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Capacitated Multiple Allocation Hub Covering Flow Problem

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ABSTRACT

The aim of the Capacitated Multiple Allocation Hub Covering Flow Problem is to find the optimal design for hub-and-spoke networks while taking into account hub opening and demand routing costs. Every network node has the potential to be a hub and demand from an origin to a destination must be sent through at least one hub. The network is incomplete in the sense that the maximum allowed or coverage distance between any opened hub and demand origin/destination is predefined. It is assumed that there is a cost saving to route demand via hubs due to consolidation. Another important issue is the consideration of capacity restrictions imposed on network links and opened hubs. The problem is developed as a mixed-integer linear optimization problem. According to the results obtained from computational experiments, we show that taking into account both flow related costs and capacities of network components concurrently is very important to have a cost effective design.

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1. Introduction

Locating economically attractive hub facilities through which demand flows are to be routed from origins to destinations is the subject of hub location problems (HLP). As for example, flows can be associated with passengers or freight, and origins/destinations with cities in the physical world. Flow form an origin to a destination can be assured with a direct trip (bypassing hubs) or via paths that visit hubs. At hubs, traffic arriving from several origins can be aggregated or they can be disaggregated to depart several destinations. On the one hand, fewer links would be required to connect origins and destinations with this redirection compared to direct connections. On the other hand, economies of scales can be achieved through consolidation of flows at hubs and thus costs related to flows can be reduced. The main decisions for HLP are to locate hub nodes and route flows so as to satisfy the demand. The network for a HLP consists of nodes so called as hub, origin and destination and arcs connecting hubs to other hubs, origins to hubs, hubs to destinations, and occasionally origins directly to destinations. Conform to the traditional location theory, locations of facilities and linking of points in the network to facilities are two distinctive decisions. Within this context, the generic HLP can be regarded as a network design problem with location. Campbell and O'Kelly [1] summarize the differentiating characteristics of HLP as in the following:

1. Demand is related with origin-destination (OD) node pairs and not with individual nodes,
2. Demand flows pass through hub nodes,
3. Location of hub nodes must be identified,
4. Routing flows via hub nodes is a requirement or has a benefit,
5. Problem objective is formulated based on the hub locations and flow routing.

This definition covers a wide range of problems which have not been viewed as HLP. Campbell [2] defines two additional important features of HLP that formed a base for further research:

1. Hub nodes that can be visited on a path linking an OD node pair can be at most two,
2. Flow from an origin directly to a destination is not allowed.

HLP can be classified according to the following characteristics [3]:

- *Solution space for locating hub nodes*: all network nodes, a subset of network nodes or the continuous space (the domain is a plane or a sphere).

- *Number of hubs to locate*: single or multiple, specified in advance or determined at the optimum solution.
- *Node allocation*: To a single hub node or single allocation (S), or to multiple hub nodes or multiple allocation (M).
- *Capacity of a hub/link*: capacitated or uncapacitated.
- *Cost of establishing hub nodes*: N/A, fixed or variable.
- *Cost of allocating nodes*: N/A, fixed or variable.
- *Objective*: Minimizing the maximum path cost that is due to the routing of demand flows from origins to destinations (minmax), or minimizing the total cost of establishing hubs and assigning other nodes to hubs (minsum).

The main application areas of HLP are in transportation and telecommunications. Freight or passengers are carried by vehicles on infrastructures such as roadways, railways, airways or waterways in transportation hub location problems, and hub nodes are located by taking into consideration the associated distance based travel time and/or cost. Meanwhile, hub facilities such as routers, switches and concentrators are located to provide communication among a set of nodes in telecommunications hub location problems. As the electronic data is moved using physical links (cables) or though the air (microwaves), there may not be significant distance based traveling costs in this type of network. Both transportation and communication networks often have very similar abstract models, but the operations, relevant costs, service measures, and constraints are quite different. Routing protocols for transportation and telecommunication usually differ, as large communication networks require packet switching while individual traffic units (passengers/shipments) are usually not divisible in transportation networks.

In this paper, the design of incomplete hub-and-spoke networks are investigated where every network node has the potential to be a hub and the demand from an origin to a destination must be sent from at least one hub. Meanwhile, the subnetwork formed by hubs is assumed to be complete. The aim to minimize the total cost due to the design (opened hubs) and operations (transportation). Another important issue is the consideration of capacity restrictions imposed on network links and opened hubs. As we will show in the sequel, taking into account both flow related costs and capacities of network components concurrently for the hub covering problem (HCP) is very important to have a cost effective design. We provide a detailed literature survey for HLP and HCP in the next section. We introduce the notation and mathematical formulation of the problem in Section 3. Based on two well-known benchmark data set from the literature, numerical trials are carried out to determine the response of our mathematical model to variations of parameters. Results of these experiments together with some practical insights are provided in Section 4. The last section of the paper includes some concluding remarks and perspectives.

2. Literature Survey

The first study including similar concepts to HLP is due to Hakimi [4]. Application of the hub location for aviation is discussed by Toh and Higgins [5]. O'Kelly [6, 7] authored pioneering papers introducing the mathematical formulation and solution methods for HLP. Many papers have been published with a significantly increasing trend since then. There exist comprehensive literature surveys on hub location problems [1, 3, 8–11], so we refer the interested reader to these works and the references therein. HLP has different types such as hub covering problem (HCP), hub center problem, hub median problem, and hub arc location problem. In this study, we are focused on a particular case of HCP designed as the hub covering flow problem (HCFP).

HCP was first proposed by Campbell [2] in analogy to the set covering problem. In this work, hub set cover and maximal hub cover problems with single and multiple allocation are provided and three different coverage cases are enlisted. Let us denote two hub nodes as k and l. Then an OD node pair (i, j) is covered if the length(s) given in the following cases do(es) not exceed a preselected particular value: (1) length of the path (i, k, l, j); (2) length of each link on the path (i, k, l, j); and (3) lengths of links (i, k) and (l, j). In our study, we take into consideration the last covering case.

Yetis Kara and Tansel [12] give a novel formulation for SHCP that is different from the hub set cover problem introduced in Campbell [2]. In terms of average CPU times and storage requirements, the linearization of this model performs pretty better than the best performing linearization of the Campbell [2]'s model. In their numerical experiments, they used the data set that is created based on the Civil Aeronautics Board (CAB) Survey of 1970 airline data in the US. Ernst et al.[13] suggest a new compact formulation for the uncapacitated p-SHCP based on the coverage radius concept. Auxiliary variables are included to this formulation to determine the distance between each hub and the furthest node assigned to it. Despite being slightly weaker than the others, it was shown empirically to perform better. Wagner [14] proposes new model formulations for S/MHCP which include quantity dependent and/or independent transportation times. The author uses AP and CAB data sets to experimentally show that models with quantity independent transportation times perform better. AP data set represents distribution divisions of the Australian Post [15].

Weng and Wang [16] give a new model for multiple allocation HCP and propose to solve large instances with scatter search and genetic algorithm. Qu and Weng [17] propose an evolutionary algoritm based on route reconnecting to solve a new reformulation of maximal MHCP. A single allocation HCP for cargo delivery applications is investigated by Alumur Alev and Yetis Kara [18]. In this study, the assumption of completely connected hub network is relaxed and the objective is to design a network constrained by a specified path traveling time. A closely related model for the SHCP over incomplete hub networks is introduced in Calik et al. [19]. The model objective is locate hub nodes, establish interhub links and allocate non-hub nodes to hub nodes such that the travel time on the path joining any OD pair is less than a specified time bound. An efficient tabu-search based heuristic is proposed and its performance is tested on TR and CAB data sets.

All non-hub nodes must be covered by hub nodes in HCP, but in most of the literature, the cost of transportation between OD node pairs is not accounted for. This can be an important shortcoming. For example, transportation costs should not be overlooked by passenger airlines when their operational hub locations are to be identified, given that the cost efficiency is a key for the survival of these companies. In consequence, Lowe and Sim [20] come up with HCFP which aims to design hub-and-spoke networks at a minimum cost by locating hub nodes and routing demand flows through these nodes given coverage constraints. Design costs they consider are fixed hub opening costs and variable transportation costs. Alumur et al. [21] consider transportation travel times and costs together while formulating multimodal HCP. In their model, different transportation modes between hub nodes and different types of service time commitments for paths joining OD node pairs are allowed.

Capacities of hubs and links of the each route is a less considered issue in the literature, especially for HCP. Campbell [22] initially propose a mixed integer linear optimization model for the capacitated MHLP with four indexed variable. Aykin [23] presents the capacitated hub-and-spoke network design problem with fixed capacity. Flows between OD node pairs can bypass hubs in the associated mathematical model. As the solution procedure, the author provides a branch-and-bound procedure and a heuristic methodology dividing the solutions set on the basis of hub placements. Bryan [24] extends HLP model proposed in O'Kelly and Bryan [25] in many different ways. The base multiple allocation model unequivocally represents scale economies by permitting interhub expenses to be a component of streams with per unit cost diminishing as streams increment.In the extended models, the effect of imposing minimum and maximum flow limits on interhub links is studied. A new formulation for the capacitated SHLP is investigated by Ernst and Krishnamoorthy [26]. They develop heuristic approach for its algorithm based on random descent and simulated annealing.

Ebery et al.[27] provide formulations with three indexed variable for the capacitated MHLP. They incorporate the upper bound obtained from an efficient heuristic in a linear optimization based branch and bound solution scheme to solve large instances. Marín[28] also consider new formulations for capacitated MHLP, and obtain better computational results with the given resolution techniques. As a natural extension of the uncapacitated one-stop (no interhub flows) hub-and-spoke model, Sasaki and Fukushima [29] give a new formulation for one-stop capacitated model. More precisely, arc and hub capacity constraints are included to the model, and a branch-and-bound based exact solution method is used to solve this model. A capacitated SHLP encountered in the design of telecommunications networks is investigated by Carello et al. [30]. In this problem, the subnetwork formed by hub nodes is required to be fully connected and the traffic passing through each hub node is limited by capacity constraints. The goal is to limit the amount of fixed expenses of opening and preparing hubs and linking expenses of introducing on each edge the capacity expected to route the traffic on the edge itself (modular capacity). A local search methodology is introduced and various metaheuristic procedures have been developed based on this approach.

The same problem is investigated by Yaman and Carello [31]. Objective of the associated model minimizes the total fixed cost of adding hubs and determining the needed capacity on each link. Capacity on an edge is adjusted by setting the number of edges of modular capacity. Moreover, hub capacity restricts the amount of flow that can pass through a hub rather than the incoming flow. They adapt the branch and cut algorithm and develop a two-level heuristic for this problem. In a later study, Yaman [32] focus on the star p-HLP with modular link capacities where each node is assigned to a single hub and each hub is served to a single central hub.

Rodríguez-Martín and Salazar-González[33] develop a formulation of capacitated MHLP for incomplete hub networks where both hubs and links are capacitated. The authors propose an efficient nested two level algorithm depending on Benders decomposition to solve the problem. Another approach to include capacity related issues to HLP is given in [34]. In this work, authors formulate a bi-objective SHLP such that one of the objectives minimize the inflow processing times at the hubs instead of using hard capacity constraints for hubs to limit the inflow. Mohammadi et al. [35] provide a new multiobjective model for the capacitated SHCP and solve it by multi-objective imperialist competitive algorithm. Contreras et al. [36] consider a capacitated HLP in which hub capacities are not parameters but decision variables and implement Benders decomposition algorithm to solve it. Sedehzadeh et al.[37] study a multi-product multi-mode capacitated SHCP using queue approach. One of the objective is to minimize the sum of hub node opening costs and transportation costs, while the second one minimizes the maximum transfer time for each product and between each OD node pair by considering transportation and waiting times. This multi-objective programming problem is solved by multi-objective parallel simulated annealing algorithm. Karimia et al. [38] present a tabu search algorithm to solve the multi-modal capacitated p-SHCP over fully interconnected networks. Allocations of non-hub nodes to hubs and locations of hubs are to be found such that the travel time between any OD node pair is less than or equal a given time bound. Merakli and Yaman [39] consider a capacitated MHLP with hose demand uncertainty. Both the feasibility of the solutions and the total cost are affected by the demand uncertainty, since hub nodes are capacitated and transportation cost is a function of demand. Hoff et al. [40] solve the capacitated modular SHLP (capacity of interhub edges is increased in a modular fashion) via different combination of heuristics which is not taken into account before.

Demir et al. [41] introduce a multi-objective linear optimization model for the capacitated MHLP. They use both link and hub capacities in their model. For solving the model, they develop a multi-objective evolutionary algorithm which is a NSGA-II based heuristic. Danach et. al [42] combine vehicle capacity idea with p-SHLP. For solving the novel model, they develop a hybrid hyper-heuristic algorithm which is a usage of Lagrangian relaxation within a reinforced learning framework. Taherkhani et al. [43] consider profit based modelling framework with capacitated HLP with various demand classes with deterministic and stochastic versions. They use a novel fastened version of Benders decomposition for deterministic version. For stochastic version, sample average approximation algorithm is employed with the benefits of the improved Benders decomposition. Butun et. al. [44] formulate the capacitated directed cycle HLP considering congestion. They first linearize the model after that use a tabu search heuristics for solving the linearized model. From the survey above, multiple allocation studies are generally not studied. On the other hand, it is assumed that all possible hubs must be selected from a subset of nodes. Generally, capacity is taken as hub capacity. There is a limited number of studies investigate link capacity. Because of these, we first introduce a novel multiple allocation hub covering problem including transportation costs and we also add to the model both hub and/or link capacity constraints.

3. Mathematical Formulation

Four models of the multiple allocation hub covering flow problem (MHCFP) is introduced in this section. These mathematical models are: uncapacitated MHCFP or UMHCFP, link capacitated MHCFP or MHCFP-1, hub capacitated MHCFP or MHCFP-2 and lastly, link and hub capacitated MHCFP or MHCFP-3. These models are based on the formulation of Ebery et al. [27] and Lowe and Sim[20]. A common notation for these problems is given below:

$\mathcal{V}$	set of nodes
h_{ij}	demand flow originating from node $i \in \mathcal{V}$ destined for node $j \in \mathcal{V}$
H	total amount of flow to be sent
O_i	total demand originating from node $i \in \mathcal{V}$
D_j	total demand destined for node $j \in \mathcal{V}$
ω_{ik}	flow capacity of the links connecting nodes $i, k \in \mathcal{V}$
Γ_k	flow capacity of hub $k \in \mathcal{V}$
f_i	cost of opening a hub at node $i \in \mathcal{V}$
c_{ij}	unit flow cost for the link connecting nodes $i, j \in \mathcal{V}$
d_{ij}	length of the link connecting nodes $i, j \in \mathcal{V}$
$\mathbf{A}$	node coverage matrix (A_{ij} is 1 if node $j \in \mathcal{V}$ can be covered by node $i \in \mathcal{V}$ and 0 otherwise)
$\mathbf{B}$	path coverage matrix ($B_{ikj} = A_{ik}A_{kj}$)
α	interhub flow cost discount factor with $\alpha \in (0, 1)$

It is clear that $O_i = \sum_{j\in\mathcal{V}} h_{ij}$, $D_j = \sum_{i\in\mathcal{V}} h_{ij}$ and $H = \sum_{i,j\in\mathcal{V}} h_{ij}$. Hub opening decision is related with the binary decision variable, x_k, which equals to 1 if a hub is set to node k, 0 otherwise. Variable z_{ik} denotes the amount of flow sent from node i to hub node k, q_{ilj} the amount of flow sent from node i to node j through hub node l, and y_{ikl} to the amount of flow sent from node i via hub nodes k and l.

We introduce first the formulation of model **UMHCFP** below:

$$\begin{aligned}
\min \quad & \sum_{k \in \mathcal{V}} f_k x_k + \alpha \sum_{i,k,l \in \mathcal{V}} c_{kl} B_{ikl} y_{ikl} \\
& + \sum_{i,k \in \mathcal{V}} c_{ik} A_{ik} z_{ik} + \sum_{i,l,j \in \mathcal{V}} c_{lj} B_{ilj} q_{ilj} && (1) \\
\text{s.t.} \quad & \sum_{k \in \mathcal{V}} A_{ik} z_{ik} = O_i && i \in \mathcal{V}, \quad (2) \\
& \sum_{l \in \mathcal{V}} B_{ilj} q_{ilj} = h_{ij} && i, j \in \mathcal{V}, \quad (3) \\
& \sum_{l \in \mathcal{V}} B_{ikl} y_{ikl} + \sum_{j \in \mathcal{V}} B_{ikj} q_{ikj} - \sum_{l \in \mathcal{V}} B_{ilk} y_{ilk} = A_{ik} z_{ik} && i, k \in \mathcal{V}, \quad (4) \\
& \sum_{i \in \mathcal{V}} B_{ilj} q_{ilj} \le D_j x_l && l, j \in \mathcal{V}, \quad (5) \\
& A_{ik} z_{ik} \le O_i x_k && i, k \in \mathcal{V}, \quad (6) \\
& q_{ilj}, y_{ikl}, z_{ik} \ge 0 \quad x_k \in \{0, 1\} && i, k, l, j \in \mathcal{V}. \quad (7)
\end{aligned}$$

The objective in Eq.(1) is to minimize the total cost of opening hubs and routing demand through network links by considering interhub flow cost discount factor. Eq.(2) ensures that all the demand originating from node $i \in \mathcal{V}$ is transported through hubs. Eq.(3) guarantees that the demand originating form node $i \in \mathcal{V}$ destined for node $j \in \mathcal{V}$ is transported through hubs. Eq.(4) corresponds to the flow conservation constraints at each hub. Eq.(5) and Eq.(6) together ensure no demand is transported directly between non-hub nodes. Finally, Eq.(7) shows the type of decision variables.

We also make use of the following constraints in this study:

$$\begin{aligned}
& z_{ik} \le \omega_{ik}(1 - x_i) + H x_i && i, k \in \mathcal{V}, \quad (8) \\
& \sum_{i \in \mathcal{V}} q_{ilj} \le \omega_{lj}(1 - x_j) + H x_j && l, j \in \mathcal{V} \quad (9) \\
& \sum_{i \in \mathcal{V}} z_{ik} \le \Gamma_k && k \in \mathcal{V}. \quad (10)
\end{aligned}$$

Constraints in Eq.(8) guarantee that the amount of flow on the link connecting node i and hub node k does not exceed the link capacity ω_{ik}. In a similar fashion, constraints in Eq.(9) do not allow an amount of flow to be transported from hub node l to node j surpassing the link capacity ω_{lj}. Constraints in Eq.(10) restrict the inflow to any hub according to its capacity. When Eqs.(8-10) are added to UMHCFP given in Eqs.(1-7), the following variant models can be built:

MHCFP-1 : Eqs.(1-7), Eq.(8), Eq.(9),
MHCFP-2 : Eqs.(1-7), Eq.(10),
MHCFP-3 : Eqs.(1-7), Eq.(8), Eq.(9), Eq.(10).

To compare numerical results, we also associate mathematical models **UMHCP**, **MHCP-1**, **MHCP-2** and **MHCP-3** with UMHCFP, MHCFP-1, MHCFP-2, MHCFP-3 respectively where transportation related costs in the objective function Eq.(1) are omitted.

4. Computational Experiments

We have applied well-known and openly available datasets in our numerical study (CAB dataset with 25 nodes [45] and TR data set with 81 nodes [46]). These benchmark network data sets are all available through OR library [45]. TR data set is complete such that *unit flow costs*, *hub opening costs*, *network links' lengths* and *demand flows* are all provided. CAB data set only includes links lengths and demand flows between nodes. Accordingly, we assume that unit flow costs are proportional to link lengths such that $c_{ij} = d_{ij}/25,000$ for all $i, j \in \mathcal{V}$ and hub opening costs f_i for all $i \in \mathcal{V}$ are set all equal to $10,000$, $20,000$ or $30,000$ for a given instance. In order to obtain reasonable results in terms of the number of opened hubs, original hub opening costs for TR data set are all multiplied with 300. CAB data set has symmetrical structure such that $h_{ij} = h_{ji}$ and $d_{ij} = d_{ji}$ for all $i, j \in \mathcal{V}$. TR data set has not this symmetrical structure.

Interhub flow cost discount factor α is set to 0.2, 0.5 or 0.8 for CAB data set, and 0.4, 0.6 or 0.8 for TR data set. The hub or *node coverage radius* is obtained by multiplying the *coverage ratio* Δ with length of the longest link of the network. Δ should be selected such that the existing network does not contain disconnected sub-networks. Hence, Δ is set to 0.6, 0.7 or 0.8 for CAB data set and 0.55, 0.65 or 0.75 for TR data set. Then, each element A_{ij} of the *node coverage matrix* is fixed to 1 if the node coverage radius is greater than or equal to the length of the link connecting nodes i and j, and 0 otherwise.

Link capacities, ω_{ik} for all $i, k \in \mathcal{V}$, are not included in the original CAB and TR data sets, so we develop a procedure to identify them which we briefly expose here. First, UMHCFP is solved to optimality to obtain optimum link flows. As there is no upperbound on the amount of flow that can be sent between hubs, we exclude interhub flow amounts among the optimum link flows and calculate the average (μ) and standard deviation (σ) of the remaining link flow values. Finally, assuming that the link flows are normally distributed, all of links' capacities ω_{ik} $i, k \in \mathcal{V}$ are set equal to $\omega_p = \mu + \zeta_p \times \sigma$ where ζ_p is the z-score corresponding to probability p with $p = \{0.70, 0.80, 0.90\}$. Original data sets do not contain also *hub capacities*, so we simply designate all Γ_k $k \in \mathcal{V}$ equal to a fraction (10%, 20% or 30%) of the total demand H where we denote these fractions as $\Gamma_{0.10}$, $\Gamma_{0.20}$ and $\Gamma_{0.30}$ respectively.

To demonstrate the usefulness of proposed MHCFP models, namely UMHCFP, MHCFP-1, MHCFP-2 or MHCFP-3, we introduce two indicators: the number of opened hubs (NH) and the percent of cost reduction (IMP). For a given setting of parameters, NH is obtained by solving one of proposed MHCFP models optimally. Let us denote the optimum objective function value of a MHCFP model as $v^{\star}_{MHCFP}$, and the value of (objective) function in Eq.(1) calculated by using the optimum solution of the associated MHCP model as v_{MHCP}. Then, IMP is calculated such that

$$\mathrm{IMP} = 100 \times \frac{v_{MHCP} - v^{\star}_{MHCFP}}{v_{MHCP}}.$$

Table 1 summarizes execution times to reach optimum solutions for four models and two data sets. For each model and data set, several instances are formed by the combinations of model parameters and solved to optimality. For example, 27 different instances are formed by varying $\alpha = \{0.20, 0.50, 0.80\}$, $\Delta = \{0.60, 0.70, 0.80\}$ and $f = \{10,000, 20,000, 30,000\}$ for UMHCFP and CAB data set. All instances have been solved with GAMS 24.9.4 [47] via solver CPLEX 12.7.1 on dual Intel Xeon E5-2670 (2.6 GHz) processor and 32 GB of RAM workstation running Windows Server 2012 R2-64 bits. While the network size for TR data set is more than the triple compared to CAB data set, the solution times reported for it in average CPU times are much more smaller except for MHCFP-1 interestingly. In fact, it can observed that there is a great discrepancy in execution times for CAB data set by just looking to maximum and minimum CPU times in Table 1. This can be mainly attributed to varying hub opening costs. As hub opening costs are included in the original TR data set, we did not need to generate them and setup experiments with different cost sets accordingly. This is exactly what we have done in case of CAB data set since hub opening costs were not provided. Accordingly, the number of instances solved for CAB data set is three times more than TR data set as three different cost sets are considered.

Table 1: Solution time statistics (CPU times in seconds) for models and data sets

Statistics for CAB data set

	Num. of Instances	Mean Time	Max. Time	Min. Time
UMHCFP	27	31.53	111.33	3.73
MHCFP-1	81	65.02	196.09	6.69
MHCFP-2	81	82.08	739.16	4.19
MHCPF-3	243	107.04	1127.27	6.73

Statistics for TR data set

	Num. of Instances	Mean Time	Max. Time	Min. Time
UMHCFP	9	26.54	39.27	16.80
MHCFP-1	27	109.50	332.97	47.03
MHCFP-2	27	36.45	72.73	18.11
MHCFP-3	81	68.87	125.19	35.81

Results for UMHCFP given in Tables 2 and 6 indicate that both the optimum number of hubs NH and the cost improvement ratio IMP significantly increase as α and Δ decrease, and reach their highest values when these two model parameters attain their lowest values. We have to first note that the optimum solution of UMHCP requires only one hub to be opened for all combinations of α and Δ for both CAB and TR data sets. Then, this outcome can be easily interpreted as follows. For UMHCFP, the decrease in α favors more flows between hubs and this stimulates the opening of new hubs to benefit from this cost reduction. In fact, α is one of the most influential factor on NH for all models. Meanwhile, IMP is more affected by the decrease in the coverage ratio Δ. For two data sets, the added cost for increasing the number of hubs is overshadowed by the cost saving due to the increasing interhub flows, and the total cost is reduced accordingly. For CAB data set, increasing hub opening costs obviously decreases NH as the total hub opening cost becomes consequential compared to the total flow cost. In fact, the single objective of UMHCP is to set as few as possible nodes to hubs while routing all the flow demand without any regard for transportation costs. This approach is insufficient as it leads to design hub-and-spoke networks resulting in higher total cost.

Table 2: Summary of UMHCFP Results (CAB data set)

f_i	α	Δ					
		0.80		0.70		0.60	
		NH	IMP	NH	IMP	NH	IMP
10,000	0.80	6	31.67	6	31.43	5	42.62
	0.50	7	42.74	7	42.11	8	50.61
	0.20	9	56.34	9	54.89	10	60.42
20,000	0.80	4	24.79	4	24.38	4	36.28
	0.05	5	32.67	5	32.13	5	41.92
	0.20	5	44.45	5	43.59	5	50.09
30,000	0.08	3	20.08	3	19.89	3	32.17
	0.50	4	25.64	4	24.86	4	35.63
	0.20	5	36.44	5	35.60	5	43.39

The effects of imposing bounds on the flow passing thru network links and hubs are shown in Tables3-5 and 7-9. The results given in those tables are obtained by solving MHCFP-1, MHCFP-2 and MHCFP-3 respectiveley to optimality for several combinations of α, Δ, ω and Γ. Compared with the results of the unbounded model UMHCFP, it is clear that IMP rises further as bounds become more restrictive. In other words, it becomes more critical to consider transportation costs in hub covering problems and thus the *usefulness* of our proposed models increases.

Table 3: Summary of MHCFP-1 Results (CAB data set)

ω		$\omega_{0.90}$						$\omega_{0.80}$						$\omega_{0.70}$					
Δ		0.50		0.50		0.50		0.50		0.50		0.50		0.50		0.50		0.50	
f_i	α	NH	IMP	NH	IMP	NH	IMP	NH	IMP	NH	IMP	NH	IMP	NH	IMP	NH	IMP	NH	IMP
10,000	0.80	6	50	7	64	7	59	7	65	7	54	7	59	7	52	7	56	7	57
	0.50	7	54	7	66	8	61	8	65	7	55	8	61	9	56	8	56	8	58
	0.20	10	61	10	70	11	64	10	68	10	59	11	66	11	62	11	59	12	60
20,000	0.80	4	46	4	60	4	55	4	61	4	49	5	56	5	61	5	42	5	57
	0.50	5	46	5	57	5	54	6	59	6	47	6	56	6	60	7	44	6	57
	0.20	6	51	7	57	7	55	7	60	7	49	7	58	7	62	7	50	7	58
30,000	0.80	3	50	3	58	4	54	3	59	3	47	4	57	4	59	4	54	4	58
	0.50	4	48	4	57	4	52	4	55	4	43	4	53	4	57	5	49	5	55
	0.20	5	49	5	58	5	53	5	54	5	42	5	50	6	56	6	46	6	53

Table 4: Summary of MHCFP-2 Results (CAB data set)

Γ		$\Gamma_{0.30}$						$\Gamma_{0.20}$						$\Gamma_{0.10}$					
Δ		0.80		0.70		0.60		0.80		0.70		0.60		0.80		0.70		0.60	
f_i	α	NH	IMP	NH	IMP	NH	IMP	NH	IMP	NH	IMP	NH	IMP	NH	IMP	NH	IMP	NH	IMP
10,000	0.80	6	61	7	65	7	67	7	64	7	64	7	61	10	61	10	66	10	58
	0.50	7	62	7	67	8	68	8	66	8	66	8	62	12	63	12	66	12	58
	0.20	9	66	9	70	10	70	10	70	10	69	11	65	12	67	12	67	13	60
20,000	0.80	4	57	4	62	4	64	5	57	5	60	6	56	10	51	10	61	10	53
	0.50	6	56	6	61	5	63	6	57	6	60	6	56	10	52	10	59	10	51
	0.20	6	58	6	63	5	63	6	59	6	62	6	58	10	56	10	58	10	50
30,000	0.80	4	55	4	54	4	62	5	59	5	57	5	53	10	50	10	54	10	48
	0.50	4	53	4	51	4	60	5	58	5	55	6	52	10	50	10	52	10	46
	0.20	5	53	5	52	5	59	6	59	6	57	6	53	10	51	10	50	10	45

Table 5: Summary of MHCFP-3 Results (CAB data set)

Γ			$\Gamma_{0.30}$						$\Gamma_{0.20}$						$\Gamma_{0.10}$					
Δ			0.80		0.70		0.60		0.80		0.70		0.60		0.80		0.70		0.60	
ω	f_i	α	NH	IMP	NH	IMP	NH	IMP	NH	IMP	NH	IMP	NH	IMP	NH	IMP	NH	IMP	NH	IMP
$\omega_{0.90}$	10,000	0.80	6	71	7	70	7	61	7	56	7	59	7	57	10	68	10	68	10	59
		0.50	7	72	7	72	8	63	8	59	8	61	8	58	12	69	12	68	12	59
		0.20	10	74	10	76	11	66	10	65	10	65	11	62	12	71	12	70	13	61
	20,000	0.80	4	58	4	54	4	58	5	52	6	55	6	56	10	63	10	63	10	53
		0.50	6	58	6	53	6	57	6	53	6	55	6	55	10	62	10	62	10	52
		0.20	6	60	7	54	7	58	6	56	7	58	7	55	10	62	10	62	10	51
	30,000	0.80	4	66	4	51	4	61	5	49	5	52	6	51	10	60	10	55	10	50
		0.50	4	64	4	49	4	58	6	48	6	51	6	51	10	58	10	54	10	48
		0.20	5	63	5	50	5	57	6	51	6	53	6	51	10	57	10	53	10	47
$\omega_{0.80}$	10,000	0.80	7	64	7	65	7	67	7	67	7	60	7	63	10	67	10	65	10	56
		0.50	8	66	7	67	8	67	9	68	8	61	8	63	12	68	12	64	12	57
		0.20	10	70	10	71	11	68	10	71	10	64	11	66	12	70	12	66	13	60
	20,000	0.80	4	57	5	56	5	62	6	64	6	57	6	49	10	62	10	59	10	51
		0.50	6	56	6	56	6	61	6	63	7	56	6	47	10	61	10	58	10	50
		0.20	7	59	7	59	7	60	7	64	7	57	7	45	10	61	10	57	10	50
	30,000	0.80	4	52	4	63	4	60	5	51	5	48	6	53	10	55	10	54	10	50
		0.50	4	50	5	61	5	57	6	48	6	45	6	52	10	55	10	52	10	48
		0.20	6	51	5	62	5	55	6	46	6	45	6	52	10	56	10	51	10	47
$\omega_{0.70}$	10,000	0.80	7	56	7	58	7	64	7	68	7	54	7	62	10	67	10	64	10	67
		0.50	9	58	8	60	8	65	9	69	9	55	8	63	12	68	12	64	12	65
		0.20	11	64	11	64	12	68	11	71	11	59	12	65	12	71	12	67	13	64
	20,000	0.80	5	52	5	54	5	63	6	62	6	62	6	57	10	61	10	59	10	62
		0.50	6	51	7	53	6	61	7	61	7	61	6	56	10	61	10	59	10	58
		0.20	7	54	7	55	7	62	7	61	7	61	7	57	10	62	10	59	10	55
	30,000	0.80	4	56	4	58	4	59	5	52	5	47	6	46	10	57	10	49	10	53
		0.50	5	54	5	56	5	56	6	51	6	43	6	44	10	56	10	47	10	49
		0.02	6	55	6	57	6	55	6	52	6	41	6	42	10	57	10	47	10	47

Table 6: Summary of UMHCFP Results (TR data set)

α	Δ					
	0.75		0.65		0.55	
	NH	IMP	NH	IMP	NH	IMP
0.80	7	42.97	7	76.85	11	79.33
0.60	7	44.61	15	77.77	15	80.35
0.40	21	47.96	24	79.68	25	82.01

Table 7: Summary of MHCFP-1 Results (TR data set)

ω	α	Δ					
		0.75		0.65		0.55	
		NH	IMP	NH	IMP	NH	IMP
$\omega_{0.90}$	0.80	8	79.13	8	78.10	8	87.90
	0.60	10	76.18	10	75.15	10	85.44
	0.40	25	73.00	25	72.08	25	82.12
$\omega_{0.80}$	0.80	9	77.62	9	75.83	9	87.28
	0.60	13	74.74	13	73.00	13	84.80
	0.40	26	71.75	26	70.22	26	81.48
$\omega_{0.70}$	0.80	9	85.46	9	85.43	9	82.40
	0.60	13	83.25	13	83.22	13	80.16
	0.40	28	80.53	28	80.51	28	77.65

Table 8: Summary of MHCFP-2 Results (TR data set)

Γ	α	Δ					
		0.75		0.65		0.55	
		NH	IMP	NH	IMP	NH	IMP
$\Gamma_{0.30}$	0.80	9	92.11	12	93.00	11	90.95
	0.60	13	90.63	16	91.82	17	89.83
	0.40	24	88.61	26	90.40	28	88.62
$\Gamma_{0.20}$	0.80	12	95.74	13	93.98	14	93.44
	0.60	14	94.82	18	92.78	18	92.20
	0.40	26	93.41	28	91.13	29	90.49
$\Gamma_{0.10}$	0.80	18	97.21	17	96.81	17	94.53
	0.60	23	96.67	25	96.21	24	93.59
	0.40	33	95.87	33	95.37	34	92.31

Table 9: Summary of MHCFP-3 Results (TR data set)

Γ		$\Gamma_{0.30}$						$\Gamma_{0.20}$						$\Gamma_{0.10}$					
Δ		0.75		0.65		0.55		0.75		0.65		0.55		0.75		0.65		0.55	
ω	α	NH	IMP	NH	IMP	NH	IMP	NH	IMP	NH	IMP	NH	IMP	NH	IMP	NH	IMP	NH	IMP
$\omega_{0.90}$	0.80	10	93	12	92	14	92	13	96	13	93	14	92	18	97	17	97	17	93
	0.60	13	91	17	91	18	90	15	95	18	92	18	90	25	96	28	97	25	92
	0.40	26	89	28	90	30	89	27	94	29	91	30	88	34	95	34	96	34	91
$\omega_{0.80}$	0.80	11	95	12	95	14	89	13	96	13	94	14	90	19	96	18	96	18	94
	0.60	14	94	17	94	18	88	16	95	19	93	18	88	25	96	28	96	24	92
	0.40	26	92	28	93	30	87	27	94	29	91	30	87	34	95	34	95	34	91
$\omega_{0.70}$	0.80	13	95	13	94	14	89	14	96	14	94	14	89	20	96	19	97	19	93
	0.60	16	94	19	93	18	88	17	95	20	93	19	87	25	96	28	96	25	92
	0.40	27	92	28	91	30	87	27	93	30	91	30	85	35	95	34	95	34	91

Link capacities have less consequence for IMP compared to hub capacities. This can be observed when results in Tables 3-4 for CAB data set and Tables 7-8 for TR data set are contrasted. We can also derive the conclusion that hub capacities are a dominant factor for IMP according to the results presented in Tables 4-5 and Tables 8-9. Moreover, IMP is seriously decreased when the cost of opening hubs increases and the capacities of hubs decrease simultaneously according to Table 4-5. From Tables 4-5, it is apparent that the rise of hub opening costs decreases NH, as expected. The other most influential factor on the value of NH is the capacity of hubs, while link capacities have a marginal effect on it. The decrease in Δ has no or slightly increasing consequence on NH for all models. As α is reduced, NH can dramatically increase but IMP is very slightly effected or worsened by this decrease. For some particular combination of parameters values, proposed models are solved optimally and the real locations of opened hubs are plotted on the maps in Figures 1 and 2. The black dots represent hubs that are common for all instances while white dots correspond to additional hubs specific to the instance. It is apparent that high accessibility and total demand flowing in/out of a node are determinants for hub locations. Moreover, cost savings resulting from interhub transfers encourage the opening of several hubs.

5. Conclusion

In MHCFP, the aim is to find the optimal design for hub-and-spoke networks while considering hub opening and demand routing costs and coverage constraints. Flow demand associated with a specific origin-destination node pair must be routed by visiting at least one hub node. In this study, it is assumed that a hub covers a nonhub node if the distance between these two is less than a predefined value, while there is no limit on the distance between hubs.

Our formulations of MHCFP are linear programming problems with continuous and integer variables. The results obtained from our computational experiments reveal that the number of hubs in the network decreases depending on the increase in hub opening costs, as expected. In fact, the increase in hub opening costs also affect how hub nodes are located such that hubs hubs have a tendency to be placed exactly at or near at nodes having high demands.

We can also derive from our numerical experiments that not taking into account flow costs can be consequential in the network design. For example, the total cost of designing a network and making it operational depending on the solution of UMHCFP could be as low as 80% below that based on MHCP. For other models, this percent can be even larger. This is attributed to the interhub flow cost discount factor which models the cost reduction due to the consolidated shipments.

There are several future research directions for MHCFP.Exact or heuristic methods to solve larger versions of MHCFP can be the initial point of a stream of work. In another line of research, MHCFP can be considered in a stochastic environment, where all costs and flow demands and even the network structure can depends on scenarios and the objective can be to design a resilient network to changing conditions.

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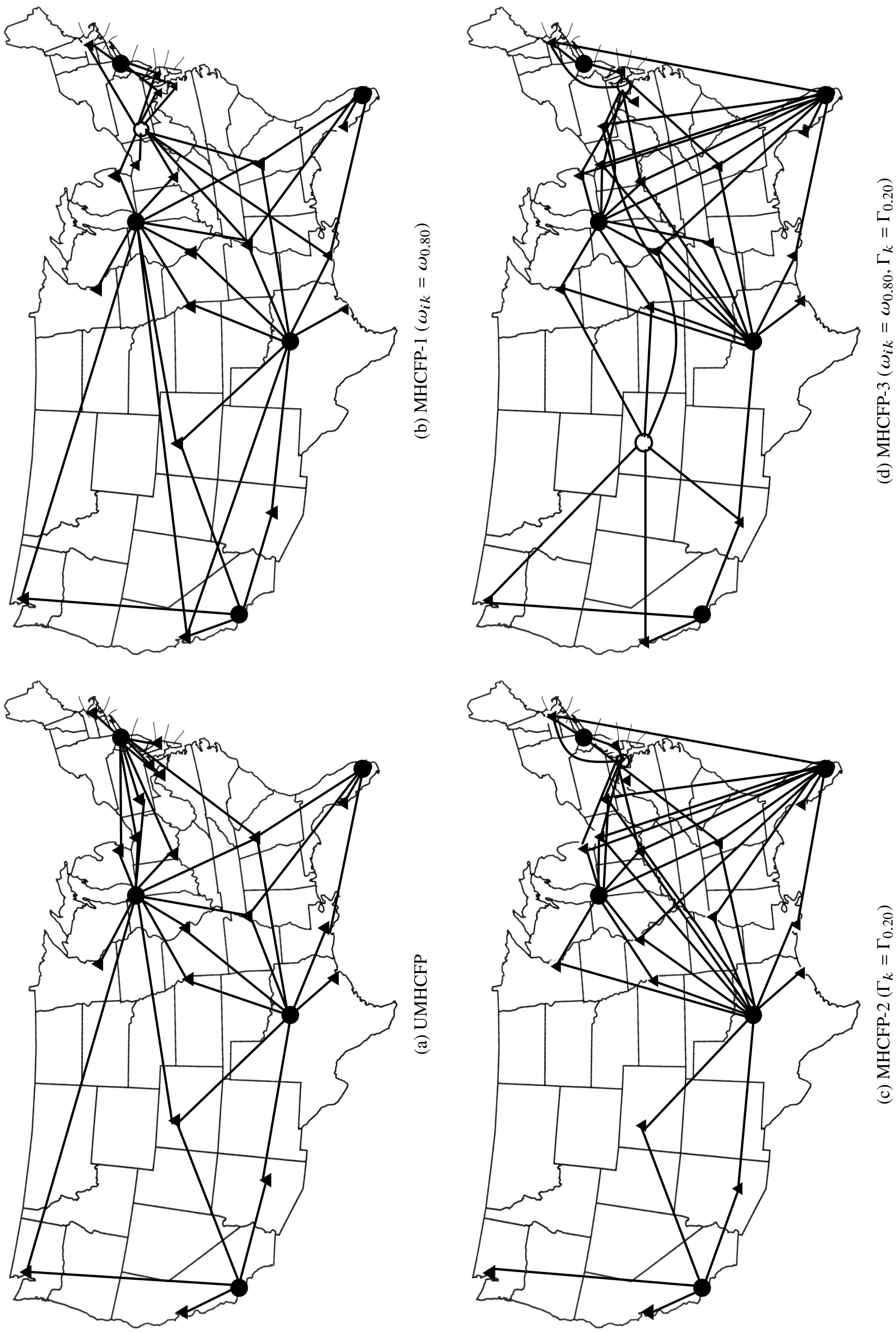


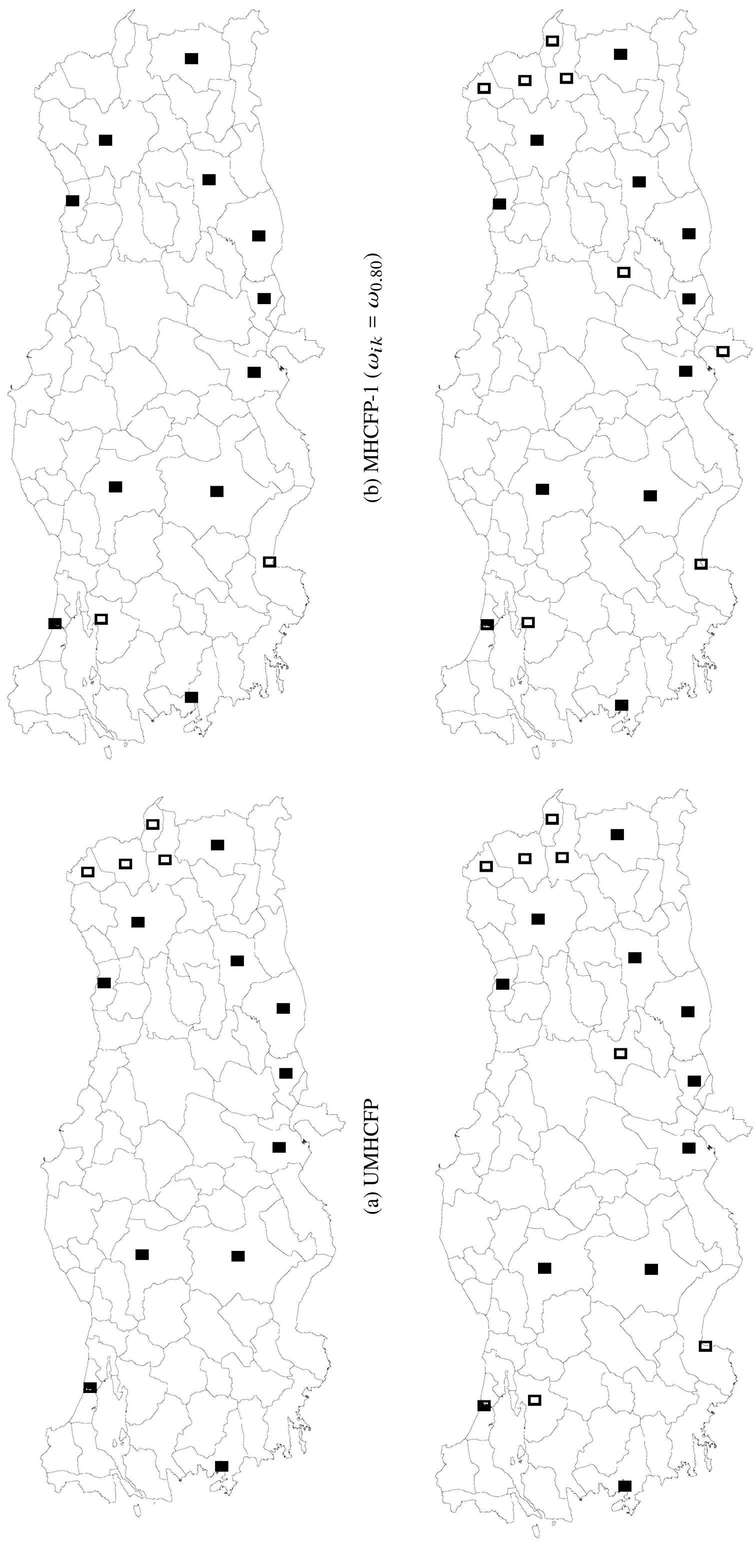
(a) UMHCFP

(b) MHCFP-1 ($\omega_{ik} = \omega_{0.80}$)

(c) MHCFP-2 ($\Gamma_k = \Gamma_{0.20}$)

(d) MHCFP-3 ($\omega_{ik} = \omega_{0.80}, \Gamma_k = \Gamma_{0.20}$)

Figure 1: Optimum solution for models with CAB data set ($\alpha = 0.50$, $\Delta = 0.70$ and $f_i = 20,000$)



(a) UMHCFP

(b) MHCFP-1 ($\omega_{ik} = \omega_{0.80}$)

(c) MHCFP-2 ($\Gamma_k = \Gamma_{0.02}$)

(d) MHCFP-3 ($\omega_{ik} = \omega_{0.80}, \Gamma_k = \Gamma_{0.20}$)

Figure 2: Optimum placement of hubs for models with TR data set ($\alpha = 0.60$ and $\Delta = 0.65$)

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