



Schema of Analogical Reasoning - Thinking Process in Example Analogies Problem

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ABSTRACT

Purpose: Analogical reasoning has been studied using a variety of tasks that generally required consideration of the relationship between the object and its integration to conclude an analog schema. The aim of this study was to describe the model of analogical reasoning schema based on some analogical problems.

Research Method: In this study used a qualitative research approach with design research is the case study. Data were taken from 16-year-old high school students, West Nusa Tenggara Indonesia. The researcher selected 4 students who answered two analogical problems correctly and used in analyzing qualitative data.

Findings: The findings showed that there is a difference in the schema. The first type is that students can directly map between target problem and source problem, followed by structuring, applying, and verifying. The second type is that the student can not directly map between the target problem with the source problem, but the student needs to do a representation of the target problem so as to find a form of problem that has similarities with the source problem. Then the student can map between target problem with source problem, followed by doing the process as the first schema type.

The Implication for Research and Practice: Findings of this study may have an impact on the way teachers teach mathematics on the analogical problems they use. Teachers can consider mathematical problems used in learning so that students easily understand the theory from the concepts being taught. And then, teachers need to develop an analogical problem that can develop critical and creative thinking to enhance the creativity of high school students with their analogical reasoning.

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Introduction

Analogical reasoning Allows one to make connections to transfer solutions from known problems to new problems of unknown solutions (Gentner & Loewenstein, 2002; Trench, Oberholzer, & Minervino, 2009). Additionally, resolving target problems needs to see the similarity between target problems and source problems. Holyoak & Hummel (2001) say that the similarities of the specific problems between the source and the target problems that can be identified by students through analogical reasoning targets help students solve problems. A reasoner should look at the similarity of the relational structure between known problems (source problems) and the new problems (target problems); namely "alignment structure" or "mapping" between two problems to be found (Bassok, 2001; Gentner, Holyoak, & Kokinov, 2001).

Problem-solving using analogical reasoning applies to certain cases. Gentner (1983) argues that analogical reasoning applies to certain cases such as a case known to be used to inferring new information about the case at hand. Someone did analogical reasoning has the purpose of obtaining a new conclusion or knowledge by comparing between analogical objects or with pre-existing knowledge (Amir-Mofidi, Amiripour, & Bijan-Zadeh, 2012).

DiMaggio (1997) describes a pattern of thought or behavior that organizes categories of information and the relationships among them. It can also be described as a mental structure of preconceived ideas, a framework representing some aspect of the world, or system of organizing and perceiving new information. Schema of analogical reasoning describe the pattern of thought or behavior of a person associated with the analogical reasoning process in solving an analogical problem.

Analogical reasoning in solving analogies problems concerns the existence of adaptation, memory, and reasoning (Gust, Krumnack, Kühnberger, & Schwering, 2008). Magdas (2015) said that the schema of analogical reasoning in solving the problem occurs: (1) recognition is the process of recognizing target problems to source problems, and (2) analogical reasoning. Magdas (2015) made a schema of analogical reasoning that uses analogical problems. Furthermore, Magdas stated that in order to solve the target problem, it is necessary to recognize the source problem. A source problem that has similarities to target problems. Solving source problems is known resolution steps need to be added with new completion steps to resolve the target issue. Schema of analogical reasoning illustrated by Magdas is adding source problems solving step with a new step to target problems that can be seen in Figure 1.

In schema Figure 1, to solve the target problem (problem P), through recognition, we identify an analogue, marked as source problem (basic problem BP), solved previously. Solving steps 1, 2, 3, ..., n of basis problem BP through analogy are transformed in the analogue steps 1, 2, ..., n for solving target problem P. However sometimes we need to add new steps 1, 2, ..., k for solving target problem P (Magdas, 2015). Target problem-solving steps being faced are hardly sufficient only by using source-solving steps but needing additional knowledge. This means that at every step of solution, the source problem needs to add a new step to solving the target problem.

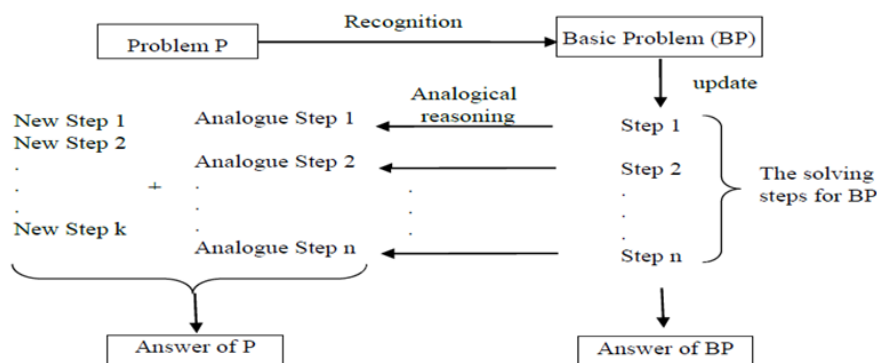


Figure 1. Scheme of an Analogical Reasoning to Solve Problems (Magdas, 2015)

A source problem with target problems can be similar or very different. Similar source problems with target problems can directly use solve source problem procedure to resolve target problem. However, the different source problem with target problem, resolving source problem cannot be used directly in resolving the target problem. Some researchers use analogical problem in their research. The researchers who used the analogy problem were English (2004), Bernardo (2001), and Assmus, Forster, & Fritzlar (2014). English (2004) in research, used analogical problems about Algebra and Combinatorics. Bernardo used analogical problems about statistics concepts is conjunction with problems with independent events. While Assmus et al. used analogical problems about Algebra concepts is Arithmetic series. There is a similarity between analogical problems used by the researchers, namely (1) analogical problems consists of source problem with target problems, (2) the problem used between source problem with target problem is the same, and (3) solve procedure between source problem with target problem is the same.

This study uses two analogical problems. The first problem is analogical problems that have similarities between source problems with the target problem. This problem used in this study has characteristics such as analogical problems used by English, Bernardo, and Assmus, at al. This first analogical problem is adapted from analogical problems used (Bernardo, 2001).

The second analogical problem is a problem developed by the researcher. This analogical problem is an analogical problem different from that used by previous researchers. Analogies problems used in this research has characteristics such as (1) mathematical concepts between source problem with target problem are different; (2) procedures in solving source problem are used to solve target problem; and (3) theory mathematics context of source problem with target problem is different.

The schema of analogical reasoning described by Magdas is based on examples of solving geometric problems theoretically. But empirically, the schemes produced by students in solving the problem cannot be explained. In addition, Magdas does not explain analogical reasoning stages in the schema of analogical reasoning he made.

While, the process of resolving analogical problems using analogical reasoning components are structuring, mapping, applying, and verifying (Ruppert, 2013). Structuring: identify every mathematical object that exists on the target problem by encoding its objects or characteristics and making conclusions from identical relationships between source problem with target problem. Mapping: Looks for identical linkages of character codes between source problem and target problem then builds conclusions from the similarity/identity relationship of character codes between source problem and target problem, then the resulting relationship is mapped to the target problem. Applying: resolving target problem based on source problem-solving steps. Verifying: checking the answer of the target problem by checking the suitability of the target problem with source problem.

This research will describe the schema of analogical reasoning appear empirically based on the answer of the student in solving two analogical problems. Analogies' problem consists of source problems with the target problem. In this research given two problems are a similar source problem with the target problem and different source problems with the target problem. How does the schema of analogical reasoning based on the answers from two problems analogy given to the students? Do both of the given analogy problems have the same or different schemas?

Method

Research Design

This study aims to describe the schema of analogical reasoning based on analogies problem. In this study, researchers explore processes and activities students analogical reasoning in solving analogies problems. This study applies the descriptive qualitative approach. Creswell (2013) said that such research is qualitative research with research design is a case study. Case study research involves a detailed description of the setting or individuals, followed by an analysis of the data for themes or issues.

Subject

Subjects of the study were taken from 16-year-old high school students in Mataram city, West Nusa Tenggara, Indonesia. All subjects haven't completed the probability theory learning. Then, all subjects have completed algebraic lessons on quadratic equations at the first level and have not completed learning on trigonometric equations.

Research Instruments

There are two analogical problems used in this research: (1) similar analogies problem between source problem with target problem and (2) different analogies problem between source problem with target problem. The first problem relates to the problem of conjunction with independent events. The second problem are the quadratic equations problem with trigonometric equations problem. Instruments have been declared valid and reliable by mathematicians and mathematics educators. Analogies problem in this research there are on Table 1 and Table 2.

Table 1

Conjunction with Independent Events Problems

<i>Source problem</i>	<i>Target problem</i>
There are 76 books in the Science section of the library, six of which are new. In the History section, there are 120 books, 15 of which are new. The principal randomly picks a book from each of the two sections. What is the probability that the principal picks a new book from both sections?"	There are 24 schools in District A, 8 of which are public schools. In District B, there are 32 schools, 12 of which are public schools. For each district, a school is randomly chosen to host the district sports fest. What is the probability that a public school is chosen to host the sports fest in both districts?

(Bernardo, 2001)

Table 2

Quadratic Equations Problem and Trigonometric Equations Problem

<i>Source problem</i>	<i>Target problem</i>
Find the roots of the quadratic equation $x^2 + 5x + 6 = 0$	Find the value of x that satisfies $\cos 2x + 6 \sin x + 7 = 0$

(Saleh et al., 2017)

Data Collection

The subject was worked the problem individually. The subject was asked to carefully read the instructions and answer all questions about the task. Steps of data collected: (1) the subject is working on source problem; furthermore, the results of source problem answers are taken and collected from the students, (2) the subject is working on target problem; and then the results of target problem answers are taken and collected from the students as well, (3) selecting student answer sheets that answer correctly from source problem with target problem, and (4) checking answers to students.

Then, data of the research result analyzed to know the occurrence of analogical reasoning. Based on the results of the analysis and student answers of the analogical problems is described and described schema of analogical reasoning that occurred.

Results

The first analogies problem, the source problems, 18 students gave true answers and 17 students gave false answers. While the target problems, 15 students gave true answers and 20 students gave false answers. The second analogies problem, the source problems, 26 students gave true answers and 9 students gave false answers. While the target problems, 16 students gave true answers and 19 students gave false answers. Based on the answer sheets and the interviews found 12 students using analogical

reasoning to answer conjunction with independent events problem. While there are 10 students using analogical reasoning to answer quadratic equation problems and trigonometric equation problems

This study aimed to describe the model schema of analogical reasoning based on some analogical problems. To describe the schema of analogical reasoning on the first analogical problem is represented by subject S1 and subject S2. While describing the schema of analogical reasoning on the second analogical problem is represented by subject S3 and subject S4.

Conjunction with Independent Events Problems

The results showed that 12 students did analogical reasoning. The student answers the source problem correctly and uses the solution of source problem to resolve the target problem. Furthermore, the completion steps used in source problems are directly applied for the target problem. This can be seen from the result of 2 student's answers in solving the source problem with the target problem seen in Figure 2(a), Figure 2(b), Figure 3(a), and Figure 3(b).

Based on Figure 1(a) with Figure 1(b), subject S1 identifies for resolving source problem. Then subject S1 starts by identifying what is known from problems. subject S1 to write the probability of a new book in the Science section $\frac{3}{38}$ and the probability new book in the History section $\frac{3}{24}$. Then the subject S1 determines to the probability chosen a new book from Science with History section obtained $\frac{3}{304}$. Next, subject S1 completes target problems. Subject S1 does the mapping process from the target problem to the source problem. Furthermore, in the structuring process, subject S1 identifies the problem as it did in resolving source problem. The subject S1 identifies the target problem by stating that the selected the probability a public school in area A is $\frac{1}{3}$ and area B is $\frac{3}{8}$. In the applying process, subject S1 solves the target problem equally by resolving the source problem. In the verifying process, the answer to the target problem like that result of the source problem answer. The answer to the target problem is $\frac{3}{24}$.

Based on Figure 2(a) and 2(b), the subject S2 identifies for resolving source problem. The subject S2 starts by identifying what is known from problems. The subject S2 to write the probability of a new book in the Science section $\frac{3}{38}$ and the probability of a new book in the History section $\frac{1}{8}$. Then the subject S2 determines to choose a new book from the Science and History section obtained $\frac{3}{304}$. Next, the subject S2 completes the target problem. The subject S2 does a mapping process from the target problem to source problem. Furthermore, in the structuring process, subject S2 identifies the problem as it did in resolving source problem. The subject S2 identifies the target problem by stating that the probability chosen a public school in area A is $\frac{1}{3}$ and area B is $\frac{3}{8}$. In the applying process, subject S2 solves the target problem equally

by resolving source problem. In the verifying process, the answer to the target problem like that result of the source problem answer. The answer to the target problem is $\frac{1}{8}$.

peluang buku baru di bag sains = $\frac{6}{76} = \frac{3}{38}$
 peluang buku baru di bag sejarah = $\frac{15}{120} = \frac{1}{8}$
 Maka, peluang memilih sebuah buku baru di bag sains & sejarah adalah:
 $\frac{3}{38} \cdot \frac{1}{8} = \frac{3}{304}$

2(a)

The probability that a new book from the Science section = $\frac{6}{76} = \frac{3}{38}$.

The probability that a new book from History section = $\frac{15}{120} = \frac{1}{8}$.

Then, the probability that chosen a new book from Science and History section is

$$\frac{3}{38} \times \frac{1}{8} = \frac{3}{304}$$

Figure 2(a). Source Problem with Target Problem Answered the Subject S1.

Figure 2(b). Source Problem with Target Problem Answered the Subject S2

$P(A) = \frac{8}{24} = \frac{1}{3}$
 $P(B) = \frac{12}{32} = \frac{3}{8}$
 Peluang memilih sekolah negeri utk menjadi tuan rumah pekan olahraga baru smp dari kedua daerah tsb adalah
 $\frac{1}{3} \cdot \frac{3}{8} = \frac{3}{24}$

2(b)

$$P(A) = \frac{8}{24} = \frac{1}{3}$$

$$P(B) = \frac{12}{32} = \frac{3}{8}$$

The probability that chosen a public school to host the sportsfest from A and B districts is

$$\frac{1}{3} \times \frac{3}{8} = \frac{3}{24}$$

Based on the work of the subject of S1 and the subject of S2, solving analogical problems begins by recognizing the similarity between target problems and source problems. Then, they do the mapping of target problems to source problems. Source problems solving steps are mapped one-to-one to problem-solving step that starts with structuring, applying, and verifying processes. An analogical problem-solving process between target problems and source problems using analogical reasoning stages can be seen in Figure 4.

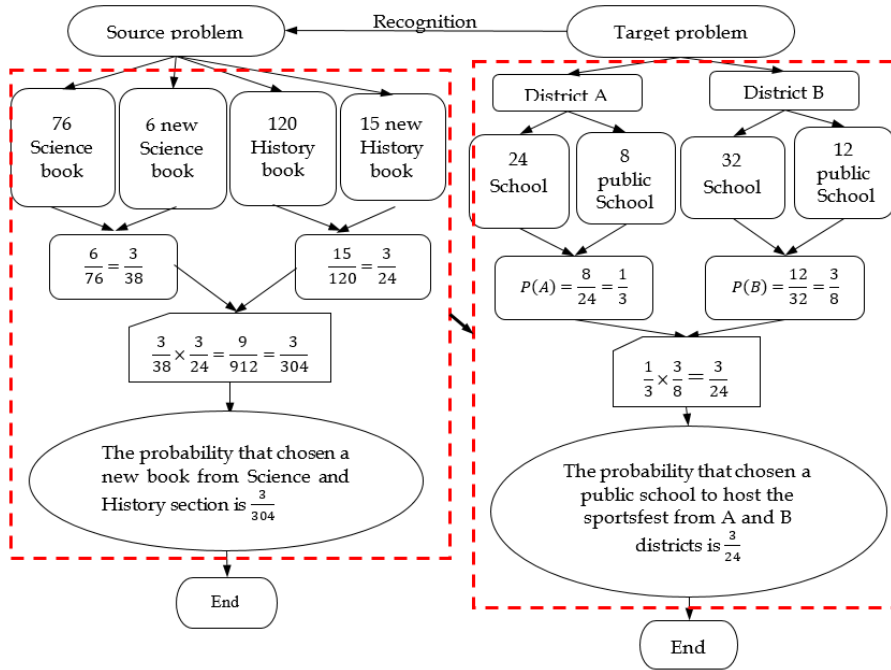


Figure 4. Process of Analogical Reasoning in Analogical Reasoning in Solving of Conjunction with Independent Events Problems

Description of the coding on the process of analogical reasoning in solving of the conjunction of independent events problems Figure 4 can see Table 3.

Tabel 3

Description of the Coding on the Process of Analogical Reasoning in Solving of Conjunction of Independent Events Problems

Term	Code
Start/End	
Structuring process	
Mapping process	
Applying process	
Verifying process	
Activity process	

The source problem has been answered. Then students are given a similar target problem as the previous source problem. In solving the target problem, students use analogical reasoning stages. Structuring process: (a) students identify problems targets, (b) the students see the similarities between the properties of target problem

and source problems, (c) the students make the assumption that the target problems have a common with source problems. Mapping process: the student declares that solving the target problem equals solving the source problem. Applying process: students solving target problems equals solving source problems. Verifying process: the student checks the solution of the target problem by observing the source problem. In general, the schema of analogical reasoning in solving analogical problem-related conjunction with independent events problems based on the Magdas scheme can be illustrated in Figure 5.

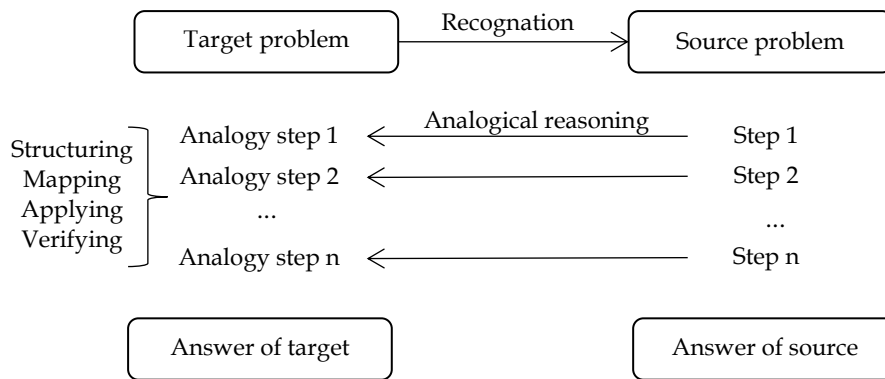


Figure 5. Schema of Analogical Reasoning for the First Analogical Problem

Glossary

—————> : Process of analogical reasoning

Quadratic Equation Problems and Trigonometric Equation Problems

The results showed that 10 students did analogical reasoning. The student answers source problems correctly and uses the concept of source problem to solve the target problem. Furthermore, some of the solving steps used in source problems; students apply to target problems. This can be seen from the result of the student's answer on source problem with the target problem seen in Figure 6 and Figure 7.

Based on Figure 6, the subject S3 solves quadratic equations problems (source problems) by using factorization and obtaining correct answers. Furthermore, the subject S3 solves trigonometric equations problems(target problems) beginning with a change $\cos 2x = 2 \cos^2 x - 1$. The subject S3 substitutes $\cos 2x = 2 \cos^2 x - 1$ into the trigonometric equation so that obtain $\cos^2 x - 12 \cos x - 13 = 0$. The subject S3 performs the mapping process by making the conclusion that new trigonometric equations obtain is a form of a quadratic equation on trigonometric equations. That is a process of solving target problems using the solving process by way of solving quadratic equation problems. Structuring process, the subject S3 identifies the problem as it did in resolving source problem. Applying process, the subject S3 solves

trigonometric quadratic equation problems like solving quadratic equation problems (source problems). And finally, the verifying process, the subject S3 finds the value of x that satisfies for the value $\cos x = -1$ so that the subject S3 obtains the value $x = 180$.

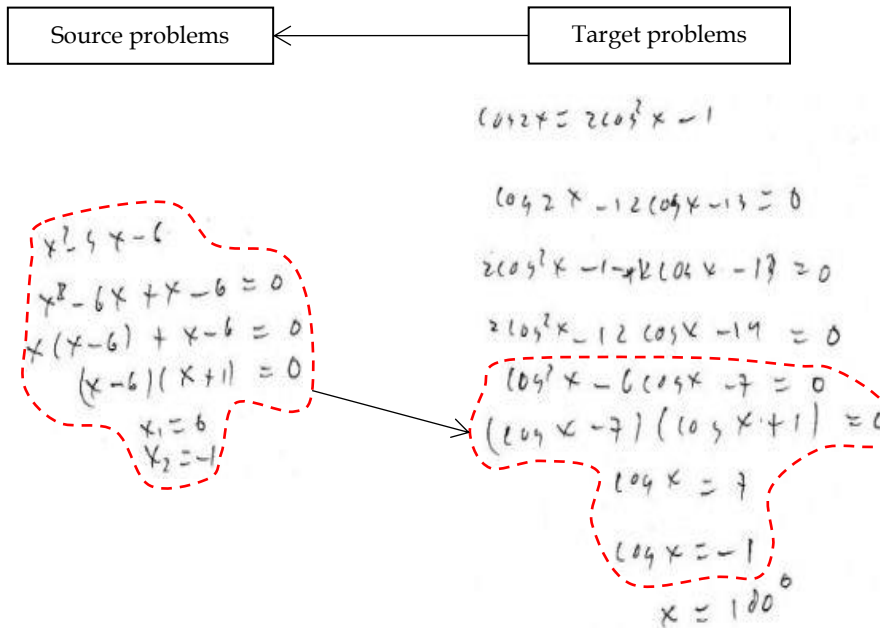


Figure 6. Source Problem with Target Problem Answered the Subject S3

Glossary

—————> : Process of analogical reasoning

Based on Figure 7, the subject S4 solves quadratic equations problems (source problem) by using factorization and obtaining correct answers. Furthermore, the subject S4 solves trigonometric equations problems (target problem) beginning with a change $\cos 2x = 2 \cos^2 x - 1$. The subject S4 substitutes $\cos 2x = 2 \cos^2 x - 1$ into the trigonometric equation, so that obtain $2 \cos^2 x - 12 \cos x - 14 = 0$ then $\cos^2 x - 6 \cos x - 7 = 0$. Then, the subject S4 gives $\cos x = A$ obtained $A^2 - 6A - 7 = 0$. The subject S4 performs the mapping process by making the conclusion that the new trigonometric equation obtained is a form of a quadratic equation on trigonometric equations. And then, the subject S4 obtains a new quadratic equation. That is a process of solving target problems using the solving process by way of solving quadratic equation problems. Structuring process, the subject S4 identifies the problem as it did in resolving source problem. Applying process, the subject S4 solves trigonometric quadratic equation problems like solving quadratic equation problems (source problem). And finally, verifying process, the subject S4 finds the value of x that satisfies for the value $\cos x = -1$. The subject S4 obtains value $x = \pi, 540, \dots$. The subject S4 obtains the value $x = \pi$.

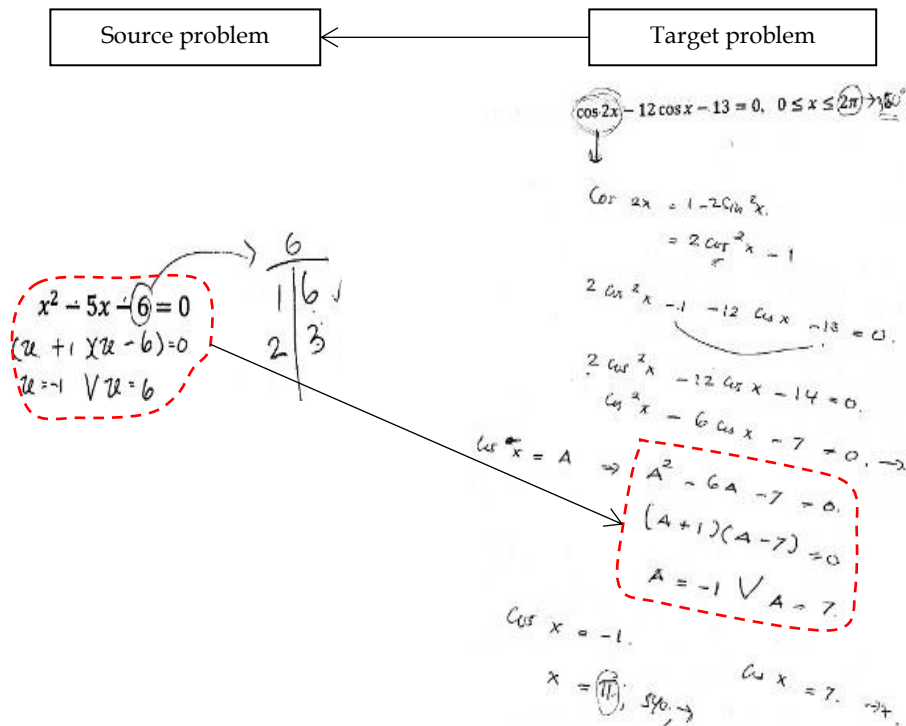


Figure 7. Source Problem with Target Problem Answered the Subject S4

Glossary

→ : Process of analogical reasoning

Based on the work of subject S3 and subject S4, solving analogical problems starts by recognizing the similarity between target problem with source problem. Target problems do not explicitly resemble source problems. Before the structuring process, target problems needs to be representation modified to form problems similar to source problems: that is, there is one stage in solving target problems is representation. The aim of changing target problems like source problems. And then, structuring proses, to identify new target problems and to find similarity with source problems, then mapping from target problem to source problem. Applying process, solving source problem steps are mapped one-to-one to problem-solving steps. And finally, the verifying process to determine the answer to target problems. An analogies problem-solving process between target problem with source problem using analogical reasoning stages there is in Figure 8.

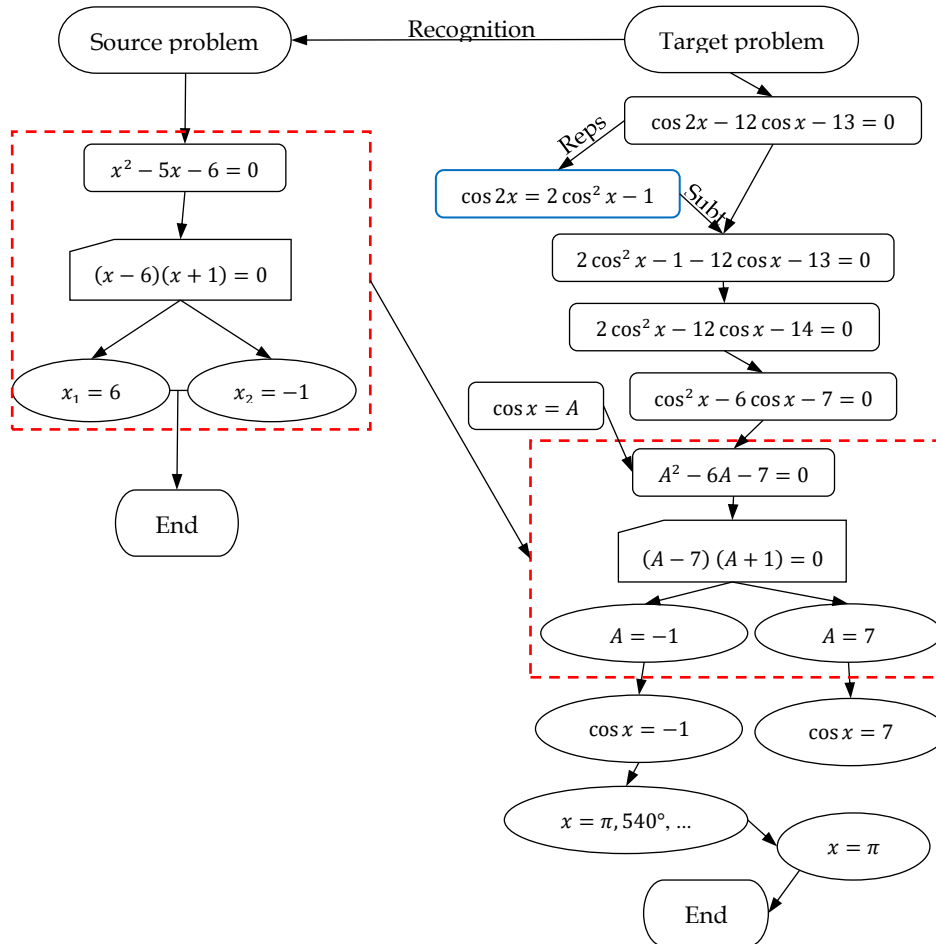


Figure 8. Process of Analogical Reasoning in Analogical Reasoning in Solving of Quadratic Equation and Trigonometric Equation Problems


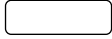
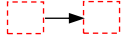



Description of the coding on the process of analogical reasoning in solving of quadratic equation and trigonometric equation problems Figure 8 can see Table 4.

Tabel 4

Description of the Coding on the Process of Analogical Reasoning in Solving of Quadratic Equation and Trigonometric Equation Problems

Term	Code
Start/End	

Tabel 4 Continue

<i>Term</i>	<i>Code</i>
Representation process	
Structuring process	
Mapping process	
Applying process	
Verifying process	
Activity process	
Representation	Reps
Substitution	Subt

The source problem has been answered. Then students are given target problem that is different from the previous source problem. In solving the target problem, students need to do representation before using the analogical reasoning stage. Representation: the student performs a representation of a target problem such as changing $\cos 2x = 2 \cos^2 x - 1$. Structuring process: (a) students identify problems targets, (b) the students see the similarities between the properties of target problem and source problems, (c) the students make the assumption that the target problems have a common with source problems. Mapping process: the student declares that solving the target problem equals solving the source problem. Applying process: students solving target problems equals solving source problems. Verifying process: the student checks the solution of the target problem by observing the source problem. In general, the schema of analogical reasoning in solving analogical problem related quadratic equation and trigonometric equation problems based on the Magdas scheme can be illustrated in Figure 9.

Discussion, Conclusion and Recommendations

This research resulted in two schema model of analogical reasoning from two different analogical problems. First, schema model of analogical reasoning from two similar problems between source problem with target problem. Second, the schema model of analogical reasoning from two different problems between source problem with target problem.

A difference in the analogical reasoning process done by students in resolving the given analogical problems. In the first analogical problems, the student directly mapp source problem procedure to target problem. Students transferring analogical problems information between source problems and target problems direct. While the second analogical problem, students do not directly map procedure solve source problem to solve the target problem. Students are transferring analogical problems information between source problem with target problem indirect because the target

problem must be presented in the form of a quadratic equation. Students need one stage before using analogical reasoning in solving analogies problem.

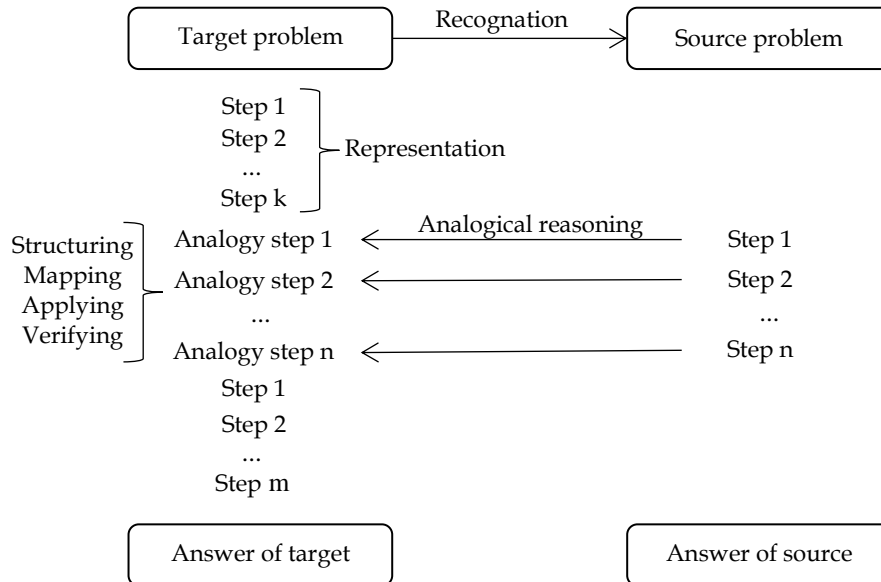


Figure 9. Schema of Analogical Reasoning for Quadratic Equation and Trigonometric Equation Problems

Glossary

—————> : Process of analogical reasoning

So there is a difference between schema; the first is that students can directly map between target problem with source problem, followed by structuring, applying, and verifying. The second type is that the student can not directly map between the target problem with the source problem, but the student needs to do a representation of the target problem so as to find a form of problem that has similarities with the source problem. Then the student can map between target problem with source problem, followed by doing the process as the first schema type.

According to the results of the research, subprocesses analogical problem the first in analogical reasoning include: (1) the student identified targets problem and sought the similarity between source problem with target problem, (2) the student does one-to-one mapping features of target problem to source problem, (3) the student applies the problem-solving procedures of source problem to solve target problem, and (4) the student checked the results obtained from target problem with source problem. This analogical reasoning process is consistent with English, Thagard and Kristayulita et al. (English, 2004; Thagard, 2005; Kristayulita, et al., 2018), the analogical reasoning process in solving analogies problem. They said analogical reasoning process in

solving the analogies problem are: (1) the student identifies target problem; (2) students remembered the known source problem solution; (3) students compare between source and target problem, see relevant relation between its components; and (4) students adapting the source problem-solving steps to resolve the target problems.

In solving the second analogical problem, students do not immediately seek a one-on-one relationship between source problem with target problem. The problem faced is different from the previous analogical problem. Students need to represent target problems to a form that has a resemblance to source problems. In this case, one needs to have the ability to represent problems in order to find specific information between target problems and source problems. The ability to represent this problem is also a characteristic of mapping success between base problems and analog problems (Gentner & Markman, 1997; Markman & Gentner, 2000; Eveleth, 2016). According to the results of this study, subprocesses analogical problem the second in analogical reasoning include: (1) representation of the target problem, (2) structuring on new problems of the target problem, (3) mapping of new problems and related source problems, (4) applies the problem-solving procedures of source problems to solve new problems from target problems, and (5) verify the result based on target problems.

The Schema of analogical reasoning depicted in solving the analogical problem the first involved recognition and analogical reasoning process. This is in line with the schema of analogical reasoning described by Magdas (Magdas, 2015). However, schema of analogical reasoning in solving the second analogical problem occurs recognition, representation, and analogical reasoning process. This means that the given analogical problem can give a different schema of analogical reasoning. The more difficult an analogical problem is given, the greater the cognitive load used in solving it. Strategies that are implemented in problem-solving can be difficult because problem-solvers need to pay attention to information other than problems to be solved (target problems) (Voskoglou, 2012). Thus the solver can have no solution, either because it has never solved a similar problem before or because it fails to recognize the similarity with the previous problem. However, if the solver is aware of an analogy, the solver must know how to use it to determine the solution procedure for target problems.

By identifying similarities between different objects or situations, humans can solve new problems, learn, and form new concepts, or communicate specific ideas to others. This identification of similarities allows us to connect knowledge domains and transfer solutions that differ from domain to domain. In analogical reasoning, the similarity is usually relational, ie relating the components of an object or situation rather than to the component itself (Gentner & Holyoak, 1997; Krawczyk, 2012).

This suggests that the process of analogical reasoning can depend on the instrument used. Therefore, teachers need to provide a variety of problems to develop analogical reasoning abilities, in which the problems provided do not only bring up the first schema or the second schema. However, students need to be given analogical problems that can lead to both schemas. Furthermore, teachers need to develop an analogical problem that can develop critical and creative thinking to enhance the

creativity of high school students with their analogical reasoning. Therefore, future research should focus on developing an analogical problem-related instrument capable of enhancing creativity and critical thinking of students.

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