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Research Article

An Investigation of the Large Angles from 100g Measured at the Resection Point in the Cassini Method

Hüseyin İNCE[1], Nuri ERDEM[2], Yener TÜREN[3], F. Engin TOMBUŞ[4], İ. Murat OZULU[5]

[1] Professor , Hitit University, Vocational School of Technical Sciences / Department of Architecture and City Planning, Çorum, Turkey (*ORCID Number:* https://orcid.org/0000-0001-6118-5502*)*
[2] Assistant Professor, Osmaniye Korkut Ata University, Faculty of Engineering, Department of Geomatics Engineering, Osmaniye, Turkey, (*ORCID Number:* https://orcid.org/0000-0002-1850-4616*)*
[3]Ph. D., Trakya Üniversity, Vocational School of Technical Sciences / Department of Architecture and City Planning, Edirne, Turkey (*ORCID Number:* https://orcid.org/0000-0003-3381-3780*)*
[4]Ph. D., Hitit University, Vocational School of Technical Sciences / Department of Architecture and City Planning, Çorum, Turkey, (*ORCID Number:* https://orcid.org/0000-0002-2607-3211)
[5]Lec., Hitit University, Vocational School of Technical Sciences / Department of Architecture and City Planning, Çorum, Turkey, (*ORCID Number:* https://orcid.org/0000-0002-0963-3600*)*

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Abstract

In the measurement information literature, it is explained how the coordinate of the resection point is obtained by taking into account the situation of α and β angles measured at the P resection point less than 100g. When the α and β angles measured between the A, B, C triangulation points at the P resection point are less than 100g, the perimeter beam angles at the auxiliary points formed in the circles drawn according to the Cassini method for the solution of the problem are equal to the angles between the triangulation directions viewed from the P point. However, when the angles α and β measured between the specified triangulation points are greater than 100g at the P resection point, the perimeter beam angles at the auxiliary points formed in the circles drawn according to the method are not equal to the angles between the triangulation directions viewed from the P point. Edge lengths calculated with these angles are marked (-). In this case, the positions of the auxiliary points to be calculated as a requirement of the method and indirectly the P resection point are negatively affected. In the research conducted in the literature, it was seen that this issue was not included in the theory of the Cassini method. In this study; the solution of the mentioned problem has been researched and this problem has been solved. Numerical applications on the subject were made and the findings and opinions obtained from the study were stated.

GERİDEN KESTİRME NOKTASINDA ÖLÇÜLEN 100^g'DAN BÜYÜK AÇILARIN CASSİNİ YÖNTEMİNDE İNCELENMESİ

Anahtar Kelimeler
Geriden kestirme yöntemleri,
Cassini yöntemi,
Geriden kestirme noktasında
ölçülen 100^g'dan büyük açılar

Özet

Ölçme bilgisi literatüründe geriden kestirme yöntemlerinde, P geriden kestirme noktasında ölçülen α ve β açılarının 100^g'dan küçük durumu dikkate alınarak kestirme noktasının koordinatının nasıl elde edildiği açıklanmıştır. P geriden kestirme noktasında, koordinatları bilinen A, B, C nirengi noktaları arasında ölçülen α ve β açıları 100^g'dan küçük olduğunda; problemin çözümü için Cassini yöntemine göre çizilen dairelerde oluşturulan yardımcı noktalardaki çevre kiriş açıları, P noktasından bakılan nirengi doğrultuları arasındaki açılara eşit olmaktadır. Ancak P geriden kestirme noktasında, belirtilen nirengi noktaları arasında ölçülen α ve β açıları 100g'dan büyük olduğunda, yönteme göre çizilen dairelerde oluşturulan yardımcı noktalardaki çevre kiriş açıları, P noktasından bakılan nirengi doğrultuları arasındaki açılara eşit olmamaktadır. Bu açılarla hesaplanan kenar uzunlukları (-) işaretli olmaktadır. Bu durumda, yöntemin gereği olarak koordinatı hesaplanacak yardımcı noktaların ve dolaylı olarak P kestirme noktasının konumları olumsuz yönde etkilenmektedir. Literatürde yapılan araştırmada, belirtilen bu konunun Cassini yönteminin teorisinde yer almadığı görülmüştür. Bu çalışmada; belirtilen problemin çözümü araştırılmış ve bu problem çözülmüştür. Konuyla ilgili sayısal uygulamalar yapılmış ve çalışma sonucunda elde edilen bulgular ve kanaatler belirtilmiştir.

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[1] Corresponding Author Email: nurierdem@osmaniye.edu.tr

INTRODUCTION

The so-called resection method, which is called the 'three-point resection problem' in surveying, is a method that is often used in measurements made with the classical method, where it is not possible to work with satellites. In the surveying literature, three different resection methods (Kaestner, Collin and Cassini methods) explain how the coordinate of the resection point is calculated, taking into account the state of the angles α and β measured at the P resection point less than 100g [1-5].

When the angles α and β measured at the resection point P are greater than 100g, in the Cassini method, the edge lengths calculated with these angles are marked (-) and in this case the position of the auxiliary points to be calculated and indirectly the resection point is negatively affected. The subject can be explained more clearly as follows. When the angles of α, β, whose coordinates are known between A, B, and C triangulation points, and the angles α and measured at the resection point P are less than 100g, the perimeter beam angles at the auxiliary points D and E formed in circles drawn according to the method are equal to the angles measured at point P (Fig. 1).

However, when the angles α and β measured between the specified triangulation points are greater than 100g at the back shortcut point, the perimeter beam angles at the D and E auxiliary points formed in the circles drawn according to the method are not equal to the angles measured at the point P (Figure 2). In this case, as required by the method, the AD and CE edge lengths calculated with these angles are marked (-) and the positions of the auxiliary points to be calculated and the coordinates of the P shortcut point indirectly are adversely affected.

In the research carried out [1-11], this issue was not disclosed. In this study; the solution of the mentioned problem has been researched and this problem has been solved. Numerical applications on the subject were made and the findings and opinions obtained from the study were stated.

EXAMINATION OF THE ANGLE PROBLEM GREATER THAN 100^G MEASURED AT THE RESECTION POINT IN CASSINI METHOD

In Figure 1, when the angles α and β measured between points A, B, C at the P resection point are less than 100^g; in the first auxiliary circle drawn according to the Cassini method, the perimeter beam angle between the DA and DB directions is equal to the α angle between the PA and PB directions at the P point, and similarly, the angle between the EB and EC directions at the E point on the second auxiliary circle is it appears to be equal to the angle β between the PC directions.

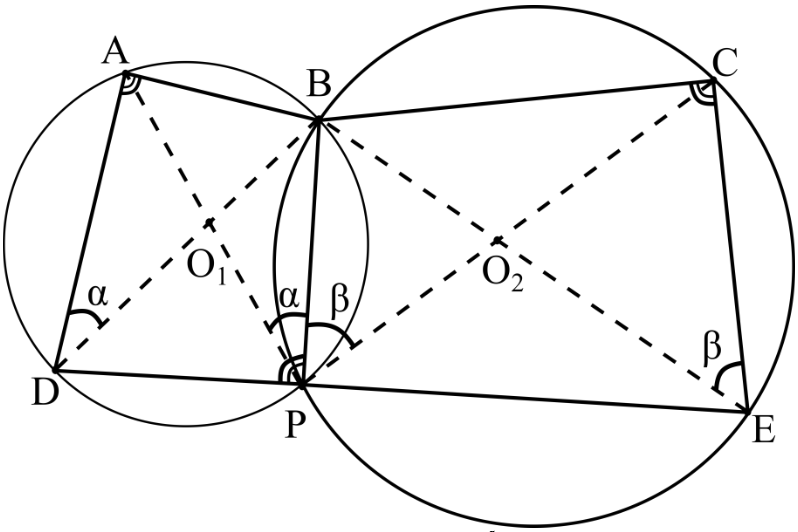

Figure 1- The position of angles smaller than 100^g measured at points P, D and E

In Figure 2, when the angles α β measured between points A, B, C at the P resection point are greater than 100^g; In the circle passing through points A, B, P, D, it is observed that the angle of the α perimeter beam seeing the AB arc at point P is not equal to the angle α' perimeter beam that shows the AB arc at point D.

Similarly, in the second circle passing through points B, C, E, P, it is seen that the angle of β perimeter beam seeing the BC arc at point P is not equal to the perimeter beam angle that sees the BC arc at point E.

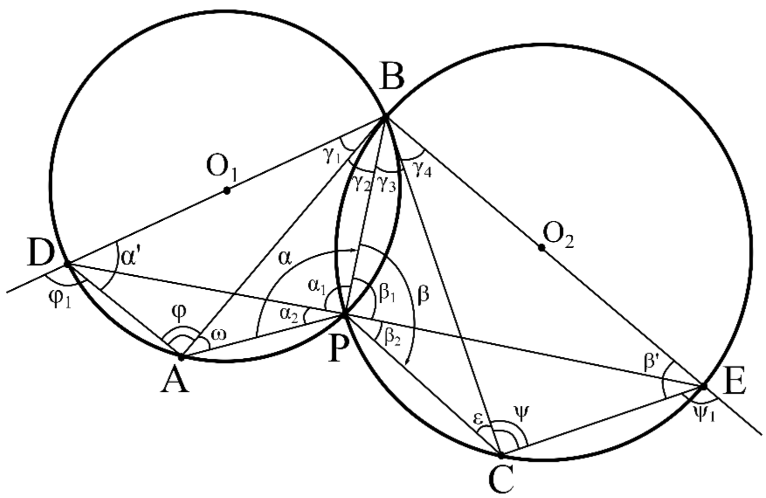

Figure 2- The position of angles greater than 100g measured at points P, D and E

SOLUTION OF THE ANGLE PROBLEM GREATER THAN 100^g IN CASSINI METHOD

In Figure 2, the AD arc between PA and PD directions at point P is seen by the α_2 perimeter beam angle at point P and by the perimeter beam angle γ_1 at point B. Similarly, the β_2 perimeter beam angle at point P and the γ_2 perimeter beam angle at point B see the CE arc. The diameter of the DB passing through the center of O_1 is seen with the angle α_1 at the point P, and the diameter of the BE passing through the center of the O_2 is seen with the angle β_1 at the point P.

Perimeter angles that see the same arc are equal to each other. The perpendicular angles that see the diameter are right angles, and the following equations are written according to the principle of analytical geometry.

$$\alpha = \alpha_1+\alpha_2 \quad (1)$$
$$\beta=\beta_1+\beta_2 \quad (2)$$
$$\alpha_1=\varphi=100^g \quad (3)$$
$$\beta_1=\psi=100^g \quad (4)$$
$$\gamma_1=\alpha_2 \quad (5)$$
$$\gamma_4=\beta_2 \quad (6)$$

The angle φ_1 at the D point between the extension of BD in the DBA triangle and the DA directions, and the angle at the E point between the extension of BE in the CBE triangle and the EC direction is the outer angle

The following equations are written according to the analytical principle of “An external angle is equal to the sum of two internal angles not adjacent to it”:

$$\varphi_1=\varphi+ \gamma_1 \quad (7)$$
$$\psi_1=\psi+ \gamma_4 \quad (8)$$

If the above equations (3), (4), (5) and (6) are written in places φ, ψ, γ_1 and γ_4;

$$\varphi_1= \alpha_1+\alpha_2=\alpha \quad (9)$$
$$\psi_1= \beta_1+\beta_2=\beta \quad (10)$$

relations are obtained.

In Figure 2, the following relations are written between $\varphi_1 = \alpha$ and α 'and between $\psi_1 = \beta$ and β'.

$$\alpha'=200^g-\alpha \quad (11)$$
$$\beta'=200^g-\beta \quad (12)$$

Since tanα’ and tanβ’ are required in the calculation of the AD edge in the DBA triangle and the CE edge in the CBE triangle, the following relations are written for tanα’ and tanβ’.

$$\tan\alpha'=\left|\tan\alpha\right| \quad (13)$$
$$\tan\beta'=\left|\tan\beta\right| \quad (14)$$

NUMERICAL APPLICATIONS

Numerical Application: 1 - Calculate the P coordinate using the Cassini method, taking into account the angle measurements and the coordinates of the points given in Table 1, in accordance with Figure 2, and the coordinates of the points.

Table 1- Angle measurements at point P and coordinates of points

Point Number	Y	X		Station Number	Point of View	Horizontal Angle
A	22681.33	19456.17		P	A	0.0027
B	25712.24	20711.75			B	124.5011
C	28852.52	18487.10			C	248.4005

Solution: 1- The angles of α and β are obtained from the direction angles in the table as follows.

$\alpha=124.5011-0.0027=124^g.4985$, $\beta=248.4005-124.5011=123.8994$.

$\Delta Y_{AB}=3030.91$, $\Delta X_{AB}=1255.58$ $(AB)=\arctan\left(\frac{3030.91}{1255.58}\right)=74.9976$

$\Delta Y_{BC}=3140.28$, $\Delta X_{BC}=-2224.65$ $(BC)=200-\arctan\left(\frac{3140.28}{2224.65}\right)=139.2385$

$AB=\sqrt{(3030.91^2+1255.58^2)}=3280.685m$, $BC=\sqrt{(3140.28^2+2224.65^2)}=3848.432m$

$(AD)=(AB)-100^g+400^g=374^g.9976$, $(CE)=(BC)+100^g\pm200^g=39.2385$

$AD=\frac{AB}{|tan\alpha|}=1328.724.m$, $CE=\frac{BC}{|tan\beta|}=1516.672m$

$Y_D=Y_A+AD*\sin(AD)=22172.803m$ $X_D=X_A+AD*\cos(AD)=20683.732m$

$Y_E=Y_C+CE*\sin(CE)=29729.257m$ $X_E=X_C+CE*\cos(CE)=19724.689m$

$\Delta Y_{DE}=7556.454$, $\Delta X_{DE}=-959.043$ $(DE)=200-\arctan\left(\frac{8237.982}{943.263}\right)=108.0368$

$(PB)=(DE)+100^g\pm200^g=8.0368$ $(BP)=(PB)\pm200^g=208.0368$

$\gamma_2=(BA)-(BP)=66.9608$, $\gamma_3=(BP)-(BC)=68.7983$, $\omega=200-(\alpha+\gamma_2)=8.5407$, $\varepsilon=200-(\beta+\gamma_3)=7.3023$

$AP=\frac{AB}{sin\alpha}\sin\gamma_2=3073.483m$, $BP=\frac{AB}{sin\alpha}sin\omega=473.432m$, $BP=\frac{BC}{sin\beta}sin\varepsilon=473.436m$

$CP=\frac{BC}{sin\beta}sin\gamma_3=3649.555m$, $(AP)=(AB)+\omega=83.5383$, $(CP)=(CB)-\varepsilon=331.9362$

$A\rightarrow Y_P=Y_A+AP*\sin(P)=25652.632m$, $X_P=X_A+AP*\cos(AP)=20242.084$

$B\rightarrow Y_P=Y_B+BP*\sin(BP)=25652.632m$, $X_P=X_B+BP*\cos(BP)=20242.084m$

$C\rightarrow Y_P=Y_C+CP*\sin(CP)=25652.630m$, $X_P=X_C+CP*\cos(CP)=20242.081m$

On average $Y_P=25652.631m$, $X_P=20242.083m$

Numerical Application: 2 - Calculate the P coordinate using the Cassini method, taking into account the angle measurements and the coordinates of the points given in Table 2, in accordance with Figure 3, and the coordinates of the points.

Table 2- Angle measurements at point P and coordinates of points

Point Number	Y	X		Station Number	Point of View	Horizontal Angle
A	400054.49	4503729.22		P	B	0.0000
B	406030.12	4509529.88			A	119.4197
C	396233.14	4510980.99			C	227.5372

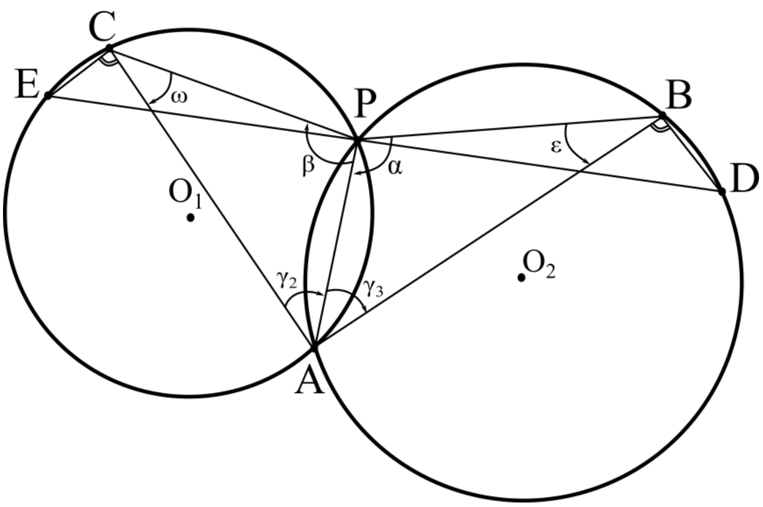

Figure 3. Calculation of the coordinate of the P point according to the values given in Table 2

Solution: 2- The angles of α and β are obtained from the direction angles in the table as follows.

$\alpha=119^g.4197$, $\beta=227.5372-119.4197=108.1175$

$\Delta Y_{AB}=5975.63$, $\Delta X_{AB}=5800.66$ (AB)=arctan$\left(\frac{5975.63}{5800.66}\right)=50.9458$

$\Delta Y_{AC}=-3821.35$, $\Delta X_{AC}=7251.77$ (BC)=200-arctan$\left(\frac{3821.35}{7251.77}\right)=369.1254$

AB=$\sqrt{(5975.63^2+5800.66^2)}$=8328.012m, BC=$\sqrt{(3821.35^2+7251.77^2)}$=8197.001m

(BD)=(AB)-100^g=$150^g.9458$, (CE)=(CA)+100^g=269.1254

BD=$\frac{AB}{|tan\alpha|}$=2622.254 m, CE=$\frac{BC}{|tan\beta|}$=1050.896 m

$Y_D=Y_B+BD*\sin(AD)$=407856.580 m $X_D=X_B+BD*\cos(AD)$=4507648.325 m

$Y_E=Y_C+CE*\sin(CE)$=395303.432 m $X_E=X_C+CE*\cos(CE)$=4510491.076 m

$\Delta Y_{ED}=12553.148$, $\Delta X_{ED}=-2842.751$ (ED)=200-arctan$\left(\frac{12553.148}{2842.751}\right)=114.1776$

(PA)=(ED)+$300^g\pm200^g$=214.1776 (AP)=(PB)$\pm200^g$=14.1776

γ_2=(AP)-(AC)=45.0522, γ_3=(AB)-(AP)=36.7682, ω=200-(β+γ_2)=46.8303, ε=200-+γ_3)=43.8121

BP=$\frac{AB}{sin\alpha}\sin\gamma_3$=4766.967m, AP=$\frac{AB}{sin\alpha}sin\varepsilon$=5545.528m, AP=$\frac{AC}{sin\beta}sin\omega$=5545.524m

CP=$\frac{AC}{sin\beta}sin\gamma_2$=5372.249, (BP)=(BA)+ε=294.7579, (CP)=(CA)-ω=122.2951

A→ $Y_P=Y_A+AP*\sin(AP)$=401279.300m, $X_P=X_A+AP*\cos(AP)$=4509137.797

B→ $Y_P=Y_B+BP*\sin(BP)$=401279.302m, $X_P=X_B+BP*\cos(BP)$=4509137.798m

C→ $Y_P=Y_C+CP*\sin(CP)$=401279.301m, $X_P=X_C+CP*\cos(CP)$=4509137.796m

On average Y_P=401279.301m, X_P=4509137.797m

Numerical Application: 3 - Calculate the P coordinate using the Cassini method, taking into account the angle measurements and the coordinates of the points given in Table 3, in accordance with Figure 4, and the coordinates of the points.

Table 3- Angle measurements at point P and coordinates of points

Point Number	Y	X		Station Number	Point of View	Horizontal Angle
A	406707.28	4512012.20		P	B	0.0000
B	402322.02	4514791.36			A	119.2603
C	402920.16	4508688.25			C	227.4050

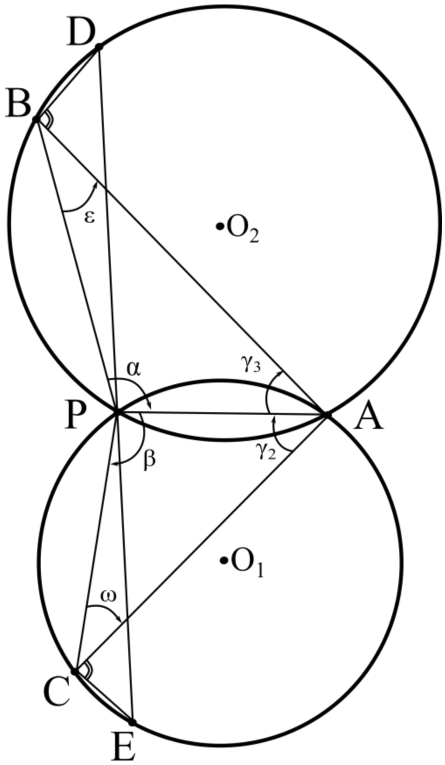

Figure 4. Calculation of the coordinate of the P point according to the values given in Table 3

Solution: 3- The angles of α and β are obtained from the direction angles in the table as follows.

$\alpha=119^g.2603$, $\beta=227.4050-119.2603=108.1447$

$\Delta Y_{AB}=-4385.26$, $\Delta X_{AB}=2779.16$ (AB)=400-arctan$\left(\frac{4385.26}{2779.16}\right) = 335.9605$

$\Delta Y_{AC}=-3787.12$, $\Delta X_{AC}=-3323.95$ (AC)=200+arctan$\left(\frac{3787.12}{3323.16}\right) = 254.1407$

AB=$\sqrt{(4385.26^2 + 2779.16^2)}$=5191.7452m, AC=$\sqrt{(3787.12^2 + 3323.95^2)}$=5038.937m

(BD)=(AB)-100^g=35.9605, (CE)=(CA)+100^g=154.1407

BD=$\frac{AB}{|tan\alpha|}$=1620.456 m, CE=$\frac{AC}{|tan\beta|}$=648.205 m

$Y_D=Y_B$+BD*sin(BD)=403189.462 m $X_D=X_B$+BD*cos(BD)=4516160.091 m

$Y_E=Y_C$+CE*sin(CE)=403347.753 m $X_E=X_C$+CE*cos(CE)=4508201.078 m

$\Delta Y_{DE}=158.291$, $\Delta X_{DE}=-7959.013$ (ED)=200-arctan$\left(\frac{158.291}{7959.013}\right) = 198.7340$

(PA)=(DE)+$100^g \pm 200^g$=98.7340 (AP)=(PA)$\pm 200^g$=298.7340

γ_2=(AP)-(AC)=44.5933, γ_3=(AB)-(AP)=37.2266, $\omega=200-(\beta+\gamma_2)=47.2620$, $\varepsilon=200-(\alpha+\gamma_3)=43.5131$

BP=$\frac{AB}{sin\alpha}\sin\gamma_3$=3002.164m, AP=$\frac{AB}{sin\alpha}sin\varepsilon$=3434.643m, AP=$\frac{AC}{sin\beta}sin\omega$=3434.646m

CP=$\frac{AC}{sin\beta}sin\gamma_2$=3274.747, (BP)=(BA)+ε=179.4736, (CP)=(CA)-ω=6.8787

A→ $Y_P=Y_A$+AP*sin(AP)=403273.315m, $X_P=X_A$+AP*cos(AP)=4511943.898

B→ $Y_P=Y_B$+BP*sin(BP)=403273.317m, $X_P=X_B$+BP*cos(BP)=4511943.897m

C→ $Y_P=Y_C$+CP*sin(CP)=403273.312m, $X_P=X_C$ +CP*cos(CP)=4511943.898m

On average Y_P=403273.315m, X_P=4511943.898m

CONCLUSION AND SUGGESTIONS

- When the angles α and β measured between points A, B, C, whose coordinates are known at the resection point P are greater than 100g; it was observed that the angle of the perimeter beam between the DA and DB directions on the first auxiliary circle drawn according to the Cassini method is equal to the difference of the perimeter beam angle between the PA and PB directions from 200^g at the point P.

- Similarly, at the point E on the second auxiliary circle, the perimeter beam angle between the EB and EC directions is equal to the difference of the perimeter beam angle PB between the PB and PC directions at point P from 200^g.
- In the calculation of AD and CE edges created in auxiliary circles drawn according to the method, in case of using angles of α and β greater than 100^g, the specified edge values are marked (-).
- To save edge values from the (-) signed state, their absolute values should be used instead of tanα and tanβ in the edge calculation of AD and CE.
- The fact that this issue mentioned here is included in the surveying literature to be published from now on will facilitate the practitioners and students.

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