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RESEARCH ARTICLE

Modelling Local Geometric Geoid using Soft Computing and Classical Techniques: A Case Study of the University of Mines and Technology (UMaT) Local Geodetic Reference Network

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INFORMATION

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ABSTRACT

Geoid determination for national heighting is one of the major research focuses in geodetic sciences. Many studies in the past and recent years have suggested various mathematical techniques for local geometric geoid modelling. This study considered an empirical evaluation of soft computing techniques such as Backpropagation Artificial Neural Network (BPANN), Multivariate Adaptive Regression Spline (MARS), Generalized Regression Neural Network (GRNN), Adaptive Neuro-Fuzzy Inference System (ANFIS), and conventional methods such as Polynomial Regression Model (PRM), and Multiple Linear Regression (MLR). The motive is to apply and assess for the first time in our study area the working efficiency of the aforementioned techniques. Each model technique was assessed based on performance criteria indices such as mean error (*ME*), mean square error (*MSE*), minimum and maximum error value (r_{min} and r_{max}), correlation coefficient (*R*), coefficient of determination (R^2) and standard deviation (*SD*). The statistical analysis of the results revealed that ANFIS, GRNN, MARS, BPANN, MLR and PRM, successfully estimate the geoid heights with a good precision for the study area. However, ANFIS outperforms BPANN, MARS, MLR, PRM, and GRNN in estimating a local geoid height. In terms of *ME* and *SD*, ANFIS achieved 0.0445 m and 0.0013m as compared to BPANN, MARS, MLR, PRM, and GRNN which achieved 0.1462 m, 0.0059 m, 0.1423 m, 0.0148 m, 0.3117 m, 0.0102 m, 0.1798 m, 0.0208 m, 0.0878 m and 0.0023, respectively. The main conclusion drawn from this study is that, the method of using soft computing is promising and can be adopted to solve some of the major problems related to height issues in Ghana.

1. Introduction

The determination of a geoid for national heighting has been one of the major research areas in geodesy, geophysics, oceanography and has drawn the attention of many researchers. The essence is to convert ellipsoidal height into orthometric height and vice versa (Odera et al., 2014; Abeho et al., 2014; Erol, 2011). This is because for practical applications such as structural health monitoring, road and building construction, extraction of metallic minerals, oceanographic studies for sea surface topography interpretation, the orthometric heights referenced to the geoid are mostly used (Yilmaz et al., 2017; Gucek and Basic, 2009; Rummel and Sanso, 1993). However, determining a geoid to fit an area of interest is still a challenging task (Al-Krargy et al., 2017). In view of that, several scholars have proposed different methodologies to estimate the geoid with a good precision.

Several studies have suggested that the geoid could be obtained either by a gravimetric or geometric approach (Yilmaz et al., 2017; Al-Bayari and Al-Zoubi, 2007). The presented study considered the latter approach in determining the local geoid due to unavailability of gravimetric data for the study area. The various geometric approaches that have been applied in the past and recent years include the Least Squares Collocation (LSC) (Ramouz et al., 2020; Doganalp and Selvi, 2015; Ophaug and Gerlach, 2017; Darbeheshti, 2009), Moving Least Squares (MLS) (Kiani, 2020), Polynomial Regression Model (PRM) (Peprah et al., 2017; Soycan, 2014; Erol, 2011; Sanlioglu and Maras, 2011; Kutoglu, 2006), Geostatistical Kriging (Erol and Celik, 2004) and the use of Earth Gravitational Model (EGM) (Pavlis et al., 2008; Pavlis et al., 2012; Kotsakis et al., 2008; Featherstone and Olliver, 2013; Peprah et al., 2017).

Even though these techniques have been utilized, they exhibit some practical drawbacks as elaborated by several scholars. The efficiency of LSC for global and regional modelling does not hold without modifying the cross-variance function (Ophaug and Gerlach, 2017; Darbeheshti, 2009). The PRM has a few problems associated with it. The best fit is obtained when the order is high (Chen and Hill, 2005; Tusat, 2011), but rather creates higher distortions when using the derived unknown parameters (Poku-Gyamfi, 2009).

Hence, there is the need to keep the order as low as possible. Increase in distance introduce noise and increase variance when using the Kriging method (Erol and Celik, 2005). EGM accuracy cannot satisfy any civil engineering works (Al-Krargy et al., 2017) and its values need to be validated using independent datasets (Abeho et al., 2014). In view of that, researchers have tried to evaluate the performance of soft computing techniques for local geoid modelling.

Notably, the Artificial Neural Network (ANN) is one of the commonly used soft computing methods (Veronez et al., 2011; Kaloop et al., 2017; Akyilmaz et al., 2009). ANN techniques which is most widely used in geoscientific discipline can form linear relationship between nonlinear variables (Ziggah et al., 2016). ANN have been used to solve some of the problems related to height issues in geodesy. Notable among them are GPS heights transformation (Yilmaz et al., 2017; Fu and Liu, 2014; Wu et al., 2012; Liu et al., 2011), and geoid modelling (Kao et al., 2017; Zaletnyik et al., 2007; Akcin and Celik, 2013; Ahmadi et al., 2016; Kavzoglu and Saka, 2005; Pikridas et al., 2011; Kutoglu, 2006).

The authors concluded that, the results achieved by ANN models' techniques are encouraging and provides promising testaments in the future for solving some of the problems related to height issues (Akcin and Celik, 2013; Veronez et al., 2011; Akyilmaz et al., 2009). Multivariate Adaptive Regression Spline (MARS) invented by Friedman (1991) is a soft computing technique which has been extensively used to solve some problems in geoscientific discipline.

Some of the areas of application include slope stability analysis (Samui, 2013), prediction of river water pollution (Kisi and Parmar, 2015), modelling of reservoir induced earthquakes (Samui and Kim, 2012), regional spatio-temporal mapping (Durmaz and Karslioglu, 2011) and modelling of the ionosphere (Durmaz et al., 2010; Nohutcu et al., 2010). These studies have shown that, the MARS model has the ability to train independent variables (input) with several basis functions to yield an optimal dependent variable (output) (Samui, 2013).

The local geoid was established in the University of Mines and Technology (UMaT), Tarkwa which happens to be the study area. This study for the first time in Ghana, applied and assessed the performance of soft computing techniques namely Adaptive Neuro-Fuzzy Inference System (ANFIS), MARS, Generalized Regression Neural Network (GRNN), Backpropagation Artificial Neural Network (BPANN) and conventional techniques such as Multiple Linear Regression (MLR) and Polynomial Regression Model (PRM) as effective tools for modelling local geoid in the study area. Each model technique was assessed based on performance criteria indices such as *ME*, *MSE*, minimum residual value (r_{min}), maximum residual value (r_{max}), and *SD*. This study will therefore create the opportunity for researchers in Ghana to know the performance of using soft computing techniques in solving some of the problems related to heights issues in the country. The authors were motivated to embark on this study since the aforementioned techniques is yet to be conducted in Ghana.

2. Resources and Methods Used

The study area (Fig. 1a to 1d) is found in a mining community in the Southwestern part of Ghana with geographical coordinates between longitude 001° 59′ 55" West (W) to 002° 00' 15″ W and latitude 005° 17′ 45" North (N) to 005° 18' 00″ N. The area has an average altitude of about 78 m above Mean Sea Level (MSL) (Peprah et al., 2017). The type of coordinate system used in the study area is the Ghana projected grid derived from the Transverse Mercator 01° North West (NW). The horizontal geodetic datum of the study area is the War Office 1926 ellipsoid, and the vertical datum is the MSL which approximate the geoid (Ziggah, 2014).

Primary data collected from the study area was used in this present study. The data consists of 328 control points collected with the Differential Global Positional System (DGPS) receivers and Total Station instruments. The data comprise of three-dimensional geographical coordinates namely latitude, longitude, and ellipsoidal heights denoted as (φ, λ, h) obtained from the GPS while eastings, northings, and orthometric heights denoted as *(E, N, H)* was recorded using the Total Station instrument for the selected controls of the study area.

Fig. 2 shows the control points distribution map of the study area. The fast-static GPS survey technique was adopted during the field work in collection of data points. The reference receiver (base station) was a Continuous Operating Reference Station (CORS) located at UMaT campus. Because observations period was short, the rovers were allowed to occupy each point for a minimum period of five minutes to acquire more satellites position for better output results. After data collection, there was a post-processing of the raw GPS data using specified processing software to get the real coordinates of each fixed position.

However, it is worth acknowledging that, one of the contributing factors affecting the estimation accuracy of models is related to the quality of datasets used in model-building (Devi and Karthikiyan, 2015; Dreiseitl and Ohno-Machado, 2002; Ismail et al., 2012). Therefore, to ensure that the obtained field data from the GPS receivers are reliable, several factors such as checking of overhead obstruction, observation period, observation principles and techniques as suggested by many researchers (Yakubu et al., 2018; Yakubu and Dadzie, 2019) were performed on the field. In addition, all potential issues relating to DGPS survey work were also considered. The mean values of the ellipsoidal heights and orthometric heights data used for the estimation of the local geometric geoid heights were 108.8500 and 73.6558 m, respectively.

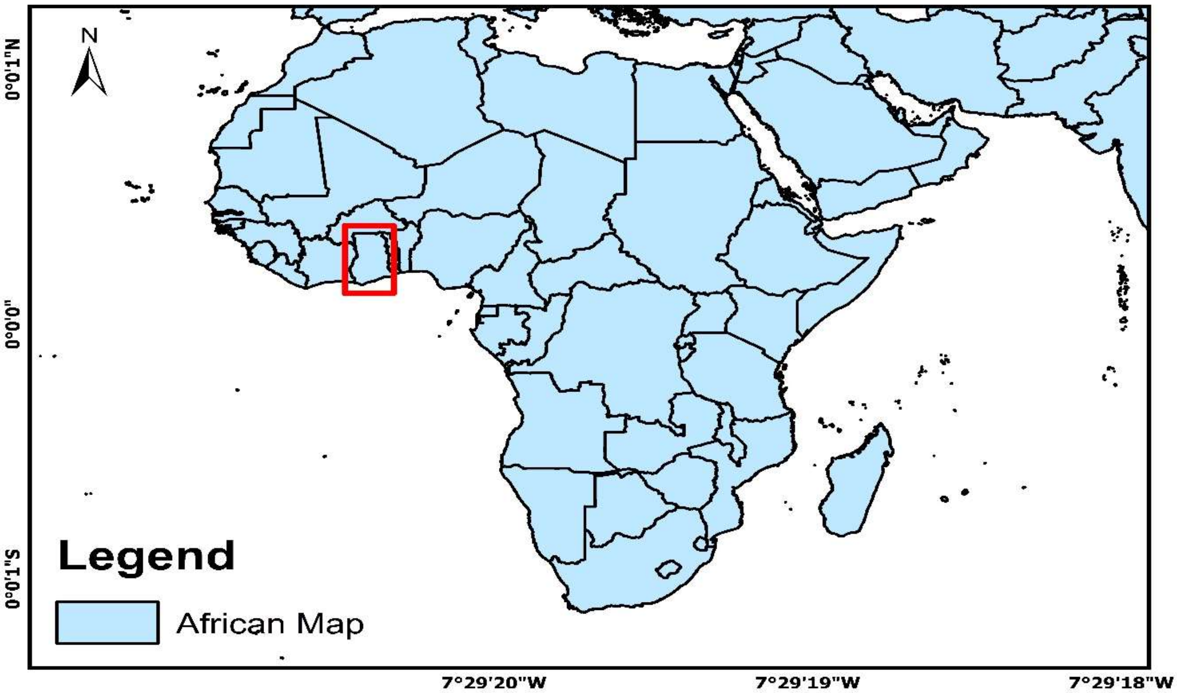

Fig. 1a. African map showing the location of Ghana

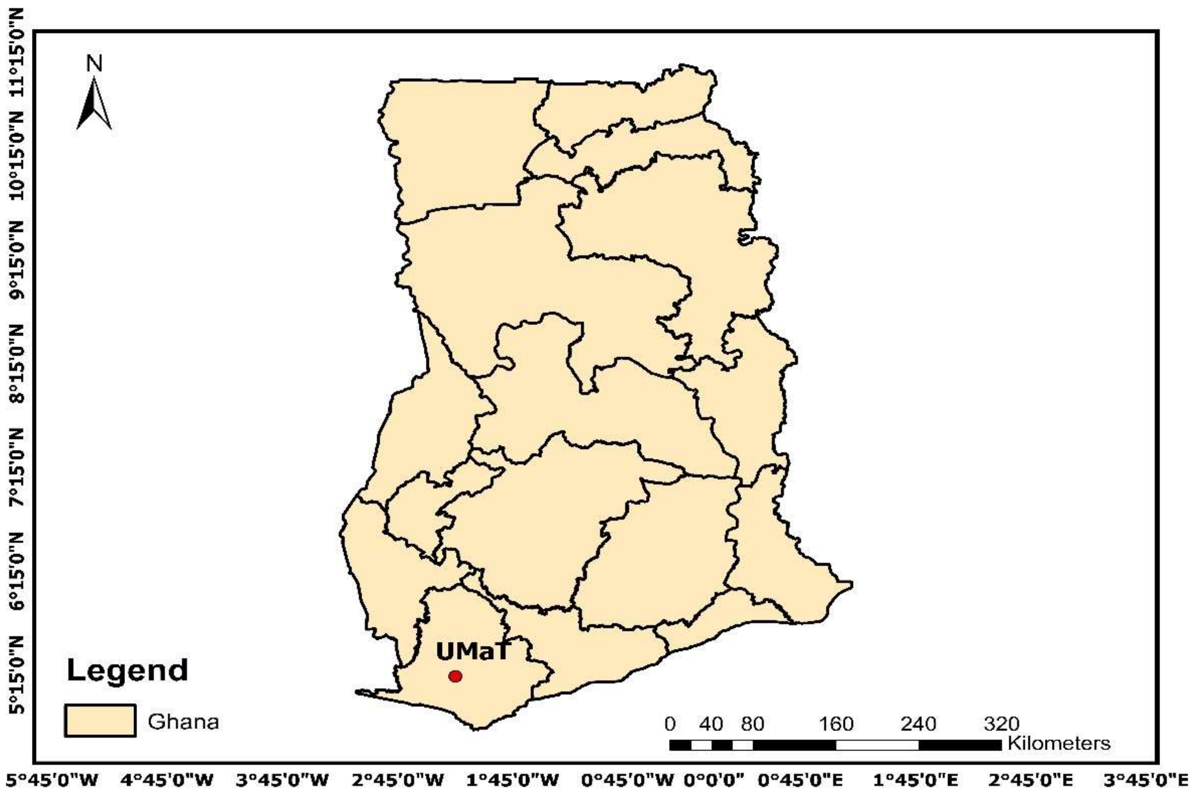

Fig. 1b. Regional Map of Ghana showing the Location of the study area

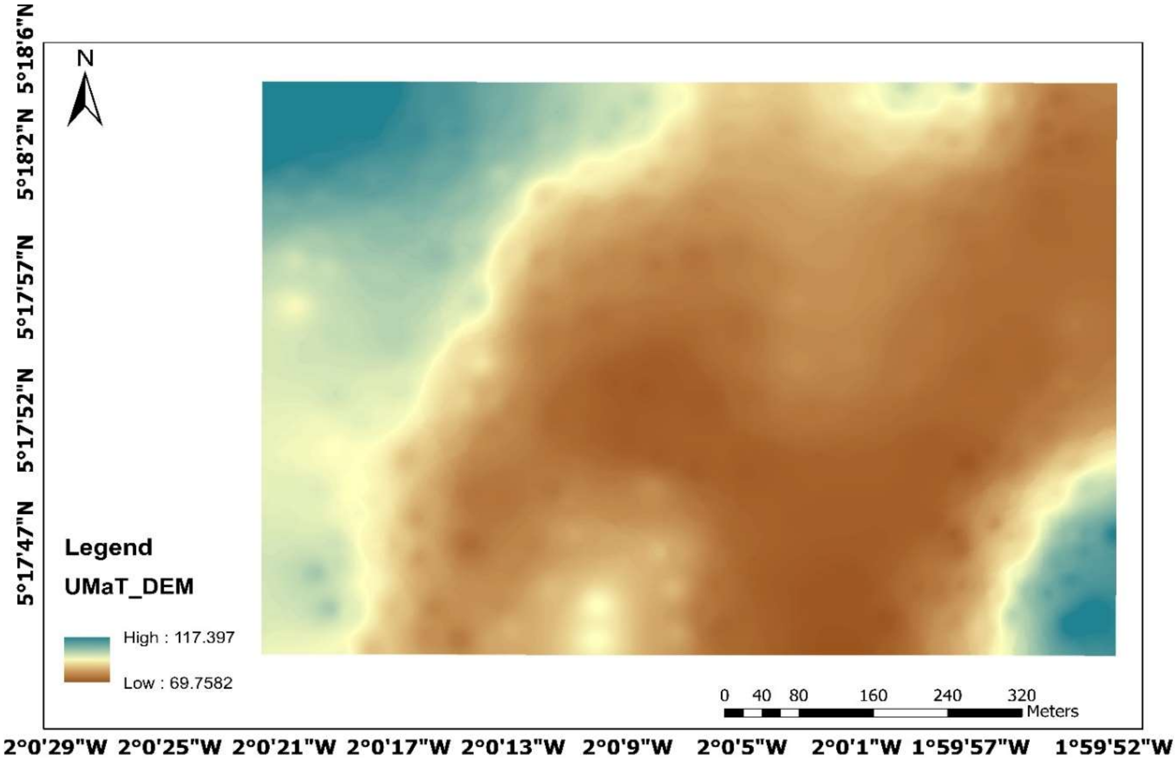

Fig. 1c. Digital Elevation Model of the study area

Fig. 1d. An aerial photo of the study area

2.1. Methods

2.1.1. NGPS/Levelling Estimated Geoidal Heights Computation

The geometric technique for estimating geoidal heights using GPS obtained ellipsoidal height *(h)* collocated with orthometric height *(H)* was done according to Eq. 1. The estimated geoidal heights (*N*) derived from GPS ellipsoidal heights and orthometric heights are referred to as NGPS/Levelling.

$$N_{GPS/Levelling} = h - H \tag{1}$$

where $N_{GPS/Levelling}$ is the estimated geoidal heights, h is the ellipsoidal height from GPS measurements and H is the orthometric height obtained from levelling procedure. The statistical analysis of the NGPS/Levelling estimated geoidal heights; thus, maximum value, minimum value, mean value and SD is given by Table 1. Fig. 3 is the histogram graph analysis of the computed local geometric geoidal heights.

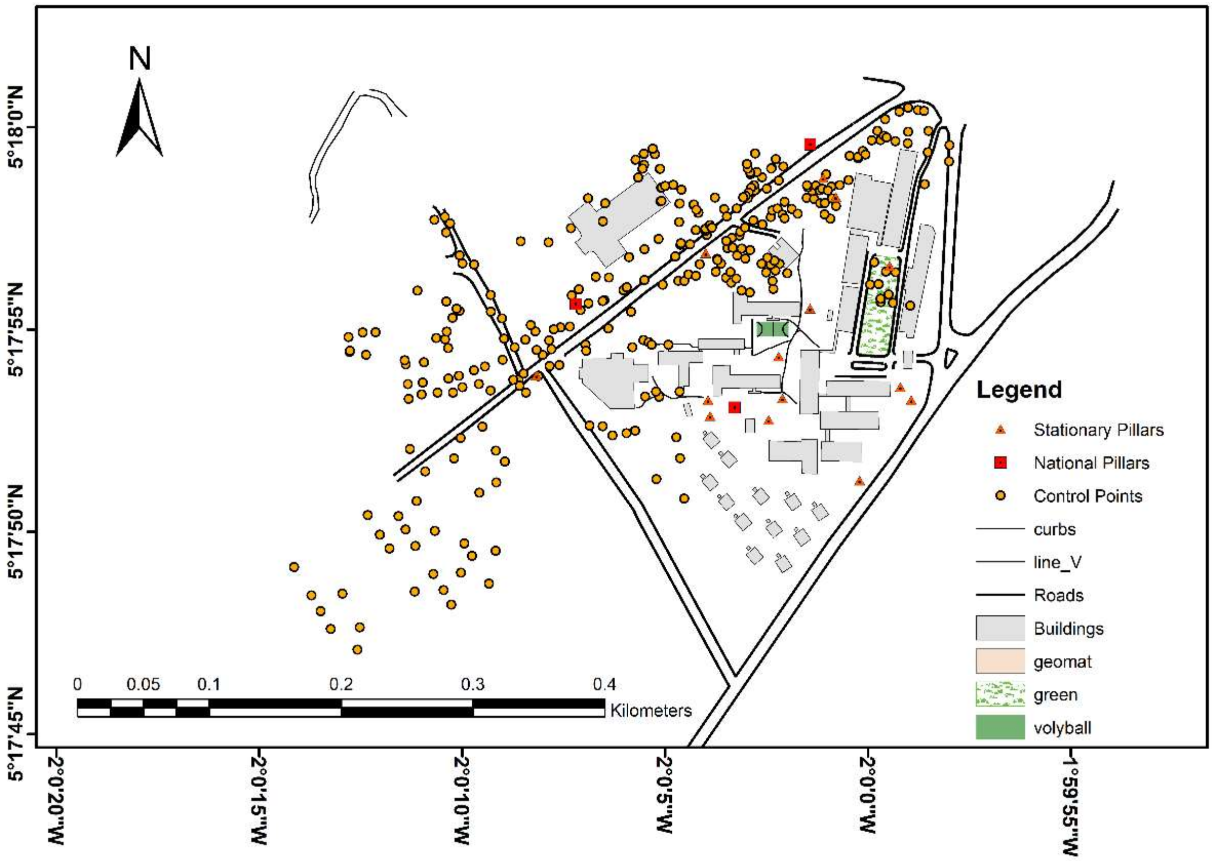

Fig. 2. Control points distribution map of the study area

2.1.2. Polynomial Regression Model (PRM)

In this study, a PRM was adopted to model and predict the geoidal height values produced by Eq. 1. The horizontal geographical coordinates (φ, λ) were used as the independent variables. The general expression of an m-degree polynomial interpolation is given by Eq. 2 (Yilmaz et al., 2017) as;

$$Z(\varphi,\lambda) = \sum_{i=0}^{m}\sum_{j=0}^{m-1} a_{ij}\varphi^{i}\lambda^{j} \quad (2)$$

where $Z_{(\varphi,\lambda)}$ is the geoidal height information of the point with known horizontal coordinates (φ, λ) and $a_{i,j}$ is the unknown polynomial coefficients to be estimated, $(i, j = 0, \ldots, m)$. The Simple Planar (SP) polynomial model was adopted in this study due to its efficiency and performance in estimating local geoid heights as recommended by (Peprah et al., 2017; Dawod et al., 2010; Dawod, 2008). The general SP polynomial model is denoted by Eq. 3 given as;

$$N_{Undulations} = a_0 + a_1\lambda + a_2\varphi \quad (3)$$

where $N_{Undulation}$ is the estimated geoidal height, φ, λ are the geographical coordinates of the horizontal position, $a_{i,j}$ are the unknown parameters that can be determined using least square approach. The least squares approach has been successfully and frequently applied in geodetic sciences. Therefore, the mathematical backgrounds and theories of the method will not be repeated here. A more comprehensive detail on them can be found in the work of Ghilani (2010).

2.1.3. Multiple Linear Regression (MLR)

MLR is a nonlinear regression model in which observational data are modelled by a function *Z(f(x))* and depends on one or more independent variables (Tiryaki, 2008). MLR was adopted in this study to estimate a local geoid height produced by Eq. 1. MLR fits a linear combination of the components of multiple input parameters to a single output parameter (Ziggah et al., 2016; Sheta et al., 2015) defined by Eq. 4 as;

$$Z(x) = \alpha_0 + \sum_{i=1}^{M}\beta_i x_i \quad (4)$$

where α_0 is the intercept (values when all the independent variables are zero) (Ziggah et al., 2016) with β_i values denoting the regression coefficients. In Eq. 4, i is an integer varying from 1 to M, where M is the total number of observations and x is the three-dimensional geographical coordinates (φ, λ, h).

2.1.4. Adaptive Neuro-Fuzzy Inference System (ANFIS)

ANFIS is an integrated model between ANN and Fuzzy logic algorithms (Kaloop et al., 2017). The detail theoretical concept can be found in the following available literature (Acar et al., 2006; Tusat, 2011; Eldessouki and Hassan, 2015; Akyilmaz et al., 2009; Erol and Erol, 2013). In the ANFIS model formulation, the dataset was divided into training data (70 %) and testing data (30 %). The training data are the inference points which were used to estimate the ANFIS model parameters, whiles the test data was used to validate the estimated model parameters (Akyilmaz et al., 2009). The input variables (independent variables) were the three-dimensional geographical coordinates of longitude, latitude and ellipsoidal heights denoted as $\varphi_{i,j}, \lambda_{i,j}, h$ and the output variable (dependent variables) were the estimated geoidal heights by Eq. 1 ($N_{i,j}$) was used in this stage.

Table 1. Statistical analysis of NGPS/Levelling estimated geoidal heights (units in meters)

PCI	max	min	mean	SD
N	46.8177	21.4702	35.5774	0.0022

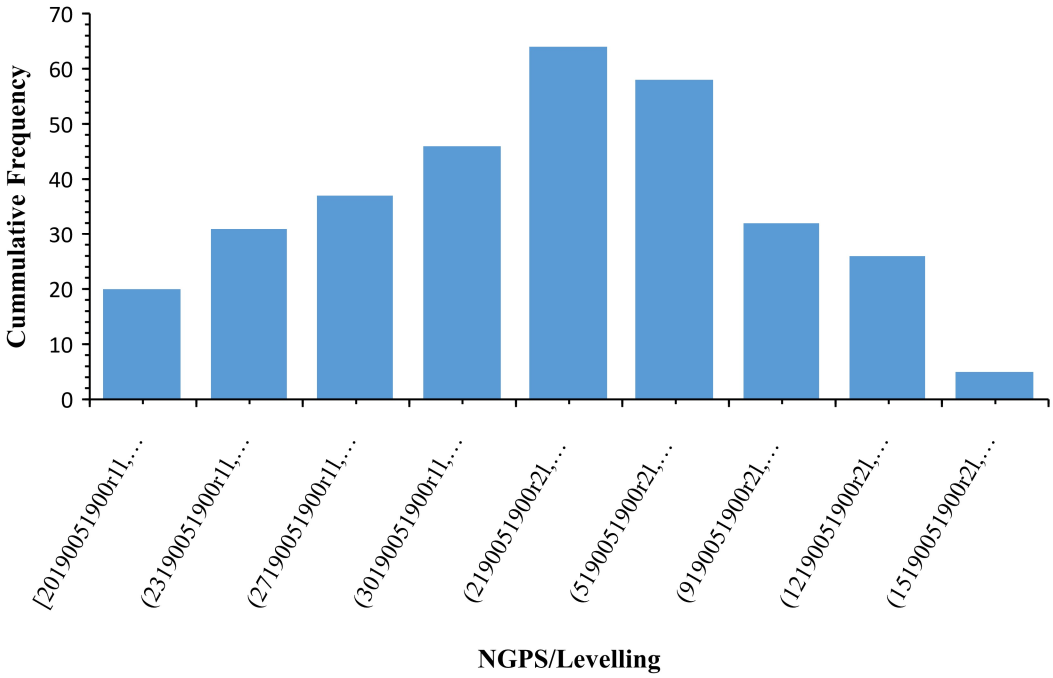

Fig. 3. Histogram graph analysis of NGPS/Levelling estimated geoidal heights

Numerous types and membership functions are available hence, for geodetic purpose, membership functions of the Gaussian type are the most appropriate ones to be used (Akyilmaz et al., 2009). The optimum number of member functions for each input variable is obtained by trial and error approach. The output member functions are the first order polynomials of the input variables (Yilmaz and Arslan, 2008).

The number of the output member functions depend on the number of fuzzy rules which are the number of all combinations of the input membership functions (Tusat, 2011). There are several mathematical methods that can be used for the ANFIS training. The hybrid training algorithm proposed by Jang (1993) was adopted in this study because of its rapid convergence to the global optimum solution (Akyilmaz et al., 2009). After the parameters of the ANFIS model are estimated after the training step, they are used to calculate the geoid heights at the test data points according to Eq. 5 (Kaloop et al., 2017) given as;

$$y(i) = \sum_{i=1}^{n} w_i (f_i x_i) \quad (5)$$

where f_i are the estimated parameters, x_i are the model inputs, w_i are normalized weights, and n is the number of points.

The best model achieved by the soft computing techniques were analyzed based on their *MSE*, correlation coefficient (R), and coefficient of determination (R^2). Their mathematical representation is found in model performance assessment section.

2.1.5. Generalized Regression Neural Network (GRNN)
GRNN which was first introduced by Specht (1991) is a different kind of Radial Basis Function Neural Network (RBFNN) which is built on Kernel regression network (Hannan et al., 2010) with one pass learning algorithm and highly parallel structure (Dudek, 2011). GRNN consist of four layers namely; input layer, pattern layer (radial basis layer), summation layer, and output layer. In this study, the input variables (independent datasets) were the latitude, longitude, and ellipsoidal height denoted as $\varphi_{i,j}, \lambda_{i,j}, h_{i,j}$ and the output variables (dependent datasets) were the estimated geoidal heights by Eq. 1 denoted as $(N_{i,j})$.

The number of input units in the first layer depends on the total number of the observational parameters. The first layer is connected to the pattern layer and in this layer, each neuron is being presented by a training pattern and its output. The pattern layer is connected to the summation layer. The summation layer consists of two different types of summation namely, single division unit and summation unit (Hannan et al., 2010).

The summation with output layer combined perform a normalization of output datasets. In training of the network, radial basis and linear activation functions are used in hidden and output layers. Each pattern layer unit is connected to two neurons in the summation layer. One neuron unit computes the sum of the weighted response of the pattern, and the other neuron unit computes unweighted outputs of pattern neurons. The output layer divides the output of each neuron unit by each other yielding the estimated output variables.

In this present study, the Gaussian function is applied, and the output neuron is a summation of the weighted hidden output layer given by Eq. 6 (Erdogan, 2009);

$$y(x) = \sum_{j=1}^{n} \kappa_j \chi_j(x) \tag{6}$$

where n is the number of hidden neurons, $x \in R^M$ is the input, $k_j$are the output layer weights of the radial basis function network, $\chi_j(x)$ is Gaussian radial basis function given by Eq. 7 (Srichandan, 2012; Idri et al., 2010);

$$\chi_j(x) = e^{\left(\frac{-\|x_i - c_j\|^2}{\sigma_j^2}\right)} \tag{7}$$

where $c_j \in R^M$ and σ are the centre and width of jth hidden neurons respectively, $\| \|$ denotes the Euclidean distance.

2.1.6. Multivariate Adaptive Regression Splines (MARS)
The MARS model developed in this study for the geoidal height estimation was constructed in a two-phase procedure namely, the forward and the backward phase. In the forward phase, basis functions are added, and potential knots are determined to improve the model's performance, resulting in overfitting. Therefore, in the backward phase the least relevant basis functions are deleted (Samui, 2013). A general MARS model can be written in the following form as denoted by Eq. 8 (Friedman, 1991);

$$Z(x) = \alpha_0 + \sum_{i=1}^{M} \beta_i \prod_{j=1}^{ki} bji(X_v(j,i)) \tag{8}$$

where $Z_{(x)}$ is the output variable (geoidal height), α_0 is a constant, β_i is a vector coefficient of the non-constant basis functions, $bji(X_v(j,i))$ is the truncated power basis function with $v(j,i)$ being the index of the independent variable used in the i^{th} term of the j^{th} product, and k_i is a parameter that limits the order of interactions (Samui and Kim, 2012; Adoko et al., 2013).

It must be known that in the forward process, the basis functions were selected according to Eq 8. In the backward elimination process, the ineffective basis functions are removed based on the generalized cross-validation (GCV) criterion (Craven and Wahba, 1979). The GCV criterion is defined by Eq. 9 as;

$$GCV = \frac{\frac{1}{N}\sum_{i=1}^{N}[y_i + (x_i)]^2}{\left[\frac{1 - M + \delta(m-1)/2}{N}\right]^2} \tag{9}$$

where M is the number of basis function, δ is the penalty factor (Craven and Wahba, 1979), N is the number of observations, y_i is the i^{th} measured element and x_i denotes the j^{th} predicted value of the model.

2.1.7. Backpropagation Artificial Neural Network (BPANN)
BPANN is an effective multilayer perceptron (MLP) model (Yilmaz et al., 2017) and is widely used due to its simple implementation (Ziggah et al., 2016). BPANN consists of one input layer with M inputs, one hidden layer with q units and one output layer with n outputs (Mihalache, 2012). The M inputs in this study were the geographical coordinates $(\varphi_{i,j}, \lambda_{i,j}, h_{i,j})$, the q units were achieved by a trial and error training in changing number of hidden neurons, and the n outputs were the estimated geoid heights (N_i) achieved by the BPANN model. The output of the model (y_i) with a single output neuron is represented by Eq. 10 (Ziggah et al., 2016; Mihalache, 2012);

$$y(i) = f\left(\sum_{j=1}^{q} W_j f\left(\sum_{i=1}^{M} w_{j,i} x_i\right)\right) \tag{10}$$

where W_j is the weight between the hidden layer and the output layer, $w_{i,j}$ is the weight between the input layer and the hidden layer, and x_i is the input parameter. In this study, the selected input and output variables were normalized into the interval [-1, 1] using Eq. 11 given as (Mueller and Hemond, 2013);

$$Z(i) = \frac{y_{\min} + (y_{\max} - y_{\min}) \times (x_i - x_{\min})}{(x_{\max} - x_{\min})} \tag{11}$$

where $Z(i)$ represents the normalized data, x_i is the measured geoid height values, while x_{min} and x_{max} represent the minimum and maximum value of the measured geoid heights with y_{max} and y_{min} values set at 1 and -1, respectively. The optimal model was obtained based on the lowest *MSE*, *R* and R^2. Their mathematical expression is represented by Eq. 12 to Eq. 14 respectively as;

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(\alpha_i - \beta_i)^2 \tag{12}$$

where α_i and β_i are the measured and predicted geoid heights from the BPANN model.

$$R = \left(\frac{\sum_{i=1}^{N}(\alpha_i - \bar{\alpha})(\beta_i - \bar{\beta})}{\sqrt{\sum_{i=1}^{N}(\alpha_i - \bar{\alpha})^2} \times \sqrt{\sum_{i=1}^{N}(\beta_i - \bar{\beta})^2}} \right) \tag{13}$$

$$R^2 = \left(\frac{\sum_{i=1}^{N}(\alpha_i - \bar{\alpha})(\beta_i - \bar{\beta})}{\sqrt{\sum_{i=1}^{N}(\alpha_i - \bar{\alpha})^2} \times \sqrt{\sum_{i=1}^{N}(\beta_i - \bar{\beta})^2}} \right)^2 \tag{14}$$

Here, *N* is the total number of test examples presented to the learning algorithm, α_i and β_i are the measured and predicted geoid heights from the BPANN learning approach, while $\bar{\alpha}$ and $\bar{\beta}$ is the mean of the calculated and predicted geoid heights. *i* is an integer varying from 1 to *N* where *N* is the total number of observations.

The present study adopted one hidden layer in the BPANN. This decision was in line with literature and conclusion made by (Hornik et al., 1989) that the BPANN with one hidden layer could be used as a global approximator for any discrete and continuous functions. Furthermore, to introduce non-linearity into the network, the hyperbolic tangent activation function was selected for the hidden units, while a linear function was applied for the output units. The hyperbolic tangent function is defined by Eq. 15 (Yonaba et al., 2010) as;

$$Z(x) = tanh(x) = \frac{2}{1+e^{-2x}} - 1 \tag{15}$$

where *x* is the sum of the weighted inputs.

2.2. Model performance assessment

In order to determine the accuracies of the models being used, statistical error analysis was carried out. The statistical indicators applied were the *ME*, *MSE* (Eq. 12), *R* (Eq. 13), R^2 (Eq. 14), r_{min}, r_{max}, and *SD*. Their mathematical expressions are given by Eq. 16 to Eq. 19, respectively, as;

$$ME = \frac{1}{n}\sum_{i=1}^{n}(\alpha_i - \beta_i) \tag{16}$$

$$r_{\max} = \alpha_i - \beta_i \tag{17}$$

$$r_{\min} = \alpha_i - \beta_i \tag{18}$$

$$SD = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(\mu - \bar{\mu})^2} \tag{19}$$

where, *n* is the total number of the observations, α_i and β_i are the measured and predicted geoid heights from the various techniques, μ denote the residual between the measured and estimated geoid height, $\bar{\mu}$ is the mean of the residual and *i* is an integer varying from 1 to *n*.

3. Results and Discussions

The optimal model achieved by the BPANN model after successive iterative training was [3 15 1]. Thus, 3 input variables (independent dataset), 15 hidden neurons and 1 output variable (dependent dataset). The network was allowed to train for 5000 epochs for each iterative training. The optimal ANN architecture was selected based on the lowest *MSE* and highest *R*. The optimal model for the GRNN was achieved by varying the spread parameter between 0 and 1 until the best results was achieved. In the MARS model formulation, 21 basis functions were used in the forward training, 2 basis functions were used in the final model formulation thus, 19 basis functions were removed during the backward training due to overfitting. The optimal MARS equation for estimating the geoid heights is given by Eq. 20 and the number of basis functions used is tabulated by Table 2.

$$N_{i,j} = 0.556571 + 1.01427 \text{ X } BF1 + 3122.18 \text{ X } BF2 \tag{20}$$

ANFIS model was trained for specific epoch by varying the number of membership functions. The summarized results of the training and testing by all the soft computing techniques is represented by Table 3 below. Based on the statistical results given in Table 3 below, it can be observed that soft computing techniques provide satisfactory results in estimating local geoid heights with much better accuracy for the study area. The r_{min} and r_{max} are very quiet encouraging. The *MSE* of both training and testing are quite good and very much encouraging. In addition, the *R* and R^2 which ranges from (0 to 1) shows how closely the estimated values from the models corresponds to the actual data (dependent variables).

As seen from Table 3 below, *R* and R^2 conform that soft computing techniques have successfully provided good approximations with much better accuracy. However, ANFIS have proven to be a powerful realistic alternative tool in computing local geoid heights for the study area with much better accuracy as compared to the other soft computing techniques. The soft computing techniques (ANFIS, BPANN, MARS, and GRNN) have been compared to the conventional techniques (PRM and MLR) using all the data points. Table 4 is the estimated unknown parameters of the PRM model using least square approach and Eq. 21 is the optimal equation used by the MLR model for estimating the geoid height.

$$N_{i,j} = -12757.769 + (2529.703 \text{ X } \varphi_{i,j}) - (361.947 \text{ X } \lambda_{i,j}) + (1.0360 \text{ X } h_{i,j}) \quad (21)$$

The statistical analysis of all the models is represented by Table 5. The r_{max} value of the conventional techniques and the soft computing techniques was about 1.2 m. When comparing their statistical analysis, the PRM model had a r_{min} value of 0.0478 m and *SD* of 0.02077 m. This situation agrees with the recommendations of Poku-Gyamfi (2009) and Chen and Hill (2005) that, the defects of the PRM model are due to the increase in order and there are distortions in the estimated values using the transformed parameters estimated by the least squares approach and the order must be kept low.

Hence, the results achieved by Peprah et al., (2017) for the study area was quite precise. The soft computing techniques outperforms the conventional techniques in estimating local geoid heights. The outlined models did not achieve a millimeter accuracy and this situation agrees with Akyilmaz et al., (2009) that geoid heights vary in mountainous areas. For example, the study area under consideration and for proper estimation of the geoid requires high resolution data.

Additionally, the inability of the soft computing techniques may be attributed to the orthometric heights and ellipsoidal heights used in the NGPS/Levelling in estimating the geoid height. After comparing the soft computing techniques to the conventional techniques in terms of their statistical analysis, the soft computing was much better as compared to the conventional methods in estimating local geoid heights for the study area.

Table 2. Basis functions used by the MARS model

Basis Functions	Equation
BF1	*max (0, h-79.7706);*
BF2	*max (0, Y-5.29744);*

Table 3. Model results for soft computing techniques (units in meters)

Training Results				
PCI	**ANFIS**	**BPANN**	**GRNN**	**MARS**
r_{max}	1.2990	1.2643	1.2811	-1.2908
r_{min}	5.2600 x 10^{-05}	-0.0028	5.7400 x 10^{-08}	-0.0344
ME	0.0465	0.1552	0.0182	0.0998
MSE	0.3744	0.5339	0.4863	0.6966
SD	0.0014	0.0093	0.0039	0.0257
R	0.9675	0.9609	0.9291	0.8081
R^2	0.9361	0.9233	0.8760	0.6530
Testing Results				
PCI	**ANFIS**	**BPANN**	**GRNN**	**MARS**
r_{max}	-1.1718	-1.2384	1.2965	1.2983
r_{min}	-0.0067	0.0206	0.1702	0.1767
ME	0.0267	0.1307	0.3225	0.2076
MSE	0.4229	0.4580	0.7084	0.6380
SD	0.0547	0.0077	0.0005	0.0289
R	0.7700	0.9123	0.9320	0.9016
R^2	0.5929	0.8323	0.8560	0.8129

Table 4. PRM coefficients results (units in meters)

PCI	VALUE	SD
a_0	-1.5048	2.0819
a_1	1.6406	320.9703
a_2	-3.5740	243.0151
SD	3.1515	

Table 5. Statistical analysis of all the models (units in meters)

PCI	r_{max}	r_{min}	*ME*	*MSE*	*SD*
ANFIS	1.2990	5.6200 10^{-05}	0.0445	0.3794	0.0013
BPANN	1.2643	-0.00281	0.1462	0.5061	0.0059
GRNN	1.2965	0.5740 x 10^{-08}	0.0878	0.5371	0.0023
MARS	1.2983	-0.0344	0.1423	0.6735	0.0148
MLR	1.2749	0.0285	0.3117	0.7008	0.0102
PRM	1.2922	0.0478	0.1798	0.7736	0.0208

4. Conclusions

Geoid studies have become obligatory in establishing a vertical geodetic reference network for measuring vertical distance. The geoid enables the standard forward transformation of GPS heights (*h*) which has no physical meaning to physical meaningful heights (*H*) for engineering and mapping purposes. This study assesses and compares the performance of soft computing techniques namely, ANFIS, BPANN, MARS, GRNN to conventional techniques namely, PRM and MLR in estimating local geoid heights for UMaT local geodetic reference network. After comparing the soft computing techniques to the conventional methods based on their statistical analysis, it was revealed that the soft computing techniques outperform the conventional methods in estimating a local geoid height for the study area. We conclude that, utilizing soft computing techniques in estimating local geoid heights for the study area have proven to be an alternative realistic technique to the conventional techniques. These techniques will help in converting GPS heights to orthometric heights for geodetic purposes. However, more work should be done in Ghana utilizing other soft computing techniques which were not considered in this study to evaluate its effectiveness for larger engineering projects since the classical techniques of obtaining vertical distances are costly, time consuming and laborious. This study will create the opportunity for geodesist in Ghana to know the efficiency of soft computing techniques in solving some problems related to height in geodesy.

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Conflict of Interest

The Principal Author and co-author, declare that there is no any potential conflict of interests in this work. The work does not infringe any copyright, proprietary right or any other right of any third party; and the Author and co-author are the sole owners of the work.

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