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Secondary School Students'Attitudes towards the Concept of Equality and Preservice Teachers' Professional Noticing

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ABSTRACT

With the understanding of importance of pedagogical content knowledge, professional noticing has become one of the research subjects of the mathematics educators for the last 10 years. This qualitative study which explores secondary school students' solution processes regarding the concept of equation ,one of the main components of algebraic learning field and the pre-service teachers' noticing about this is quite rich in terms of its results. The 6th, 7th, and 8th graders' attitudes towards equations were revealed and it was found as stated in literature that students were dominantly "computational thinkers". However, the differences in this general concept of thinking seem to make contribution to literature by involving discrete attitudes. In addition to this, the secondary findings of the study include some evidence revealing that pre-service teachers' level of noticing accumulates at middle levels but they can be improved.

Keywords:
equal sign, equations, relational thinking, preservice teacher noticing

1. Introduction

The algebraic learning field accepted as the gate keeper in mathematics education in the world (National Research Council [NRC], 1998) started to take part more in education curricula due to the re-design of the curricula in Turkey (MEB, 2005). The basic concept for every level of algebra education is the concept of equality and equal sign (Martha W. Alibali, Eric J. Knuth, and Shanta Hattikudur, 2017). Especially, during the transition from arithmetic to algebra, correct interpretation of equality makes this transition process easy (Kieran, 1981; Knuth, Stephens, McNeil, & Alibali, 2006; MacGregor & Stacey, 1997). Before the symbolic introduction of the equal sign, it is known that students can use the equal sign without its symbolic meaning and this develops their problem solving skills (Sherman & Bisans, 2009; Driver & Powell; Alibali, Knuth, & Hattikudur, 2007). Despite its importance, the studies reveal that teachers and preservice teachers fail to notice their students' relational thinking approaches and the misconceptions they exhibit (Asquith, Stephens, Knuth, & Alibali, 2007; Stephens, 2006) and most secondary school teachers do not care these subjects (Falkner, Levi & Carpenter, 1999; Knuth et al., 2006). In addition to this, it is emphasized in the studies carried out that teachers and preservice teachers' knowledge about the students' perception of equality is important to make correct educational decisions (Carpenter, Franke & Zeringue, 2005; Matthews, Rittle-Johnson, McEldon & Taylor; 2013, Knuth et al., 2006). Thus, it will be significant to determine how teachers and prospective teachers evaluate the solutions and approaches. This study, using the framework of noticing, will explore the student attitudes towards the concept of equality through the eyes of the preservice teachers.

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1.1. Theoretical Framework

Noticing of Preservice Teacher

Teacher noticing, one of the subject matters paid attention by the mathematics educators in the last ten years (Schack, Fisher, & Wilhelm, 2017; Sherin, Jacobs, & Philipp, 2011), can refer to doing observations and noticing some points to understand the world (Sherin and et al., 2011). However, this process of noticing becomes meaningful after some professional skills and provides special awareness for the field of study (Goodwin, 1994). The concept of noticing in mathematics courses was first discussed by Mason (2002) and included the concepts "what will be noticed first" and "what will be reflected retrospectively". Classroom is a complex environment and many learning activities are actualized at any time. Thus, it is important that teachers must develop a specific point of view about the learning environment and determine what is considerable. When a student solves a question and makes an explanation, the teacher must analyse these processes and give feedback to his/her student. The feedback must include more than "yes" or "no" (Ball, 1990) because the teacher's ability to recognize his/her student's mathematical thinking requires more than seeing what is true or false about his /her student's answers (Callejo & Zapatera, 2017).

The components of the structure introduced by Mason (2002) and developed by Van Es and Sherin (2002) include the following:

"a) identifying what is important in a teaching situation; (b) making connections between specific events and broader principles of teaching and learning (c) using what one knows about the context to reason about a situation".

This process of noticing became more detailed with the contributions made by Jacobs, Lamb, and Philipp(2010) and included three constructs: attending, interpreting, and responding. The first component, that is attending, involves teacher's identifying mathematical strategies used by his/her student. Interpreting means to analyse student's strategies in detail and discuss them in terms of important components. Responding, the last component, involves teachers' guidance to ameliorate the existing state.

Mason (2002), Van Es&Sherin (2002) and Jacobs et. al.(2010) based their studies on general observation of classroom events, but this study rather than in-class observations explores a specific case via clinical interviews rather than in-class observations as with the studies carried out by Van den Kieboom, Magiera and Moyer (2017).

There are many studies in literature exploring the teachers and pre-service teachers' noticing of different subjects (e.g. Schack, Fisher, Thimas, Eisenhardt, Tassell&Yoder, 2013; Bartell, Webel, Bowen & Dyson, 2013; Van den Kieboom, Magiera, & Moyer, 2017; Callejo &Zapatera, 2017; Fernández, Llinares & Valls, 2012). This study specifically explored preservice teachers' noticing of the concept of equality, one of the main components of algebra, in terms of student thinking.

1.2. Student Thinking About the Equal Sign

According to the studies carried out about students' understanding and perceptions of equal sign Alibali, 1999; Falkner, Levi, & Carpenter, 1999; Kieran, 1981; McNeil & Alibali, 2005), it is revealed that students interpreted the equal sign as an "operator signal" . For example, in the equation of $5+4 = \square +3$, some students say that 9 should go into the box; on the other hand, there are some students who say that the equation beginning with the unknown as in $\square =5+4$ cannot be solved. This case results from the fact that students usually view the equality as a sign which separates the quantities on both sides. In fact, in order to construct the meaning of equal sign correctly, it is important to explain that equality represents the sameness on both sides of the quantities (Carpenter et al., 2003; McNeil & Alibali, 2005). In this stage, discussing equality as a relationship makes sense. Relational thinking is an approach investigating the relationship between the quantities by using the properties of equality, numbers, and operations rather than following a sequence of procedures to solve an equation (Carpenter, Levi, Franke, & Zeringue; 2005). In the light of the relevant literature, Van den Kieboom and et. al. (2017) summarize student thinking process about the concept of equal sign from simple to complex as in Table 1.

Table 1. Student Thinking about the Equal Sign

Rigid Operational Thinking (ROT) Students regard the equal sign as an operator and focus on finding the result.	**Sample student attitude** In the expression of " ☐ =5+4, the number that goes into the box cannot be calculated because the unknown should come after the equal sign ". "With the expression of 5+4= ☐+11, we add 5 and 4 and write 9 into the box and then add 9 and 11and the result is 20".
Flexible Operational Thinking (FOT) Students regard the equal sign as an operator but they do not pay attention if the equal sign is either to the left or right of the equation.	**Sample student attitude** " With the expression of ☐=5+4 or 5+4= ☐ , 9 goes into the box. It is not important if the box comes before the equal sign."
Computational Thinking (CT) Students know that the equal sign on both sides of the equation refers to the equality of numbers. In order to find the unknown number, they add the numbers on both sides and compare them.	**Sample student attitude** "With the expression of 15+4= +☐, if you add 15 and 4, you get 19. Because the right side is equal to the left side, 8 must go into the box to get 19."
Relational Thinking (RT) Students recognise that the equal sign represents equivalence between the two sets of quantities and compare the quantities on both sides of the equation without calculating them.	**Sample student attitude** "With the expression of 15+4=☐+11, both sides must be equal to each other. 11 is 4 less than 15 and, ☐4 less than 4. So I need 4+4=8 to make both sides same".

The studies carried out reveal that students with relational thinking skills at early periods are more successful in advanced algebra topics, algebraic reasoning, and solving problems that require equations (Alibali, Knuth, Hattikudur, McNeil & Stephens, 2007; Jacobs, Franke, Carpenter, Levi & Battey, 2007; Kieran, 1992; Knuth and et al, 2006; NRC, 1998). Undoubtedly, teachers who notice whether or not their students have this skill will be able to make correct educational decisions for the future.

1.3. Teacher Professional Noticing About Equal Sign

For the last ten years, the concept of professional noticing has become a highly increased issue among mathematics education researchers (Schack, Fisher, & Wilhelm, 2017; Sherin, Jacobs, & Philipp, 2011; Fisher, Thomas, Jong, Schack, Dueber, 2019). In addition to this, considering the studies carried out in the field of algebra, it is viewed that relational thinking is accepted as an important stage for students' transition to algebra (Carpenter et al., 2005), but unfortunately students lack this skill (Kieran, 1981; McNeil&Alibali, 2005). Moreover, there are studies which reveal that teachers and pre-service teachers can develop students' relational thinking skills with some activities (Carpenter et al. 2005, Matthews, Rittle-Johnson, McEldon, & Taylor, 2013; Knuth et al., 2006). In order to do this, teachers must understand and analyse correctly what is going on in the learning environment. Thus, this study aimed at revealing prospective teachers' professional noticing related to students' attitudes towards the equal sign. Because there are not any studies encountered about pre-service teacher noticing related to the equality and relational thinking in Turkey, the contributions this study will make is important.

2. Method

This case study carried out to explore preservice teachers' professional noticing related to their students' attitudes towards the equal sign consisted of two phases (Hitchock & Hughes 1995: 317).

In the first phase, the preservice teachers were asked to implement a task prepared before for the 6th, 7th, and 8th grade secondary school students and observe their students throughout the process and make an effort to understand what their students did as far as possible. Moreover, the candidates were asked to decide whether or not the problem was solved appropriately according to their opinions and also guide their students during the solution process, if required. The whole process was video-recorded. (During the process only worksheets were video-recorded but students were not included in the recording).

In the second phase, one of the researchers watched the problem–solving process video with each preservice teacher and asked them to answer the questions given below throughout the process:

Can you please explain what your students did for each question and how they solved them?

When you analyse the students' ways of solution, do you think there are differences between the ways (or methods) of solution? Can you categorize the ways of solutions? Can you compare three students' ways of solutions among themselves? What are the similarities and differences?

Have you interfered with your students' solutions (although they solve problems correctly or incorrectly) for getting better solutions? Do you think of interfering? Why?

2.1. Participants

The participants of the study were selected by convenience sampling, one of the purposeful sampling methods. Convenience sampling is prompt and practical for the researchers and thanks to it, the researcher chooses a case on the basis of proximity and ease of access (Yıldırım & Şimşek, 2008). In addition to this, the study has two different types of participants. The first participants of the study are 8 preservice teachers in their final year of studies in the department of elementary mathematics education at a state university located in Aegean Region, Turkey. The preservice teachers took major area courses such as algebra education and special teaching methods and they still continue their internship programs. The second group of the study participants are 24 6^{th}, 7^{th}, and 8^{th} grade secondary school students studying at a state school with high socio-economic background in the same region. The students were chosen among those from moderate and high levels of academic achievement. Due to confidentiality, the preservice teachers were given codes as PT1, PT2, PT3,... , PT8 instead of using their names, and the students were called PT 1.6, PT 1.7, PT 1.8. The first number represents the prospective teachers, and the second number indicates the grade of the student who was examined by the prospective teacher. For example, PT1.6 means the first teacher candidate's 6th grade student.

2.2. Data Collection and Instruments

The data was collected in two phases. In the first phase of data collection, "Diagnostic Clinical Interview Protocol" consisting of different types of equation questions was used. The clinical interview protocol, designed by the researchers, consisted of eight tasks in which prospective teachers follow student approaches to equality. 7th and 8th grade students were asked the same questions. The same questions were asked differently for 6th grade students. For example, since the equation solutions did not take place in the 6th grade of the Turkish curriculum, boxes were used instead of x to investigate the meaning of the equation. This interview form was adapted from the form used by Van den Kieboom, and et al. (2017) in their studies. The interview form was examined by two expert educators and they were in agreement about its convenience and practicality for its validity and reliability. In the first phase, while students solved questions, the process was video- recorded and then the whole process was watched with each preservice teacher and the researcher. During this watching together, the researcher and the candidate's talks were voice-recorded. Thus, the data collection tools include the interview form implemented with the secondary school students, video recordings consisting of the solution process, and voice-recordings of the semi-structured interviews carried out with the researcher and the candidate. Each teacher (8) was responsible for three students, so 24 interview recordings (6^{th}, 7^{th} and 8^{th} grades, 8.3=24) were analyzed.

2.3. Procedure of Data Analysis

The data analysis was carried out in two phases as in the data collection procedure. The first phase aimed at determining secondary school students' attitudes towards equality and the second phase was intended for determining the preservice teachers' noticing of these attitudes. These phases were explained in-detail below.

2.3.1. Phase I

In the first phase, firstly, video-recordings were examined and then the students' solutions were analyzed according to the framework expressed in Table 1.After that, firstly discrete solutions and student attitude profiles were identified. Primarily, it was decided that under which categories the attitudes fell into in literature called ROT, FOT, CO, and RT. When there were hesitations about some points, ideas were exchanged with the other researcher. The disintegration of some categories within themselves was mentioned in the next section (findings).

2.3.2. Phase II

In the second phase, some research was carried out about the components of noticing in order to determine the preservice teachers' noticing. Considering the attending construct, it was investigated whether or not the candidates recognised the ways of thinking (ROT, FOT, CT, RT) identified in the first stage correctly. For this purpose, the video-recordings were analysed and the researcher asked the candidates the question "Can you please explain the solution process of the students?" Then, the indicators for this were turned into a statement. The scores one can get according to the rubric were given in Table 2.

Table 2. For attending

Demonstration	Score
S/he recognises the entire mathematical thinking in their students' solutions.	3
S/he recognises most of the mathematical thinking in their students' solutions.	2
S/he recognises less than half of the mathematical thinking in their students' solutions.	1
S/he recognises too little mathematical thinking in their students' solutions.	0

For interpreting, the second construct, the preservice teachers were asked to create a profile for each student with whom he interviewed by considering their solutions. "For example, my 6th grade student uses Rigid operational thinking with all/ most questions, so he is a rigid operational thinker" and etc. In addition to this, considering the grades (6th, 7th, 8th), they were asked to explain whether or not there was a difference between the students' ways of solutions. The scores one can get according to the rubric were given in Table 3.

Table 3. For interpreting

Demonstration	Score
S/he can determine his/her three students' thinking profile and explain their similarities and differences.	3
S/he can determine his/her two students' thinking profile and explain their similarities and differences.	2
S/he can determine only one student's thinking profile.	1
S/he cannot determine any of his/her students' thinking profile .	0

For responding, the last construct, how the preservice teachers guided their students was explored. Although their solution was correct, whether or not they did something was questioned by the preservice teachers in order to encourage students for a more meaningful solution. The scores one can get according to the rubric were given in Table 4.

Table 4. For responding

Demonstration	Score
Although his/her students' problem-solution is true, s/he can interfere for a better way of solution.	3
Although his/her students' problem-solution is true, s/he thinks of interfering.	2
If his/her students' solution is false, s/he thinks it is right to interfere.	1
S/he does not have any idea whether she should interfere or not.	0

As observed in the tables, the maximum score one can get from the indicators is 9 and the minimum score is 0. According to this, the following levels of noticing were determined: the points between 0 and 3 are low, 4-6 are medium, and 7-9 are high.

The data were analysed by the same researcher at different times and by another researcher reciprocally to provide reliability for the analysis of the findings. When there were incompatibilities, they discussed and came to an agreement.

3. Findings

3.1. Findings Phase 1

The distribution of student thinking for the equal sign which occurred after watching the video-recordings for each student was presented in Table 5 according to the theoretical framework:

Table 5. The distribution of student thinking for the equal sign

	QUESTION							
STUDENT	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8
PT-1.6	CT	CT	CT	CT	CT	CT	CT	CT
PT-1.7	CT	CT	CT	CT	CT	CT	CT	CT
PT-1.8	RT	RT	RT	RT	RT	RT	CT	CT
PT-2.6	CT	CT	CT	CT	CT	CT	CT	CT
PT-2.7	CT	CT	CT	CT	CT	CT	CT	CT
PT-2.8	CT	CT	CT	CT	CT	CT	CT	CT
PT-3.6	CT	CT	CT	CT	CT	CT	CT	CT
PT-3.7	CT	CT	CT	CT	CT	CT	CT	CT
PT-3.8	CT	CT	CT	CT	CT	CT	CT	CT

Continiued Table 5. The distribution of student thinking for the equal sign

PT-4.6	CT	CT	CT	CT	CT	CT	CT	CT
PT-4.7	CT	CT	CT	CT	CT	CT	CT	CT
PT-4.8	CT	CT	CT	CT	CT	CT	CT	CT
PT-5.6	CT	CT	CT	CT	CT	CT	CT	CT
PT-5.7	CT	CT	CT	CT	CT	CT	CT	CT
PT-5.8	CT	CT	RT	CT	CT	CT	CT	CT
PT-6.6	-	-	-	-	-	-	-	-
PT-6.7	-	-	-	-	-	-	-	-
PT-6.8	CT	CT	CT	CT	CT	CT	CT	CT
PT-7.6	CT	CT	CT	CT	CT	CT	CT	CT
PT-7.7	CT	CT	CT	CT	CT	CT	CT	CT
PT-7.8	RT	CT	RT	CT	CT	CT	CT	CT
PT-8.6	CT	CT	CT	CT	CT	CT	CT	CT
PT-8.7	RT	RT	RT	CT	CT	CT	CT	CT
PT-8.8	CT	CT	RT	CT	CT	CT	CT	CT

According to Table 5, out of 192 questions, 176 of them were answered (because two students solved the questions without an explanation, the solutions ways of 16 questions could not be determined). CT was identified as a solution construct for 164 questions (%93) and RT for 12 questions (%6,8). The constructs ROT and FOT were not encountered. That is, students were not interested in on which side the equal sign was. The fact that they passed this stage reveals that their levels about their attitudes towards equality are high, which is a promising condition.

In addition, it was detected that CT, which has the highest share in thinking profile, reveals some differences in itself. Considering the explanations made about CT in literature, as stated in Table 1, "in order to find an unknown in the equation, the numbers in the unknown part are added, and the number (s) including the unknown in the equation are subtracted from the total". However, some differences were observed with some students' thinking processes after watching the video-recordings. The information obtained from a video-recording was given in Figure 1:

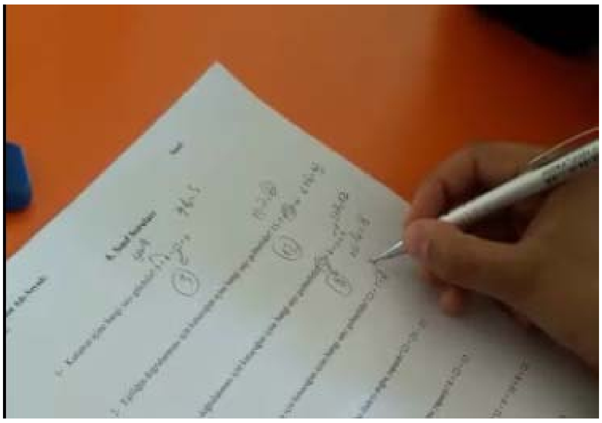

Figure.1. Sample Student's (PT 4.6) Solution

PT4.6: "5 plus 7 makes 12 on the right side. So, 4 plus what number is 12 on the left side? Hımm, 8 must go into the box".

In fact, the students' solution is a typical CT but it is understood from the students' verbal statements that he actualized mathematical reasoning beyond a routine operation. He did not subtract 4 from 12, but he thought that to get 12, what number he would add with 4. We decided to use "beyond computational thinker (BCT)" for the students who approached this way, but this description is different from the structure "basic relational thinker" (Matthews, P., Rittle-Johnson, B., McEldoon, K., & Taylor, R., 2012) encountered in literature because a lower level of relational thinking is mentioned here (eg: $4 + 5 + 8 = \square + 8$)

In addition to this, it was observed that some students called CT found the answer with a routine operation. Some students who knew how to solve equation algorithms isolated an unknown in the equation and reached the solution without reasoning. A section from a sample student solution was given Figure 2:

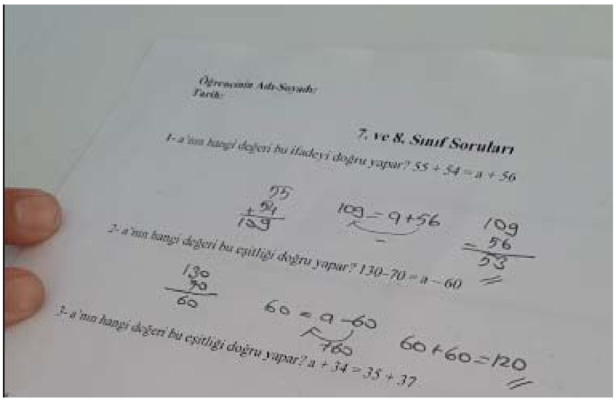

Figure.2. A Sample Student Solution (5.7)

PT 5.7:“In the equation, 56 moves to the other side as – 56.”

The difference between this student and the CT student is that he applied the algorithms he learned in the lesson without questioning the equality. We decided to use the term “routine computational thinker (RCT)" for students who think this way.

In addition to such students who think this way, according to the findings obtained, CT students appeared as in the literature. A solution by a student having this way of thinking is presented in Figure 3:

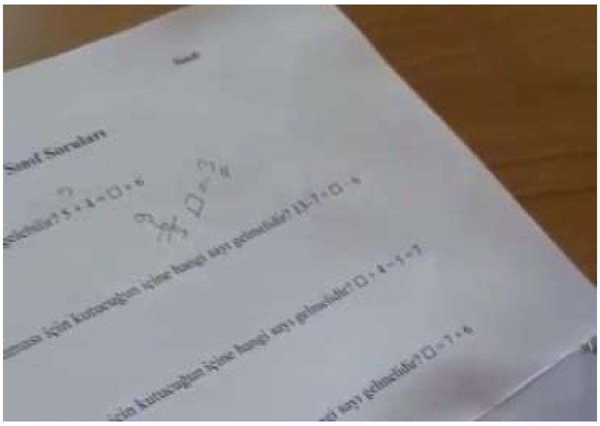

Figure.3.A Sample Student Solution

The solution procedure of PT.7.6 reflects a typical CT thinking.

It is possible to identify student profile based on CT, BCT and RCT. Considering the dominant ways of thinking students used while solving the questions, each student reflects a dominant thinking profile (Table 6):

Table 6. Students’ thinking profile

Student	CT	RCT	BCT	RT
PT-1.6	✓			
PT-1.7	✓			
PT-1.8				✓
PT-2.6			✓	
PT-2.7		✓		
PT-2.8		✓		
PT-3.6			✓	
PT-3.7		✓		
PT-3.8		✓		
PT-4.6			✓	
PT-4.7		✓		
PT-4.8	✓			
PT-5.6			✓	
PT-5.7		✓		
PT-5.8		✓		
PT-6.6	-	-	-	-
PT-6.7	-	-	-	-

Continiued Table 6. Students' thinking profile

PT-6.8	-	-	-	-
PT-7.6	✓			
PT-7.7			✓	
PT-7.8				✓
PT-8.6	✓			
PT-8.7				✓
PT-8.8		✓		

When Table 6 is examined, it is viewed that student profiles are predominantly BCT for the 6th grade and RCT for the 7th and 8th grades. This is partially an expected outcome. Solving equations is taught in the 7th grade in secondary schools in Turkish National Education Curriculum (MEB, 2018). That is, the reason why the 6th graders do not use the algorithm for solving equations is that they have not learnt the algorithm yet. The dominant profile of the 7th and 8th grades is RCT but it is not RT for the 6th grades. On the other hand, there is one RT profile in the 7th grade and two RT profiles in the 8th grade.

3.2. Findings Phase 2

The second section of the findings, based on the student responses, examined how preservice teachers discussed the student responses. PT6 was excluded because of lack of student explanations. According to this, the scores preservice teachers got from the components of noticing were graded as in Table 7:

Table 7. The scores of preservice teachers from the components of noticing

Noticing Constituent		Demonstration	Score
Attending	PT1	Three different solutions were exhibited and she saw three of them.	3
	PT2	Although three different solutions were exhibited, he said that there was only one method of solution.	1
	PT3	Although two different solutions were exhibited, he said that there was only one method of solution.	2
	PT4	Three different solutions were exhibited and she saw three of them.	3
	PT5	Although four different solutions were exhibited, he said that there were three methods of solution.	2
	PT7	Although three different solutions were exhibited, she said that there were two methods of solution.	2
	PT8	Although three different solutions were exhibited, she said that there were two methods of solution.	2
Interpreting	PT1	There were two different profiles, she saw both of them. She mostly explained the differences between the profiles.	3
	PT2	Although there were two different profiles, he said that there was only one profile. He explained very few differences between the profiles.	1
	PT3	Although there were two different profiles, he said that there was only one profile. He partially explained the differences between the profiles.	2
	PT4	There were three profiles and she saw three of them. She mostly explained the differences between the profiles.	3
	PT5	There were two different profiles and he said there was only one profile. He partially explained the differences between the profiles.	2
	PT7	There were three profiles and she saw two of them. She partially explained the differences between the profiles.	2
	PT8	There were three profiles and she saw two of them. She partially explained the differences between the profiles.	2
Responding	PT1	He does not think of interfering provided that the solution is false.	1
	PT2	He does not think of interfering provided that the solution is false.	1
	PT3	He does not think of interfering provided that the solution is false.	1
	PT4	She does not think of interfering provided that the solution is false.	1
	PT5	He does not think of interfering provided that the solution is false.	1
	PT7	She does not think of interfering provided that the solution is false.	1
	PT8	She does not think of interfering provided that the solution is false.	1

When Table 7 was examined, it was observed that only two preservice teachers could get the full marks from the components of attending and interpreting. One candidate could get 1 from each stage. The rest of the preservice teachers got 2 in both of the stages. Considering the last stage, all the candidates could get only 1 from this stage.

On the other hand, considering the total scores, one student (PT2) was ranked in the low level of noticing, four students(PT3, PT5, PT7, PT8) were ranked as middle, and two students(PT1, PT4) were ranked in the high level of noticing. The candidates usually got higher scores from the first component (attending), but the fall began later. It was observed that the per-service teachers got the lowest scores in the last stage. All the candidates had a tendency to not to suggest an alternative solution provided that students do not make mistakes. The reason for this could be that they do not know how to asses students' solutions. In addition to this, the candidates mostly mixed CT, RCT and BCT ways of thinking with each other. Examples from the preservice teachers' views were given below:

PT2 (low): *"They directly move to the other side as minus but they do not show them with arrows. They did not use a different thing, they only added or subtracted. To me, different from them, in order to isolate the unknown, they could add the same number to both sides or subtract from both sides. Three students' solution methods are the same; all of them actually use the algorithms for solving equations. The 6th grade student also solves equations. But all of them solved the questions correctly".*

However, the students with whom the preservice teacher interviewed were identified as CT, RCT and BCT. But, the candidate interpreted all the solutions as RCT and stated that all of the candidate profiles were RCT. The candidate could not distinguish these three ways of thinking.

PT3 (middle): *"They take from one side and add to the other side. The 6th grade student always used the same method, he thinks as he was thought the first time. The 8th grade student collected the unknown to the one side and known to the other side. This student also did the same things. The 7th grade student isolated the unknown. All of them used the same way. One student solved it in his mind immediately and did not show how he moved it to the other side. I could not see any differences between these three students. I did not interfere as the solutions were correct".*

In fact, the students with whom the PT3 interviewed were identified as RCT and BCT. The preservice teacher asserted that there was not a difference between them and three of them were RCT.

PT4 (high): *"They usually transfer the sign as plus if it is minus or minus if it is plus. But of course there were some who thought logically. For example, one of them said that if 4 plus 4 is 9, then if I add something and 6, the result is 9. He thought what this thing was. The thing here is the box. Therefore, the thing is equal to 3. The 7th grade student transferred this directly to the other side but the 6th grade student using reasoning thought how to get this number by adding what numbers. He said that this is what they say while teaching addition in the 1st grade. The 7th grade student solved it in his mind immediately, he applied the rules he learned directly without thinking. The 8th grade student is more logical. He comprehends that this is equality and one of the equation is equal to the other side of the equation. He knows what is unknown very well. When I look at these three children, I see three different ways of thinking. As a different solution, they could continue with 1 less and 1 more. But I did not interfere because the student solved it correctly".*

Considering the explanations of CPT4, it draws attention that they include more detail. The teacher candidate picked up all of the solution methods and she could identify her students' profiles. Especially, the other candidates frequently confused CT and BCT, but PT4 did not make this mistake.

As observed from the explanations, the mistakes mostly resulted from not being able to distinguish CT, RCT and BCT. The preservice teachers did not make the same mistake when it came to RT.

4. Results and Discussion

When the studies carried out about the concept of equality (Matthews, et. al.,2012; Van den Kieboom, et. al., 2017) were examined, it is viewed that there are some discrete approaches exhibited by the students while solving equations under different names. It was concluded in this study that two more approaches could be added to these approaches and it was decided to use the expressions "routine computational thinking" and "beyond computational thinking". While doing that, the evidence showing that these two approaches resulted from different types of thinking under the roof of "computational thinking" was used (see Figure.1, Figure.2, Figure.3). Firstly, it is viewed that nearly all of the beyond computational thinkers are the 6th grade students who have not learned the solution of algebraic equations yet. Although students did not know the algorithm for solving equations, it was observed that they were frequently accused of knowing this algorithm (being a RCT) and this condition leads to the assumption that the preservice teachers will teach these rules untimely in the future in spite of the flexible transition from arithmetic to algebra and curriculum rules. Hence, it could be useful to determine preservice teachers' noticing. In addition, the preservice teachers must analyse the students' reasons for having such approaches.

After this theoretical implication, the results of the study will be discussed in two phases as in the findings.

Considering the findings about the secondary school students, it is viewed that the dominant approach profile for the 6th graders is BCT. It is pleasing that they do not have the misconceptions about equality in the literature. Of course, the high academic achievement of the chosen students has a big share. However, it draws attention that the relational thinking approach whose importance is frequently emphasized during the transition process from arithmetic to algebra (Stephens, 2006) has not been used by the 6th grade students. However, this approach was mostly preferred by the 8th graders. This exhibits that the 8th graders looked for a solution via reasoning rather than routine operations. The 7th grade students are predominantly in the RCT category. In addition, the 7th graders were the groups who applied the algorithms for solving equations which they learned regularly. However, this is the grade that the shift to equation solving was ensured (MEB, 2018) and also it could be the period to acquire relational thinking. Considering the advantages of the relational thinking for the future topics, it is important that this process should be reconsidered effectively.

The findings of Phase 2 in the study are about the professional noticing of the pre-service teachers. The findings obtained as a result of analysis carried out according to the framework developed by Jacobs and et al.(2010) reveal that the noticing levels of pre-service teachers accumulate mostly at middle level (4 pre-service teachers out of 7) . When the pre-service teachers examined their students' solutions, they had difficulties distinguishing CT, BCT, and RCT. RT became a more specific approach for the noticing levels of pre-service teachers and it was easily recognized contrary to the findings obtained from the earlier studies (Asquith, Stephens, Knuth, & Alibali, 2007; Stephens, 2006).

Based on the attending stage, the candidates demonstrated similar performance while determining the student profiles. However, the biggest fall was experienced at the responding stage because the candidates did not guide their students and stated that they did not think of guiding them. According to them, it was enough for students to solve the questions. This attitude of candidates contradict with the suggestions offered about professional noticing (Hines & McMahon, 2005; Holt, Mojica & Confrey, 2013) and transition process from arithmetic to algebra (Sherman and Bisans, 2009; Driver and Powell; Alibali, Knuth, and Hattikudur, 2007). The pre-service teachers must be more knowledgeable and experienced about the features of relational thinking, the transition from arithmetic to algebra, and period before algebra.

This study did not investigate the pre-service teachers' gains about the concept of equality after this study just like the studies carried out by Van den Kieboom and et al. (2017). Because they questioned how pre-service teachers strengthened their knowledge during the study. However, as supported with the earlier studies (van Es & Sherin, 2002), periodic repetition of such studies and studying the detailed student responses in faculty courses naturally will develop pre-service teacher noticing about this and different subjects. Thus, the professional noticing of teachers is a process which can be developed and improved (Star and Strickland, 2008; van Es, 2011; van Es and Sherin, 2002). Based on this result, new studies can be designed in order to improve student approach awareness of prospective teachers.

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Examining Mathematics Teachers' Use of Curriculum and Textbook

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ABSTRACT

The aim of this study is to determine how mathematics teachers interpret curriculum and textbooks and to what extent / how they use these materials. For this purpose, case study design was used and 45 mathematics teachers constituted the study group of the study. The data obtained through the structured interview form developed by the researcher to determine teachers' use of textbooks and curricula were analyzed deductively. According to the findings obtained from the analysis, it was seen that teachers working in both secondary and high school levels did not like the textbooks sufficiently and did not prefer to use them in their lessons. The reasons for this are that the textbooks contain various errors, are not interesting, are not suitable for student level; Even if the curriculums are updated, they are still dense and have uneven distribution of the content according to class levels. Teachers prefer to use supplementary resources as well as textbooks. On the other hand, there are also teachers who express positive opinion about the textbook and curriculum. One of the interesting findings of the study is that teachers' perceptions of curriculum are generally acquisition-oriented.

Keywords:
Curriculum, textbook, mathematics

1. Introduction

"*…teachers make a difference*" (Wright, Horn, & Sanders, 1997, s.57)

While the term "curriculum" is used as the program in which the content of a particular course is drawn in our country, it is used as the resources used by teachers in addition to the content of the course by the researchers and teachers in other countries (Stein, Remillard, & Smith, 2007). Oliva (2009) argues that the curriculum and the teaching are part of a cyclic process, that they are closely related to each other and that they cannot be isolated from each other even though they can be studied and studied as two different terms. Therefore, it will not be wrong to discuss and interpret the textbooks and the curriculum as an important part of the teaching process. Teachers implement the program materials with a specific interpretation or adaptation (Brown, 2009). Components such as teachers' knowledge, skills, past experiences and beliefs affect teachers' interpretation and usage levels of the program (Ball & Cohen, 1996; Brown, 2009; Peterson, Fennema, Carpenter, & Loef, 1989; Stein, Remillard, & Smith, 2007). Therefore, it is very difficult to reflect the curriculum in the classroom environment as prepared by the program developers. Stein, Remillard and Smith (2007) depict the stage from the preparation of the program to the reflection of students' learning, as shown in Figure 1 (p.322).

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