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NEAR SOFT GROUPOID

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Abstract

In this article, firstly some concepts on the near soft set obtained by combining the near set and the soft set are given. In the previous studies in the literature, the definition of a soft element with binary operation in the set of all non-empty soft elements of a soft set and the definition of the concept of soft groupoid depending on the set of soft elements are given. In this study, starting from the concept of soft element, the concept of near soft groupoid is defined by using the near soft element with binary operation in the set of all non-empty soft near elements of a near soft set. In addition, properties related to the defined near soft groupoid are given with theorem and example.

Keywords: Near set; Soft Set; Near Soft Set; Near Soft Element; Near Soft Group, Near Soft Groupoid.

1 Introduction

The notion of near sets has been given by Peters [1, 2] and the concept of soft theory has been given by Molodtsov [4]. Then it was studied by many scientists [3, 4, 5, 6, 7]. The definition of soft element given by Wardowski[8] with a binary operation on the set of all nonempty soft elements of a given soft set. Then J.Ghosh[9, 10] defines soft groupoid based on the set of soft elements. In the rough set theory, which is another concept, the concepts of group and groupoid have been studied[11, 12]. Feng and Li [5] have investigated the problem of combining soft sets with rough sets, and introduced the notion of rough soft sets. Afterwards, Tasbozan [13, 14]combine near sets approach with soft set theory and introduced the notion of near soft sets. In this paper, we introduce the concept of near soft element and define near soft groupoid using the near soft element with a binary operation on the set of all nonempty near soft elements of a given near soft set.

2 Preliminary

In this section, we recall some descriptions and results presented and discussed in [13]. Also, we present the concepts of near soft sets, their fundamental properties, and operations such as near soft point, near soft elements. Then we define a binary composition on near soft sets and this form is called near soft groupoid over near soft set.

A nearness approximation space (NAS) is denoted by $NAS = (O, F, \sim_{B_r}, N_r, \upsilon_{N_r})$ which is defined with a set of perceived objects O, a set of probe functions F representing object features, an indiscernibility relation $\sim_{B_r}= \{(x, x^{'}) \in O \times O | \forall i \in B_r, i(x) = i(x^{'})\}$ defined relative to $B_r \subseteq B \subseteq F$, a collection of partitions (families of neighbourhoods) $N_r(B)$, and a neighbourhood overlap function Nr . The relation $\sim_{B_r}$ is the usual indiscernibility relation from rough set theory restricted to a subset

$B_r \subseteq B$. The subscript r denotes the cardinality of the restricted subset B_r, where we consider $\binom{|B|}{r}$, i.e., $|B|$ functions $i \in F$ taken r at a time to define the relation $\sim_{B_r}$ .This relation defines a partition of O into non-empty, pairwise disjoint subsets that are equivalence classes denoted by $[x]_{B_r}$, where $[x]_{B_r} = \{x\prime \in O | x \sim_{B_r} x^{'}\}$. These classes contitue a new set called the quotient set $O/\sim_{Br}$, where $O/\sim_{B_r} = \{[x]_{B_r} | x \in O\}$. And the overlap function υ_{N_r} is defined by $\upsilon_{N_r} : P(O) \times P(O) \to [0,1]$, where $P(O)$ is the powerset of O.

Definition 1 *Let $NAS = (O, F, \sim_{Br}, N_r, \upsilon_{N_r})$ be a nearness approximation space and $\sigma = (F, B)$ be a soft set over O . The lower and upper near approximation of $\sigma = (F, B)$ with respect to NAS are denoted by $N_r * (\sigma) = (F_*, B)$ and $N_r^*(\sigma) = (F^*, B)$, which are soft sets over with the set-valued mappings given by*

$F_*(\phi) = N_r * (F(\phi))$=$\cup\{x \in O : [x]_{Br} \subseteq F(\phi)\}$ and

$F^*(\phi) = N_r^*(F(\phi))$=$\cup\{x \in O : [x]_{Br} \cap F(\phi) \neq \emptyset\}$ where all $\phi \in B$. The operators N_r* and N_r^* are called the lower and upper near approximation operators on soft sets, respectively. If $Bnd_{N_r(B)}(\sigma) \geqslant 0$, then the soft set σ is called a near soft set [13].

The collection of all near soft sets on O will be denoted $NSS(O)$.

Definition 2 *Let O be an initial universe set, E be the universe set of parameters and $B \subseteq E$.For a near soft set (F, B) over O , the set*

$$Supp(F, B) = \{\phi \in B : F(\phi) \neq \emptyset\}$$

is called the support of the near soft set (F, B).

1. *A near soft set (F, B) is called non-null near soft set (with respect to the parameters of B) if $Supp(F, B) \neq \emptyset$. Otherwise (F, B) is called null near soft set.*

2. *A near soft set (F, B) is called full null near soft set if $Supp(F, B) = B$. A collection of all full near soft sets on O will be denoted by $NS_f(O)$.*

Definition 3 *Let O be an initial universe set, E be the universe set of parameters and $B \subseteq E$ and $(F, B) \in NSS(O)$. We say that $(\phi, \{x_k\})$ is a nonempty near soft element of (F, B) if $\phi \in B$ and $x_k \in F(\phi)$. The pair $(\phi, \emptyset)$, where $\phi \in B$ will be called an empty near soft element of (F, B). Then $(\phi, \{x_k\})$ is a near soft element of (F, B) and denoted by F_B.*

Example 4 *Let $X = \{x_1, x_2, x_3\} \subseteq O = \{x_1, x_2, x_3, x_4, x_5\}, B = \{\phi_1, \phi_2\} \subseteq F = \{\phi_1, \phi_2, \phi_3\}$ denote a set of perceptual objects and a set of functions respectively. Let $(F, B) = \sigma$ defined by $(F, B) = \{(\phi_1, \{x_1, x_2\}), (\phi_2, \{x_3\})\}$. For $r = 1$*

$$[x_1]_{\phi_1} = \{x_1, x_2\},\ [x_3]_{\phi_1} = \{x_3, x_4\}$$
$$[x_1]_{\phi_2} = \{x_1, x_3\},\ [x_2]_{\phi_2} = \{x_2, x_4\}$$

$N_*(\sigma) = \{(\phi_1, \{x_1, x_2\})\}$, $N^*(\sigma) = \{(\phi_1, \{x_1, x_2\}), (\phi_2, O)\}$, *then σ is a near soft set. For $r = 2$*

$$[x_1]_{\phi_1,\phi_2} = \{x_1\},\ [x_2]_{\phi_1,\phi_2} = \{x_2\}, [x_3]_{\phi_1,\phi_2} = \{x_3\}, [x_4]_{\phi_1,\phi_2} = \{x_4\}$$

$N_*(\sigma) = \{(\phi_1, \{x_1, x_2\}), , (\phi_2, \{x_3\})\}$, $N^*(\sigma) = \{(\phi_1, \{x_1, x_2\}), , (\phi_2, \{x_3\})\}$, *then σ is a near soft set. Hence all the near soft elements of (F, B) are*

$$(\phi_1, \{x_1\}), (\phi_1, \{x_2\}), (\phi_2, \{x_3\})$$

Near Soft Groupoid

Let $(F, \circ)$ and $(O, *)$ be two groupoids, $(O, *)$ be a group with "$*$" operation, $(F, \circ)$ be a group with "$\circ$" operation and $B \subseteq F$. Also let $(F, B) \in NS_f(O)$, i.e., (F, B) be a full near soft set on O.,*i.e.,* for each parameter $\phi \in B$, there exists at least one nonempty near soft element of (F,B). We define a binary composition $*$ on (F,B) by

$$(\phi_i,\{x_a\}) * (\phi_j,\{x_b\}) = (\phi_i \circ \phi_j,\{x_a * x_b\})$$

for all $(\phi_i,\{x_a\}),(\phi_j,\{x_b\}) \in (F,B)$. (F,B) is said to be closed under the binary composition $*$ if and only if $(\phi_i \circ \phi_j,\{x_a * x_b\}) \in (F,B)$ for all $(\phi_i,\{x_a\}),(\phi_j,\{x_b\}) \in (F,B)$ i.e., if and only if $\phi_i \circ \phi_j \in B$ and $x_a * x_b \in F(\phi_i \circ \phi_j)$ for all $(\phi_i,\{x_a\}),(\phi_j,\{x_b\}) \in (F,B)$.

Definition 5 *If (F,B) is closed under the binary composition $*$, then the algebraic system $((F,B),*)$ is said to be a near soft groupoid over O.*

Theorem 6 *Let $(F,B) \in NS_f(O)$, then $((F,B),*)$ forms a near soft groupoid over O if and only if*

1. B is a subgroupoid of F i.e., $\phi_i \circ \phi_j \in B$ for all $\phi_i,\phi_j \in B$
2. for $\phi_i,\phi_j \in B$, $x_a \in F(\phi_i)$, $x_b \in F(\phi_j)$ then $x_a * x_b \in F(\phi_i \circ \phi_j)$.

Proof. Suppose $((F,B),*)$ is a near soft groupoid over (F,O). Let $\phi_i,\phi_j \in B$. Since $(F,B) \in NS_f(O)$, there exist some $x_a,x_b \in O$ such that $(\phi_i,\{x_a\}),(\phi_j,\{x_b\}) \in (F,B)$. Hence $(\phi_i,\{x_a\}) * (\phi_j,\{x_b\}) \in (F,B)$. This implies $(\phi_i \circ \phi_j,\{x_a * x_b\}) \in (F,B)$, $\phi_i \circ \phi_j \in B$ and $x_a * x_b \in F(\phi_i \circ \phi_j)$ by definition (near soft element). Therefore B is a subgroupoid of F and for $\phi_i,\phi_j \in B$, $x_a \in F(\phi_i)$, $x_b \in F(\phi_j)$ then $x_a * x_b \in F(\phi_i \circ \phi_j)$. Conversely, suppose that the given two conditions hold. Now let $(\phi_i,\{x_a\})$, $(\phi_j,\{x_b\}) \in (F,B)$. This implies that $\phi_i,\phi_j \in B$, $x_a \in F(\phi_i)$, $x_b \in F(\phi_j)$ by hypothesis (1), $\phi_i,\phi_j \in B$ then $\phi_i \circ \phi_j \in B$,by hypothesis (2), $x_a \in F(\phi_i)$, $x_b \in F(\phi_j)$ then $x_a * x_b \in F(\phi_i \circ \phi_j)$. Therefore $(\phi_i \circ \phi_j,\{x_a * x_b\}) \in (F,B)$. So (F,B) is closed under the binary composition $*$. Hence $((F,B),*)$ forms a near soft groupoid over O. ■

Example 7 *Let $O = \{0,1,4,5\}$ be the set of objects which $(O,\cdot)$ be a group with "$\cdot$" operation being multiplication of O integers modulo 4 and $F = \{\phi_1,\phi_2\}$ be a set of quotient function*

$$\phi_i : \mathcal{O} \to \mathcal{O}/\sim_{\phi_i}$$

$$\phi_1 : 0 \to \phi(0) = \bar{0} = \{0,4\}$$

$$\phi_2 : 1 \to \phi(1) = \bar{1} = \{1,5\}$$

which $(F,+)$ be a group with "$+$" operation being addition the classes of residues of integers modulo 4 .

Take $B = \{\phi_1\} \subset F$ and define a near soft set $\sigma = (F,B) = \{\phi_1,\{0,4\}\}$ with $[0]_{\phi_1} = \{0,4\}$, $[1]_{\phi_1} = \{1,5\}$,

$$N_*(\sigma) = \{\phi_1,\{0,4\}\}, N^*(\sigma) = \{\phi_1,\{0,4\}\}$$

Hence all the near soft elements of F_B are $\{\phi_1,\{0\}\},\{\phi_1,\{4\}\}$. Then the binary composition "$\cdot$" is given by

$$\begin{aligned} \{\phi_i,\{x_a\}\} \cdot \{\phi_j,\{x_b\}\} &= \{\phi_i + \phi_j,\{x_a \cdot x_b\}\} \\ \{\phi_1,\{0\}\} \cdot \{\phi_1,\{0\}\} &= \{\phi_1,\{0\}\} \\ \{\phi_1,\{0\}\} \cdot \{\phi_1,\{4\}\} &= \{\phi_1,\{0\}\} \\ \{\phi_1,\{4\}\} \cdot \{\phi_1,\{4\}\} &= \{\phi_1,\{0\}\} \end{aligned}$$

Hence $(F_B,\cdot)$ is a near soft groupoid over O.

Definition 8 *Let $(F_B,*)$ be a near soft groupoid over (F,O) where the binary composition $*$ is defined. Then $*$ said to be*

1. commutative if $(\phi_i,\{x_a\}) * (\phi_j,\{x_b\}) = (\phi_j,\{x_b\}) * (\phi_i,\{x_a\})$

2. associative if $[\ (\phi_i,\{x_a\})*(\phi_j,\{x_b\})]*(\phi_k,\{x_c\})=(\phi_i,\{x_a\})*[(\phi_j,\{x_b\})*(\phi_k,\{x_c\})]$ for all $(\phi_i,\{x_a\}),(\phi_j,\{x_b\}),(\phi_k,\{x_c\})\in F_B$

Definition 9 *A near soft element $(\phi,\{x\})\in F_B$ is said to be a near soft identity element in a near soft groupoid $(F_B,*)$ if for all $(\phi_i,\{x_a\})\in F_B$*

$$(\phi,\{x\})*(\phi_i,\{x_a\})=(\phi_i,\{x_a\})=(\phi_i,\{x_a\})*(\phi,\{x\})$$

Definition 10 *Let $(F_B,*)$ be a near soft groupoid over (F,O)*

1. *If the composition $\circ$ on B and the composition $*$ on O are associative (commutative) then the composition $*$ on F_B is associative (commutative).*
2. *If F_B contains the near soft identity element $(\phi,\{x\})$ then ϕ is the identity element of B and x is the identity element of $\cup_{\phi_i\in B}F(\phi_i)$.*
3. *If $*$ is associative then near soft groupoid $(F_B,*)$ is called near soft semigroup.*
4. *If the soft semigroup $(F_B,*)$ contains near soft identity element then called near soft monoid.*

Definition 11 *Let $(F_B,*)$ be a near soft groupoid with near soft identity element $(\phi,\{x\})$. A near soft element $(\phi_i,\{x_a\})\in F_B$ is said to be invertible if there exists a near soft element $(\phi_i',\{x_a'\})\in F_B$ such that*

$$(\phi_i,\{x_a\})*(\phi_i',\{x_a'\})=(\phi,\{x\})=(\phi_i',\{x_a'\})*(\phi_i,\{x_a\})$$

Then $(\phi_i',\{x_a'\})$ is called the near soft inverse of $(\phi_i,\{x_a\})$ and denoted by $(\phi_i,\{x_a\})^{-1}$.

Theorem 12 *Let $(F_B,*)$ be a near soft groupoid with near soft identity element $(\phi,\{x\})$. If a near soft element $(\phi_i,\{x_a\})\in F_B$ is invertible then ϕ_i is invertible in F and $\{x_a\}\in F(\phi_i)$ is invertible in O.*

Proof. Suppose $(\phi_i,\{x_a\})\in F_B$ is invertible. then there exist a near soft element $(\phi_i',\{x_a'\})\in F_B$ such that

$$(\phi_i,\{x_a\})*(\phi_i',\{x_a'\})=(\phi,\{x\})=(\phi_i',\{x_a'\})*(\phi_i,\{x_a\})$$
$$(\phi_i\circ\phi_i',\{x_a*x_a'\})=(\phi,\{x\})=(\phi_i'\circ\phi_i,\{x_a'*x_a\})$$
$$\phi_i\circ\phi_i'=\phi=\phi_i'\circ\phi_i \text{ and } x_a*x_a'=x=x_a'*x_a$$

Since $(\phi,\{x\})$ is the near soft identity element of F_B then ϕ is the identity element of B and x is the identity element of $\cup_{\phi_i\in B}F(\phi_i)$. Also ϕ_i is invertible in $B\subseteq F$ and $\{x_a\}\in F(\phi_i)$ is invertible in $\cup_{\phi_i\in B}F(\phi_i)\subseteq O$. ■

Remark 13 *Converse of this theorem is not necessarily true. In a near soft groupoid $(F_B,*)$ with near soft identity element, if ϕ_i is invertible in B and $\{x_a\}\in F(\phi_i)$ is invertible in O then $(\phi_i,\{x_a\})\in F_B$ is not necessarily invertible in F_B.*

Example 14 *Let $O=\{0,1,4,5\}$ be the set of objects which $(O,\cdot)$ be a group with "$\cdot$" operation being multiplication of O integers modulo 4 and $F=\{\phi_1,\phi_2\}$ be a set of quotient function*

$$\phi_i:\mathcal{O}\to\mathcal{O}/R$$

$$\phi_1:0\to\phi(0)=\bar{0}=\{0,4\}$$

$$\phi_2:1\to\phi(1)=\bar{1}=\{1,5\}$$

which $(F,+)$ be a group with "$+$" operation being addition the classes of residues of integers modulo 4 . $\sigma=(F,B)=\{\phi_1,\{0,4\}\}$ is a near soft set , it is all the near soft elements are $\{\phi_1,\{0\}\},\{\phi_1,\{4\}\}$. Hence $(F_B,*)$ is a near soft groupoid with near soft identity element $\{\phi_1,\{0\}\}$. The near soft inverse of $\{\phi_1,\{4\}\}$ is $(\{\phi_1,\{4\}\})^{-1}=(\{\phi_1,\{0\}\})\in F_B$. Therefore $(\{\phi_1,\{4\}\})$ is invertible in F_B.

Conclusion

As a result, the definition of near soft groupoid was given to the concept of near soft set obtained with the help of near set and soft set concepts in this study by applying the definition of soft element used in previous studies. This new concept can be used in new studies in set theory.

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