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Series Expansions and Polynomial Approximations of Monomolecular Growth Model for Some Populations of *Salmo platycephalus*, 1968 from Zamanti Stream, Seyhan River, Turkey

Seyhan Nehri'nin Zamanti Irmağı'ndan Yakalanan Salmo platycephalus, 1968 Adlı Alabalığa Ait Popülasyonlar için Monomoleküler Büyüme Modellerinin Seri Açılımları ve Polinom Yaklaşımları

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Abstract

In this study, firstly the series expansions of monomolecular growth model from first degree polynomial to (n-1)th degree polynomial, were given with respect to (t-r) where t is time, n is the number of data, r is integer number: $t_0 \le r \le t_{n-1}$, t_0 and t_{n-1} are initial and final values of time, respectively. Secondly, monomolecular growth model's series expansions having m-th degree polynomials, studied on the data taken for *Salmo platycephalus*, 1968 from Zamanti Stream of Seyhan River, Turkey, were given with R^2 with respect to (t-l), respectively where t is time; n is the number of data points; m, *l* are integer numbers $1 < m \le n-1,\ 1 \le l \le 10$ Finally, for each data set, polynomial approximations having m-th degree were given with R^2. For each purpose, the tables and the graphs were used for analyzing the differences.

Keywords: Growth models, Monomolecular growth model, Polynomial approximation, *Salmo platycephalus*, 1968, Series expansions

Öz

Bu çalışmada ilk olarak monomoleküler büyüme modelinin birinci dereceden (n-1) inci dereceye kadar seri açılımları (t-r) ye göre verildi ki burada t zamanı, n veri noktalarının sayısını, r $t_0 \le r \le t_{n-1}$ arasında olmak üzere tam sayıyı, t_0 ve t_{n-1} sırasıyla başlangıç ve bitiş zamanını göstermektedir. İkinci olarak m-inci dereceden polinomlara sahip olan monomoleküler büyüme modelinin seri açılımları sırasıyla (t-l) ye göre R^2 değerleri ile verilmiştir ki bunlar Seyhan Nehri'nin Zamanti Irmağı'ndan yakalanan *Salmo platycephalus*, 1968 adlı alabalığa ait elde edilen ve üzerinde çalışılan verilerdir. Burada t zamanı, n veri noktalarının sayısını, m ve *l*, $1 < m \le n-1,\ 1 \le l \le 10$ arasında olmak üzere tam sayıyı göstermektedir. Son olarak, her bir veri seti için m-inci dereceye sahip polinom yaklaşımları R^2 ile verilmiştir. Her bir amaç için tablolar ve grafikler farkları analiz etmek için kullanılmıştır.

Anahtar Kelimeler: Büyüme modelleri, Monomoleküler büyüme modeli, Polinom yaklaşımı, *Salmo platycephalus*, 1968, Seri açılımları

1. Introduction

In mathematics, a series expansion is a method for calculating a function that cannot be expressed by only elementary operations such as addition, subtraction, multiplication and division. The so called resulting series often can be limited to a finite number of terms, thus yielding an approximation of the function. The fewer terms of the sequence are used, the simpler this approximation will be. Actually, there are some kinds of series such as divergent series, Taylor series and power series. A Taylor series is a representation of a function as an infinite sum of terms that are calculated from the values of the function's derivatives at a single point. If the Taylor series is centered at zero, then that series is also called a Maclaurin series (Bethard 1997). The general idea behind Taylor series is that if a function satisfies certain criteria, then the function can be expressed as an infinite series of polynomials. In its most general terms, the value of a function, f(x), in the vicinity of the point x_0, is given by:

$$f(x) = \sum_{r=0}^{\infty} (x - x_0)^r \quad (1)$$

where x_0 is the initial point of the series, a_r are the coefficient of the series.

A polynomial function is a function such as quadratic, cubic, and so on, involving only non-negative integer powers of x.

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The degree of a polynomial is the highest power of x in its expression. Actually polynomial is the special condition of a Maclaurin series when n is finite. In its most general terms, the value of a polynomial, p(x) is given by:

$$p(x)=\sum_{r=0}^{n} a_r x^r \qquad (2)$$

where a_r are the coefficient of the polynomial, n is the degree of the polynomial. A central problem of mathematical analysis is the approximation to more general functions by polynomials and the estimation of how small the discrepancy can be made.

Growth models have generally had sigmoidal shape. These models have one inflection point. For these growth models, growth rate increased continually until the inflection point and the highest growth rate occured at inflection point. After that point the growth rate decreased continually (Figure 1).

The monomolecular or Brody function (Brody 1945) is of decaying exponential type with no inflection point. Fabens (1965) described a similar function based on the work of Bertalanffy. For these functions the highest growth rate occurs at birth and decreases continually (Figure 2) (Fitzpatrick 2014).

In this study the series expansions of only one of the growth models were presented to investigate the series expansions. For that reason, series expansions of Monomolecular growth model w.r.t. (t-r) were given below where t is time, r is integer number $t_0 \le r \leqslant t_{n-1}, t_0$ and t_{n-1} is initial and final values of time, respectively and n is the number of data points.

Monomolecular Growth Model (M.G.M.):

Series expansions of Monomolecular growth model, $y=a(1-b\exp(-ct))$, with respect to (w.r.t.) (t-1) and (t-2) are given in Tables 1, 2, respectively.

Table 1. Series expansions of Monomolecular growth model w.r.t. (t-1)

Degrees of series expansion of M.G.M. w.r.t. (t-1)	$y=a(1-b\exp(-ct))$
1	$a(1-be^{(-c)})+abe^{(-c)}c(t-1)$
2	$a(1-be^{(-c)})+abe^{(-c)}c(t-1)-\frac{1}{2}abe^{(-c)}c^2(t-1)^2$
3	$a(1-be^{(-c)})+abe^{(-c)}c(t-1)-\frac{1}{2}abe^{(-c)}c^2(t-1)^2+\frac{1}{6}abe^{(-c)}c^3(t-1)^3$
4	$a(1-be^{(-c)})+abe^{(-c)}c(t-1)-\frac{1}{2}abe^{(-c)}c^2(t-1)^2+\frac{1}{6}abe^{(-c)}c^3(t-1)^3-\frac{1}{24}abe^{(-c)}c^4(t-1)^4$
5	$a(1-be^{(-c)})+abe^{(-c)}c(t-1)-\frac{1}{2}abe^{(-c)}c^2(t-1)^2+\frac{1}{6}abe^{(-c)}c^3(t-1)^3-\frac{1}{24}abe^{(-c)}c^4(t-1)^4$ $+\frac{1}{120}abe^{(-c)}c^5(t-1)^5$
6	$a(1-be^{(-c)})+abe^{(-c)}c(t-1)-\frac{1}{2}abe^{(-c)}c^2(t-1)^2+\frac{1}{6}abe^{(-c)}c^3(t-1)^3-\frac{1}{24}abe^{(-c)}c^4(t-1)^4$ $+\frac{1}{120}abe^{(-c)}c^5(t-1)^5-\frac{1}{720}abe^{(-c)}c^6(t-1)^6$
7	$a(1-be^{(-c)})+abe^{(-c)}c(t-1)-\frac{1}{2}abe^{(-c)}c^2(t-1)^2+\frac{1}{6}abe^{(-c)}c^3(t-1)^3-\frac{1}{24}abe^{(-c)}c^4(t-1)^4$ $+\frac{1}{120}abe^{(-c)}c^5(t-1)^5-\frac{1}{720}abe^{(-c)}c^6(t-1)^6+\frac{1}{5040}abe^{(-c)}c^7(t-1)^7$
8	$a(1-be^{(-c)})+abe^{(-c)}c(t-1)-\frac{1}{2}abe^{(-c)}c^2(t-1)^2+\frac{1}{6}abe^{(-c)}c^3(t-1)^3-\frac{1}{24}abe^{(-c)}c^4(t-1)^4$ $+\frac{1}{120}abe^{(-c)}c^5(t-1)^5-\frac{1}{720}abe^{(-c)}c^6(t-1)^6+\frac{1}{5040}abe^{(-c)}c^7(t-1)^7-\frac{1}{40320}abe^{(-c)}c^8(t-1)^8$
9	$a(1-be^{(-c)})+abe^{(-c)}c(t-1)-\frac{1}{2}abe^{(-c)}c^2(t-1)^2+\frac{1}{6}abe^{(-c)}c^3(t-1)^3-\frac{1}{24}abe^{(-c)}c^4(t-1)^4$ $+\frac{1}{120}abe^{(-c)}c^5(t-1)^5-\frac{1}{720}abe^{(-c)}c^6(t-1)^6+\frac{1}{5040}abe^{(-c)}c^7(t-1)^7-\frac{1}{40320}abe^{(-c)}c^8(t-1)^8$ $+\frac{1}{362880}abe^{(-c)}c^9(t-1)^9$

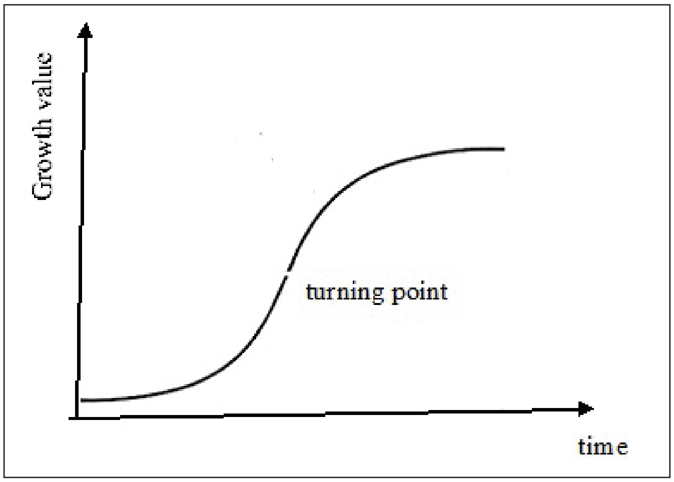

Figure 1. Sigmoidal function.

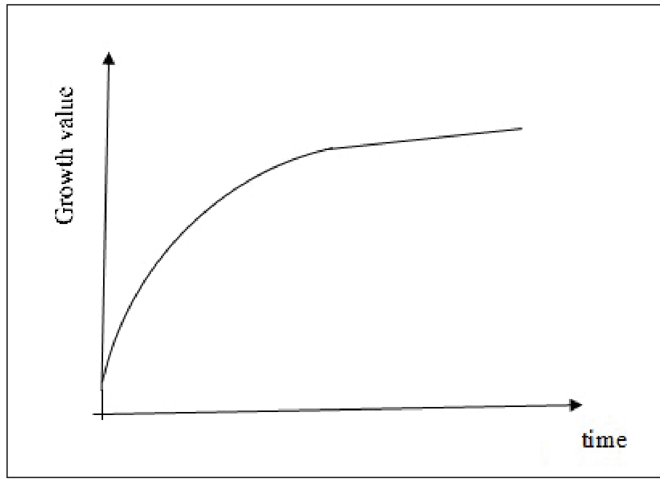

Figure 2. Increasing function by decreasing rate.

Table 2. Series expansions of Monomolecular growth model w.r.t. (t-2)

Degrees of series expansion of M.G.M. w.r.t.(t-2)	$y=a(1-b\exp(-ct))$
1	$a(1-be^{(-2c)})+abe^{(-2c)}c(t-2)$
2	$a(1-be^{(-2c)})+abe^{(-2c)}c(t-1)-\frac{1}{2}abe^{(-2c)}c^{2}(t-2)^{2}$
3	$a(1-be^{(-2c)})+abe^{(-2c)}c(t-2)-\frac{1}{2}abe^{(-2c)}c^{2}(t-2)^{2}+\frac{1}{6}abe^{(-2c)}c^{3}(t-2)^{3}$
4	$a(1-be^{(-2c)})+abe^{(-2c)}c(t-2)-\frac{1}{2}abe^{(-2c)}c^{2}(t-2)^{2}+\frac{1}{6}abe^{(-2c)}c^{3}(t-1)^{3}-\frac{1}{24}abe^{(-c)}c^{4}(t-2)^{4}$
5	$a(1-be^{(-2c)})+abe^{(-2c)}c(t-2)-\frac{1}{2}abe^{(-2c)}c^{2}(t-2)^{2}+\frac{1}{6}abe^{(-2c)}c^{3}(t-2)^{3}-\frac{1}{24}abe^{(-2c)}c^{4}(t-2)^{4}$ $+\frac{1}{120}abe^{(-2c)}c^{5}(t-2)^{5}$
6	$a(1-be^{(-2c)})+abe^{(-2c)}c(t-2)-\frac{1}{2}abe^{(-2c)}c^{2}(t-2)^{2}+\frac{1}{6}abe^{(-2c)}c^{3}(t-2)^{3}-\frac{1}{24}abe^{(-2c)}c^{4}(t-2)^{4}$ $+\frac{1}{120}abe^{(-2c)}c^{5}(t-2)^{5}-\frac{1}{720}abe^{(-2c)}c^{6}(t-2)^{6}$
7	$a(1-be^{(-2c)})+abe^{(-2c)}c(t-2)-\frac{1}{2}abe^{(-2c)}c^{2}(t-2)^{2}+\frac{1}{6}abe^{(-2c)}c^{3}(t-2)^{3}-\frac{1}{24}abe^{(-2c)}c^{4}(t-2)^{4}$ $+\frac{1}{120}abe^{(-2c)}c^{5}(t-2)^{5}-\frac{1}{720}abe^{(-2c)}c^{6}(t-2)^{6}+\frac{1}{5040}abe^{(-2c)}c^{7}(t-2)^{7}$
8	$a(1-be^{(-2c)})+abe^{(-2c)}c(t-2)-\frac{1}{2}abe^{(-2c)}c^{2}(t-2)^{2}+\frac{1}{6}abe^{(-2c)}c^{3}(t-2)^{3}-\frac{1}{24}abe^{(-2c)}c^{4}(t-2)^{4}$ $+\frac{1}{120}abe^{(-2c)}c^{5}(t-2)^{5}-\frac{1}{720}abe^{(-2c)}c^{6}(t-2)^{6}+\frac{1}{5040}abe^{(-2c)}c^{7}(t-2)^{7}-\frac{1}{40320}abe^{(-2c)}c^{8}(t-2)^{8}$
9	$a(1-be^{(-2c)})+abe^{(-2c)}c(t-2)-\frac{1}{2}abe^{(-2c)}c^{2}(t-2)^{2}+\frac{1}{6}abe^{(-2c)}c^{3}(t-2)^{3}-\frac{1}{24}abe^{(-2c)}c^{4}(t-2)^{4}$ $+\frac{1}{120}abe^{(-2c)}c^{5}(t-2)^{5}-\frac{1}{720}abe^{(-2c)}c^{6}(t-2)^{6}+\frac{1}{5040}abe^{(-2c)}c^{7}(t-2)^{7}-\frac{1}{40320}abe^{(-2c)}c^{8}(t-2)^{8}$ $+\frac{1}{362880}abe^{(-2c)}c^{9}(t-2)^{9}$

Similarly, the remainder series expansions of monomolecular growth model w.r.t. (t-r) could easily be shown in a similar manner where t is time, r is integer number $3 \leq r \leq t_{n-1}, t_{n-1}$ final value of time and n is the number of data points.

2. Material and Methods

In this study, the mean fork length measurements of the flathead trout (*Salmo platycephalus*), which is listed in the IUCN red list of threatened species, which were sampled from the Zamanti Stream of Seyhan River were used (Kara et al. 2011). For the presentation of monomolecular growth model, the measurements of the fork lengths (cm) in the age-structured female of *S. platycephalus* from Zamanti Stream of the River Seyhan in Table 1 in their articles were used in this study. The mean values of fork lengths (cm) of female *S. platycephalus* from Zamanti Stream of the River Seyhan were given in Table 3.

While the degrees of Taylor series approximations in the neighborhood of (t-l) were increasing, these expansions did not show uniform convergence and regular decreasing of Sum of Squared Errors (SSE) and regular increasing of coefficient of determination (R^2) were not found. However, R^2 of the series expansion having m-th degree polynomial with respect to (t-l) generally increased uniformly or kept on the same value while l was increasing where t was time, m and l were integer numbers $1 < m \leq n-1, 1 \leq l \leq 10$.

Since we got the same Sum of Squared Errors (SSE) and R^2 of all series expansions for 10 and higher degrees, we did not make a table for them.

The highest R^2 of series expansions of Monomolecular growth model in this study was found when the degree of series expansions of Monomolecular growth model was 2.

3. Results and Discussions

By using Table 3, the series expansions of Monomolecular growth model were given in the following tables. Since this monomolecular growth model is a nonlinear model, we have started to fit the model by using second degree polynomial and then we have found the third, fourth, fifth, sixth seventh, eighth and ninth degree polynomials respectively.

For each degree polynomial, we got the series expansions w.r.t. t-l where l is an integer, $1 \leq l \leq 10$, respectively. While l was increasing, R^2 of the series expansions generally increased uniformly or kept on the same value (Table 4-11).

Table 3. The mean values of fork lengths (cm) of female *S. platycephalus* from Zamanti Stream of the River Seyhan

Age (year)	1	2	3	4	5	6	7	8	9	10
Mean values of fork lengths (cm)	13.68	18.03	21.63	25.55	28.29	30.85	33.37	36.03	38.30	40.0

Table 4. Second degree series expansions of Monomolecular growth model with respect to (t-l) where t is time and l is integer number: $1 \leq l \leq 10$ and their values of R^2

Series expansion of M.G.M. w.r.t.(t-l)	$y = a(1 - b\exp(-ct))$	R^2
t-1	9.79727273845771 + 4.12509090365373 t-0.137272726609751 $(t-1)^2$	0.9992
t-2	10.2090909150553 + 3.85054545160926 t-0.137272726777321 $(t-2)^2$	0.9992
t-3	10.8954545349028 + 3.57600000498158 t-0.137272728086516 $(t-3)^2$	0.9992
t-4	11.8563636344694 + 3.30145454831520 t-0.137272728611029 $(t-4)^2$	0.9992
t-5	13.0918181926290 + 3.02690909103175 t-0.137272728633673 $(t-5)^2$	0.9992
t-6	14.6018181608905 + 2.75236363834126 t-0.137272726043862 $(t-6)^2$	0.9992
t-7	16.3863636528725 + 2.47781817983988 t-0.137272727819698 $(t-7)^2$	0.9992
t-8	18.4454544770987 + 2.20327273595877 t-0.1372s72725828273 $(t-8)^2$	0.9992
t-9	20.7790908890516 + 1.92872727527316 t-0.137272726977466 $(t-9)^2$	0.9992
t-10	23.3872726530827 + 1.65418182738655 t-0.137272726423662 $(t-10)^2$	0.9992

Table 5. Third degree series expansions of monomolecular growth model with respect to (t-l) where t is time and l is integer number: $1 \leq l \leq 10$ and their values of R^2

Series expansion of M.G.M. w.r.t.(t-l)	$y = a(1 - b\exp(-ct))$	R^2
t-1	9.30969569350388 + 4.49124637333656 t-0.264754403595291 $(t\text{-}1)^2$ + 0.0104046684589583 $(t\text{-}1)^3$	0.9994
t-2	9.90212997421744 + 4.03908724875305 t-0.226261426324639 $(t\text{-}2)^2$ + 0.00844980229271098 $(t\text{-}2)^3$	0.9995
t-3	10.9333701475896 + 3.61011181030404 t-0.187541516079389 $(t\text{-}3)^2$ + 0.00649505649713633 $(t\text{-}3)^3$	0.9995
t-4	12.1222055671860 + 3.26302883940577 t-0.163539869996117 $(t\text{-}4)^2$ + 0.00546430742614732 $(t\text{-}4)^3$	0.9995
t-5	13.4668711472498 + 2.96147203457142 t-0.146123569535799 $(t\text{-}5)^2$ + 0.00480664059959031 $(t\text{-}5)^3$	0.9995
t-6	14.9558879932095 + 2.69022271374810 t-0.132336919741597 $(t\text{-}6)^2$ + 0.00433992824886336 $(t\text{-}6)^3$	0.9995
t-7	16.5745477080990 + 2.44158367308336 t-0.120836950190006 $(t\text{-}7)^2$ + 0.00398691190809018 $(t\text{-}7)^3$	0.999446
t-8	18.3076296454555 + 2.21106253551317 t-0.110901677760591 $(t\text{-}8)^2$ + 0.00370837758847302 $(t\text{-}8)^3$	0.999435
t-9	20.1400594056770 + 1.99579058304295 t-0.102097323548924 $(t\text{-}9)^2$ + 0.00348194964756294 $(t\text{-}9)^3$	0.999426
t-10	22.0569833586811 + 1.79382728743223 t-0.0941418928069199 $(t\text{-}10)^2$ + 0.00329377528645510 $(t\text{-}10)^3$	0.999417

Table 6. Fourth degree series expansions of monomolecular growth model with respect to (t-l) where t is time and l is integer number: $1 \leq l \leq 10$ and their values of R^2

Series expansion of M.G.M. w.r.t.(t-l)	$y = a(1 - b\exp(-ct))$	R^2
t-1	9.47458051718407 + 4.33327412545796 t - 0.203271975632709 $(t\text{-}1)^2$ + 0.00635693240743342 $(t\text{-}1)^3$ – 0.000149100445991981 $(t\text{-}1)^4$	0.9994
t-2	10.0400128909355 + 3.95287977204475 t - 0.188100100279496 $(t\text{-}2)^2$ + 0.00596723566371022 $(t\text{-}2)^3$ – 0.000141977202882983 $(t\text{-}2)^4$	0.9995
t-3	10.9336304059838 + 3.59575147209418 t - 0.172638556825480 $(t\text{-}3)^2$ + 0.00552579371036201 $(t\text{-}3)^3$ – 0.000132651694489416 $(t\text{-}3)^4$	0.9995
t-4	12.0880279736779 + 3.26607004861609 t - 0.157590051719838 $(t\text{-}4)^2$ + 0.00506921652656824 $(t\text{-}4)^3$ – 0.000122296534169474 $(t\text{-}4)^4$	0.9995
t-5	13.4422622614041 + 2.96490288366247 t - 0.143427502005405 $(t\text{-}5)^2$ + 0.00462554742121356 $(t\text{-}5)^3$ – 0.000111880681773408 $(t\text{-}5)^4$	0.9995
t-6	14.9464597878095 + 2.69109761592105 t - 0.130383012950361 $(t\text{-}6)^2$ + 0.00421134978913197 $(t\text{-}6)^3$ – 0.000102019427100373 $(t\text{-}6)^4$	0.9995
t-7	16.5617122581950 + 2.44236626531178 t - 0.118512896042437 $(t\text{-}7)^2$ + 0.00383379750144952 $(t\text{-}7)^3$ – 0.0000930152146281465 $(t\text{-}7)^4$	0.9995
t-8	18.2579754418125 + 2.21606856771308 t - 0.107771580124842 $(t\text{-}8)^2$ + 0.00349408968411455 $(t\text{-}8)^3$ – 0.0000849620746941819 $(t\text{-}8)^4$	0.9995
t-9	20.0117513733793 + 2.00964019135257 t - 0.0980657209117066 $(t\text{-}9)^2$ + 0.00319025122318377 $(t\text{-}9)^3$ – 0.0000778383830690623 $(t\text{-}9)^4$	0.9995
t-10	21.8042068092383 + 1.82077682414404 t - 0.0892863825311034 $(t\text{-}10)^2$ + 0.00291892193111522 $(t\text{-}10)^3$ – 0.0000715683483730906 $(t\text{-}10)^4$	0.9995

Table 7. Fifth degree series expansions of monomolecular growth model with respect to (t-l) where t is time and l is integer number: 1 ≤ *l* ≤ 10 and their values of R^2

Series expansion of M.G.M. w.r.t.(t-l)	$y = a(1 - b\exp(-ct))$	R^2
t-1	$9.40868716575077 + 4.37303095721827\, t - 0.212935552259530\, (t-1)^2 + 0.00691229947979656\, (t-1)^3 - 0.000168289948266227\, (t-1)^4 + 0.327781014352497\, 10^{-5}\, (t-1)^5$	0.9995
t-2	$10.0175259368578 + 3.96533238396250\, t - 0.192490416732080\, (t-2)^2 + 0.00622941651054628\, (t-2)^3 -$ t230.000151198293611204 $(t-2)^4 + 0.293586713326831\, 10^{-5}\, (t-2)^5$	0.9995
t-3	$10.9320133143665 + 3.59809125169093\, t - 0.174403923630450\, (t-3)^2 + 0.00563571561512445\, (t-3)^3 - 0.000136585045651801\, (t-3)^4 + 0.264817829283496\, 10^{-5}\, (t-3)^5$	0.9995
t-4	$12.0921991722074 + 3.26571009222068\, t - 0.158190064203080\, (t-4)^2 + 0.00510845435467667\, (t-4)^3 - 0.000123726035000751\, (t-4)^4 + 0.239730152005650\, 10^{-5}\, (t-4)^5$	0.9995
t-5	$13.4460918097456 + 2.96426666812477\, t - 0.143554380714029\, (t-5)^2 + 0.00463472926683421\, (t-5)^3 - 0.000112226018129886\, (t-5)^4 + 0.217396588584642\, 10^{-5}\, (t-5)^5$	0.9995
t-6	$14.9485909328579 + 2.69068136242790\, t - 0.130301186300534\, (t-6)^2 + 0.00420671615955950\, (t-6)^3 - 0.000101858977743396\, (t-6)^4 + 0.197308322280839\, 10^{-5}\, (t-1)^5$	0.9995
t-7	$16.5610214114641 + 2.44231720646971\, t - 0.118286119279272\, (t-7)^2 + 0.00381921615998837\, (t-7)^3 - 0.0000924859917984839\, (t-7)^4 + 0.179170977931247\, 10^{-5}\, (t-7)^5$	0.9995
t-8	$18.2507330484769 + 2.21679685700694\, t - 0.107391435257727\, (t-8)^2 + 0.00346834377426474\, (t-8)^3 - 0.0000840109491106890\, (t-8)^4 + 0.162794463982452\, 10^{-5}\, (t-8)^5$	0.9995
t-9	$19.9905818126677 + 2.01193019802790\, t - 0.0975136870889489\, (t-9)^2 + 0.00315084462307295\, (t-9)^3 - 0.0000763571412520676\, (t-9)^4 + 0.148034288393489\, 10^{-5}\, (t-9)^5$	0.9995
t-10	$21.7582831304007 + 1.82569414366509\, t - 0.0885581823926250\, (t-10)^2 + 0.00286376981449239\, (t-10)^3 - 0.0000694558424373283\, (t-10)^4 + 0.134762620215244\, 10^{-5}\, (t-10)^5$	0.9995

Table 8. Sixth degree series expansions of monomolecular growth model with respect to (t-l) where t is time and l is integer number: 1 ≤ *l* ≤ 10 and their values of R^2

Series expansion of M.G.M. w.r.t.(t-l)	$y = a(1 - b\exp(-ct))$	R^2
t-1	$9.42059358006101 + 4.36592196100265\, t - 0.211260184656285\, (t-1)^2 + 0.00681503487230725\, (t-1)^3 - 0.000164884477828826\, (t-1)^4 + 0.319140154535179\, 10^{-5}\, (t-1)^5 - 0.514756551465048\, 10^{-7}\, (t-1)^6$	0.9995
t-2	$10.0209045784593 + 3.96348444989825\, t - 0.191867873061657\, (t-2)^2 + 0.00619207352832331\, (t-2)^3 - 0.000149875695582871\, (t-2)^4 + 0.290212627788763\, 10^{-5}\, (t-2)^5 - 0.468295682190231\, 10^{-7}\, (t-2)^6$	0.9995
t-3	$10.9322710659896 + 3.59778942945332\, t - 0.174196517984154\, (t-3)^2 + 0.00562279097073498\, (t-3)^3 - 0.000136121169354206\, (t-3)^4 + -0.263626698453412\, 10^{-5}\, (t-3)^5 - 0.425472861588243\, 10^{-7}\, (t-3)^6$	0.9995
t-4	$12.0918513636333 + 3.26572955597728\, t - 0.158129811098948\, (t-4)^2 + 0.00510453314837866\, (t-4)^3 - 0.000123583237476608\, (t-4)^4 + -0.239360836985466\, 10^{-5}\, (t-4)^5 - 0.386336174793578\, 10^{-7}\, (t-4)^6$	0.9995
t-5	$13.4458939526682 + 2.96429409260662\, t - 0.143537300900973\, (t-5)^2 + 0.00463358360232053\, (t-5)^3 - 0.000112183889823033\, (t-5)^4 + -0.217287114526625\, 10^{-5}\, (t-5)^5 - 0.350716505238147\, 10^{-7}\, (t-5)^6$	0.9995
t-6	$14.9485675920296 + 2.69067986274207\, t - 0.130289513064148\, (t-6)^2 + 0.00420596480702418\, (t-6)^3 - 0.000101831718120799\, (t-6)^4 + -0.197237956877166\, 10^{-5}\, (t-6)^5 - 0.318358663611811\, 10^{-7}\, (t-6)^6$	0.9995
t-7	$16.5609049411139 + 2.44232047196420\, t - 0.118264680759751\, (t-7)^2 + 0.00381782676918929\, (t-7)^3 - 0.0000924354664421007\, (t-7)^4 + -0.179040401210935\, 10^{-5}\, (t-7)^5 - 0.288989591870664\, 10^{-7}\, (t-7)^6$	0.9995

Table 8. Cont.

Series expansion of M.G.M. w.r.t.(t-l)	$y = a(1 - b\exp(-ct))$	R^2
t-8	$18.2499111001385 + 2.21687889478561\, t - 0.107350899063766\, (t-8)^2 + 0.00346559767936362\, (t-8)^3 - 0.0000839096415117805\, (t-8)^4 + -0.162530763003705\, 10^{-5}\, (t-8)^5 - 0.262348168484488\, 10^{-7}\, (t-8)^6$	0.9995
t-9	$19.9878173338958 + 2.01222719962025\, t - 0.0974462204345593\, (t-9)^2 + 0.00314602177420548\, (t-9)^3 - 0.0000761762715857846\, (t-9)^4 + -0.147559674260086\, 10^{-5}\, (t-9)^5 - 0.238195974223881\, 10^{-7}\, (t-9)^6$	0.9995
t-10	$21.7514354619634 + 1.82642517953706\, t - 0.0884578616411097\, (t-10)^2 + 0.00285614155775822\, (t-10)^3 - 0.0000691646659206809\, (t-10)^4 + 0.133991986466501\, 10^{-5}\, (t-10)^5 - 0.216317732451095\, 10^{-7}\, (t-10)^6$	0.9995

Table 9. Seventh degree series expansions of monomolecular growth model with respect to (t-l) where t is time and l is integer number: $1 \leq l \leq 10$ and their values of R^2

Series expansion of M.G.M. w.r.t.(t-l)	$y = a(1 - b\exp(-ct))$	R^2
t-1	$9.41904506770986 + 4.36683428199201\, t - 0.211468985186246\, (t-1)^2 + 0.00682708629149100\, (t-1)^3 - 0.000165304762742446\, (t-1)^4 + 0.32020294947077410^{-5}\, (t-1)^5 - 0.51687323436616310^{-7}\, (t-1)^6 + 0.71514806695216010^{-9}\, (t-1)^7$	0.9995
t-2	$10.0205116339910 + 3.96369663453140\, t - 0.191936817487321\, (t-2)^2 + 0.00619619264606282\, (t-2)^3 - 0.000150021256251393\, (t-2)^4 + 0.29058331285483410^{-5}\, (t-2)^5 - 0.46903720967933610^{-7}\, (t-2)^6 + 0.64892892821809810^{-9}\, (t-2)^7$	0.9995
t-3	$10.9322420861768 + 3.59782016586369\, t - 0.174216429537601\, (t-3)^2 + 0.00562402842491599\, (t-3)^3 - 0.000136165526157091\, (t-3)^4 + 0.26374049507282710^{-5}\, (t-3)^5 - 0.42570153831870810^{-7}\, (t-3)^6 + -0.58896139992308010^{-9}\, (t-3)^7$	0.9995
t-4	$12.0918805814757 + 3.26572793698589\, t - 0.158134634259336\, (t-4)^2 + 0.00510484707337331\, (t-4)^3 - 0.000123594668703907\, (t-4)^4 + 0.23939039759613010^{-5}\, (t-4)^5 - 0.38639586333698210^{-7}\, (t-4)^6 + 0.53457841740580010^{-9}\, (t-4)^7$	0.9995
t-5	$13.4459125996961 + 2.96429113892640\, t - 0.143538080714358\, (t-5)^2 + 0.00463363856642493\, (t-5)^3 - 0.000112185941828499\, (t-5)^4 + 0.21729248604140110^{-5}\, (t-5)^5 - 0.35072743013731510^{-7}\, (t-5)^6 + 0.48523036191591910^{-9}\, (t-5)^7$	0.9995
t-6	$14.9485767158895 + 2.69067804969645\, t - 0.130289062741515\, (t-6)^2 + 0.00420593856679296\, (t-6)^3 - 0.000101830799466637\, (t-6)^4 + 0.19723562872524410^{-5}\, (t-6)^5 - 0.31835401995598010^{-7}\, (t-6)^6 + 0.44044178895711510^{-9}\, (t-6)^7$	0.9995
t-7	$16.5608993762395 + 2.44232015967991\, t - 0.118263060995659\, (t-7)^2 + 0.00381772267943115\, (t-7)^3 - 0.0000924316921171480\, (t-7)^4 + 0.17903066147093210^{-5}\, (t-7)^5 - 0.28896995006590910^{-7}\, (t-7)^6 + 0.39978929397593610^{-9}\, (t-7)^7$	0.9995
t-8	$18.2498382397470 + 2.21688593414659\, t - 0.107347219417489\, (t-8)^2 + 0.00346534910048724\, (t-8)^3 - 0.0000839004805160185\, (t-8)^4 + 0.16250693195277910^{-5}\, (t-8)^5 - 0.26229987768726310^{-7}\, (t-8)^6 + 0.36289208453007710^{-9}\, (t-8)^7$	0.9995
t-9	$19.9875256215116 + 2.01225811292155\, t - 0.0974391540006104\, (t-9)^2 + 0.00314551719165224\, (t-9)^3 - 0.0000761573607483125\, (t-9)^4 + 0.14751007844537210^{-5}\, (t-9)^5 - 0.23809498986874510^{-7}\, (t-9)^6 + 0.32940639793983510^{-9}\, (t-9)^7$	0.9995
t-10	$21.7506021842729 + 1.82651338857708\, t - 0.0884459365159449\, (t-10)^2 + 0.00285523363259758\, (t-10)^3 - 0.0000691300195734262\, (t-10)^4 + 0.13390034501308610^{-5}\, (t-10)^5 - 0.21613020538759210^{-7}\, (t-10)^6 + 0.29902152130687310^{-9}\, (t-10)^7$	0.9995

Table 10. Eighth degree series expansions of monomolecular growth model with respect to (t-l) where t is time and l is integer number: $1 \le l \le 10$ and their values of R^2

Series expansion of M.G.M. w.r.t.(t-l)	$y = a(1 - b\exp(-ct))$	R^2
t-1	$9.41922024676317 + 4.36673180771694\, t - 0.211445922743992\, (t-1)^2 + 0.00682575744878544\, (t-1)^3 - 0.000165258441065070\, (t-1)^4 + 0.32008582254110810^{-5}\, (t-1)^5 - 0.51663994252229310^{-7}\, (t-1)^6 + 0.71476409873120810^{-9}\, (t-1)^7 - 0.86525782355933010^{-11}\, (t-1)^8$	0.9995
t-2	$10.0205506484247 + 3.96367569302154\, t - 0.191930151391726\, (t-2)^2 + 0.00619579499202496\, (t-2)^3 - 0.000150007210845364\, (t-2)^4 + 0.29054755148702610^{-5}\, (t-2)^5 - 0.46896567615744510^{-7}\, (t-2)^6 + 0.64881085273515210^{-9}\, (t-2)^7 - 0.78542226227850910^{-11}\, (t-2)^8$	0.9995
t-3	$10.9322447145886 + 3.59781750675566\, t - 0.174214763812198\, (t-3)^2 + 0.00562392503661828\, (t-3)^3 - 0.000136161821731124\, (t-3)^4 + 0.26373099323225310^{-5}\, (t-3)^5 - 0.42568244599807710^{-7}\, (t-3)^6 + 0.58892978987005010^{-9}\, (t-3)^7 - 0.71293404995893310^{-11}\, (t-3)^8$	0.9995
t-4	$12.0918785937902 + 3.26572802445820\, t - 0.158134287864910\, (t-4)^2 + 0.00510482457229754\, (t-4)^3 - 0.000123593849881221\, (t-4)^4 + 0.23938828082527910^{-5}\, (t-4)^5 - 0.38639158995072010^{-7}\, (t-4)^6 + 0.53457131987752610^{-9}\, (t-4)^7 - 0.64713024437990810^{-11}\, (t-4)^8$	0.9995
t-5	$13.4459118095858 + 2.96429125345161\, t - 0.143538021394658\, (t-5)^2 + 0.00463363455753615\, (t-5)^3 - 0.000112185794071383\, (t-5)^4 + 0.21729210165626910^{-5}\, (t-5)^5 - 0.35072665121452610^{-7}\, (t-5)^6 + 0.48522906500199810^{-9}\, (t-5)^7 - 0.58739858838489510^{-11}\, (t-5)^8$	0.9995
t-6	$14.94857675957222.69067802301110\, t - 0.130289022260140\, (t-6)^2 + 0.00420593599490003\, (t-6)^3 - 0.000101830706568713\, (t-6)^4 + 0.19723538946587810^{-5}\, (t-6)^5 - 0.31835353801578710^{-7}\, (t-6)^6 + 0.44044098971557210^{-9}\, (t-6)^7 - 0.53317997752395610^{-11}\, (t-6)^8$	0.9995
t-7	$16.5608990035354 + 2.44232013845985\, t - 0.118262949324838\, (t-7)^2 + 0.00381771550277572\, (t-7)^3 - 0.0000924314318855288\, (t-7)^4 + 0.17902998993397910^{-5}\, (t-7)^5 - 0.28896859580060510^{-7}\, (t-7)^6 + 0.39978704632399510^{-9}\, (t-7)^7 - 0.48396598848376710^{-11}\, (t-7)^8$	0.9995
t-8	$18.2498329291325 + 2.21688642041203\, t - 0.107346929585536\, (t-8)^2 + 0.00346532962787820\, (t-8)^3 - 0.0000838997641307609\, (t-8)^4 + 0.16250506998394510^{-5}\, (t-8)^5 - 0.26229610659104710^{-7}\, (t-8)^6 + 0.36288580784210310^{-9}\, (t-8)^7 - 0.43929491496881110^{-11}\, (t-8)^8$	0.9995
t-9	$19.9874998221771 + 2.01226079245202\, t - 0.0974385116066112\, (t-9)^2 + 0.00314547152792012\, (t-9)^3 - 0.0000761556516757550\, (t-9)^4 + 0.14750559922553410^{-5}\, (t-9)^5 - 0.23808587329716210^{-7}\, (t-9)^6 + 0.32939117485103610^{-9}\, (t-9)^7 - 0.39874784041695310^{-11}\, (t-9)^8$	0.9995
t-10	$21.7505171206564 + 1.82652227680403\, t - 0.0884447077546145\, (t-10)^2 + 0.00285514040503485\, (t-10)^3 - 0.0000691264656140825\, (t-10)^4 + 0.13389094952720110^{-5}\, (t-10)^5 - 0.21611098593044410^{-7}\, (t-10)^6 + 0.29898932188418410^{-9}\, (t-10)^7 - 0.36194498599365010^{-11}\, (t-10)^8$	0.9995

Table 11. Ninth degree series expansions of monomolecular growth model with respect to (t-l) where t is time and l is integer number: $1 \le l \le 10$ and their values of R^2

Series expansion of M.G.M. w.r.t.(t-l)	$y = a(1 - b\exp(-ct))$	R^2
t-1	$9.41920304506965 + 4.36674182830374\, t - 0.211448152867675\, (t-1)^2 + 0.00682588576855397\, (t-1)^3 - 0.000165262911593648\, (t-1)^4 + 0.320097122916790\; 10^{-5}\, (t-1)^5 - 0.516662445672942\; 10^{-7}\, (t-1)^6 + 0.714801130186454\; 10^{-9}\, (t-1)^7 - 0.865309792681080\; 10^{-11}\, (t-1)^8 + 0.931119633686037\; 10^{-13}\, (t-1)^9$	0.9995
t-2	$10.0205473509913 + 3.96367746770462\, t - 0.191930711395338\, (t-2)^2 + 0.00619582837350173\, (t-2)^3 - 0.000150008389569884\, (t-2)^4 + 0.290550552208174\; 10^{-5}\, (t-2)^5 - 0.468971677909314\; 10^{-7}\, (t-2)^6 + 0.648820758707661\; 10^{-9}\, (t-2)^7 - 0.785436194048728\; 10^{-11}\, (t-2)^8 + 0.845170951065854\; 10^{-13}\, (t-2)^9$	0.9995

Table 11. Cont.

Series expansion of M.G.M. w.r.t.(t-l)	$y = a(1 - b\exp(-ct))$	**R^2**
t-3	$10.9322445178849 + 3.59781770598524\,t - 0.174214886984482\,(t-3)^2 + 0.00562393267758302\,(t-3)^3 - 0.000136162095456300\,(t-3)^4 + 0.26373169526657310^{-5}\,(t-3)^5 - 0.42568385652833610^{-7}\,(t-3)^6 + 0.58893212510327210^{-9}\,(t-3)^7 - 0.71293734147309010^{-11}\,(t-3)^8 + 0.76715824561667610^{-13}\,(t-3)^9$	0.9995
t-4	$12.0918787174007 + 3.26572801886179\,t - 0.158134309234571\,(t-4)^2 + 0.00510482596073845\,(t-4)^3 - 0.000123593900410824\,(t-4)^4 + 0.239388411456062\;10^{-5}\,(t-4)^5 - 0.386391853676775\;10^{-7}\,(t-4)^6 + 0.534571757897587\;10^{-9}\,(t-4)^7 - 0.647130863188878\;10^{-11}\,(t-4)^8 + 0.696346973740612\;10^{-13}\,(t-4)^9$	0.9995
t-5	$13.4459118804441 + 2.96429124416527\,t - 0.143538025368748\,(t-5)^2 + 0.00463363482863193\,(t-5)^3 - 0.000112185804092434\,(t-5)^4 + 0.217292127762806\;10^{-5}\,(t-5)^5 - 0.350726704161731\;10^{-7}\,(t-5)^6 + 0.485229153208740\;10^{-9}\,(t-5)^7 - 0.587398713267672\;10^{-11}\,(t-5)^8 + 0.632072030877860\;10^{-13}\,(t-5)^9$	0.9995
t-6	$14.9485768174329 + 2.69067801454809\,t - 0.130289022213010\,(t-6)^2 + 0.00420593600508614\,(t-6)^3 - 0.000101830707098784\,(t-6)^4 + 0.197235391041589\;10^{-5}\,(t-6)^5 - 0.318353541445270\;10^{-7}\,(t-6)^6 + 0.440440995686250\;10^{-9}\,(t-6)^7 - 0.533179986235962\;10^{-11}\,(t-6)^8 + 0.573729815761673\;10^{-13}\,(t-6)^9$	0.9995
t-7	$16.5608990050852 + 2.44232013366502\,t - 0.118262942867430\,(t-7)^2 + 0.00381771509336005\,(t-7)^3 - 0.0000924314171076010\,(t-7)^4 + 0.179029951886744\;10^{-5}\,(t-7)^5 - 0.288968519178368\;10^{-7}\,(t-7)^6 + 0.399786919273055\;10^{-9}\,(t-7)^7 - 0.483965809205626\;10^{-11}\,(t-7)^8 + 0.520772759884638\;10^{-13}\,(t-7)^9$	0.9995
t-8	$18.2498326499232 + 2.21688644123361t0.107346909934507\,(t-8)^2 + 0.00346532832659808\,(t-8)^3 - 0.0000838997164784451\,(t-8)^4 + 0.162504946411790\;10^{-5}\,(t-8)^5 - 0.262295856656074\;10^{-7}\,(t-8)^6 + 0.362885392219417\;10^{-9}\,(t-8)^7 - 0.439294327289145\;10^{-11}\,(t-8)^8 + 0.472703895869845\;10^{-13}\,(t-8)^9$	0.9995
t-9	$19.9874980045536 + 2.01226097277797\,t - 0.0974384613159564\,(t-9)^2 + 0.00314546799911813\,(t-9)^3 - 0.0000761555201086904\,(t-9)^4 + 0.147505255043794\;10^{-5}\,(t-9)^5 - 0.238085173542263\;10^{-7}\,(t-9)^6 + 0.329390007216954\;10^{-9}\,(t-9)^7 - 0.398746185388712\;10^{-11}\,(t-9)^8 + 0.429071966371191\;10^{-13}\,(t-9)^9$	0.9995
t-10	$21.7505097623127 + 1.82652302766214\,t - 0.0884445967195379\,(t-10)^2 + 0.00285513206254474\,(t-10)^3 - 0.0000691261484329618\,(t-10)^4 + 0.133890112050843\;10^{-5}\,(t-10)^5 - 0.216109274027377\;10^{-7}\,(t-10)^6 + 0.298986455205285\;10^{-9}\,(t-10)^7 - 0.361940912525508\;10^{-11}\,(t-10)^8 + 0.389467584765789\;10^{-13}\,(t-10)^9$	0.9995

The research for the second degree expansions of Monomolecular growth model in the neighborhood of (t-l), where t is time and l is integer number, $1 \le l \le 10$ was done and for each one the same R^2 was found (0.9992). Moreover, for the third, fourth, ... and ninth degree series expansions of Monomolecular growth model, research for expansion in the neighborhood of (t-l) was done and for each one the same R^2 was generally found (0.9995). Therefore, the highest value of R^2 (0.9995) is the same for all the degree series expansions except the second degree series expansions.

Since the number of data points is 10 and the only ninth degree polynomial for monomolecular model is unique, all series expansions are actually the same function. For that reason, R^2 is the same for all series expansions of ninth degree polynomial.

For each degree of polynomial, we got the approach equations. While the degree of polynomial was increasing, R^2 of the polynomial approximations generally increased uniformly (Table 12).

For ninth degree polynomial, the model is not of full rank. R^2 of ninth degree polynomial is a little bit lower than that of eighth degree polynomial.

Actually, for ninth degree polynomial, the degree of freedom is zero and values for the items of ninth degree polynomial are not available.

For fitting a regression, we need to have this inequality: $n>k$ where n is the number of observation and k is the rank of the model.

Table 12. Polynomial approximations and their values of R^2

Degree of Polynomial	Polynomial Approximations	R^2
1	$12.6800000000000 + 2.88963636363636\ t$	0.9850
2	$9.66000000000002 + 4.39963636363636\ t - 0.137272727272727\ t^2$	0.9992
3	$8.91800000000002 + 5.05775058275058\ t - 0.279965034965035\ t^2 + 0.00864801864801866\ t^3$	0.9995
4	$8.32000000000002 + 5.82441724941724\ t - 0.560495337995337\ t^2 + 0.0469813519813518\ t^3 - 0.00174242424242424\ t^4$	0.9996
5	$10.1679999999998 + 2.77801724941758\ t + 1.03873543123526\ t^2 - 0.310557109557070\ t^3 + 0.0337960372960334\ t^4 - 0.00129230769230755\ t^5$	0.9998
6	$9.47899999999632 + 4.13746724942440\ t + 0.111094900927539\ t^2 - 0.0174008595555784\ t^3 - 0.0129302884617817\ t^4 + 0.00235144230771121\ t^5 - 0.000110416666667237\ t^6$	0.9998
7	$1.014000000067652 + 3.2432115197044\ t - 15.7561517995567\ t^2 + 6.47046653464774\ t^3 - 1.45687990195190\ t^4 + 0.180381372547998\ t^5 - 0.0115215686273895\ t^6 + 0.000296393557421459\ t^7$	0.9999
8	$-14.6769999984934 + 62.5769579448088\ t - 53.7547543667714\ t^2 + 25.4042929220851\ t^3 - 6.87401965850935\ t^4 + 1.10402234470847\ t^5 - 0.104142054732675\ t^6 + 0.00533260387458217\ t^7 - 0.000114459325390350\ t^8$	0.99998
9	$4.91204830231449 + 9.86267281963855\ t + 2.63678740146010\ t^2 - 6.77458419190916\ t^3 + 4.08939448182362\ t^4 - 1.23561364198412\ t^5 + 0.211386733603548\ t^6 - 0.0207811769095135\ t^7 + 0.00109583759166933\ t^8 - 0.0000240439380477455\ t^9$	0.99996

Since degree of freedom for SSE is n-2, we could say that the smallest SSE and the best approximation may be in n-2 especially when n is large.

As it was seen in Table 12, as the degree of the approximation of function increases, a better approach is provided. Even significantly better results than series expansions of monomolecular growth model were found. It shows that it can be directly used (n-2)th degree polynomial approximation instead of using series expansions of any model. For example, in the following figure (Figure 3) it can be seen that how ninth degree polynomial deviates from the data points while eighth degree polynomial is fitting to the data set.

Lagrange interpolation function of ninth degree polynomial:

$-211.658898\ x^2 + 0.000077077833\ x^9 - 0.0039298117\ x^8 + 0.0861827183\ x^7 - 1.06246897\ x^6 + 8.06832009\ x^5 - 38.83971500\ x^4 + 117.307851\ x^3 + 207.6025850\ x - 67.82000300$

The graphs of ninth degree polynomial approximation and ninth degree Lagrange interpolation function were compared and the following figure (Figure 4) was found.

As it was seen in Figure 4, there are deviations especially at the endpoints between ninth degree polynomial approximation and ninth degree Lagrange interpolation polynomial.

Although degree of Lagrange interpolation polynomial is nine and R^2 of Lagrange interpolation polynomial is 1, we could say that the result of ninth degree polynomial approximation is much better. Because the problem at the endpoints was clearly seen.

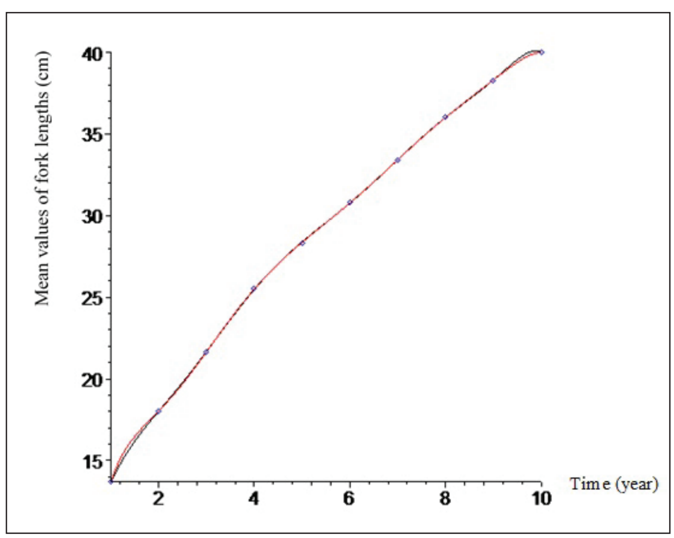

Figure 3. Eighth degree (red) and ninth degree (black) polynomial approximation.

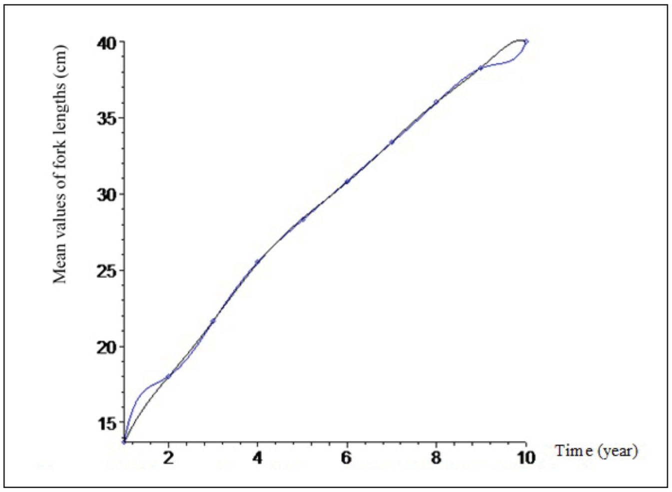

Figure 4. Ninth degree polynomial approximation (black) and ninth degree Lagrange interpolation polynomial (blue).

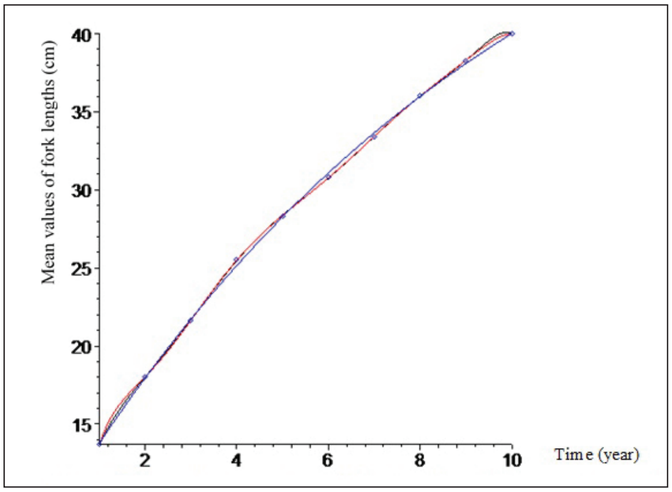

Figure 5. Monomolecular growth model (blue) and its ninth degree series expansion w.r.t. (t-l) (green) with eighth degree (red) and ninth degree (black) polynomial approximations.

Figure 5 showing Monomolecular growth model and its ninth degree series expansion w.r.t. (t-1) with eighth and ninth degree polynomial approximations is presented below. In Figure 5, the graphics of Monomolecular growth model and its ninth degree series expansion overlap. It also seems that eighth degree polynomial shows better approach.

4. Conclusion

While the degrees of Taylor series approximations in the neighborhood of (t-l) were increasing, these expansions did not show uniform convergence and regular decreasing of Sum of Squared Errors (SSE) and regular increasing of coefficient of determination (R^2) were not found. However, R^2 of the series expansion having m-th degree polynomial with respect to (t-l) generally increased uniformly or kept on the same value while l is increasing. Since we got the same Sum of Squared Errors (SSE) and R^2 of all series expansions for 10 and higher degrees, we did not make a table for them.

The highest R^2 of series expansions of Monomolecular growth model in this study was found as 0.9995 when the degree of series expansions of Monomolecular growth model was from 3 to 9.

Therefore, the highest value of R^2 (0.9995) is the same for all the degree series expansions except second degree series expansions.

Since the number of data points is 10 and the only ninth degree polynomial for monomolecular model is unique, all series expansions are actually the same function. For that reason, R^2 is the same for all series expansions of ninth degree polynomial.

In polynomial approximation, for each degree of polynomial, we got the approach equations. While the degree of polynomial was increasing, R^2 of the polynomial approximations generally increased uniformly.

As the degree of the approximation of function increases, a better approach is provided. Even significantly better results than series expansions of monomolecular growth model were found. It shows that it can be directly used (n-2)th degree polynomial approximation instead of using series expansions of any model for avoiding the deviation.

It can be said that if there are too many data points especially much more than 10, polynomial approximation can be much more problematic. However, high degree series expansions of Monomolecular growth model do not have any problem. For that reason, polynomial approximation should be used especially when the number of data points is 10 or fewer. Nevertheless, if researcher decides to do polynomial approximation of any model, he can do (n-1) th degree polynomial approximation. Although R^2 of (n-1) th degree polynomial is closer or equal to one, in order to see whether there is any deviation particularly at endpoints or not he must draw the graph of the polynomial function. If there is any deviation for (n-1)th degree polynomial approximation, (n-2)th degree series expansion and its R^2 should be used.

5. Acknowledgement

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6. References

Bethard, GL. 1997. A micro computer simulation to evaluate management strategies for rearing dairy replacement, (Ph. D. Thesis, unpublished), April 18, 1997 Blacksburg, Virginia.

Brody, B. 1945. Bioenergetics and Growth. Reinhold Publishing Corporation, New York.

Fabens, AJ. 1965. Properties and fitting of the Von Bertalanffy growth curve. *Growth Dev. Aging*, 29:265-289.

Fitzpatrick, P. 2014. Advanced Calculus (Just the facts 101). Cram 101 Publishing, Irland.

Kara, C., Alp, A., Can, F. 2011. Growth and Reproductive Properties of Flathead Trout (Salmo platycephalus Behnke, 1968) Population from Zamanti Stream, Seyhan River. Turkey, *Turk. J. Fish Aquat. Sci.*, 11: 367-375.