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# Sensitivity of Hurwitz Stability of Linear Differential Equation Systems with Periodic Coefficients

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**Abstract**

By using Hurwitz stability of a linear differential equation system (in short, LDES) with constant coefficients, and using Schur stability of a linear difference equation system (in short, LDIES) with constant coefficients, we have obtained two new continuity theorems for sensitivity of Hurwitz stability of a LDES with periodic coefficients. Our approach to the theorems is based on Floquet theory. Also, we have determined stability regions and supported the obtained results by a numerical example.

***Keywords:*** *differential equations, Floquet theory, Hurwitz stability, sensitivity, Schur stability*
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## 1. Introduction

The linear differential equation system with periodic coefficients

$$x'(t) = A(t)x(t), \;\; A(t+T) = A(t), \;\; T > 0, \;\; t \geq 0$$

is used in many application area of mechanical, physics, medicine and engineering sciences (for example [14, 13, 12]). In this paper, by using the Hurwitz and Schur stability concepts, we obtained new continuity theorems which show the sensitivity of Hurwitz stability of the above mentioned system. Also, we determined stability regions and supported the obtained results by a numerical example.
The content of this paper is organized as follows. In section 2, we recall some preliminary facts on the stability of linear differential and linear difference equation systems. Section 3 is devoted to prove the main results, and to give a numerical example. In Section 4, conclusions are given.

## 2. Preliminaries

In this section, we give some basic concepts and theorems about the linear differential and the linear difference equation systems, respectively.

### 2.1. The linear differential equation system with constant coefficients

We consider a system of linear differential equations

$$x'(t) = Ax(t), \;\; t \geq 0, \tag{2.1}$$

where $A$ is an $N \times N$ matrix whose elements are constant and, $x(t) = (x_1(t), x_2(t), ..., x_N(t))^{\top}$, $x_1(t), x_2(t), ..., x_N(t)$ are differentiable functions. Hurwitz stability of the system (2.1) (or equivalently, Hurwitz stability of the matrix $A$) is described as the following in the literature:

- According to the spectral criterion, it is necessary for Hurwitz stability of the system (2.1) that the real parts of all eigenvalues of the matrix $A$ are negative ($Re\lambda_i(A) < 0, \;\; i = 1, 2, ..., N$),
- According to Lyapunov's second theorem, the system (2.1) is Hurwitz stable if and only if there exists a solution $H = H^* > 0$ of the Lyapunov matrix equation $A^*H + HA = -I$,

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[1, 4, 7, 10]. Since the eigenvalue problem is an ill-posed problem for non-symmetric matrices [4, 17], we will study with the stability parameters which reveal the quality of Hurwitz and Schur stability without eigenvalue calculation.
The parameter $\kappa(A)$ which reveals the quality of the Hurwitz stability of the system (2.1) is defined as

$$\kappa(A) = 2\|A\|\|H\|; \ H = \int_0^\infty e^{tA^*} e^{tA} dt, \ A^*H + HA = -I, \ H = H^* > 0,$$

where $\|A\| = \max_{\|x\|=1} \|Ax\|$ is the spectral norm of the matrix $A$, $A^*$ is the adjoint of the matrix $A$, and $I$ is the identity matrix. If there is a positive definite matrix $H = H^* > 0$ providing the above Lyapunov matrix equation, then $\kappa(A) < \infty$. Otherwise, $\kappa(A) = \infty$ is chosen [4, 5, 9].
Let $B$ is an $N \times N$ matrix whose elements are constant. We consider the following system

$$y'(t) = (A+B)y(t), \ t \geq 0. \tag{2.2}$$

While the system (2.1) is Hurwitz stable, Hurwitz stability of the perturbed system (2.2) is preserved under which conditions. In literature, this problem is known as the sensitivity problem, and theorems which search for an answer to this problem are known as continuity theorems.

The following theorem is a continuity theorem which shows the sensitivity of Hurwitz stability of the system (2.1).

**Theorem 2.1.** *Let $A$ be a Hurwitz stable matrix (Let the system (2.1) be a Hurwitz stable). If $\|B\| < \frac{\|A\|}{\kappa(A)}$, then the matrix $A+B$ is Hurwitz stable (the system (2.2) is Hurwitz stable) and the inequality*

$$\kappa(A+B) \leq \frac{\kappa(A)(\|A\| + \|B\|)}{\|A\| - \|B\|\kappa(A)}$$

*holds [9, Theorem 2].*

### 2.2. The linear difference equation system with constant coefficients

We consider a system of linear difference equations

$$x(n+1) = Ax(n), \ n \in \mathbb{Z}, \tag{2.3}$$

where $A$ is an $N \times N$ matrix whose elements are constant, and $x(n)$ is an $N$ dimensional column vector. Schur stability of the system (2.3) (or equivalently, Schur stability of the matrix $A$) is described as the following in the literature:

- According to the spectral criterion, it is necessary for the Schur stability of the system (2.3) that all eigenvalues of the matrix A lie in the unit disc $(|\lambda_i(A)| < 1, \ i = 1, 2, ..., N)$,
- By using Lyapunov's second theorem, the system (2.3) is Schur stable if and only if there exists a solution $X = X^* > 0$ of the discrete-Lyapunov matrix equation $A^*XA - X + I = 0$,

[1, 4, 6, 8, 10]. The parameter $\omega(A)$ which reveals the quality of Schur stability of the system (2.3) is defined as

$$\omega(A) = \|X\|; \ X = \sum_{k=0}^{\infty} (A^*)^k A^k, \ A^*XA - X + I = 0, \ X = X^* > 0.$$

If there is a positive definite matrix $X = X^* > 0$ providing the discrete-Lyapunov matrix equation, then $\omega(A) < \infty$. Otherwise, $\omega(A) = \infty$ is chosen [1, 4, 5, 8, 11].
Let $B$ is an $N \times N$ matrix whose elements are constant. We consider the following system which called as the perturbed system of the system (2.3)

$$y(n+1) = (A+B)y(n), \ n \in \mathbb{Z}. \tag{2.4}$$

The following theorem is a continuity theorem which shows the sensitivity of Schur stability of the system (2.3).

**Theorem 2.2.** *Let $A$ be a Schur stable matrix (Let the system (2.3) be a Schur stable). If $\|B\| < \sqrt{\|A\|^2 + \frac{1}{\omega(A)}} - \|A\|$, then the matrix $A+B$ is Schur stable (the system (2.4) is Schur stable) and the inequality*

$$\omega(A+B) \leq \frac{\omega(A)}{1 - (2\|A\| + \|B\|)\|B\|\omega(A)}$$

*holds [8, Corollary 1].*

## 3. Sensitivity of Hurwitz Stability of the LDES with Periodic Coefficients

In this section, by using Floquet-Lyapunov theorem, we will obtain new continuity theorems for sensitivity of Hurwitz stability of a LDES with periodic coefficients. To investigate the Hurwitz stability of a LDES with periodic coefficients via Schur stability of a LDIES with constant coefficients, is an interesting result.
Consider a system of linear differential equations

$$x'(t) = A(t)x(t), \;\; A(t+T) = A(t), \;\; T > 0, \;\; t \geq 0, \tag{3.1}$$

where $A(t)$ is a continuous, $T$-periodic, (real or complex) $N \times N$ coefficient matrix, and $x(t) = (x_1(t), x_2(t), ..., x_N(t))^\top$, $x_1(t), x_2(t), ..., x_N(t)$ are differentiable functions.
By Floquet-Lyapunov theorem, the fundamental matrix $X(t)$ of the system (3.1) with $X(0) = I$ has the representation

$$X(t) = F(t)e^{Mt} \tag{3.2}$$

where $M$ is a constant $N \times N$ matrix satisfies the equation $X(T) = e^{MT}$, and $F(t)$ is a continuously differentiable, $T$-periodic ($F(t+T) = F(t)$ for all $t$), $N \times N$ matrix function. Clearly, $F(0) = I$ and $\det F(t) \neq 0$ for all $t$ [16, p.196]. Conversely, if $F(t)$ and $M$ are matrices with the above properties, then the matrix $X(t)$ is the fundamental matrix of the system (3.1) [2, p.42].
Hurwitz stability of the system (3.1) depends on Hurwitz stability of the constant matrix $M$ (see [2, p.42]), in other words, Hurwitz stability of the system (3.1) depends on Hurwitz stability of the system

$$u'(t) = Mu(t), \;\; t \geq 0. \tag{3.3}$$

Moreover, Hurwitz stability of the system (3.1) depends on the Schur stability of the monodromy matrix $X(T) = e^{MT}$ (see [2, p.9] and [18, p.84]), in other words, Hurwitz stability of the system (3.1) depends on the Schur stability of the system

$$y(n+1) = e^{MT}y(n), \;\; n \in \mathbb{Z}. \tag{3.4}$$

Consider the perturbed system of the system (3.1)

$$w'(t) = [A(t) + C(t)]w(t), \;\; C(t+T) = C(t), \;\; t \geq 0, \tag{3.5}$$

where $C(t)$ is a continuous $N \times N$ matrix function of $t$.
In this section, unless stated otherwise, the matrices $F(t)$ and $X(T)$ will be considered as matrices satisfying the properties in the Floquet-Lyapunov theorem given above.
We will give the following lemma and remark that are necessary for the proof of Theorem 3.3 and Theorem 3.5. We denote that it is omitted because the proof of lemma is easy.

**Lemma 3.1.** *Let $P$ be a square matrix. Then the matrix $C(t)$ defined as $C(t) := F(t)PF^{-1}(t)$ has the period $T$. In particular, if $P$ is a diagonal matrix whose all terms are equal, then $C(t) = P$.*

**Remark 3.2.** *Let $A$ be an $N \times N$ arbitrary matrix. Then, $\|A\| = \sigma_{\max}(A)$ and $\|A^{-1}\| = \frac{1}{\sigma_{\min}(A)}$, where $\sigma_{\max}(A)$ and $\sigma_{\min}(A)$ are the largest and smallest singular values of the matrix $A$, respectively* [15, p.312].

Now, we give two new continuity theorems. In the following first theorem, the sensitivity of Hurwitz stability of the system (3.1) is examined with the sensitivity of Hurwitz stability of the system (3.3).

**Theorem 3.3.** *Let the system (3.1) be Hurwitz stable and the matrix $\delta M$ be a perturbation matrix satisfying $\|\delta M\| < \frac{\|M\|}{\kappa(M)}$. Then, for the perturbation matrix $C(t)$ with $C(t) := F(t)\delta MF^{-1}(t)$, the perturbed system (3.5) is Hurwitz stable and, furthermore*

$$\|C(t)\| < \frac{\sigma_{\max}(F(t))}{\sigma_{\min}(F(t))} \frac{\|M\|}{\kappa(M)}, \;\; t \geq 0$$

*where $\sigma_{\max}(F(t))$ and $\sigma_{\min}(F(t))$ are the largest and smallest singular values of the matrix $F(t)$, respectively.*

*Proof.* Suppose $C(t)$ is a matrix satisfying the hypotheses of Theorem 3.3. We will show that the perturbed system (3.5) is Hurwitz stable for the matrix $C(t)$, and the mentioned inequality holds.
Firstly, we will determine the perturbed system which has the fundamental matrix $Y(t) := F(t)e^{(M+\delta M)t}$. Suppose that $Y(t)$ is a fundamental matrix of the system

$$y'(t) = B(t)y(t). \tag{3.6}$$

Since $Y(t)$ is a matrix solution of this system, the equality $Y'(t) = B(t)Y(t)$ holds. Thus, we get

$$\begin{aligned} F'(t)e^{(M+\delta M)t} + F(t)(M+\delta M)e^{(M+\delta M)t} &= B(t)F(t)e^{(M+\delta M)t} \\ F'(t)F^{-1}(t) + F(t)(M+\delta M)F^{-1}(t) &= B(t). \end{aligned} \tag{3.7}$$

On the other hand, using the fundamental matrix $X(t) = F(t)e^{Mt}$ of the system (3.1), we obtain the following equalities

$$\begin{aligned} F'(t)e^{Mt} + F(t)Me^{Mt} &= A(t)F(t)e^{Mt} \\ F'(t)F^{-1}(t) + F(t)MF^{-1}(t) &= A(t). \end{aligned} \tag{3.8}$$

If the equality (3.8) is used in the equality (3.7), then

$$A(t)+F(t)\delta MF^{-1}(t)=B(t)$$

is found. This indicates that $Y(t)$ is a fundamental matrix for the perturbed system (3.5) with $C(t):=F(t)\delta MF^{-1}(t)$. We also denote that $C(t+T)=C(t)$ from Lemma 3.1.
Now, under the hypotheses we will show that the system (3.5) is Hurwitz stable. Since the system (3.1) is Hurwitz stable from hypotheses, the matrix $M$ is Hurwitz stable, that is, the system (3.3) is Hurwitz stable. By Theorem 2.1, the matrix $M+\delta M$ ($\|\delta M\|<\frac{\|M\|}{\kappa(M)}$) is Hurwitz stable. So, the perturbed system (3.5) is Hurwitz stable for the matrix $C(t)$ mentioned.

Moreover, from the hypotheses and Remark 3.2, we have for every $t\geq 0$,

$$\begin{aligned}\|C(t)\| &= \|F(t)\delta MF^{-1}(t)\|\\ &\leq \|F(t)\|\|\delta M\|\|F^{-1}(t)\|\\ &< \frac{\sigma_{\max}(F(t))}{\sigma_{\min}(F(t))}\frac{\|M\|}{\kappa(M)}.\end{aligned}$$

This completes the proof. □

**Remark 3.4.** *Note that the matrix $Y(t)$ mentioned in the proof of Theorem 3.3 satisfies the conditions of Floquet-Lyapunov theorem. In fact, according to Floquet-Lyapunov theorem, $F(T)=I$. Since the monodromy matrix $Y(T)$ of the perturbed system (3.5) equals to the matrix $F(T)e^{(M+\delta M)T}$, then $Y(T)=e^{(M+\delta M)T}$ is obtained.*

In the following second theorem, the sensitivity of Hurwitz stability of the system (3.1) is examined with the sensitivity of Schur stability of the system (3.4).

**Theorem 3.5.** *Let the system (3.1) be Hurwitz stable and $\delta X(T)$ be a matrix providing the following conditions*

$$\|\delta X(T)\|<\sqrt{\|X(T)\|^2+\frac{1}{\omega(X(T))}}-\|X(T)\| \ \text{ and } \ \delta X(T)+X(T) \ \text{ is a non-singular matrix,}$$

*then for the perturbation matrix $C(t)$ satisfying $C(t):=F(t)(Q-M)F^{-1}(t)$ (where the matrix $Q$ is defined as $e^{QT}:=\delta X(T)+X(T)$), the perturbed system (3.5) is Hurwitz stable and, the inequality*

$$\|C(t)\|\leq\frac{\sigma_{\max}(F(t))}{\sigma_{\min}(F(t))}\|Q-M\|,\ \ t\geq 0$$

*holds (where $\sigma_{\max}(F(t))$ and $\sigma_{\min}(F(t))$ are the largest and smallest singular values of the matrix $F(t)$, respectively).*

*Proof.* Assume that the hypotheses hold. By a similar approach to Theorem 3.3, firstly, we will determine the perturbed system which has the fundamental matrix $Y(t)=F(t)e^{Qt}$. Let $Y(t)$ be the fundamental matrix of the system

$$y'(t)=D(t)y(t). \tag{3.9}$$

In this case, since $Y(t)$ is a matrix solution of the system (3.9), we get

$$\begin{aligned}F'(t)e^{Qt}+F(t)Qe^{Qt} &= D(t)F(t)e^{Qt}\\ F'(t)F^{-1}(t)+F(t)QF^{-1}(t) &= D(t).\end{aligned} \tag{3.10}$$

Also, $X(t)=F(t)e^{Mt}$ is a fundamental matrix of the system (3.1), then the equality (3.8) holds. Therefore, from (3.8) and (3.10)

$$A(t)+F(t)(Q-M)F^{-1}(t)=D(t)$$

is obtained. This shows that $Y(t)$ is a fundamental matrix for the perturbed system (3.5) with $C(t)=F(t)(Q-M)F^{-1}(t)$. By Lemma 3.1, we see that $C(t+T)=C(t)$.
Now, we will show that the perturbed system (3.5) is Hurwitz stable. By hypotheses, since the system (3.1) is Hurwitz stable, the monodromy matrix $X(T)=e^{MT}$ is Schur stable, that is, the system (3.4) is Schur stable. By Theorem 2.2, the matrix $\delta X(T)+X(T)=e^{QT}$ is Schur stable. Finally, the perturbed system (3.5) is Hurwitz stable for the matrix $C(t)$ mentioned.

Furthermore, from Remark 3.2, we have for every $t\geq 0$

$$\begin{aligned}\|C(t)\| &= \|F(t)(Q-M)F^{-1}(t)\|\\ &\leq \|F(t)\|\|Q-M\|\|F^{-1}(t)\|\\ &= \frac{\sigma_{\max}(F(t))}{\sigma_{\min}(F(t))}\|Q-M\|.\end{aligned}$$

The proof is completed. □

Sensitivity of Hurwitz stability of the system (3.1) given by Theorem 3.3 and Theorem 3.5 can be summarized with the following diagram.

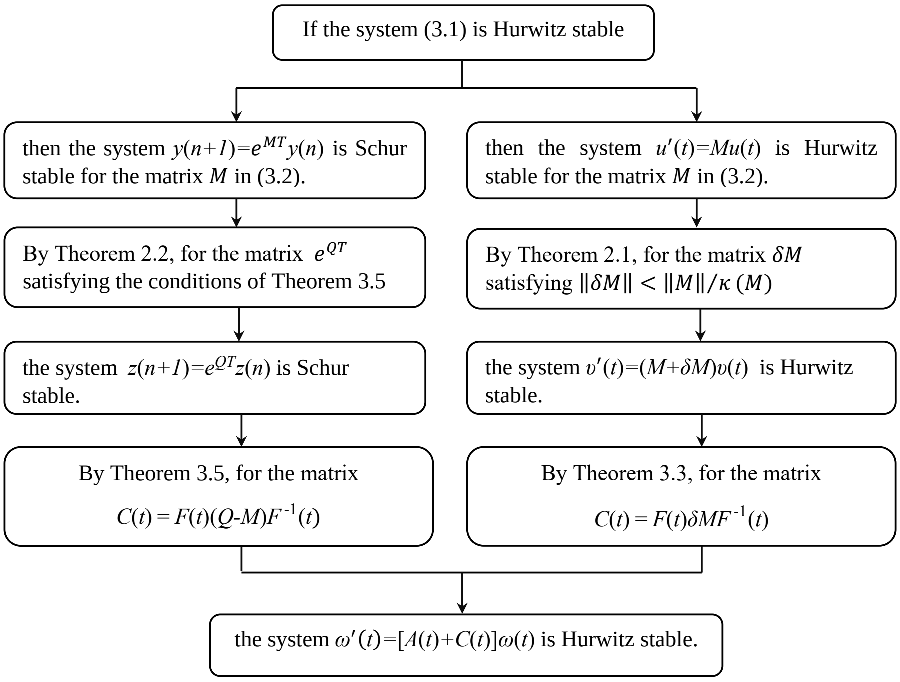


**Figure 3.1:** The diagram shows sensitivity of the system (3.1).

**Example 3.6.** *We consider the system* (3.1) *for the matrix* $A(t)=\begin{bmatrix}-1 & \cos t\\ -\cos t & -1\end{bmatrix}$.

*The fundamental and monodromy matrices of this system are, respectively,*

$$X(t)=\begin{bmatrix}e^{-t}\cos(\sin t) & e^{-t}\sin(\sin t)\\ -e^{-t}\sin(\sin t) & e^{-t}\cos(\sin t)\end{bmatrix} \text{ and } X(T)=X(2\pi)=\begin{bmatrix}e^{-2\pi} & 0\\ 0 & e^{-2\pi}\end{bmatrix}.$$

*The Floquet representation of the fundamental matrix* $X(t)$ *is*

$$X(t)=F(t)e^{Mt},$$

*where* $F(t)=\begin{bmatrix}\cos(\sin t) & \sin(\sin t)\\ -\sin(\sin t) & \cos(\sin t)\end{bmatrix}$ *and* $M=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$ (see [3, p.161]).

*Let us now examine the sensitivity of Hurwitz stability of this system by Theorem* 3.3 *and Theorem* 3.5*, respectively.*

*By Theorem* 3.3*: The condition number for the given system is* $\kappa(M)=1$*, and thus the system is Hurwitz stable. On the other hand, we can see from Theorem* 3.3 *that the perturbed system* (3.5) *is Hurwitz stable for* $C(t):=F(t)\delta MF^{-1}(t)$ *with* $\|\delta M\|<1$.
*Now, we take these matrices* $\delta_i M$ *as* $(i=1,2,3,\ldots)$

$$\delta_i M=\begin{bmatrix}1-1.10^{-i} & 0\\ 0 & -1+1.10^{-i}\end{bmatrix}.$$

$\|\delta_i M\|=1-1.10^{-i}<1$. *Thus the system persists its Hurwitz stability if it are perturbed with the matrices*

$$C_i(t)=F(t)\delta_i MF^{-1}(t),$$

$$C_i(t)=\begin{bmatrix}(1-1.10^{-i})\cos(2\sin t) & -(1-1.10^{-i})\sin(2\sin t)\\ -(1-1.10^{-i})\sin(2\sin t) & -(1-1.10^{-i})\cos(2\sin t)\end{bmatrix}, \quad i=1,2,\ldots.$$

*We note that* $C_i(t+2\pi)=C_i(t)$*. For the matrices* $C_i(t)$*, the Floquet representations of the fundamental matrices of the perturbed system* (3.5) *are*

$$Y_i(t)=F(t)e^{(M+\delta_i M)t}$$

*where,*

$$M+\delta_i M=\begin{bmatrix}-1.10^{-i} & 0\\ 0 & -2+1.10^{-i}\end{bmatrix}.$$

*The parameters* $\kappa(M+\delta_i M)$ *which reveal the quality of the Hurwitz stability of the perturbed system* (3.5) *can be calculated as*

$$\kappa(M+\delta_i M)=19.10^{i-1}+\sum_{j=0}^{i-2}9.10^{j}, \quad i=1,2,3,4,\ldots.$$

*For the matrices* $\delta_1 M$*,* $\delta_2 M$*,...,* $\delta_{10} M$*, respectively, we have that*

- $\|C_1(t)\| = 0,9$ *and* $\kappa(M+\delta_1 M) = 19$,
- $\|C_2(t)\| = 0,99$ *and* $\kappa(M+\delta_2 M) = 199$,

$\vdots$

- $\|C_{10}(t)\| = 0,9999999999$ *and* $\kappa(M+\delta_{10} M) = 19999999999$.

*Since* $\kappa(M+\delta_i M) < \infty$ *for* $i = 1,2,3,...$, *the matrices* $M+\delta_i M$ *are Hurwitz stable. Note that, while the norms of perturbation matrices* $C_i(t)$ *approache 1, the values* $\kappa(M+\delta_i M)$ *which reveal the quality of stability of perturbed system increase, so the quality of stability of the system gradually decreases.*
*By Theorem 3.5: The condition number of the given system is* $\omega(X(2\pi)) = 1$, *and thus the perturbed system (3.5) is Hurwitz stable from Theorem 3.5 for the matrix* $C(t) := F(t)(Q-M)F^{-1}(t)$ *satisfying the conditions* $\|\delta X(2\pi)\| < 0,998128$ *and* $\delta X(2\pi) + X(2\pi) = e^{Q2\pi}$.
*Now, we take the perturbation matrices* $\delta_i X(2\pi)$ *as* $(i = 1,2,3,...)$

$$\delta_i X(2\pi) = \begin{bmatrix} 0.99 - \frac{1}{10^{i+1}} & 0 \\ 0 & 0.99 - \frac{1}{10^{i+1}} \end{bmatrix}.$$

*Then* $\|\delta_i X(2\pi)\| < 0,998128$ *and,*

$$Q_i = \begin{bmatrix} \frac{1}{2\pi}\ln(e^{-2\pi} + 0.99 - \frac{1}{10^{i+1}}) & 0 \\ 0 & \frac{1}{2\pi}\ln(e^{-2\pi} + 0.99 - \frac{1}{10^{i+1}}) \end{bmatrix}.$$

*If the system (3.5) is perturbed with the matrices* $C_i(t)$, *then the system persists its Hurwitz stability, where* $C_i(t) = F(t)(Q_i - M)F^{-1}(t) = Q_i - M$ *and*

$$C_i(t) = \begin{bmatrix} \frac{1}{2\pi}\ln(e^{-2\pi} + 0.99 - \frac{1}{10^{i+1}}) + 1 & 0 \\ 0 & \frac{1}{2\pi}\ln(e^{-2\pi} + 0.99 - \frac{1}{10^{i+1}}) + 1 \end{bmatrix}.$$

*For the matrices* $\delta_1 X(2\pi)$, $\delta_2 X(2\pi)$,..., $\delta_7 X(2\pi)$, *the parameters* $\omega(\delta_i X(2\pi) + X(2\pi))$ *which reveal the quality of the Hurwitz stability of the perturbed system (3.5) are as the following.*

- $\|C_1(t)\| = 0.9970$ *and* $\omega(\delta_1 X(2\pi) + X(2\pi)) = 27.8263$,
- $\|C_2(t)\| = 0.9985$ *and* $\omega(\delta_2 X(2\pi) + X(2\pi)) = 54.9977$,

$\vdots$

- $\|C_7(t)\| = 0.9987$ *and* $\omega(\delta_7 X(2\pi) + X(2\pi)) = 61.729$.

*Note that, while the norms of the perturbation matrices* $C_i(t)$ *approache 1, the values* $\omega(\delta_i X(2\pi) + X(2\pi))$ *which reveal the quality of stability of the perturbed system (3.5) increase, so the quality of stability of the system gradually decreases.*

## 4. Conclusions

In this paper, using some theorems which exist in the literature dealing with sensitivity of the linear differential and linear difference equation systems with constant coefficients, and using the Floquet-Lyapunov theory of the LDES with periodic coefficients, two new continuity theorems for the LDES with periodic coefficients are obtained. Moreover, the stability regions are determined and the obtained results are supported by a numerical example.

## References


[1] Ö. Akın and H. Bulgak, *Linear Difference Equations and Stability Theory*, Selçuk University, Research Center of Applied Mathematics, Konya, 1998. in Turkish
[2] K. Aydın, Condition number for asymptotic stability of periodic ordinary differential equation systems, Ph.D. thesis, Institute of Science and Technology, Selçuk University (1995), in Turkish.
[3] D. Betounes, *Differential Equations: Theory and Applications*, Springer, 2nd ed., 2010.
[4] H. Bulgak, Pseudoeigenvalues, spectral portrait of a matrix and their connections with different criteria of stability, in *Error Control and Adaptivity in Scientific Computing*, eds. H. Bulgak and C. Zenger, NATO Science Series, Series C: Mathematical and Physical Sciences, (Kluwer Academic Publishers, Dordrecht) **536** (1999): 95-124.
[5] A. Ya. Bulgakov, *An effectively calculable parameter for the stability quality of systems of linear differential equations with constant coefficients*, Sib. Math. J. **21** (1980), 339-347.
[6] A.Ya. Bulgakov and S.K. Godunov, *Circle dichotomy of the matrix spectrum*, Sib. Math. J. **29** (1988), no. 5, 59-70.
[7] A.Ya. Bulgakov, *Matrix Computations with Guaranteed Accuracy in Stability Theory*, Selçuk University, The Research Center of Applied Mathematics, Konya, 1995.
[8] A. Duman and K. Aydın, *Sensitivity of Schur stability of linear difference equation systems with constant coefficients*, Sci. Res. Essays **6** (2011), no. 28, 5846-5854.
[9] A. Duman and K. Aydın, *Sensitivity of Hurwitz stability of linear differential equation systems with constant coefficients*, Int. J. Geom. Methods Mod. Phys. **14** (2017), no. 6.
[10] S.N. Elaydi, *An Introduction to Difference Equations*. Springer, Verlag, New York, 1999.
[11] S.K. Godunov, *Modern Aspects of Linear Algebra*, RI: American Mathematical Society, Translation of Mathematical Monographs 175. Providence, 1998.
[12] P.T. Jianjun and J. Wang, *Some results in Floquet theory, with application to periodic epidemic models*, Appl. Anal. **94** (2014), no. 6, 1128-1152.
[13] G.L. Kenneth and P.W. Likins, *Infinite determinant methods for stability analysis of periodic-coefficient differential equations*, AIAA Journal **8** (1970), no. 4, 680-686.
[14] Z. Li, Y. Li, C. Wu and B. Chen, *Study on bifurcation behaviors and stabilization in current mode controlled Buck converter*, Power System Protection and Control **44** (2016) no. 18, 54-60.
[15] A. Neubauer, J. Freudenberger, V. Kühn, *Coding Theory: Algorithms, Architectures, and Applications*, England, 2007.
[16] W. Walter, *Ordinary Differential Equations*, Springer-Verlag, New York, 1998.
[17] J.H. Wilkinson, *The Algebraic Eigenvalue Problem*, Clarendon Press, Oxford, 1965.
[18] J.R. Wilson, *Linear System Theory*, New Jersey, Prentice Hall, Second Edition, 1996.