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Influence of traffic and road surface materials on elastic behavior of layered pavements

Trafik ve yol yüzeyi malzemelerinin katmanlı kaplamaların elastik davranışına etkisi

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Influence of Traffic and Road Surface Materials on Elastic Behavior of Layered Pavements

Highlights

- *Elastic Layered Pavements*
- *Road Surface Materials*
- *Traffic Road Categories*
- *Anisotropic behaviour of pavements*
- *Hashin-Shtrikman bounds*

Graphical Abstract

The figure shows a list of pavement sections for heavy traffic categories T3 and T4, depending on the esplanade category (see [3]). MB: bituminous mixture, AA: artificial aggregate, RC: rich concrete, CSS: Cement stabilized soil, AA: artificial aggregate (minimum thickness in cm).

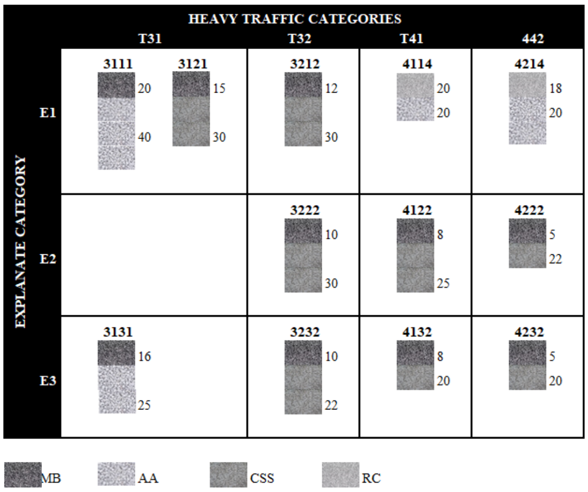

Figure. List of pavement sections for heavy traffic categories depending on the esplanade type.

Aim

Following the Spanish Highway Administration (SHA) classification of the pavement sections, the aim of this work is to determine the effective elastic behavior of any of the cases covered by this document from their composition and the type of traffic.

Design & Methodology

Using micromechanical theory and the Voigt/Reuss and Hashin-Shtrikman bounds, all the elastic moduli that characterize the stiffness of the composite medium are determined.

Originality

Although numerous experimental and laboratory works have been developed to study the mechanisms that govern different types of pavements, there are few works where they are explicitly determined. That is why the main novelty of this work consists in presenting analytical methods capable of quantifying the performance of the pavement based on the materials that compose it and the traffic loads to which it is subjected.

Findings

Analogously to what field and laboratory test show, this shows that the improved performance is due to lateral restraint proper ties conferred by the material inclusions.

Conclusion

Following the SHA classification of the pavement sections, the proposed scheme easily allows the prediction of pavement performance as function of the elastic properties of the mixture components and the traffic category.

Declaration of Ethical Standards

The author(s) of this article declare that the materials and methods used in this study do not require ethical committee permission and/or legal-special permission.

Influence of Traffic and Road Surface Materials on Elastic Behavior of Layered Pavements

Research Article

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ABSTRACT

In this paper the overall elastic properties of different layered road pavements have been presented. Using micromechanical theory and the Voigt/Reuss and Hashin-Shtrikman bounds, all the elastic moduli that characterize the stiffness of the composite medium are determined. Following the Spanish Highway Administration classification of the pavement sections, this work presents in detail the effective elastic behavior of any of the cases covered by this document from their composition and the type of traffic. Although numerous experimental and laboratory works have been developed to study the mechanisms that govern different types of pavements, there are few works where they are explicitly determined. That is why the main novelty of this work consists in presenting analytical methods capable of quantifying the performance of the pavement based on the materials that compose it and the traffic loads to which it is subjected. The proposed scheme is implemented in some scenarios by showing the dependence of the different elastic modulus on the volume fraction of the different materials that made the layered composite pavement.

Keywords: Pavements, layered materials, elastic properties, traffic loads, composite materials.

1. INTRODUCTION

Traffic is one of the most important variables in road design. The structural section of pavement is basically conditioned by the average daily intensity of heavy vehicles, since those are the ones whom transmit the most axle load and shorten the life of the infrastructure. The high cost of construction and maintenance of road networks leads to numerous studies that evaluate the resistance of different pavement sections ([11], [17], [19]), numerical studies where new reinforcing materials are introduced ([15], [18]) or research evaluating sections of recycled pavements [29]. In the last decades, many authors have focused on the development of finite element models (FEM) that faithfully represent the properties of pavements [2], [5]. Several works have also been conducted to determine the influence of the reinforcement shape ([4, 24]) or even randomly oriented geosynthetic fibers as stiffeners of soils ([22, 23]). A large number of laboratory tests have also been developed, comparing both unreinforced and reinforced pavements [12] or scaling sections of the composite inside box [28]. Field and laboratory test results determine that mechanical properties of reinforced asphalt with geosynthetics improve over the unreinforced sections since surface deflections are reduced. In general, experimental data show that the improved performance is due to lateral restraint proper ties conferred by the material inclusions. However, the application of these models is very expensive due to the complex structure of these materials.

The main objective of all those studies is to provide to the designer with standards that make easier the relationship between life, traffic and layer thickness. The Spanish Government (BOE, 2003) suggests a catalogue of pavements relating to heavy traffic intensities and permissible levels of damage for each type of soil. Moreover, for each previous combination proposes different sections based in the different materials that compose it.

However, as far as the author are aware no specifications about the elastic properties of the proposed composites are available. For this reason, the aim of this work will provide explicit predictions of the anisotropic elastic performance of the pavement as function of the elastic properties of the mixture components and the traffic category stablished in (BOE, 2003).

Different micromechanical methods to address composites behaviour have been developed by several authors since the 1960s. They consist on the determination of bounds on the effective parameters involved, and particularly on the fourth-order effective elastic tensor C^*. Classical results were given by Voigt and Reuss ([20, 28]). Using a variational principle and a first order correlation function providing more information than the volume fraction, new tighter expressions for the shear and the bulk modulus were developed for the elastostatic context by Hashin and Shtrikman ([7, 8]). These classical expressions, among a general review about methods concerning to the elastic behaviour of composites, are also presented in [26]. However, although classical expressions for the Hashin-Shtrikman (HS) bounds are stated by numerous works, very few explicit expressions from first principles are written down for anisotropic materials ([16, 21]). For all the reasons above, in this work using the tensor basis proposed by [16] (see also (2.10) in [26] for the isotropic case) and the appropriate formulation for the Eshelby tensor, the main objective is to present easy-use estimations of some the effective elastic moduli characterizing different layered composites proposed in [3].

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This work is organized as follows. In section 2 the mathematical formulation that model the elastic performance of a laminated reinforced soils is introduced. Following this, in section 3, from (HS) variational formulation introduced by [27], considering the tensor shorthand notation ([9]) and a transversely isotropic (TI) basis tensor ([16]), explicit expressions for all the elastic moduli describing the TI performance of pavement are derived. In section 4 we show the implementation of the previous numerical scheme with several situations exposed in [3]. Finally, we conclude in section 5.

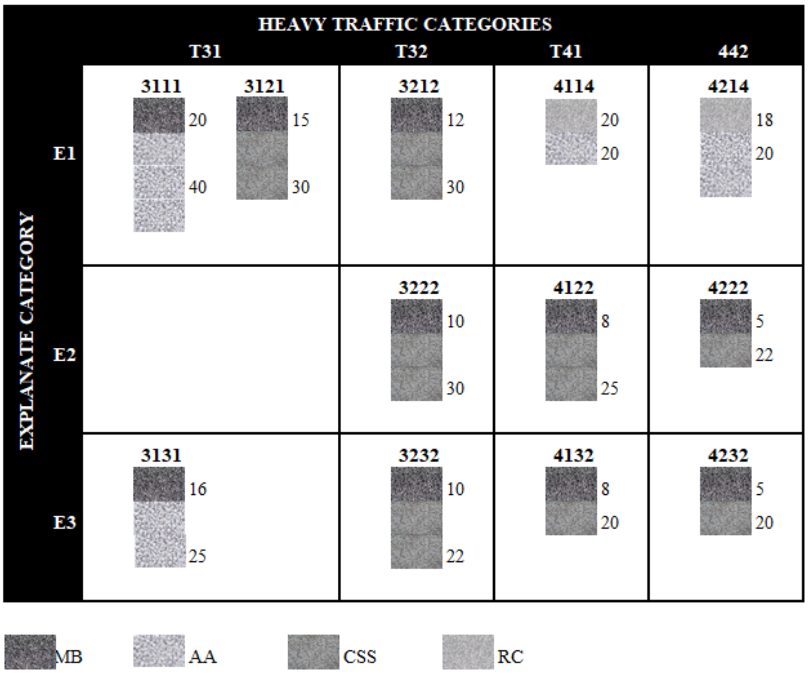

Figure 1. List of pavement sections for heavy traffic categories T3 and T4, depending on the esplanade category (see [3]). MB: bituminous mixture, AA: artificial aggregate, RC: rich concrete, CSS: Cement stabilized soil, AA: artificial aggregate (minimum thickness in cm

Table 1. Heavy traffic categories ($T_0 - T_4$) ([3])

Category	T_{00}	T_0	T_1	T_2	T_{31}	T_{32}	T_{41}	T_{42}
DAIHV	≥4000	≤4000	≤2000	≤800	≤200	≤100	≤50	≤25
NHV/D		≥2000	≥800	≥200	≥100	≥50	≥25	

Table 2. Modulus of compressibility in the second load cycle ([3])

Esplanade Category	E_1	E_2	E_3
E_{v2} (MPa)	≥60	≥120	≥300

2. STATEMENT OF THE PROBLEM

The catalog of firm sections of the Spanish Highway Administration [3] publishes the relationships in each type of structural section, between the intensities of heavy traffic and the types of esplanade. By means of a series of tables such as those indicated in Figure 1, this document lists the construction solutions according to the case as well as the minimum thickness of the layers. For the purposes of applying the standard [3], eight categories of heavy traffic are defined (see Table 1) according to parameters as DAIHV (daily average intensity of heavy vehicles) and NHV/D (number of heavy vehicles circulating per day). On the other hand, to define the structure of the pavement in each case, three categories of esplanade are established, called respectively E1, E2 and E3. These categories are determined according to the compressibility modulus in the second load cycle E_{v2} (see Table 2).

Each section is designated by three or four numbers. The first (if it is three digits) or the first two (if it is four digits) indicate the category of heavy traffic, from T00 to T42. The penultimate of the numbers represents the type of esplanade, while the last one refers to the type of pavement that is denoted as follows:

1. Bituminous mixtures on granular layer.
2. Bituminous mixtures on cement stabilized soil.

3. Bituminous mixtures on gravel cement built on cement stabilized soil.
4. Concrete pavement

Despite this, although the construction standards try to reduce the effects of traffic or environmental loads, the specific conditions or mechanisms that govern the behavior of pavements are, at least, unclear and in what refers to the standard [3] have remained practically without quantification. That is why in the following section the intention is to predict the elastic behavior of the materials described in Table 3 through the characterization of all the elastic constants that are involved in Hooke's Law, i.e.:

$$\boldsymbol{\sigma}(\mathbf{x})=\mathbf{C}(\mathbf{x})\mathbf{e}(\mathbf{x}),\ \ \mathbf{e}_{ij}(\mathbf{x})=\frac{1}{2}\left(\frac{\delta u_i}{\delta x_j}+\frac{\delta u_j}{\delta x_i}\right) \tag{1}$$

satisfying the equations of elastostatics

$$-\operatorname{div}\boldsymbol{\sigma}(\mathbf{x})=\mathbf{f}\ \text{in}\ \Omega_\varepsilon \tag{2}$$

$$\mathbf{C}(\mathbf{x})\mathbf{e}(\mathbf{x})\boldsymbol{\nu}(\mathbf{x})=\breve{\boldsymbol{u}}\quad \text{on}\ \partial\Omega_\varepsilon$$

In Eq. 2 *div* is the divergence operator, $\partial\boldsymbol{\Omega}_\varepsilon$ the boundary of the heterogeneous domain, **ν** the outer unit normal to $\partial\boldsymbol{\Omega}_\varepsilon$, **e** is the linear strain tensor, u is the displacement field and **f** is the force field. In Eq. (1) **σ** denotes the Cauchy stress tensor and **C=C(x)** (we work in Cartesian coordinates **x=** (x_1,x_2,x_3)) that in the considered case, for a transversely isotropic material is related to the engineer constants as follows

$$\boldsymbol{C}=\begin{pmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0\\ C_{12} & C_{11} & C_{13} & 0 & 0 & 0\\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & C_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & C_{44} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{C_{11}-C_{12}}{2} \end{pmatrix} \tag{3}$$

that in terms of the engineering constants (E: Young modulus, v= Poisson modulus) can be expressed as

$$\boldsymbol{C}=\begin{pmatrix} E_1\frac{1-\nu_{23}\nu_{32}}{\Delta} & E_1\frac{\nu_{21}-\nu_{31}\nu_{23}}{\Delta} & E_1\frac{\nu_{31}-\nu_{21}\nu_{32}}{\Delta} & 0 & 0 & 0\\ & E_1\frac{1-\nu_{23}\nu_{32}}{\Delta} & E_1\frac{\nu_{31}-\nu_{21}\nu_{32}}{\Delta} & 0 & 0 & 0\\ & & E_3\frac{1-\nu_{12}\nu_{21}}{\Delta} & 0 & 0 & 0\\ & & & \frac{E_1}{2(1+\nu_{31})} & 0 & 0\\ & & & & \frac{E_1}{2(1+\nu_{31})} & 0\\ sim & & & & & \frac{E_1^H\frac{1-\nu_{23}\nu_{32}}{\Delta}-E_1^H\frac{\nu_{21}-\nu_{31}\nu_{23}}{\Delta}}{2} \end{pmatrix} \tag{4}$$

where $\Delta=1-\nu_{31}\nu_{21}-\nu_{23}\nu_{32}\nu_{31}\nu_{13}-2\nu_{21}\nu_{32}\nu_{13}$ and $\mu_{13}=C_{44}$, $\mu_{12}=C_{66}$ are the in-plane and anti-plane shear modulus respectively.

3. METODOLOGY

In order to determine the effective properties of the layered pavement we will use some micromechanical methods.

The main reason is the complex geometric structure of these kind of materials, usually depending of small parameters **ε** (Figure 2) describing the size of the heterogeneities or the periodicity of its structure. It makes that the application of the usual numerical methods for the resolution of the partial differential equations (EDPs) that model them, it turns out impossible to implement. From the numerical point of view it is necessary to carry out some type of average for its study. Micromechanical techniques can be useful when looking for global information, that is, at macroscopic level, in a medium heterogeneous from which microscopic information is obtained. Key results in this scientific field are due to Voigt [25] and Reuss [20] that together with other authors ([6], [7], [8], [17]) and since the 1960s, they have studied the properties of the effective composite materials, especially fiber reinforced and laminated. Voigt-Reuss bounds estimate the isotension and isodeformation states of the composite as upper and lower dimensions of the elastic properties of the same. Specifically, for composites made of two different materials (Figure 1), in the following denoted by *phases* denoted by C^1 and C^2 the effective properties C^* of the composite can be determine by

$$C^R\ \leq C^*\ \leq C^V \tag{5}$$

where

$$C^R=(\phi_1\ (C^1)^{-1}+\phi_2\ (C^2)^{-1})\text{^}(-1)C^V=\phi_1\ C^1\ +\phi_2\ C^2 \tag{6}$$

Motivated by knowing the behavior of fiber-reinforced materials, in the early 1960s, numerous works were carried out. In particular, Hashin and Shtrikman (HS) [7] through a variational principle for elastostatics presented bounds on the effective bulk and shear moduli that are currently known as Hashin–Shtrikman bounds. For example, their expressions for bulk modulus [8] are given by

$$k^L\ \leq C^*\ \leq k^U \tag{7}$$

$$k^L=k_1+\frac{\phi_2}{\frac{1}{k_2-k_1}+\frac{3\phi_1}{3k_1+4\mu_1}},\qquad k^U=k_2+\frac{\phi_1}{\frac{1}{k_1-k_2}+\frac{3\phi_2}{3k_2+4\mu_2}}. \tag{8}$$

These estimates depend on the microstructure of the problem and therefore provide finer dimensions than the

Voigt-Reuss bounds that only depend only on the phase volume fraction (and phase material properties).

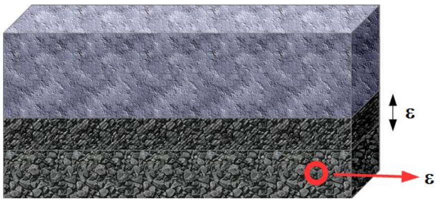

Figure 2. Heterogeneous layered composite with small size of the inhomogeneities

In general, (HS) bounds that appear in the literature are almost always merely "stated" and it is often unclear how to construct them when the material is not of isotropic medium. In this work, using the shorthand notation and basis tensor defined in [16], the effective properties for the two-phase layered composites described in Figure 1 will be deduced. Assuming that the inclusion $\boldsymbol{\Omega}_\varepsilon$ has an ellipsoidal shape (see [10], [13]), these expressions predict the behavior inside the composite through the so called concentration tensor **A** in the following way

$$\mathrm{e}|\boldsymbol{\Omega}_\varepsilon = \mathbf{A}\mathrm{e}, \quad \mathbf{A} = (\mathbf{I} + \mathbf{P}(\mathrm{C}^m - \mathrm{C}^\varepsilon))^{-1} \qquad (9)$$

where **I** is the fourth order identity tensor and **P** is the Hill Tensor (also known as **P**-tensor).

For layered materials, from Backus' expressions [1] the **P**-tensor is given by

$$\mathrm{e}|\boldsymbol{\Omega}_\varepsilon = \mathbf{A}\mathrm{e}, \quad \mathbf{A} = (\mathbf{I} + \mathbf{P}(\mathrm{C}^m - \mathrm{C}^\varepsilon))^{-1} \qquad (10)$$

$$\mathbf{P} = \left(0,0,0,\frac{\mathrm{k}}{-\mathrm{l}^2 + \mathrm{kn}},\frac{-\mathrm{l}^2}{\mathrm{n}(-\mathrm{l}^2 + \mathrm{kn})},0,\frac{1}{2\mathrm{p}}\right), \qquad (11)$$

where the parameters are k, l, n, p are defined as in Hill [9].

4. NUMERICAL RESULTS

In this section, using proper numerical software, elastic properties of composite pavements described in Figure 1 is showed. To do it, the properties defined in Table 3 empirically determine by several authors [23] and the description of the pavement according to the notation T_{ijkl} (section 2) are followed. For example, the notation T_{3111} indicates that the esplanade is subdued to heavy traffic (T_{31}) described in Table 1, with esplanade category 1 defined in Table 2 (T_{311}) and that is made of *1. bituminous mixtures on granular layer* (T_{3111}). On the other hand, T_{4232} denotes traffic (T_{42}) according Table 1 across and esplanade of type 3 (T_{423}) made of *2. Bituminous mixtures on cement stabilized soil* (T_{4232}). By simplicity, the properties of sections of pavement subjected to heavy traffic for three types of esplanade described in Figure 1 are estimated. They represent part of all those that appear in [3] for which the procedure is similar.

First, in Figure 3 using the expressions Eqs. 9 and 10 we consider the case of transversely isotropic behavior of bituminous pavements (host phase) reinforced with artificial aggregate for different volume fractions of the inclusion phase (aggregate). Effective anti-plane μ_{13} and plane μ_{12} shear moduli are presented showing the improvement of HS bounds over Voigt-Reuss ones. Analogously in Figure 4 bounds on bulk modulus for different esplanades are represented. The compressibility modulus for the anisotropic laminate are plotted depending on the volume fraction inclusion of artificial aggregate and cement stabilized soil.

For simplicity, in Tables 4 and 5 we will present the elastic modulus of one pavement of each type of the classification

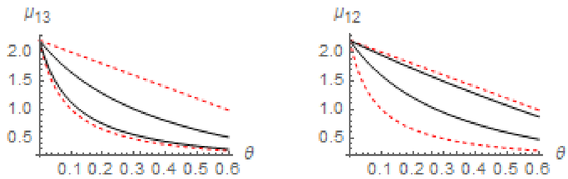

Figure 3. Hashin-Strikman (black solid lines) bounds of the effective shear μ_{13}, μ_{12} modulus of a transversely isotropic pavement, made of a bituminous mixture reinforced with artificial aggregate with the volumen fraction indicated in Figure 1 (T_{3111} material). Voigt-Reuss bounds are also plotted (red dashed lines) noting the improvement of the Hashin-Strikman bounds over Voigt-Reuss ones

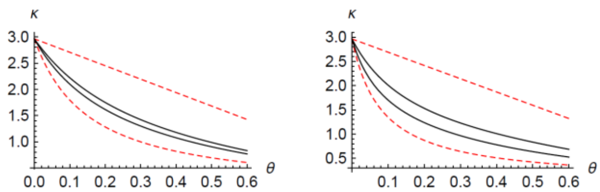

Figure 4. Hashin-Strikman (black solid lines) bounds of the bulk modulusκ of a transversely isotropic pavement, made of a bituminous mixture reinforced with artificial aggregate (on the left, T_{3111} material) and cement stabilized soil (on the right, T_{3112} material) with the volumen fraction indicated. Voigt-Reuss bounds are also plotted (red dashed lines). Note that HS bounds (solid lines

Table 3. Elastic constants of materials for pavements ([14])

Material	Young Module E (GPA)	Poisson Coefficient ν
Bituminous mixtures	6	0.35
Asphalt concrete	1.2	0.25
Cement stabilized soil	1.5	0.25
Granular layer	0.46	0.35

Table 4. HS and Voigt-Reuss bounds on the bulk modulus of pavement sections for different heavy traffic categories for a fixed volume fraction of the reinforcement (Φ=0.02)

BULK MODULUS (κ GPa)	Reuss bound	Voigt bound	HS lower bound	HS upper bound
T3111	1.1573333333333333	0.33110556869180846	0.46730435637747614	0.606110085295573
T3121	1.3761975308641978	0.37105601735896787	0.5458884306522523	0.7138774415116693
T3131	1.2968592592592594	0.3555067120246861	0.5160312966108286	0.6735979705187106
T3212	1.1330074074074075	0.5264848762861927	0.6406396923533942	0.6839809833983175
T3222	1.0407407407407407	0.5061370958427021	0.6050448942804901	0.6424530377247044
T3232	1.2201481481481482	0.5473311106506145	0.6761350287193165	0.7252347193512455
T4114	0.2735802469135802	0.2642891506769284	0.2859116015670019	0.2864751053512516
T4122	0.9111111111111111	0.29533325307973196	0.39121230406456453	0.4955381321012999
T4132	1.0205432098765432	0.3102296542456776	0.42362127710823216	0.5434875467438034
T4214	0.2717234567901235	0.2626380266039726	0.28372732559803515	0.28438773977678217
T4222	0.8869629629629627	0.47562533401963875	0.54971219972463	0.5775791582094717
T4232	0.24572839506172842	0.24099372505066424	0.251246299503611	0.25228013240392044

Table 5. Effective axial and transverse Young's modulus HS bounds on the bulk modulus of pavement sections for different heavy traffic categories for a fixed volume fraction of the reinforcement (Φ=0.02)

BULK MODULUS (κ GPa)	Effective Young's modulus (E^*_{12})	Effective Young's modulus (E^*_{13})	HS lower bound (shear modulus μ_{12})	HS upper bound (shear modulus μ_{12})
T3111	0.9020712977909856	0.9458528605461283	0.41170015926429887	0.7587108499365659
T3121	1.0700287890143534	1.0716134989985673	0.4941536380645635	0.9112116115571373
T3131	1.0062637257603282	1.0229354349635293	0.46267316900172045	0.8552526814428191
T3212	1.7606165209983542	2.153023783830408	0.9219746920124119	1.0034265832261922
T3222	1.674700583719813	2.0749610422242757	0.8771697604788169	0.9503410851606532
T3232	1.8445564595609092	2.231943160299745	0.9655788016583235	1.0542081629951727
T4114	0.6213051571300484	0.8434774413530317	0.29766913177506404	0.31287197385335597
T4122	0.7393330710597408	0.830889259564751	0.33328787135060745	0.5939254963530239
T4132	0.8086216669712566	0.879095593505824,	0.36649246012993364	0.6663044367055164
T4214	0.6111692419319678	0.8308905387342802	0.29171449606549776	0.30673806898789635
T4222	1.5377261785531957	1.955930783509008	0.8054460120158035	0.8633904753428512
T4232	0.47838306172925976	0.6822942654116554	0.21638303122320313	0.22477477595445045

5. DISCUSSION

As expected, prediction given by HS bounds show improvement over Voigt-Reuss ones (Table 4). They accord to FE numerical analysis and laboratory tests ([18, 19]). The results show the improvement of the overall stiffness of the homogenizated composite compared with unreinforced pavements. Geometrically, it was also found that the reinforcing with layers increases the load capacity of soils. Indeed, in Table 5 we observe how due to the presence of the reinforcement the decrease in the modulus of rigidity occurs, which is higher in the direction of the reinforcement.

6. CONCLUSIONS AND SUGGESTIONS

Explicit estimations for elastic properties of heterogeneous pavements are presented. The approximations are based on the implementation of the Hashin-Shtrikman bounds using transversely isotropic tensors for layered materials. Following the Spanish Highway Administration classification of the pavement sections, the proposed scheme easily allows the prediction of pavement performance as function of the elastic properties of the mixture components and the traffic category.

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DECLARATION OF ETHICAL STANDARDS)

The authors of this article declare that the materials and methods used in this study do not require ethical committee permission and/or legal-special permission.

AUTHORS' CONTRIBUTIONS

Pablo ROLDÁN-OLIDEN: Proposed the idea and analyzed the results.

Carmen CALVO-JURADO: Analyzed the results and wrote the manuscript

CONFLICT OF INTEREST

There is no conflict of interest in this study.

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