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Several Types of $\mathcal{B}^{\#}$-closed Sets in Ideal Nanotopological Spaces

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Abstract — In this paper, we made an attempt to unveil to notions of nano $\mathcal{B}_g^{\#}$-closed sets and $I_{\mathcal{B}_g^{\#}}$-closed sets are introduce and their properties are discussed with suitable examples. They are characterizations in the context of an ideal nanotopological spaces.

Keywords — *Nano $\mathcal{B}^{\#}$-set, nano $t^{\#}$-set, nano $\mathcal{B}_{gs}^{\#}$-set and nano $I_{\mathcal{B}_g^{\#}}$-closed set.*

1. Introduction

An ideal I [1] on a space (X, τ) is a non-empty collection of subsets of X which satisfies the following conditions.

1. $A \in I$ and $B \subset A$ imply $B \in I$ and
2. $A \in I$ and $B \in I$ imply $A \cup B \in I$.

Given a space (X, τ) with an ideal I on X if $\wp(X)$ is the set of all subsets of X, a set operator $(.)^{\star} : \wp(X) \to \wp(X)$, called a local function of A with respect to τ and I is defined as follows: for $A \subset X$, $A^{\star}(I, \tau) = \{x \in X : U \cap A \notin I \text{ for every } U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau : x \in U\}$ [2]. The closure operator defined by $cl^{\star}(A) = A \cup A^{\star}(I, \tau)$ [3] is a Kuratowski closure operator which generates a topology $\tau^{\star}(I, \tau)$ called the $\star$-topology which is finer then τ. We will simply write $A^{\star}$ for $A^{\star}(I, \tau)$ and $\tau^{\star}$ for $\tau^{\star}(I, \tau)$. If I is an ideal on X, then (X, τ, I) is called an ideal topological space or an ideal space.

Some new notions in the concept of ideal nano topological spaces were introduced by Parimala et al. [4, 5] and Rajasekaran et.al [6] were introduced nano $\mathcal{B}^{\#}$-set and nano $t^{\#}$-set.

In this paper, we made an attempt to unveil to notions of nano $\mathcal{B}_g^{\#}$-closed sets and $I_{\mathcal{B}_g^{\#}}$-closed sets are introduce and their properties are discussed with suitable examples. They are characterizations in the context of an ideal nanotopological spaces.

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2. Preliminaries

Definition 2.1. [7] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

1. The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.

2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \cap X \neq \phi\}$.

3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2. [8] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:

1. U and $\phi \in \tau_R(X)$.

2. The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

3. The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

Thus $\tau_R(X)$ is a topology on U called the nano topology with respect to X and $(U, \tau_R(X))$ is called the nano topological space. The elements of $\tau_R(X)$ are called nano-open sets (briefly n-open sets). The complement of a n-open set is called n-closed.

In the rest of the paper, we denote a nano topological space by $(U, \mathcal{N})$, where $\mathcal{N} = \tau_R(X)$. The nano-interior and nano-closure of a subset A of U are denoted by $n\text{-}int(A)$ and $n\text{-}cl(A)$, respectively.

Definition 2.3. A subset A of a space $(U, \mathcal{N})$ is called a

1. nano semi-open [8] if $H \subseteq n\text{-}cl(n\text{-}int(H))$.

2. nano pre-open [8] if $H \subseteq n\text{-}int(n\text{-}cl(H))$.

The complements of the above mentioned sets are called their respective closed sets.

Definition 2.4. A subset H of a space $(U, \mathcal{N})$ is called a

1. nano g-closed (briefly, ng-closed) [9] if $n\text{-}cl(H) \subseteq G$, whenever $H \subset G$ and G is n-open.

2. nano gp-closed (briefly, ngp-closed) [10] if $n\text{-}pcl(H) \subseteq G$, whenever $H \subseteq G$ and G is n-open.

3. nano gs-closed (briefly, ngs-closed) [11] if $n\text{-}scl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano semi-open.

The complements of the above mentioned sets are called their respective closed sets.

Definition 2.5. [6] A subset H of a space $(U, \tau_R(X))$ is called a

1. nano $t^{\#}$-set (briefly, $nt^{\#}$-set) if $n\text{-}int(H) = n\text{-}cl(n\text{-}int(H))$.

2. nano $\mathcal{B}^{\#}$-set (briefly, $n\mathcal{B}^{\#}$-set) if $H = P \cap Q$, where P is n-open and Q is $nt^{\#}$-set.

Remark 2.6. [6] In a space $(U, \mathcal{N})$, each n-open set is $n\mathcal{B}^{\#}$-set.

Theorem 2.7. In a space $(U, \mathcal{N})$,

1. each n-closed set is ng-closed set. [9]

2. each ng-closed set is ngp-closed set. [10]

3. each ng-closed set is ngs-closed set. [11]

A nano topological space $(U, \mathcal{N})$ with an ideal I on U is called [4] an ideal nano topological space and is denoted by $(U, \mathcal{N}, I)$. $G_n(x) = \{G_n \mid x \in G_n, G_n \in \mathcal{N}\}$, denotes [4] the family of nano open sets containing x.

In future an ideal nano topological spaces $(U, \mathcal{N}, I)$ is referred as a space.

Definition 2.8. [4] Let $(U, \mathcal{N}, I)$ be a space with an ideal I on U. Let $(.)^\star_n$ be a set operator from $\wp(U)$ to $\wp(U)$ ($\wp(U)$ is the set of all subsets of U). For a subset $A \subseteq U$, $A^\star_n(I, \mathcal{N}) = \{x \in U : G_n \cap A \notin I$, for every $G_n \in G_n(x)\}$ is called the nano local function (briefly, n-local function) of A with respect to I and $\mathcal{N}$. We will simply write $A^\star_n$ for $A^\star_n(I, \mathcal{N})$.

Theorem 2.9. [4] Let $(U, \mathcal{N}, I)$ be a space and A and B be subsets of U. Then

1. $A \subseteq B \Rightarrow A^\star_n \subseteq B^\star_n$.

2. $A^\star_n = n\text{-}cl(A^\star_n) \subseteq n\text{-}cl(A)$ ($A^\star_n$ is a n-closed subset of $n\text{-}cl(A)$).

3. $(A^\star_n)^\star_n \subseteq A^\star_n$.

4. $(A \cup B)^\star_n = A^\star_n \cup B^\star_n$.

5. $V \in \mathcal{N} \Rightarrow V \cap A^\star_n = V \cap (V \cap A)^\star_n \subseteq (V \cap A)^\star_n$.

6. $J \in I \Rightarrow (A \cup J)^\star_n = A^\star_n = (A - J)^\star_n$.

Theorem 2.10. [4] Let $(U, \mathcal{N}, I)$ be a space with an ideal I and $A \subseteq A^\star_n$, then $A^\star_n = n\text{-}cl(A^\star_n) = n\text{-}cl(A)$.

Definition 2.11. [4] Let $(U, \mathcal{N}, I)$ be a space. The set operator $n\text{-}cl^\star$ called a nano $\star$-closure is defined by $n\text{-}cl^\star(A) = A \cup A^\star_n$ for $A \subseteq X$.

It can be easily observed that $n\text{-}cl^\star(A) \subseteq n\text{-}cl(A)$.

Theorem 2.12. [5] In a space $(U, \mathcal{N}, I)$, if A and B are subsets of U, then the following results are true for the set operator $n\text{-}cl^\star$.

1. $A \subseteq n\text{-}cl^\star(A)$.

2. $n\text{-}cl^\star(\phi) = \phi$ and $n\text{-}cl^\star(U) = U$.

3. $If A \subset B$, then $n\text{-}cl^\star(A) \subseteq n\text{-}cl^\star(B)$.

4. $n\text{-}cl^\star(A) \cup n\text{-}cl^\star(B) = n\text{-}cl^\star(A \cup B)$.

5. $n\text{-}cl^\star(n\text{-}cl^\star(A)) = n\text{-}cl^\star(A)$.

Definition 2.13. [12] A subset A of a space $(U, \mathcal{N}, I)$ is $n\star$-dense in itself (resp. $n\star$-perfect and $n\star$-closed) if $A \subseteq A^\star_n$ (resp. $A = A^\star_n$, $A^\star_n \subseteq A$).

Definition 2.14. [13] A subset A of a space $(U, \mathcal{N}, I)$ is called a weakly nano I-locally closed set (briefly, $\mathcal{W}$-nI-LC) if $A = P \cap Q$ where P is n-open and Q is $n\star$-closed.

Definition 2.15. [12] A subset A of a space $(U, \mathcal{N}, I)$ is called a nano I_g-closed (briefly nI_g-closed) if $A^\star_n \subseteq B$ whenever $A \subseteq B$ and B is n-open.

Theorem 2.16. [13] For a subset A of a space $(U, \mathcal{N}, I)$, the following are equivalent,

1. A is $\mathcal{W}$-nI-LC,

2. $A = G \cap n\text{-}cl^\star(A)$ for few n-open set G,

3. $n\text{-}cl^\star(A) - A = A^\star_n - A$ is n-closed,

4. $(U - A^\star_n) \cup A = A \cup (U - n\text{-}cl^\star(A))$ is n-open,

5. $A \subset n\text{-}int(A \cup (U - A^\star_n))$.

3. On nano $I_{\mathcal{B}_g^{\#}}$-closed sets

Definition 3.1. A subset H of a space $(U, \mathcal{N})$ is called a

1. nano $\mathcal{B}_g^{\#}$-closed set if (briefly, $n\mathcal{B}_g^{\#}$-closed set) $n\text{-}cl(H) \subseteq G$ whenever $H \subseteq G$ and G is $n\mathcal{B}^{\#}$-set. The complement of $n\mathcal{B}_g^{\#}$-open if $H^c = U - H$ is $n\mathcal{B}_g^{\#}$-closed.
2. nano $\mathcal{B}_g^{\#}$-set if (briefly, $n\mathcal{B}_g^{\#}$-set) $H = P \cap Q$ where P is ng-open and $nt^{\#}$-set.
3. nano $\mathcal{B}_{gs}^{\#}$-set if (briefly, $n\mathcal{B}_{gs}^{\#}$-set) $H = P \cap Q$ where P is ngs-open and $nt^{\#}$-set.
4. nano $\mathcal{B}_{gp}^{\#}$-set if (briefly, $n\mathcal{B}_{gp}^{\#}$-set) $H = P \cap Q$ where P is ngp-open and $nt^{\#}$-set.

Remark 3.2. The diagram holds for any subset of a space $(U, \mathcal{N})$:

$$\begin{array}{ccccc} & & n\mathcal{B}^{\#}\textbf{-set} & & \\ & & \downarrow & & \\ n\mathcal{B}_{gp}^{\#}\textbf{-set} & \longleftarrow & n\mathcal{B}_g^{\#}\textbf{-set} & \longrightarrow & n\mathcal{B}_{gs}^{\#}\textbf{-set} \end{array}$$

In this diagram, none of the implications are reversible.

Example 3.3. Let $U = \{a, b, c, d, e\}$ with $U/R = \{\{e\}, \{a, b\}, \{c, d\}\}$ and $X = \{b, e\}$. Then $\mathcal{N} = \{\phi, U, \{e\}, \{a, b\}, \{a, b, e\}\}$. Let the ideal be $I = \{\phi, \{a\}, \{b\}, \{a, b\}\}$. Then

1. $\{c\}$ is $n\mathcal{B}_g^{\#}$-set but not $n\mathcal{B}^{\#}$-set.
2. $\{b, c, d, e\}$ is $n\mathcal{B}_{gp}^{\#}$-set and $n\mathcal{B}_{gs}^{\#}$-set but not $n\mathcal{B}_g^{\#}$-set.

Definition 3.4. A subset H of a space $(U, \mathcal{N}, I)$ is called a

1. $nI_{\mathcal{B}_g^{\#}}$-closed set if $H_n^{\star} \subseteq P$ whenever $H \subseteq P$ and P is $n\mathcal{B}^{\#}$-set.
2. $nI_{\mathcal{B}_{gs}^{\#}}$-closed set if $H_n^{\star} \subseteq P$ whenever $H \subseteq P$ and P is $n\mathcal{B}_{gs}^{\#}$-set.
3. $nI_{\mathcal{B}_{gp}^{\#}}$-closed set if $H_n^{\star} \subseteq P$ whenever $H \subseteq P$ and P is $n\mathcal{B}_{gp}^{\#}$-set.
4. nI_{gp}-closed set if $H_n^{\star} \subseteq P$ whenever $H \subseteq P$ and P is ngp-open.
5. nI_{gs}-closed set if $H_n^{\star} \subseteq P$ whenever $H \subseteq P$ and P is ngs-open.

The complements of the above mentioned closed sets are called their respective open sets.

Theorem 3.5. Let $(U, \mathcal{N}, I)$ be a space and $H \subseteq U$, then

1. H is $n\star$-closed $\Rightarrow$ H is $nI_{\mathcal{B}_g^{\#}}$-closed.
2. H is $nI_{\mathcal{B}_g^{\#}}$-closed $\Rightarrow$ H is nI_g-closed.
3. H is $nI_{\mathcal{B}_{gp}^{\#}}$-closed $\Rightarrow$ H is $nI_{\mathcal{B}_g^{\#}}$-closed.
4. H is $nI_{\mathcal{B}_{gs}^{\#}}$-closed $\Rightarrow$ H is $nI_{\mathcal{B}_g^{\#}}$-closed.
5. H is $nI_{\mathcal{B}_{gs}^{\#}}$-closed $\Rightarrow$ H is nI_{gs}-closed.
6. H is $nI_{\mathcal{B}_{gp}^{\#}}$-closed $\Rightarrow$ H is nI_{gp}-closed.
7. H is nI_{gp}-closed $\Rightarrow$ H is nI_g-closed.
8. H is nI_{gs}-closed $\Rightarrow$ H is nI_g-closed.

PROOF. 1. Let H be $n\star$-closed set and P be $n\mathcal{B}^{\#}$-set in U such that $H \subseteq P$. Since H is $n\star$-closed, $H_n^\star \subseteq H$, so $H_n^\star \subseteq P$. Hence H is $nI_{\mathcal{B}_g^{\#}}$-closed set.

2. Let H be $nI_{\mathcal{B}_g^{\#}}$-closed set and $H \subseteq P$ where $P \in \mathcal{N}$. Since each n-open set is $n\mathcal{B}^{\#}$-set, so P is $n\mathcal{B}^{\#}$-set. Since H is $nI_{\mathcal{B}_g^{\#}}$-closed set, we obtain that $H_n^\star \subseteq P$ and hence H is nI_g-closed set.

3. It follows from Remark 3.2 and Definition 3.4.

4. It follows from Remark 3.2 and Definition 3.4.

5. Let $H \subseteq P$ where P is ngs-open set in U. Since each ngs-open set is $n\mathcal{B}_{gs}^{\#}$-set, so P is $n\mathcal{B}_{gs}^{\#}$-set. Since H is $nI_{\mathcal{B}_{gs}^{\#}}$-closed set, we have $H_n^\star \subseteq P$. Hence H is nI_{gs}-closed set.

6. Let $H \subseteq P$ where P is ngp-open set in U. Since each ngp-open set is $n\mathcal{B}_{gp}^{\#}$-set, so P is $n\mathcal{B}_{gp}^{\#}$-set. Since H is $nI_{\mathcal{B}_{gp}^{\#}}$-closed set, we have $H_n^\star \subseteq P$. Hence H is nI_{gp}-closed set.

7. It follows from Theorem 2.7 and Definition 3.4(4).

8. It follows from Theorem 2.7 and Definition 3.4(5).

Remark 3.6. These relations are shown in the diagram.

$$\begin{array}{ccccc}
 & & n\star\textbf{-closed} & & \\
 & & \downarrow & & \\
nI_{\mathcal{B}_{gp}^{\#}}\textbf{-closed} & \longrightarrow & nI_{\mathcal{B}_g^{\#}}\textbf{-closed} & \longleftarrow & nI_{\mathcal{B}_{gs}^{\#}}\textbf{-closed} \\
\downarrow & & \downarrow & & \downarrow \\
nI_{gp}\textbf{-closed} & \longrightarrow & nI_g\textbf{-closed} & \longleftarrow & nI_{gs}\textbf{-closed}
\end{array}$$

The converses of each statement in Theorem 3.5 are not true as shown in the following Examples.

Example 3.7. In Example 3.3, Then

1. $\{c\}$ is $nI_{\mathcal{B}_g^{\#}}$-closed but not $n\star$-closed.

2. $\{d\}$ is $nI_{\mathcal{B}_g^{\#}}$-closed but not $nI_{\mathcal{B}_{gp}^{\#}}$-closed.

3. $\{a, c\}$ is $nI_{\mathcal{B}_g^{\#}}$-closed but not $nI_{\mathcal{B}_{gs}^{\#}}$-closed.

4. $\{b, c, d\}$ is nI_{gp}-closed but not $nI_{\mathcal{B}_{gp}^{\#}}$-closed.

5. $\{a, e\}$ is nI_{gs}-closed but not $nI_{\mathcal{B}_{gs}^{\#}}$-closed.

6. $\{c\}$ is nI_g-closed but not nI_{gp}-closed.

7. $\{d\}$ is nI_g-closed but not nI_{gs}-closed.

Example 3.8. Let $U = \{a, b, c, d\}$ with $U/R = \{\{b\}, \{d\}, \{a, c\}\}$ and $X = \{c, d\}$. Then $\mathcal{N} = \{\phi, U, \{d\}, \{a, c\}, \{a, c, d\}\}$. Let the ideal be $I = \{\phi, \{d\}\}$. Then
$\{b, c\}$ is nI_g-closed but not $nI_{\mathcal{B}_g^{\#}}$-closed.

Remark 3.9. The following Example shows that the family of $n\mathcal{B}^{\#}$-sets and the family of $nI_{\mathcal{B}_g^{\#}}$-closed sets are independent of a space $(U, \mathcal{N}, I)$.

Example 3.10. In Example 3.3, then

1. $\{e\}$ is $n\mathcal{B}^{\#}$-set but not $nI_{\mathcal{B}_g^{\#}}$-closed.

2. $\{a\}$ is $nI_{\mathcal{B}_g^{\#}}$-closed but not $n\mathcal{B}^{\#}$-set.

Theorem 3.11. If H is both $n\mathcal{B}^{\#}$-set and $nI_{\mathcal{B}_g^{\#}}$-closed set, then H is $n\star$-closed.

Proof. Let H be a both $n\mathcal{B}^{\#}$-set and $nI_{\mathcal{B}_g^{\#}}$-closed set. Then $H_n^{\star} \subseteq H$, whenever H is $n\mathcal{B}^{\#}$-set and $H \subseteq H$. Hence H is $n\star$-closed.

Theorem 3.12. If A and B are $nI_{\mathcal{B}_g^{\#}}$-closed sets, then $A \cup B$ is $nI_{\mathcal{B}_g^{\#}}$-closed set.

Proof. Let $A \cup B \subseteq U$, where U is $n\mathcal{B}^{\#}$-set. Since A and B are $nI_{\mathcal{B}_g^{\#}}$-closed, $A_n^{\star} \subseteq U$ and $B_n^{\star} \subseteq U$, whenever $A \subseteq U, B \subseteq U$ and U is $n\mathcal{B}^{\#}$-set. Therefore, $(A \cup B)_n^{\star} = A_n^{\star} \cup B_n^{\star} \subseteq U$. Hence $A \cup B$ is $nI_{\mathcal{B}_g^{\#}}$-closed set.

Remark 3.13. The intersection of two $nI_{\mathcal{B}_g^{\#}}$-closed sets but not $nI_{\mathcal{B}_g^{\#}}$-closed set.

Example 3.14. In Example 3.3, $H = \{c, e\}$ and $K = \{d, e\}$ are $nI_{\mathcal{B}_g^{\#}}$-closed. But $H \cap K = \{e\}$ is not $nI_{\mathcal{B}_g^{\#}}$-closed.

Theorem 3.15. If A is $nI_{\mathcal{B}_g^{\#}}$-closed set such that $A \subseteq B \subseteq A_n^{\star}$, then B is also $nI_{\mathcal{B}_g^{\#}}$-closed set.

Proof. Let G be $n\mathcal{B}^{\#}$-set in U such that $B \subseteq G$. Then $A \subseteq G$. Since A is $nI_{\mathcal{B}_g^{\#}}$-closed set, $A_n^{\star} \subseteq G$. Now $B_n^{\star} \subseteq (A_n^{\star})_n^{\star} \subseteq A_n^{\star} \subseteq G$. Therefore B is also $nI_{\mathcal{B}_g^{\#}}$-closed set.

Proposition 3.16. For any space $(U, \mathcal{N}, I)$, each singleton $\{x\}$ of U is $n\mathcal{B}^{\#}$-set.

Proof. Let $x \in U$. If $\{x\} \in \mathcal{N}$, then $\{x\}$ is $n\mathcal{B}^{\#}$-set. If $\{x\} \notin \mathcal{N}$, then $n\text{-}int(\{x\}) = \phi = n\text{-}cl(n\text{-}int(\{x\}))$, so $\{x\}$ is $n\mathcal{B}^{\#}$-set.

Corollary 3.17. For each $x \in U, \{x\}$ is $nI_{\mathcal{B}_g^{\#}}$-closed set if and only if {x} is $n\star$-closed set.

Proof. Necessity: Let $\{x\}$ be $nI_{\mathcal{B}_g^{\#}}$-closed set. Since $\{x\}$ is both $n\mathcal{B}^{\#}$-set and $nI_{\mathcal{B}_g^{\#}}$-closed set, then $\{x\}$ is $n\star$-closed.

Sufficiency: Let $\{x\}$ be $n\star$-closed set. We know that each $n\star$-closed set is $nI_{\mathcal{B}_g^{\#}}$-closed set. Therefore $\{x\}$ is $nI_{\mathcal{B}_g^{\#}}$-closed set.

Theorem 3.18. Let A be $nI_{\mathcal{B}_g^{\#}}$-closed set. Then $A_n^{\star} - A$ does not contain any non-empty complement of $n\mathcal{B}^{\#}$-set.

Proof. Let A be $nI_{\mathcal{B}_g^{\#}}$-closed set. Suppose that F is the complement of $n\mathcal{B}^{\#}$-set and $F \subseteq A_n^{\star} - A$. Since $F \subseteq A_n^{\star} - A \subseteq U - A, A \subseteq U - F$ and $U - F$ is $n\mathcal{B}^{\#}$-set. Therefore, $A_n^{\star} \subseteq U - F$ and $F \subseteq U - A_n^{\star}$. However, since $F \subseteq A_n^{\star} - A$, we have $F = \phi$.

Theorem 3.19. For a subset A of a space $(U, \mathcal{N}, I)$, the following are equivalent.

1. A is $n\star$-closed.
2. A is $\mathcal{W}$-nI-LC and $nI_{\mathcal{B}_g^{\#}}$-closed.

Proof. (1)⇒(2). Obvious.

(2)⇒(1). Since A is $\mathcal{W}$-nI-LC, by Theorem 2.16, $A = G \cap n\text{-}cl^{\star}(A)$, where G is n-open in U. So, $A \subseteq G$ and G is $n\mathcal{B}^{\#}$-set in U. Since A is $nI_{\mathcal{B}_g^{\#}}$-closed, $A_n^{\star} \subseteq G$ and $A_n^{\star} \cup A \subseteq G$. Therefore $n\text{-}cl^{\star}(A) \subseteq G \cap n\text{-}cl^{\star}(A) = A$. Hence A is $n\star$-closed set in U.

Remark 3.20. The following Example shows that the family of $\mathcal{W}$-nI-LC and the family of $nI_{\mathcal{B}_g^{\#}}$-closed sets are independent of a space $(U, \mathcal{N}, I)$.

Example 3.21. In Example 3.3, then

1. $\{e\}$ is $\mathcal{W}$-nI-LC but not $nI_{\mathcal{B}_g^{\#}}$-closed.
2. $\{c\}$ is $nI_{\mathcal{B}_g^{\#}}$-closed but not $\mathcal{W}$-nI-LC.

Theorem 3.22. Let $(U, \mathcal{N}, I)$ be a space and $A \subseteq U$. Then A is $nI_{\mathcal{B}_g^{\#}}$-open if and only if $F \subseteq n\text{-}int^{\star}(A)$ whenever F is the complement of $n\mathcal{B}^{\#}$-set and $F \subseteq A$.

Proof. Suppose A is $nI_{\mathcal{B}_g^{\#}}$-open. If F is the complement of $n\mathcal{B}^{\#}$-set and $F \subseteq A$, then $U - A \subseteq U - F$ and so $(U-A)_n^\star \subseteq U - F$ and $[(U-A) \cup (U-A)_n^\star] \subseteq [U-F] \cup [U-A]$. Hence $n\text{-}cl^\star(U-A) \subseteq U-F$. Therefore $F \subseteq n\text{-}int^\star(A)$.

Conversely, suppose the condition holds. Let G be $n\mathcal{B}^{\#}$-set such that $U - A \subseteq G$. Then $U - G \subseteq A$ and $U - G$ is the complement of $n\mathcal{B}^{\#}$-set. By assumption, $U - G \subseteq n\text{-}int^\star(A)$ which implies that $n\text{-}cl^\star(U-A) \subseteq G$ and $(U-A)_n^\star \subseteq G$. Therefore $U - A$ is $nI_{\mathcal{B}_g^{\#}}$-closed and so A is $nI_{\mathcal{B}_g^{\#}}$-open.

Theorem 3.23. Let $(U, \mathcal{N}, I)$ be a space and $A \subseteq U$. If A is $nI_{\mathcal{B}_g^{\#}}$-open and $n\text{-}int^\star(A) \subseteq B \subseteq A$, then B is $nI_{\mathcal{B}_g^{\#}}$-open.

Proof. It follows from Theorem 3.15.

Theorem 3.24. Let $(U, \mathcal{N}, I)$ be a space. Then each subset of U is $nI_{\mathcal{B}_g^{\#}}$-closed if and only if each $n\mathcal{B}^{\#}$-set is $n\star$-closed.

Proof. $(\Rightarrow)$ Let $G \subseteq U$ be any $n\mathcal{B}^{\#}$-set. Since G is both $n\mathcal{B}^{\#}$-set and $nI_{\mathcal{B}_g^{\#}}$-closed, by Theorem 3.11, G is $n\star$-closed.

$(\Leftarrow)$ Let $A \subseteq U$ and G is $n\mathcal{B}^{\#}$-set such that $A \subseteq G$, then $A_n^\star \subseteq G_n^\star \subseteq G$. Therefore A is $nI_{\mathcal{B}_g^{\#}}$-closed set.

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