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Veracity and Satisfiability Condition of State Equation of Bubble Liquid

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Abstract: Calculations have been performed to confirm the suitability of the equation of state of a gas-liquid mixture proposed in this article. The dependence of the dimensionless radius of the air bubble, of the pressure in the bubble, of gas temperature on the dimensionless time and analogical dependence for the case when the pressure went down abruptly at room temperature in the water depicted. Calculations have shown that the obtained equations of state are in good agreement with the results known in the literature.

Keywords: State equation, Gas bubble, Pressure, Gas-liquid mixture, Temperature, Volume concentration)

There are hundreds of equations of state for liquids and gases that have been published. They describe thermal, caloric, acoustic and other properties of one-component and multi-component substances [8]-[10]. Rather arbitrarily, these equations can be divided into local and general ones. The former is intended to describe the separate regions of a thermodynamic surface, while the latter allows description by a single equation of thermodynamic properties for both liquids and gases to be made. However, despite long-term investigations in this field of thermodynamics, equations of different forms and structures are used for the description of thermodynamic characteristics of a substance. Seemingly, the preferable form of the general equation of state has not been yet chosen in the literature.

1 Theoretical background

Let's follow the dependence of mean pressure of the gas -fluid mixture

$$p/p_0 \approx (1-\alpha_2)\,(p_2/p_0 - 2\sigma/p_0 R)$$

on density

$$\rho/\rho_0 \approx (1-\alpha_2)\,/(1-\alpha_{20}).$$

In the dimensionless form, we can write:

$$P \approx (1-\alpha_2)\,(P_2 - S/Y_1)\,, \Re = \rho/\rho_0 \approx (1-\alpha_2)\,/(1-\alpha_{20}).$$

$$Y_1 = R/R_0, S = \frac{2\sigma}{R_0 p_0}.$$

Here, $P = p/p_0,\ P_2 = p_2/p_0,\ Y_1 = R/R_0$
The volumetric gas content equals the ratio of total gas volume to the volume of the whole liquid. So, we get:

$$\alpha_2 = \frac{V_{gas}}{V_{mixture}} = \left(\frac{R}{R_*}\right)^3 = \alpha_{20}\left(\frac{R}{R_0}\right)^3 = \alpha_{20} Y_1^3, \alpha_{20} = \left(\frac{R_0}{R_*}\right)^3. \tag{1}$$

Here, R_* is the radius of the equivalent cell around the bubble [1]. The expression for the mean value of liquid pressure by the volume of the cell was obtained in [1].

$$p_1 = p_2 - \frac{2\sigma}{R} - 4\mu_1 \frac{\dot{R}}{R} - \rho_1^0 \left[(1-\varphi_1)\, R\ddot{R} + \frac{3}{2}(1-\varphi_2)\,\dot{R}^2\right], \tag{2}$$

$$\varphi_1 = \frac{3}{2}\frac{\alpha_2^{1/3} - \alpha_2}{1-\alpha_2}, \varphi_2 = \frac{\alpha_2^{1/3}(\alpha_2+2) - 3\alpha_2}{1-\alpha_2}.$$

Let's determine the unknowns Y_1, P_2. For this, we write a system of equations in a dimensionless form of a two-temperature, two-pressure model, which's describing the dynamics of insoluble gas bubbles in a liquid [2]-[3]-[4].

$$\frac{d\theta}{d\tau} = \frac{3\theta}{Y_1 P_2}\left[\gamma\left(1+S\right)G - \left(\gamma-1\right)P_2\dot{Y_1}\right] \tag{3}$$

$$\frac{dP_2}{d\tau} = \frac{3\gamma}{Y_1}\left[\left(1+S\right)G - P_2\dot{Y_1}\right] \tag{4}$$

$$\frac{dY_1}{d\tau} = Y_2$$

$$\frac{dY_2}{d\tau} = -\frac{3}{2}\cdot\frac{Y_2^2}{Y_1} + \frac{P_2 - P_c - S/Y_1}{Y_1}\cdot Pe_2^2 - L\cdot\frac{Y_2}{Y_1^2} \tag{5}$$

$$G = sign\ \left(1-\theta\right)\cdot\sqrt{\frac{3\left(\gamma-1\right)\theta}{Y_1}\left|\dot{Y_1}\left(1-\theta\right)\right|}, Y_1 = R/R_0, P_2 = p_2/p_0, P_c = p_c/p_0$$

$$Y_2 = \dot{R}/u_0, \tau = t/t_0, u_0 = R_0/t_0, \theta = T_2/T_0$$

$$t_0 = R_0^2/a_2, S = 2\sigma/R_0 p_0, L = 4\nu_1/a_2,$$

$$Pe_2 = \frac{R_0}{a_2}\sqrt{\frac{p_0}{\rho_1^0}}$$

Here, $a_2 = \dfrac{\lambda_2}{\rho_{20}c_{p2}}$ is the heat conductivity of gas, $\gamma = c_{p2}/c_{V2}$ is the adiabatic exponent.

In the system of equations (3)- (5) we show the Nusselt parameter which has given in the paper [1]:

$$Nu_2 = \sqrt{\frac{12\left(\gamma-1\right)TR}{a_2}\left|\frac{\dot{R}}{T_0 - T_2}\right|}$$

The system of equations (3)-(5)is closed system of equations describing dynamics and heat exchange of insoluble gas bubble in liquid. So, the initial conditions are in the following form:

For $\tau = 0 : \theta = 1, P_2 = 1 + S, Y_1 = 1, Y_2 = 0$.

We obtain the Cauchy problem for the system of differential equations (3)- (5)

Now, we consider a problem of radial motion of a bubble arising change $\Delta p = p_c - p_0$ of pressure from p_0 to p_c at the moment $\tau = 0$, in liquid far from the bubble.

We write the system of equations (3)- (5)in finite differences having denoted by h time integration step, and by P_{2i}, θ_i, Y_{1i} and Y_{2i}appropriate values of variables at partition points of the time segment. Then we get:

$$\theta_0 = 1, P_{20} = 1 + S, Y_{10} = 1, Y_{20} = 0 \tag{6}$$

$$\theta_i = \theta_{i-1} + \frac{3\theta_{i-1}h}{Y_{1i-1}P_{2i-1}}\left[\gamma\left(1+S\right)G_{i-1} - \left(\gamma-1\right)P_{2i-1}Y_{2i-1}\right], i = 1, 2, ... \tag{7}$$

$$P_{2i} = P_{2i-1} + \frac{3\gamma h}{Y_{1i-1}}\left[\left(1+S\right)G_{i-1} - P_{2i-1}Y_{2i-1}\right], i = 1, 2, ...,$$

$$G_{i-1} = sign\left(1-\theta_{i-1}\right)\cdot\sqrt{\frac{3\left(\gamma-1\right)\theta_{i-1}}{Y_{1i-1}}\left|Y_{2i-1}\left(1-\theta_{i-1}\right)\right|} \tag{8}$$

$$Y_{1i} = Y_{i1-1} + hY_{i2-1}, Y_{2i} = Y_{2i-1} - \frac{3h}{2}\cdot\frac{Y_{2i-1}^2}{Y_{1i-1}} + \frac{P_{2i-1} - P_c - S/Y_{1i-1}}{Y_{1i-1}}\cdot Pe_2^2 h - L\cdot\frac{hY_{2i-1}}{Y_{1i-1}^2} \tag{9}$$

We take the necessary thermophysical characteristics of air and water at atmospheric pressure $p_0 = 10^5 N/m^2$and room temperature $T_0 = 293^0 K$, equal to [7]:

$$\rho_1^0 = 963\frac{kq}{m^3}\frac{kg}{m^3}, \sigma = 0,06\frac{N}{m}, \mu_1 = 2,7\cdot 10^{-4}\frac{N\cdot san}{m}\frac{N\bullet sec}{m}, \nu_1 = \mu_1/\rho_1^0 = 3\cdot 10^{-7}\frac{m^2}{san}\frac{m^2}{sec},$$

$$A_{p2} = 1000\frac{C}{kq\cdot grad}\frac{J}{kg\bullet degree}, A_{V2} = 714,3\frac{C}{kq\cdot grad}\frac{J}{kg\bullet degree}, \lambda_2 = 0,0262\frac{vt}{m\cdot grad},$$

$$\rho_{20} = 1,16\frac{:3}{<^3}\frac{kg}{m^3}, \gamma = 1,34, a_2 = 2,26\cdot 10^{-5}\frac{<^2}{A5:}\frac{m^2}{sec},$$

$$L = 4\nu_1/a_2 = 5,3\cdot 10^{-2}, Pe_2 = \frac{R_0}{a_2}\sqrt{\frac{p_0}{\rho_1^0}} = 4,6\cdot 10^5 R_0.$$

The following equation is serving to the control of saving degree of the gas mass m_2 in the bubble:

$$\frac{d}{dt}\left(\frac{4}{3}\pi R^3\rho_2^0\right)=0 \quad or \quad \frac{Y_1^3 P_2}{\theta}=1+S \tag{10}$$

Choice of the step of finite-difference grid h was selected from the condition of the accuracy of fulfilment of condition (10) and was accepted to be equal to $h=0,001$. In all calculations, this condition was fulfilled within 1.

2 Main results

As can be seen from the graphs, the initial dimensionless values of the radius of the bubble and the temperature of the gas inside the bubble approximately equal to 1, and the initial dimensionless values of the pressure approximately equal to 1, 1.

The dependencies of dimensionless radius of air bubble (curve 1)and air pressure in the bubble (curve 3) and gas temperature (curve 2) on dimensionless time when the pressure in water abruptly increased at room temperature $T_0=293^0K$ from $p_0=1at$ to $p_c=1,5at$ were depicted in Fig.1. The initial radius of the air bubble in rest has accepted to 10 micrometers, and the initial velocity of the pulsation has accepted to be equal to zero.

The radius of the air bubble, the air pressure in the bubble, and the temperature of the gas gradually decrease in the period of time after the pressure abruptly increasing in the water from $p_0=1at$ to $p_c=1,5at$ at room temperature $T_0=293^\circ K$.

Thus, the bubble pulsates over a period of time, and the radius of the bubble, the pressure and temperature inside the bubble, get the maximum value in the first moments of time. In the following moments, the intensity of the pulsation decreases and the radius, pressure and temperature gradually decrease and return to normal.

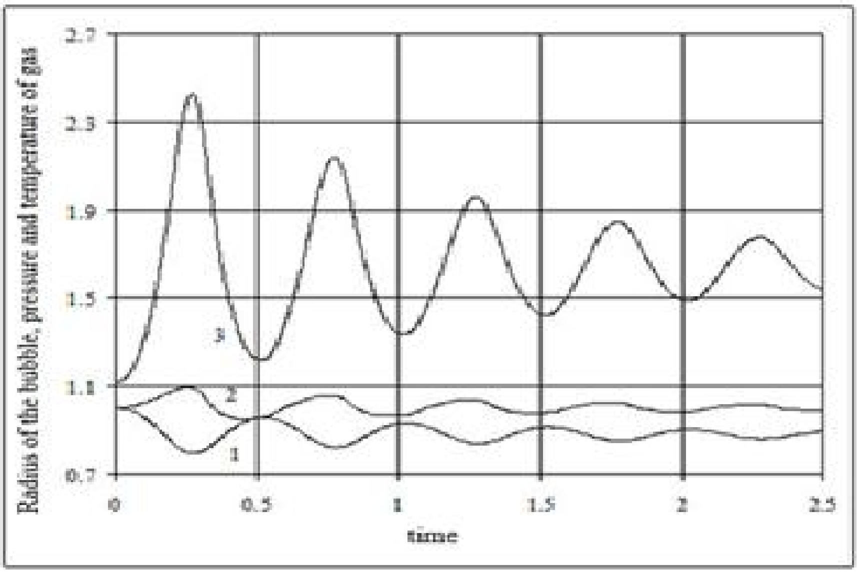

Fig. 1: Dependences of the radius of the bubble, pressure and air temperature in the bubble on time at pressure increase in liquid

Similar dependences are given in Fig. 2 for the case when the pressure in water abruptly decreased at room temperature $T_0=293^0K$ from $p_0=1at$ to $p_c=0,7at$.

The radius of the air bubble, the air pressure in the bubble, and the temperature of the gas gradually decrease in the period of time after the pressure abruptly decreasing in the water from $p_0=1at$ to $p_c=0,7at$ at room temperature $T_0=293^{F0}K$.

Thus, the bubble pulsates over a period of time, and the radius of the bubble, the pressure and temperature inside the bubble, get the maximum value in the first moments of time. In the following moments, the intensity of the pulsation decreases and the radius, pressure and temperature gradually decrease and return to normal.

We compare the results with the state equation of gas-liquid mixture [1] with the incompressible carrier phase with regard to surface tension.

$$\frac{p}{p_0}=\frac{\alpha_{20}\left(1+S\right)\rho/\rho_0}{1-\alpha_{10}\rho/\rho_0}-S\cdot\sqrt[3]{\frac{\alpha_{20}\rho/\rho_0}{1-\alpha_{10}\rho/\rho_0}} \tag{11}$$

3 Conclusion

In Fig. 3 and Fig. 4 have shown the results of calculations in Figs. 1 and 2. by the formula (11)and by the following formula (curves 2).

$$\frac{p}{p_0}=\frac{\alpha_{20}}{\rho_0/\rho}-\alpha_{10}$$

The behavior of the curves in Fig. 1 and 2 shows that the calculations of this work are in good agreement with the formula of R.I. Nigmatulin [1] (curves 1).

Based on the system of equations [2] describing the dynamics of single gas bubbles we analyzed the Veracity and satisfiability condition of Nigmatulin's state equation of bubble surround.

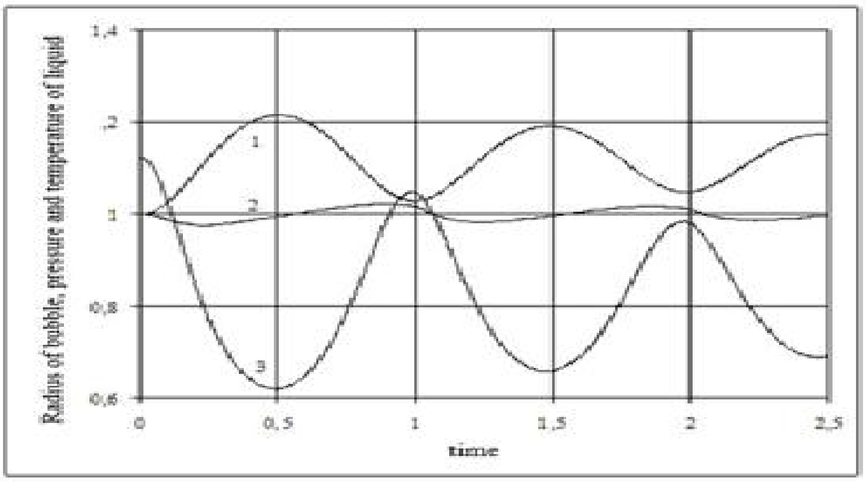

Fig. 2: Dependences of the radius of the bubble, pressure and air temperature in the bubble on time at pressure decrease in liquid

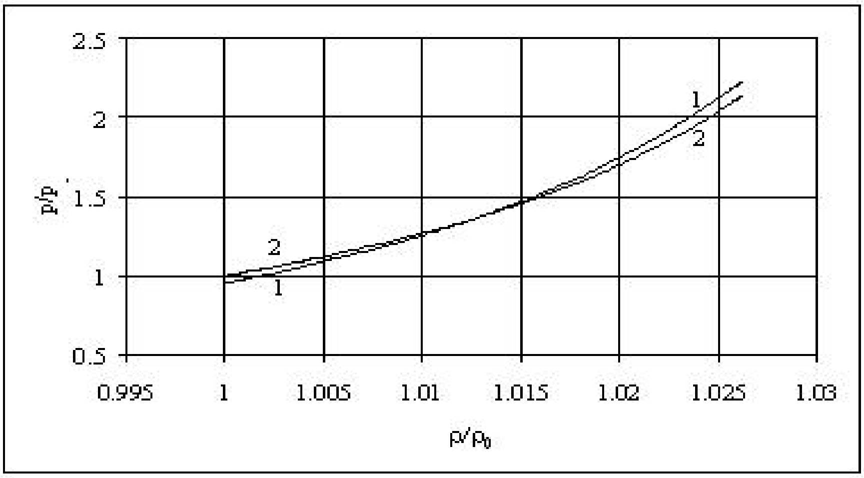

Fig. 3: Comparison of the calculation results with the state equation of [1]

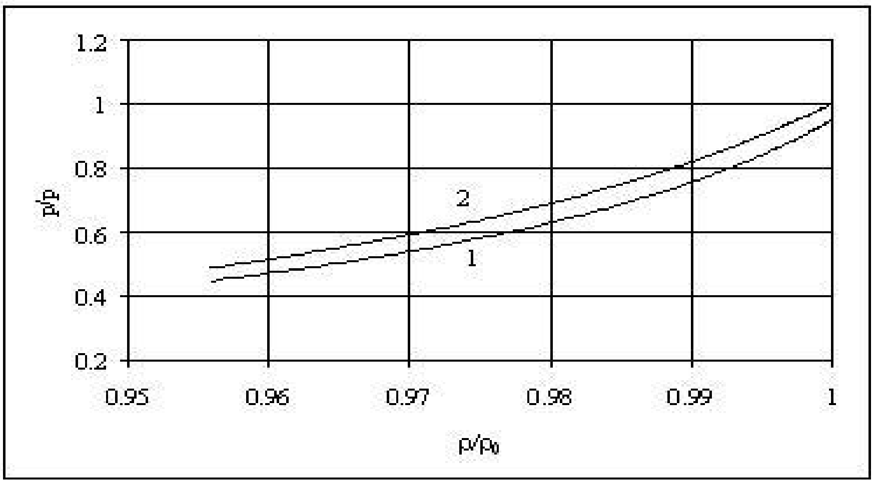

Fig. 4: Comparison of the calculation results with the state equation of [1]

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