol/L)
Valve coefficient, C _v	8.75 (ml/cm·s)	Buffer stream W _{b2}	3×10 ⁻² (mol/L)
pK ₁ (first dissociation constant of weak acid H ₂ CO ₃)	6.35	Base stream W _{b3}	5×10 ⁻⁵ (mol/L)
pK ₂ (second dissociation constant of weak acid H ₂ CO ₃)	10.25	Acid- HNO ₃ stream flow rate q ₁	16.6 (mL/s)
Reaction Invariant of Acid stream W _{a1}	3×10 ⁻³ (mol/L)	Buffer-NaHCO ₃ stream flow rate q ₂	0.55 (mL/s)
Reaction Invariant of Buffer stream W _{a2}	-3×10 ⁻² (mol/L)	Base-NaOH and NaHCO ₃ stream flow rate q ₃	15.55(mL/s)
Reaction Invariant of Base stream W _{a3}	-3.05×10 ⁻³ (mol/L)	Height of reactor tank h	14.0 (cm)
W _{a4} and W _{b4} are the effluent stream reaction invariants-Not measured			

The following equations mathematically model this pH neutralization process dynamics

$$\frac{dh}{dt} = \frac{1}{A}(q_1 + q_2 + q_3 - C_v h^{0.5}) \quad (18)$$

$$\frac{dW_{a4}}{dt} = \frac{1}{Ah} [(W_{a1} - W_{a4})q_1 + (W_{a2} - W_{a4})q_2 + (W_{a3} - W_{a4})q_3] \quad (19)$$

$$\frac{dW_{b4}}{dt} = \frac{1}{Ah} [(W_{b1} - W_{b4})q_1 + (W_{b2} - W_{b4})q_2 + (W_{b3} - W_{b4})q_3] \quad (20)$$

$$\frac{W_{b4}(1+2 \times 10^{pH-pK_2})}{1+10^{pK_1-pH}+10^{pH-pK_2}} + W_{a4} + 10^{pH-14} = 10^{-pH} \quad (21)$$

We use the input and output data from the actual benchmark pH neutralization process in training and validation to identify the LLSSVM Hammerstein model. This identification requires the generation of training and validation data for the actual model and model to be identified. Application of a persistent GBN (Generalized Binary Noise) signal with switching time (T_{sw}) =50 sec and SNR=20 excites the pH neutralization process under study. The base flow is the input to the model with a nominal flow rate of 15.55 mL/s (refer to Table 1) for 400 seconds duration at a sampling rate of 0.1 sec, while the pH is the output of this pH neutralization process.

The applied input excitation to the pH neutralization process provides 4000 input and output data samples for model identification (with 3000 data points) and model validation (with the remaining 1000 data points). The model mismatch evaluation in terms of MSE and VAF includes comparing the identified model output with the actual model output.

Figure 3 shows the test signal, which acts as a base flow rate (mL/sec) input to the pH neutralization process and the corresponding model output signal. We derived this output data from Equations (20), (21), (22), and (23) and used it to identify the Hammerstein model for the pH neutralization system. The applied LLSVM Hammerstein model fit performance depends on suitably selecting the tuning parameters p , N , γ , and δ .

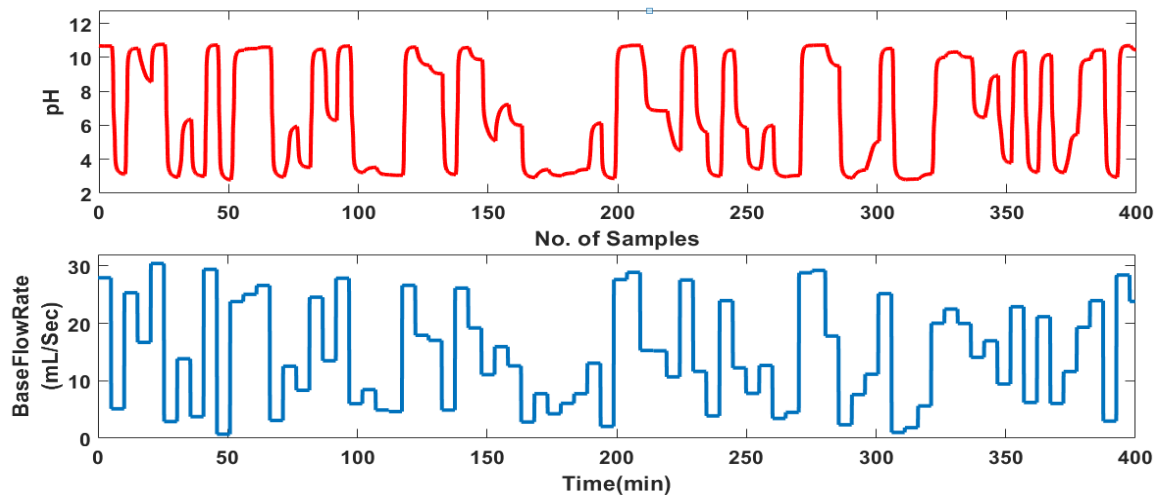


Figure 3. The pH neutralization system Input (Base flow rate)–Output (pH) data for model training

We trained and validated the LLSSVM Hammerstein model for the pH neutralization process with the well-tuned parameters. This model meets the lower value of MSE (Mean Squared Error) lesser than 0.5 and a higher value of VAF (variance accounted for) greater than 95% [28].

4. CROSS-VALIDATION OF THE LLSSVM HAMMERSTEIN MODEL

The cross-validation of the LLSSVM Hammerstein model uses different 1000 validation data points other than the 3000 data points used for training to check the model quality in predicting the process behavior and avoid over-fitting. Test results show model fitness or correctness with a parameter VAF (Variance Accounted For). The higher value of VAF (greater than 95%) shows that the estimated model output (\hat{y}) is close matches with the actual process model output (y).

The calculation of the VAF implies the following formula:

$$VAF = \max \left\{ 1, \frac{\text{variance}(y - \hat{y})}{\text{var}(y)} \right\} \times 100\%. \quad (22)$$

The best VAF values are verified by adjusting tuning parameters p , N , γ , and σ . We apply a trial and error method to tune the LLSSVM Hammerstein model for the tuned parameter combinations. We varied the tuning parameters p , N , γ , and σ till we got the best VAF and MSE performance of the LLSSVM Hammerstein model.

Initially, the Linear Laguerre model is trained and validated with the pH neutralization process input-output data. Figure 4 shows the identified model response with the Linear Laguerre model with a red-colored line. For the Linear Laguerre model, the best results of VAF=93.3827 and MSE=0.5835 are for $N=13$, with $p=1.1$. It is essential to select p and N parameters properly to meet the trade-off between MSE and VAF.

Later, we trained and validated the LLSSVM Hammerstein model. One can see the identified model response with the LLSSVM Hammerstein model in Figure 4, shown with a blue-colored line for the validation data input. This response got for $\sigma=200$, $\gamma=1$, $p=0.011$ and $N=9$ and showed $VAF=96.9270$, $MSE=0.3066$. In contrast, the LLSSVM Hammerstein model with $N=5$ responds in terms of $VAF=95.7483$ and $MSE=0.4391$.

Figure 4 shows that the identified model output with the LLSSVM Hammerstein model is comparatively better than the Linear Laguerre model with the actual plant output.

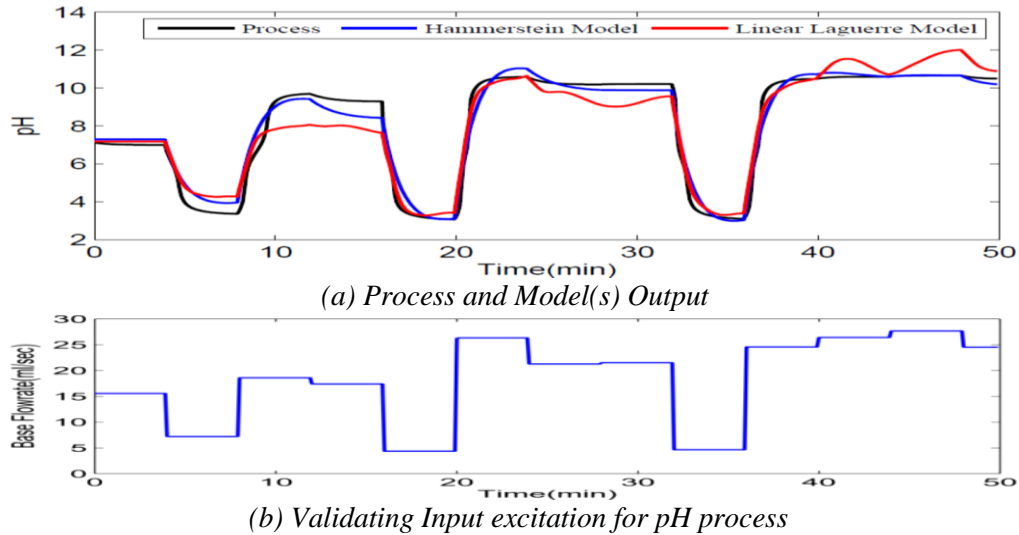


Figure 4(a, b). Comparison of performance of Hammerstein and Linear Laguerre models in system identification of pH Neutralization process

It is clear from the analysis that the LLSSVM Hammerstein model reproduces the benchmark pH neutralization system with a higher value of $VAF=96.9270$ and a smaller value of $MSE=0.3066$, showing high confidence in the estimation. Also, a lower value of MSE suggests the predictions made by the model match with the observed data or the actual process data.

5.LLSSVM HAMMERSTEIN NONLINEAR MODEL PREDICTIVE CONTROLLER OF pH NEUTRALIZATION PROCESS

A nonlinear MPC uses a plant model to predict the next control move of future control efforts for a defined control horizon (H_c) by minimizing a performance index over a finite moving horizon defined by the prediction horizon (H_p). The comparison of the LSSVM-Laguerre(LLSSVM) Hammerstein Model predicted output with the actual output of the process provides the solution for the optimization problem. Evaluating the corrected model prediction over the prediction horizon (H_p) helps to verify plant-model mismatch and unmeasured disturbances.

Equation (23) includes the terms for minimizing objective function or performance index $J(k)$ for the NMPC output. It comprises set-point variable $y_{sp}(k)$, predicted output $\hat{y}(k)$ from future values and current values of output $y(k)$, manipulated variable $u(k)$ with present and past control values along with positive output diagonal matrix Q and R as shown in Equation (23). The controller fine-tuning and normalization of variables depend on Q and R matrices

$$J(k) = \sum_{j=1}^{H_p} \{ [y_{sp}(k+j) - \hat{y}(k+j|k)]^T Q [y_{sp}(k+j) - \hat{y}(k+j|k)] \} + \sum_{j=1}^{H_c} u(k+j)^T R u(k+j). \quad (23)$$

We use the identified LLSSVM Hammerstein model to implement an NMPC for the benchmark pH neutralization process. In this process, as discussed earlier, the base flow rate is the manipulated variable, the pH is the controlled variable, and the acid flow rate is a disturbance variable.

In set-point tracking operation, NMPC is used to track the changes in the set-point variable pH by manipulating the base flow rate. Here, we use the acid flow rate as a disturbance variable in the disturbance rejection to check whether NMPC can maintain pH at the constant or desired set-point value in the presence of load or disturbances.

The following sections give details of the comparative performance analysis of the NMPC with LMPC for the set-point tracking and disturbance rejection problems. The NMPC and LMPC performance analysis checks for the different values of the prediction horizon. And we evaluate their performance in terms of parameters ISE (Integral of Squared Error), IAE (Absolute of Squared Error), TV (Total Variation), and CE (Control Effort). The total variation (TV) indicates the smoothness of manipulated variable during control action [29] and provides a trade-off between input and output performance. The calculated value of the Total Variation should be as slight as possible and as per the formula given in Equation (24)

$$TV \cong \sum_{k=1}^{\infty} |u(k+1) - u(k)|. \quad (24)$$

In Equation (24), $u(k)$ and $u(k+1)$ are the controller output present and past sample values. The control effort (CE) in Equation (25) considers the power of the control signal $u(k)$ and relates to energy consumption to maintain the output variable at the set-point value. This CE value must be as small as possible

$$CE = \int_{k=1}^n |u(k)|. \quad (25)$$

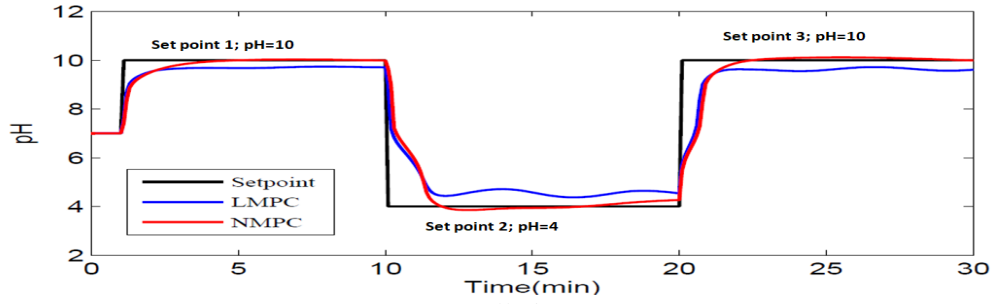
The other essential performance indices, ISE (Integral Squared Error), integrate the squared errors over time and penalize significant errors. IAE (Integral Absolute Error) integrates the absolute error over time and penalizes all errors equally, regardless of error direction. ISE and IAE, shown in Equations (26) and (27), are the cost functions to penalize system errors and achieve good under-damped system responses

$$ISE = \int_0^{\infty} e^2(k) \quad (26)$$

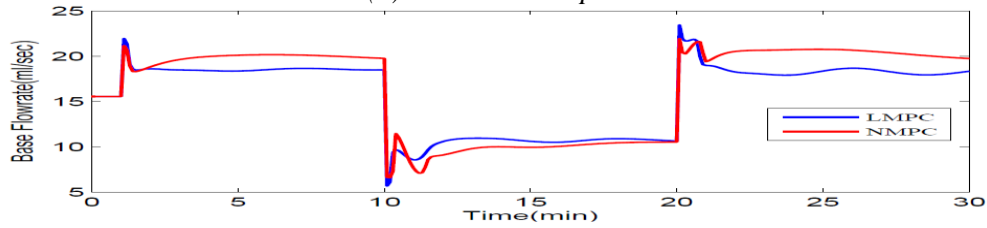
$$IAE = \int_0^{\infty} |e(k)|. \quad (27)$$

Initially, for the set-point tracking operation (pH set-point changes pH=7 to 10, from 10 to 4 and from 4 to 10), the LMPC (based on the Linear Laguerre model) tuned at $N=13$, $p=1.1$, $Q=10$, $R=1000$, giving the better values of $VAF=93.3827$ and $MSE=0.5835$ with Control horizon $H_c=2$ and sampling time $T_s=0.1$ sec and applied to maintain the pH of the pH neutralization system with different values of the prediction horizon $H_p=2, 4$ and 6 .

If we take a minimal value of H_p , it may lead to an unstable controller response and more errors. In contrast, higher values of H_p give minor errors at the cost of more computational complexity for the nonlinear plants. We selected prediction horizon H_p value on the trial and error basis and with $H_p=6$ got the smaller ISE and IAE, as shown in Table 2. The performance parameters ISE, IAE, TV, and CE for these settings are as shown in Table 2. Figures 5 to 7 show the LMPC performance for the servo problem at different values of the prediction horizon $H_p=2, 4$, and 6 .

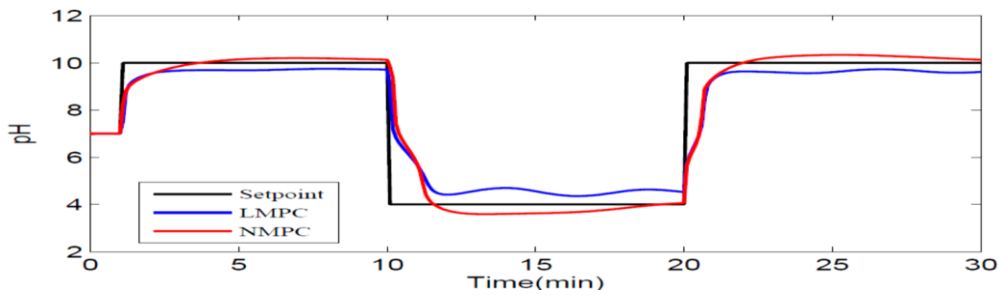


(a) Controlled output

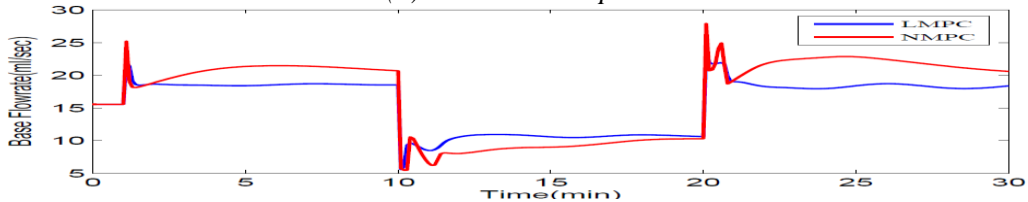


(b) Manipulated input

Figure 5(a, b). LMPC (with Laguerre model) and NMPC (with LLSSVM Hammerstein model) LMPC with $H_p=2$ for Set-Point Tracking in the pH neutralization

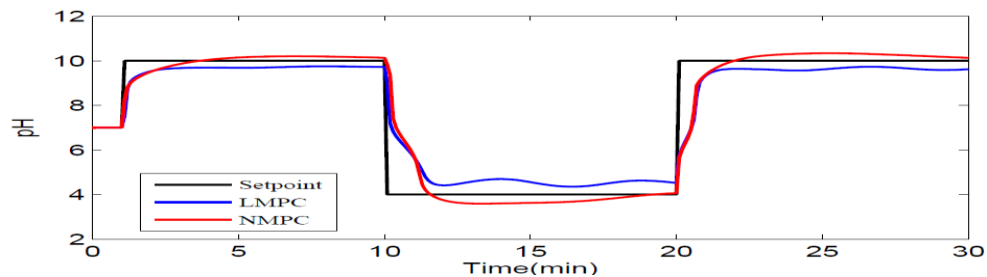


(a) Controlled output



(b) Manipulated input

Figure 6(a, b). LMPC and NMPC output with $H_p=4$ for Set-Point Tracking in the pH neutralization.



(a) Controlled output

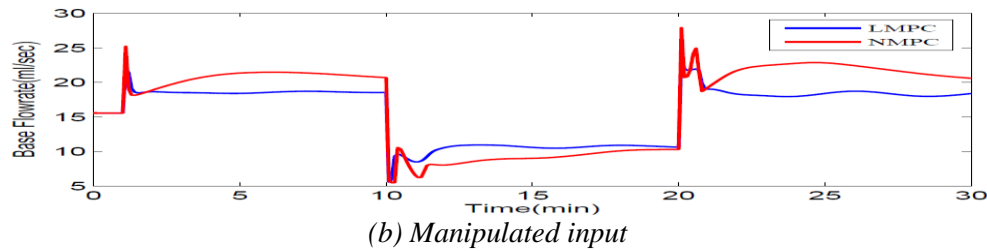


Figure 7(a, b). LMPC and NMPC output with $H_p=6$ for Set-Point Tracking in the pH neutralization

Figures 5 to 7 show that the output variable pH of the pH neutralization process is not exactly reaching the desired set-point values in the set-point tracking problem using LMPC design based on the Linear Laguerre model.

Then, we applied the NMPC based on the LLSSVM Hammerstein model with the well-tuned values per required constraints ($N=5$, $\sigma=200$, $\gamma=1$, $p=0.009$). This NMPC gives good values of $VAF=96.8948$ and $MSE=0.3082$) for the smooth and precise set-point tracking. It is clear from Figures 5 to 7 that the NMPC outperforms the LMPC in the set-point tracking/servo operation by reaching the set point steadily, unlike the LMPC.

Table 2 shows the NMPC performance with $Q=10$, $R=1000$, control horizon $H_c=2$, and sampling time $T_s=0.1$ sec with different values of the prediction horizon $H_p=2, 4$, and 6.

Table 2. LMPC and NMPC performance Evaluation for Set pointing tracking problem

Prediction Horizon	ISE		IAE		TV		CE	
H_p	LMPC	NMPC	LMPC	NMPC	LMPC	NMPC	LMPC	NMPC
2	1.2124	0.1913	0.7679	0.3288	60.2120	9.8193	5062.5	4758.3
4	1.2041	0.1868	0.7577	0.3222	61.0206	9.8959	5062.5	4760.3
6	1.2027	0.1860	0.7558	0.3209	61.1739	9.9096	5076.2	4760.7

The analysis from Tables 2 and Figures 5 to 7 shows that the NMPC does much better set-point tracking than the LMPC as ISE and IAE are smaller with the NMPC than the LMPC. Also, a smaller value of TV and CE indicates smoothness in the NMPC control action for the applied input changes and lower energy consumption to maintain the output at the set-point value, respectively.

It is clear from Table 2 that as the prediction horizon increases, the ISE and IAE values decrease. The NMPC based on the LLSSVM Hammerstein model tracks the set-point trajectory and keeps the output variable pH at the desired set-point. Thus, in the set-point tracking control problem, the NMPC performs more effectively than the LMPC.

Maintaining the output pH in a chemical process like a pH neutralization system by rejecting the disturbances is a fundamental aspect of process control. In a pH neutralization system, it is vital to keep the reactant pH at a value of 7. The efficient controller design is crucial in the disturbance rejection process by regulating the output variable at the set-point trajectory.

For the disturbance rejection control problem, we suddenly varied the acid flow rate as a disturbance from its normal flow rate of 16.6mL/sec at different intervals ($t_1=2$ min, $t_2=10$ min, and $t_3=20$ min) to the pH neutralization process. And then, we analyzed the LMPC performance in minimizing the effect of disturbance to maintain the output pH at the unchanged set-point value. The set parameters of the LMPC are $Q=10$, $R=1000$, Control horizon $H_c=2$, and sampling time $T_s=0.1$ sec with different values of the prediction horizon $H_p=2, 4$, and 6. Figures 8 to 10 show the LMPC and NMPC performance for the disturbance rejection problem at different values of the prediction horizon $H_p=2, 4$, and 6. The LMPC can't force the output variable pH to reach the set-point in the presence of disturbances.

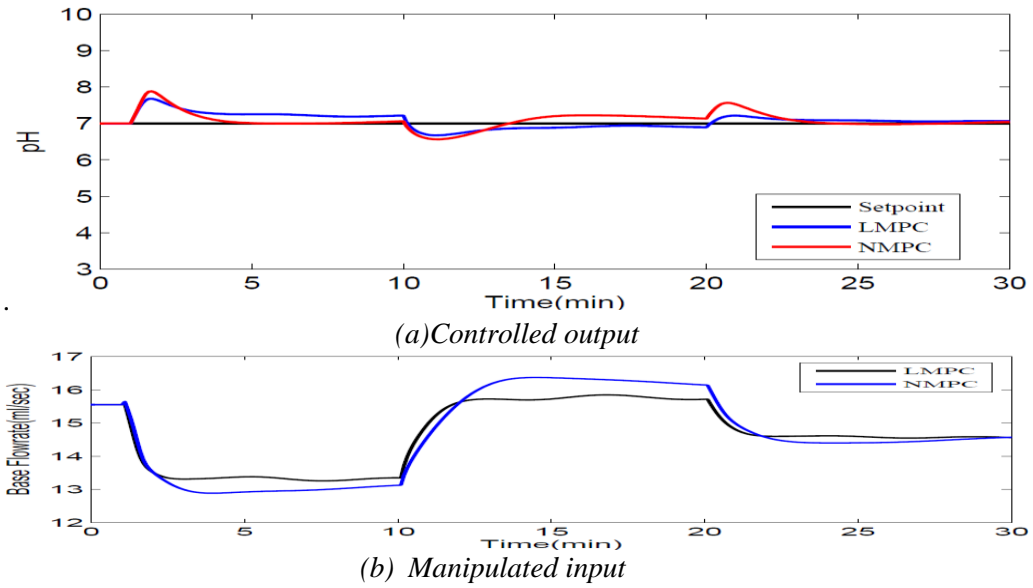


Figure 8(a, b). LMPC and NMPC output with $H_p = 2$ for Disturbance Rejection in the pH neutralization

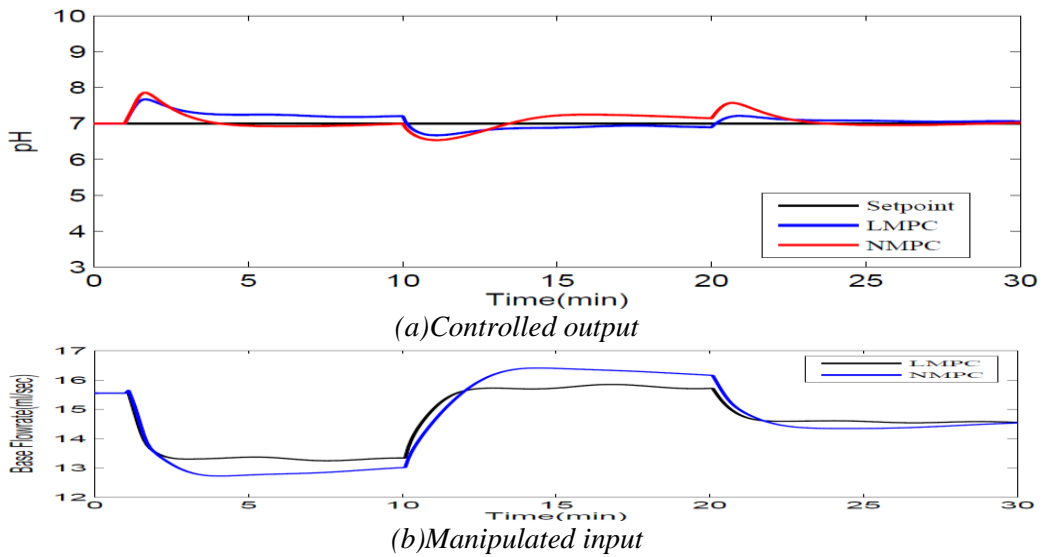


Figure 9(a, b). LMPC and NMPC output with $H_p = 4$ for Disturbance Rejection in the pH neutralization

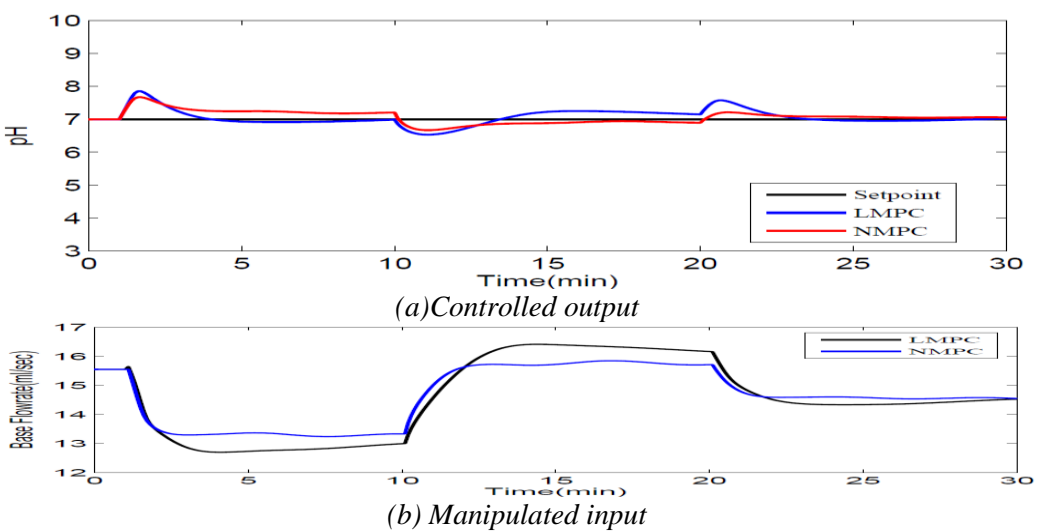


Figure 10(a, b). LMPC and NMPC output with $H_p = 6$ for Disturbance Rejection in the pH neutralization

Then, we applied NMPC using the LLSSVM Hammerstein model for the disturbance rejection control problems in the pH neutralization process. We tuned the NMPC for $Q=10$, $R=1000$, Control horizon $H_c=2$, and sampling time $T_s=0.1$ sec with different values of the prediction horizon $H_p=2, 4$, and 6. Figures 8 to 10 show the NMPC performance for the disturbance rejection problem to varying values of the prediction horizon $H_p=2, 4$, and 6. The acid flow rate fluctuation is the disturbance to the process applied at the different time intervals ($t_1=2$ min, $t_2=10$ min, and $t_3=20$ min). It is clear from Figures 8 to 10 that the NMPC rejects the effect of disturbances and regulates the pH at the set-point better than the LMPC.

Table 3 shows the improved ISE, IAE, TV, and CE performance parameters of the NMPC with the LLSSVM Hammerstein model than the LMPC with a Linear Laguerre model.

Table 3. LMPC and NMPC performance Evaluation for Disturbance Rejection problem

Prediction Horizon	ISE		IAE		TV		CE	
H_p	LMPC	NMPC	LMPC	NMPC	LMPC	NMPC	LMPC	NMPC
2	0.6402	0.1967	0.3114	0.1882	46.5141	13.4178	5085.8	4791.0
4	0.6257	0.2061	0.3928	0.2369	78.6250	14.8911	5269.3	4782.0
6	0.6400	0.2083	0.4259	0.2436	83.7660	15.1006	5361.1	4780.3

6. THE RESEARCH FINDINGS AND DISCUSSION

This research proposes an LSSVM-Laguerre (LLSSVM) Hammerstein model structure, as shown in Figure 1, to overcome the limitations of the linear Laguerre model structure used. As shown in Figure 2, this simulation studies use the pH neutralization process to check model efficacy. Table 1 specifies the pH process parameters used to model this process. The LLSSVM Hammerstein model structure implementation includes LSSVM and RBF Kernel to represent the static nonlinear part of the pH neutralization process and the linear part with Laguerre filters. This combination is novel, as earlier combinations tested for other processes but not for a pH neutralization system. The input-output data obtained from the pH neutralization process applies in training the LLSSVM Hammerstein model, as shown in Figure 3.

Training and validating the LLSSVM Hammerstein model with different tuning parameter combinations improve VAF and MSE performance. Figure 4 shows the Linear Laguerre model with $N=13$ with $p=1.1$ gives the best results of $VAF=93.3827$ and $MSE=0.5835$. Therefore, to achieve better model performance with a good model fit in terms of VAF and MSE, the LLSSVM Hammerstein model is trained and validated with different tuning parameter combinations.

As shown in Figure 4, the LLSSVM Hammerstein identifies the pH neutralization process output with more accuracy (in terms of VAF and MSE) than the Linear Laguerre model. The VAF and MSE for the LLSSVM Hammerstein model are 95.7968 and 0.4308 with $\sigma=200$, $\gamma=1$, $p=0.011$, $N=9$, whereas 95.7483 and 0.4391 for $\sigma=200$, $\gamma=1$, $p=0.011$, $N=5$. We choose $N=5$ while tuning the LLSSVM Hammerstein model with lower complexity.

Thus, the LLSSVM Hammerstein model performs better than the linear Laguerre model for a nonlinear pH neutralization process under study. We further used the identified LLSSVM Hammerstein model structure as an NMPC for model-based controller implementation of the nonlinear pH neutralization process under investigation.

Figures 5 to 7 show that for the set-point tracking problem, the NMPC provides better set-point tracking than the LMPC. It is clear from Figures 8 to 10 that the NMPC implemented with the LLSSVM Hammerstein model rejects the effects of acid flow fluctuations more effectively than the LMPC implemented from the linear Laguerre Model in the disturbance rejection problem. Tables 2 and 3 show that the NMPC based on the LLSSVM Hammerstein Model gives better ISE, IAE, and TV over the LMPC based on the Linear Laguerre model. Also, the CE parameter fluctuation is a lesser one. Thus, the LLSSVM

Hammerstein model performs better than the Laguerre model in identification and model-predictive control for the pH neutralization benchmark system under study.

7. RESULTS

This research proposes a newer kind of nonlinear Hammerstein type modeling technique to identify and model predictive control of a highly nonlinear system, such as the pH neutralization process. In the developed Hammerstein model, LSSVM implements a static nonlinear mapping tool, and the Laguerre filter in its state space form describes the linear dynamic part. In the system identification procedure, the LLSSVM model parameters tuning is on a trial and error basis for the best performance of the identified model.

From the case study, the identified Hammerstein model has higher modeling accuracy than its linear counterpart, and the resulting model predictive controller proved efficient in controlling the pH neutralization process. This LLSSVM Hammerstein model approach can be crucial in identifying and controlling MIMO nonlinear processes like Distillation columns and multi-tank liquid level systems.

CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

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