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INTERNATIONAL ADVANCED RESEARCHES
and
ENGINEERING JOURNAL

Journal homepage: www.dergipark.org.tr/en/pub/iarej

e-ISSN: 2618-575X

International Open Access

Volume 05
Issue 02

August, 2021

Research Article

Application of the Muskingum-Cunge routing method with variable parameters in a gauged creek reach

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ARTICLE INFO

Article history:
Received 23 October 2020
Revised 14 February 2021
Accepted 26 April 2021

Keywords:
Flood routing
HEC-HMS
Muskingum-Cunge
Variable parameter

ABSTRACT

In general, the Muskingum-Cunge method is used to route floods when observed flood data are not available. In this study, applicability of the Variable Parameter Muskingum-Cunge (VPMC) flood routing method was investigated in a gauged creek reach. The reach was between two stream gauging stations. Some physical characteristics of the reach, such as length, bed slope, cross sections, and Manning's coefficient (n), were determined by using digital topographical map of the reach, inflow, and also outflow data of the two stream gauging stations. The HEC-HMS hydrological model was used to route the inflow hydrograph through using the VPMC method. In conclusion, observed and computed outflow hydrographs were compared and it was seen that the VPMC flood routing method was suitable for the gauged creek reach, which was the subject of this study.

1. Introduction

Flood or flow routing is a method used to predict the time and magnitude of flood/flow in a river or a channel based on available upstream inflow data. Flood routing is classified into two types: hydrologic routing and hydraulic routing.

In hydrologic routing, flow is only time-dependent, while in hydraulic routing, flow is space and time-dependent [1]. In hydraulic routing, topographical data are needed to solve complex equations, while in hydrologic routing, equations are uncomplicated compared to those in hydraulic routing and there is less need for topographical data [2].

In 1848 Barré de Saint-Venant first put forward a solution to the hydraulic flood routing problem. In this solution, the continuity equation and the "momentum equation" statement of Newton's Second Law are solved for a differential volume of one-dimensional flow. Inertia, pressure, gravity, and friction forces acting on the control volume are taken into account and mass is maintained in the solution [3].

In hydrologic routing, continuity equation and the relation among inflow, outflow, and storage are used to solve the routing problem. The solution process is relatively simple and results are satisfactory in general [4]. The Muskingum method, proposed by McCarthy in 1938 [5], is one of the well-known hydrologic routing methods in the literature. The method establishes a linear relationship among storage, inflow, and outflow including two parameters [6]. The parameters of this linear function are determined by observed inflow and outflow data. In the absence of observed flow data, parameters of the Muskingum flood routing method may be determined by the Muskingum-Cunge flood routing method [1]. When the parameters of the Muskingum-Cunge method change with respect to space and time, the method is called the Variable Parameter Muskingum–Cunge (VPMC) method.

The Muskingum-Cunge Method has been studied by several scholars in recent years. Todini proposed a new algorithm to overcome the mass balance problem in the Muskingum-Cunge method and to eliminate contradictory values for the water volume stored in the channel. The

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DOI: 10.35860/iarej.813766

performance of the proposed model was tested by various time and space intervals, cross-section types, roughness coefficients, and bed slopes. The results were satisfactory in terms of mass balance when they are compared to the results of Saint-Venant equations. The results were also in line with the results of Muskingum equations [7].

Barati et al. investigated the performance of constant and variable parameter Muskingum-Cunge methods in terms of volume conservation and also in terms of the attenuation value and the lag time of the peak outflow. Available flow data for Karun River in Iran were used for conducting numerical experiments. Results were compared to those of kinematic wave, dynamic wave, and Muskingum model. It was concluded that with moderate values of roughness coefficients and bed slopes, Muskingum-Cunge methods were more accurate in ungauged catchments [8].

Ponce and Lugo developed a looped-rating Muskingum-Cunge model by modifying the four-point variable- parameter Muskingum-Cunge model. In the proposed model, local water surface slope and Vedernikov number were used to generate the looped ratings. In order to test the accuracy of the model, numerical experiments were made. The looped ratings calculated by the Muskingum-Cunge model and the outflow hydrographs generated by this model were compared with the results of the dynamic wave model. It was concluded that both models were capable of producing accurate looped ratings and outflow hydrographs [9].

Szel and Gaspar applied the Muskingum-scheme to the one-dimensional unsteady advection-diffusion equation. The scheme did not contain weighting parameters explicitly but contained Courant and Peclet numbers. It was shown that when negative parameters were defined in the scheme they did not affect the accuracy of the scheme. In addition, it was revealed that numerical instabilities could be eliminated by establishing a relationship between Courant and Peclet numbers [10].

Perumal and Sahoo investigated the volume conservation problem of the VPMC method and compared the method with the variable parameter Muskingum discharge hydrograph (VPMD) method. In order to analyze the problem, 6400 experiments were conducted using hypothetical data. It was concluded that the VPMC method was not as mass conservative as the VPMD method. Furthermore, the VPMC method was not successful at producing accurate peak outflow and time to peak outflow, while the VPMD method gave satisfactory results [11].

2. Variable Parameter Muskingum-Cunge Method

As mentioned above, when the parameters of the Muskingum-Cunge method change with respect to space and time, the method is called the variable parameter Muskingum–Cunge (VPMC) method. Variations of the VPMC method have been used in flood routing phenomena [7, 9, 12-18].

In this study, the VPMC method was used to model the flood routing process in a gauged creek. Basic equations of the method are described in HEC-HMS Technical Reference Manual [19] as below:

The VPMC method is solved using the continuity equation and momentum equation. The continuity equation is written as follows:

$$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x}=0 \tag{1}$$

where A is cross sectional area (m^2), Q is discharge (m^3/s), t is time (s), and x is space (m).

The finite-difference form of continuity equation is as follows:

$$\left(\frac{I_{t-1}+I_t}{2}\right)-\left(\frac{O_{t-1}+O_t}{2}\right)=\left(\frac{S_t-S_{t-1}}{\Delta t}\right) \tag{2}$$

where I_{t-1}, O_{t-1} and S_{t-1} are inflow (m^3/s), outflow (m^3/s) and storage (m^3) at time t-1 respectively, while I_t, O_t and S_t are inflow (m^3/s), outflow (m^3/s) and storage (m^3) at time t respectively. Δt is routing time step or time (s) between t and t-1.

When there is lateral inflow contribution to the system, equation 1 becomes:

$$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x}=q_L \tag{3}$$

where q_L is the lateral inflow (m^3/s).

The diffusion form of the momentum equation is written as follows:

$$S_f=S_0-\frac{\partial y}{\partial x} \tag{4}$$

where S_f is friction slope (m/m) and S_0 is channel bed slope (m/m).

After combining these two equations and after a linearization process, a convective diffusion equation is derived as [20]:

$$\frac{\partial Q}{\partial t}+c\frac{\partial Q}{\partial x}=\mu\frac{\partial^2 Q}{\partial x^2}+cq_L \tag{5}$$

where c is wave celerity (m/s), μ is hydraulic diffusivity (m^2/s). They are expressed as follows:

$$c=\frac{dQ}{\partial A} \tag{6}$$

and

$$\mu=\frac{Q}{2BS_0} \tag{7}$$

where B is top width of the water surface (m).

Storage is defined in the Muskingum model as follows:

$$S_t = K\left[XI_t + (1-X)O_t\right] \tag{8}$$

where K is a proportionality coefficient (s) and X is a weighting factor.

When Equation (2) is substituted in Equation (8), Equation (9) is obtained:

$$O_t = \left(\frac{\Delta t - 2KX}{2K(1-X)+\Delta t}\right)I_t + \left(\frac{\Delta t + 2KX}{2K(1-X)+\Delta t}\right)I_{t-1} + \left(\frac{2K(1-X)-\Delta t}{2K(1-X)+\Delta t}\right)O_{t-1} \tag{9}$$

A finite difference approximation of the partial derivatives, combined with Equation (9) yields:

$$O_t = C_1 I_{t-1} + C_2 I_t + C_3 O_{t-1} + C_4 (q_L \Delta x) \tag{10}$$

The coefficients C_1, C_2, C_3 and C_4 are expressed as follows:

$$C_1 = \frac{\frac{\Delta t}{K} + 2X}{\frac{\Delta t}{K} + 2(1-X)} \tag{11}$$

$$C_2 = \frac{\frac{\Delta t}{K} - 2X}{\frac{\Delta t}{K} + 2(1-X)} \tag{12}$$

$$C_3 = \frac{2(1-X) - \frac{\Delta t}{K}}{\frac{\Delta t}{K} + 2(1-X)} \tag{13}$$

$$C_4 = \frac{2\left(\frac{\Delta t}{K}\right)}{\frac{\Delta t}{K} + 2(1-X)} \tag{14}$$

In the above equations the parameters K and X are [21], [14]:

$$K = \frac{\Delta x}{c} \tag{15}$$

$$X = \frac{1}{2}\left(1 - \frac{Q}{BS_0 c\Delta x}\right) \tag{16}$$

Since c, Q, and B are time dependent, C_1, C_2, C_3 and C_4 must change in every time step. By using Ponce's algorithm, HEC-HMS recalculates c, Q and B at each time and distance step, Δt and Δx respectively [22].

Determination of Δt and Δx is essential for stability of the VPMC method. Therefore, minimum Δt is selected among other Δt values. Minimum Δt criteria is described in the literature [19].

After Δt is selected, Δx is calculated as follows:

$$\Delta x = c\Delta t \tag{17}$$

But Δx must ensure the criteria given below:

$$\Delta x \langle \frac{1}{2}\left(c\Delta t + \frac{Q_0}{BS_0 c}\right) \tag{18}$$

where Q_0 is reference inflow (m^3/s):

$$Q_0 = Q_B + \frac{1}{2}\left(Q_{peak} - Q_B\right) \tag{19}$$

where Q_B is base inflow (m^3/s) and Q_{peak} is peak inflow (m^3/s).

3. Study Area and Materials

Çaydere Creek is located in Isparta Province of Turkey. Catchment area of Çaydere Creek basin covers 102 km^2. Study area is located in lower reach of this creek. The creek flows from southeast toward Lake Eğirdir which is also known as the "Seven Colored Lake". In the study area, there are two stream gauging stations. Distance between upper gauging station (D09A601) and lower gauging station (D09A602) is 1764 meters in length (Figure 1). There is no lateral inflow or outflow between the stations.

The two gauging stations were installed in 2017. The inflow and outflow hydrographs chosen for this study belong to flood event occurred in 2018 spring because the two hydrographs had good hydrograph shapes during that flood event. Gauging stations are installed to measure the flow stages in 5-min intervals. 5-min interval inflow and outflow hydrographs are produced with the help of available rating curves. Afterwards, inflow and outflow hydrographs are calibrated to 30-min time steps. Elapsed time of the two hydrographs is 9 hours. They have single peak flows. Time to peak flow in the inflow hydrograph is 3 hours and lag of time to peak in the outflow hydrograph is 0.5 hours.

In this study, a 1:1000 scale digital topographical map of the study area was used. Based on the topographical map, bed slope between upper and lower gauging stations was calculated as 0.006236. The value of Manning roughness coefficient (n) is derived from the well-known Manning equation given below:

$$n = \frac{R^{2/3} J^{0,5} A}{Q} \tag{20}$$

In equation (20), R (m) is the hydraulic radius of various flows at the upper and lower gauging stations, J (m/m) is bed slopes in the vicinity of upper and lower gauging stations, A (m^2) is cross-sectional area of various flows at the upper and lower gauging stations, and Q (m^3/s) various flows in the inflow and outflow hydrographs. Thus, "n" is calculated for different values of R, J, A, and Q. As a result of the calculations, it was seen that the results are in the vicinity of 0.037. Therefore, "n" was calculated as 0.037.

Figure 1. Study area © Google Earth

Since the creek bed is lined with gravels and stones, the calibrated value of "n" is compatible with "Manning roughness coefficients for various open channel surfaces" [1].

In 2000, a bank protection project was constructed within the lower basin of Çaydere Creek including the study area. The project consists of grading the Çaydere Creek bank to a 2V:5H slope along 2500 meters of the eroded bank, and it is made of riprap (Figure 2 and 3).


Figure 2. Stream gauging station D09A601


Figure 3. Stream gauging station D09A602

4. HEC-HMS Model

In flood routing performed using the VPMC method, all of the required physical components for the HEC-HMS model are calculated based on the flow and topographical data.

In Figure 4, length is the distance between upper and lower gauging stations and it is 1764 meters. Slope is the average bed slope between the two stations and it was calculated as 0.006236. "n" roughness coefficient is the average value for the whole reach and it was calculated as 0.037. Invert is the river bed elevation where the upper gauging station is installed. Cross section is the average cross-section (eight-point) representing the cross-sectional shape of the reach.

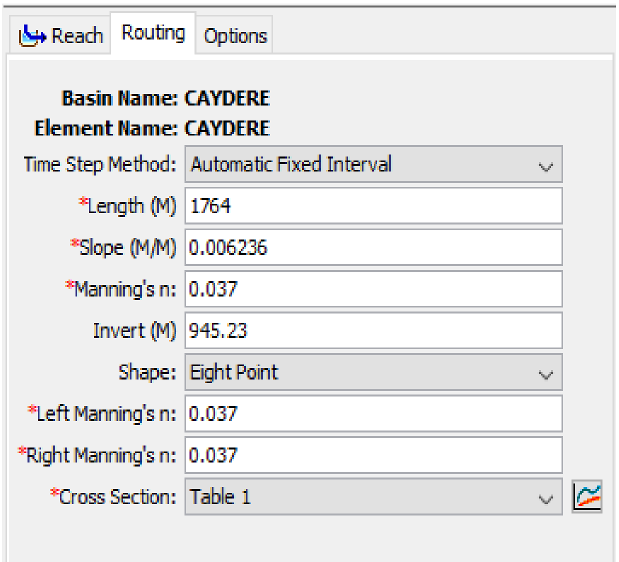

Figure 4. HEC-HMS Component editor for VPMC method

5. Results and Discussion

After the simulation, HEC-HMS gave the computed outflow hydrograph. The observed inflow hydrograph and computed outflow hydrograph are given in Figure 5. There is attenuation in the peak discharge due to routing, and lag time (0.50 hours) can be seen clearly between inflow peak discharge and computed outflow peak discharge.

In Figure 6, the observed and computed outflow hydrographs are shown. The computed outflow hydrograph has good agreement with the observed outflow hydrograph, and their times to peak are the same.

Although visual comparison of the observed and computed outflow hydrographs gives a positive opinion about the accuracy of the VPMC method in Çaydere Creek, statistical analyses are needed to support this opinion.

The root-mean-square error (RMSE) equation and the mean absolute error (MAE) equation are used to determine the difference between the observed and computed hydrographs (Equations 21 and 22).

$$RMSE = \sqrt{\frac{\sum_{i=n}^{n}(Q_c - Q_o)^2}{n}} \quad i=1,2,3,\ldots,n \qquad (21)$$

$$MAE = \frac{\sum_{i=1}^{n}|Q_c - Q_o|}{n} \quad i=1,2,3,\ldots,n \qquad (22)$$

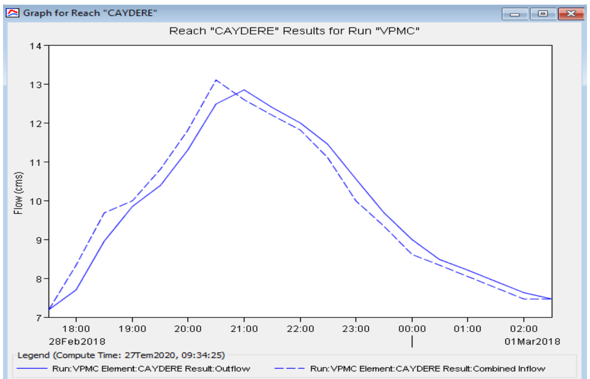

Figure 5. Inflow hydrograph and computed outflow hydrograph

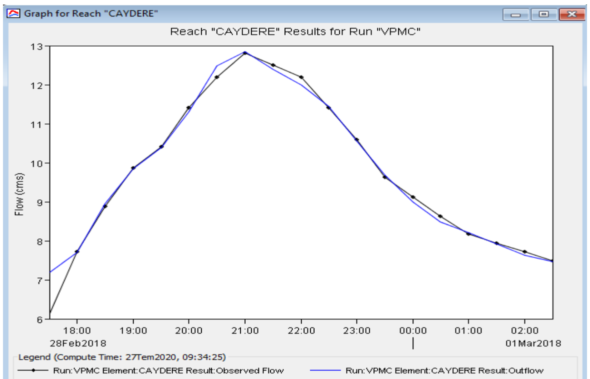

Figure 6. Observed outflow hydrograph and computed outflow hydrograph

Relative errors of 1) peak flow, 2) time to peak and 3) volume are computed as shown in Equations 23, 24, and 25.

$$\sigma_{peak} = (\frac{Q_{pc}}{Q_{po}} - 1)100 \qquad (23)$$

$$\sigma_{time} = (\frac{t_{pc}}{t_{po}} - 1)100 \qquad (24)$$

$$\sigma_{volume} = (\frac{V_c}{V_o} - 1)100 \qquad (25)$$

where:

Q_c= Computed flows in the outflow hydrograph (m^3/s)
Q_o= Observed flows in the outflow hydrograph (m^3/s)
σ_{peak}= Relative error of peak flow (%)
Q_{pc}= Computed peak outflow (m^3/s)
Q_{po}= Observed peak outflow (m^3/s)
σ_{time}= Relative error of time to peak flow (%)
t_{pc}= time to peak flow in the computed outflow hydrograph (h)
t_{po}= time to peak flow in the observed outflow hydrograph (h)
σ_{volume}= Relative error of total volume (%)
V_c=Total volume of the computed hydrograph (m^3)
V_o=Total volume of the observed hydrograph (m^3)

In order to compare the shape of the inflow and outflow hydrographs, Nash–Sutcliffe model efficiency coefficient (E) is calculated (26). If the computed flows are the same as the observed flows, E is expected to be 1. If E is between 0 and 1, it means that there are deviations between observed and computed flows. If E is negative, it means that the computed flows are too far off from the accuracy.

$$E = \frac{\sum_{i=1}^{n}(Q_o - Q_{mo})^2 - \sum_{i=1}^{n}(Q_o - Q_c)^2}{\sum_{i=1}^{n}(Q_o - Q_{mo})^2} \quad i=1,\ldots,n \qquad (26)$$

Q_{mo}= Mean of the observed outflows (m^3/s)

Results of statistical analyses are shown in Table 1 and 2. Finally, relationship between the observed and computed outflows is shown by a scatter plot in Figure 7.

Table 1. RMSE, MAE, Q_{pc}, Q_{po} and σ_{peak} values

RMSE (m^3/s)	MAE (m^3/s)	Q_{pc} (m^3/s)	Q_{po} (m^3/s)	σ_{peak} (%)
0,26	0,13	12,84	12,80	0,04

Table 2. t_{pc}, t_{po}, σ_{time}, V_c, V_o, σ_{vol} and E values

t_{pc} (h)	t_{po} (h)	σ_{time} (%)	V_c (m^3)	V_o (m^3)	σ_{vol} (%)	E
3,50	3,50	0	320634	320310	0,10	0,98

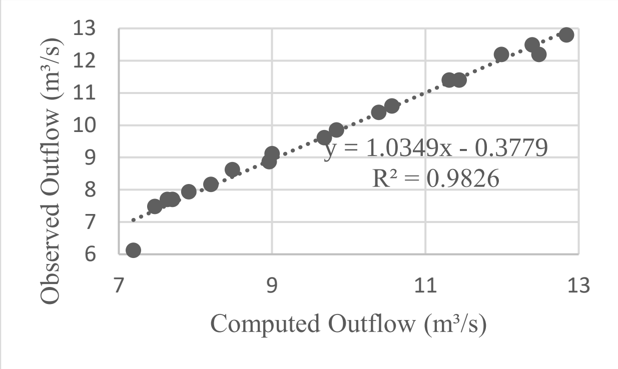

Figure 7. Scatter plot of the observed and computed outflows

According to results obtained from statistical analyses (Table 1 and 2), the three relative errors (peak, time, and volume) are acceptable. In addition, the value of the efficiency coefficient is close to 1, which indicates that the computed flow hydrograph's shape is similar to the observed hydrograph's shape.

Finally, according to the scatter plot in Figure 7, there is a positive, strong, and linear relationship between the observed and computed outflows.

6. Conclusions

In this study, the VPMC flood routing method was used in Çaydere creek reach. The reach was between two stream gauging stations. Inflow and outflow hydrographs were available and they belonged to the flood event occurred in the spring of 2018. 1/1000 scale topographical map of the study area was also available. The HEC-HMS model was used to derive the computed hydrograph. Statistical analysis was conducted to measure the accuracy of the VPMC method in the study area. Based on the results obtained from the analysis, it can be said that the VPMC method seems to produce reliable flood routing data, such as peak outflow, time to peak outflow, total volume, and hydrograph shape. Thus, it can be concluded that the VPMC method can be used in flood routing studies in the Çaydere basin when topographical and outflow data are not available. This method can also be applied to reaches that have similar physical characteristics with Çaydere reach.

Declaration

The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article. The authors also declared that this article is original, was prepared in accordance with international publication and research ethics, and ethical committee permission or any special permission is not required.

Author Contributions

H. Çakır collected hydrological and survey data, performed the analysis and wrote the manuscript. M. E. Keskin supervised and improved the study.

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