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A NEW ALTERNATIVE SELECTION CRITERION FOR FAMILY OF SOME TRANSMUTED DISTRIBUTIONS BASED ON EXPONENTIAL DISTRIBUTION

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Abstract

Although Akaike information criterion is the most popular selection criterion in the literature, it gives inconsistent results in determining the correct model according to other selection criteria in transmuted distribution families. We motivate a new extension of the Akaike information criterion in the solution of this problem. In this paper, we suggest a new selection criterion as an alternative to the Akaike information criterion for the family of transmuted distribution. We discuss special cases of this family based on exponential distribution. A Monte Carlo simulation study is considered to compare the performances of this new criterion with the Akaike information criterion. Also, a numerical example is presented.
Keywords: Akaike information criterion, Modified Akaike information criterion, Monte Carlo simulation, Transmuted family.

ÜSTEL DAĞILIMA DAYALI BAZI DÖNÜŞTÜRÜLMÜŞ DAĞILIM AİLELERİ İÇİN YENİ BİR ALTERNATİF SEÇİM KRİTERİ

Özet

Akaike bilgi kriteri literatürde en popüler seçim kriteri olmasına rağmen, dönüştürülmüş dağılım ailelerinde diğer seçim kriterlerine göre doğru modeli belirlemede uyumsuz sonuçlar vermektedir. Akaike bilgi kriterine yeni bir açılım getirmek için motive olduk. Bu çalışmada, dönüştürülmüş dağılım ailesi için Akaike bilgi kriterine alternatif olarak yeni bir seçim kriteri önerilmiştir. Üstel dağılıma dayalı bu ailenin özel durumları tartışılmıştır. Dönüştürülmüş dağılım ailesinin derecesini belirlemek için bir algoritma yürütülmüştür. Monte Carlo simülasyon çalışması, bu yeni kriter ile Akaike bilgi kriterinin karşılaştırılması için yapılmıştır. Ayrıca, bir gerçek veri örneği sunulmuştur.
Anahtar Kelimeler: Akaike bilgi kriteri, Değiştirilmiş Akaike bilgi kriteri, Monte Carlo simülasyonları, Dönüştürülmüş aile

Cite

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1. Introduction

In recent years, various families of distributions are proposed by many authors such as [1-3]. Several of these families of distributions produce better modeling performance in real-world applications than their baseline distributions. Therefore, they are preferred in the modeling of a lifetime since more flexible than their primitives. One of these families is the general transmuted family of distributions introduced by Rahman et al. [4]. This family can be summarized as follows:

Let X be a random variable with cumulative distribution function (cdf), G(x) and probability density function (pdf), g(x). Then, the cdf and pdf of the transmuted family are given respectively, by

$$F(x) = G(x) + [1 - G(x)] \sum_{i=1}^{k} \lambda_i [G(x)]^i \tag{1}$$

and

$$f(x) = g(x)\left[1 - \textstyle\sum_{i=1}^{k} \lambda_i G^i(x) \{1 - G(x)\} \sum_{i=1}^{k} i \lambda_i G^{i-1}(x)\right] \tag{2}$$

where $-1 \leq \lambda_i \leq 1$ for $i = 1,2,\dots,k$ and $-k \leq \sum_{i=1}^{k} \lambda_i \leq 1$ substituting $\lambda_i = 0$ for $i = 1,2,3,\dots,k$, the baseline distribution is obtained. Rahman et al. [4] have suggested some special models of the general transmuted family of distributions such as cubic transmuted normal distribution, cubic transmuted gamma distribution, cubic transmuted log-logistic distribution, cubic transmuted Pareto distribution, cubic transmuted Rayleigh distribution, and cubic transmuted Gumbel distribution. It is also examined cubic transmuted exponential distribution and its some statistical properties in detail in [4]. In literature, there are many papers about families of transmuted distributions recently. Pavlov et al. [5] described some of the transmuted software reliability models. Riffi [6] introduced families of higher rank transmuted distributions. Ahmad et al. [7] examined recent developments of classes of distributions. Rahman et al. [8] discussed various types of transmuted distributions. They provided a general report about suggested transmuted distributions in [8].

The main purpose of this paper to suggest a new selection criterion by modifying on Akaike Information Criterion (AIC) for a general transmuted family of distributions. We consider exponential distribution as a special sub-model of this family. A new selection criterion called modified Akaike Information Criterion (MAIC) is obtained by adding a weighting coefficient on AIC. MAIC is described as follows:

$$MAIC = -2l + 2ak \tag{3}$$

where k is denotes the number of parameters, ℓ represents the maximized log likelihood value and $a \in [0,1]$ is a weighting coefficient.

We aim to see the performances of AIC and MAIC for model selection among the general transmuted family of distributions with cdf $F(x)$ in (1) for $k = 1,2,3$ when the underlying distribution is exponential via Monte Carlo simulations.

The rest of this study is organized as follows: In Section 2, some transmuted exponential distribution of Rahman's families are examined. In Section 3, the maximum likelihood estimators (MLEs) of parameters for examined transmuted models are obtained. Section 4 presents a Monte Carlo simulation study. In Section 5, a real data application is conducted. Finally, the conclusions are given in Section 6.

2. Some Transmuted Exponential Distribution

In this section, we describe some transmuted distributions based on exponential distribution, such as transmuted exponential (order 2), cubic transmuted exponential (order 3) and quartic transmuted exponential (order 4) distributions.

2.1. Transmuted Exponential Distribution

In this subsection, we examine the transmuted family of distributions. The transmuted family of distributions is derived by taking $k = 1$ in (1)-(2). Its cdf and pdf are given as follows:

$$F_1(x) = (1+\lambda_1)G(x) - \lambda_1 G^2(x) \tag{4}$$

and

$$f_1(x) = g(x)\{1+\lambda_1 - 2\lambda_1 G(x)\}, \tag{5}$$

respectively, where $\lambda_1 \in [-1,1]$. The baseline distribution is substituted by exponential distribution then, cdf and pdf of the transmuted exponential distribution (TE) can be respectively written as

$$F_{TE}(x) = (1+\lambda_1)\left(1-e^{-\theta x}\right) - \lambda_1\left(1-e^{-\theta x}\right)^2 \tag{6}$$

and

$$f_{TE}(x) = \theta e^{-\theta x}\left[1+\lambda_1 - 2\lambda_1\left(1-e^{-\theta x}\right)\right], \tag{7}$$

where $\lambda_1 \in [-1,1]$, $\theta > 0, x > 0$.

2.2. Cubic Transmuted Exponential Distribution

In this subsection, it is provided that a special case of a general transmuted family of distributions generated by [4] called as a cubic transmuted family of distributions. Also, we introduce cubic transmuted exponential (CTE) distribution as a sub-model of this family of distributions. The cubic transmuted family of distributions are obtained by setting $k = 2$ in (1) with cdf and pdf as follows:

$$\begin{aligned} F_2(x) &= (1+\lambda_1)G(x) \\ &\quad + (\lambda_2 - \lambda_1)G^2(x) - \lambda_2 G^3(x) \end{aligned} \tag{8}$$

and

$$\begin{aligned} f_2(x) &= g(x)\left[1+\lambda_1 + 2(\lambda_2 - \lambda_1)G(x)\right. \\ &\quad \left. -3\lambda_2 g(x)G^2(x)\right], \end{aligned} \tag{9}$$

respectively, where $-2 \leq \lambda_1 + \lambda_2 \leq 1$, $\lambda_1 \in [-1,1]$, $\lambda_2 \in [-1,1]$.

Thus, the cdf and pdf of the CTE distributions are obtained by using (8) and (9) to obtain as follows:

$$\begin{aligned} F_{CTE}(x) &= (1+\lambda_1)(1-e^{-\theta x}) \\ &\quad + (\lambda_2 - \lambda_1)\left[1-e^{-\theta x}\right]^2 - \lambda_2\left[1-e^{-\theta x}\right]^3 \end{aligned} \tag{10}$$

and

$$\begin{aligned} f_{CTE}(x) &= \theta e^{-\theta x}\left[1+\lambda_1 + 2(\lambda_2 - \lambda_1)(1-e^{-\theta x})\right] \\ &\quad -3\lambda_2\theta e^{-\theta x}(1-e^{-\theta x})^2, \end{aligned} \tag{11}$$

respectively, where $-2 \leq \lambda_1 + \lambda_2 \leq 1$, $\lambda_1 \in [-1,1]$, $\lambda_2 \in [-1,1], \theta > 0, x > 0$.

2.3. Quartic Transmuted Exponential Distribution

In this subsection, we describe the quartic transmuted family of distributions by using the general transmuted distribution suggested by [4]. Further, we have provided quadratic transmuted exponential (QTE) distributions. The quartic transmuted family of distributions is obtained by setting $k = 3$ in (1) with cdf and pdf respectively given as;

$$F_3(x) = (1+\lambda_1)G(x) + (\lambda_2 - \lambda_1)G^2(x) + (\lambda_3 - \lambda_2)G^3(x) - \lambda_3 G^4(x) \tag{12}$$

and

$$f_3(x) = g(x)\left[1+\lambda_1 + 2(\lambda_2 - \lambda_1)G(x)\right] + 3(\lambda_3 - \lambda_2)g(x)G^2(x) - 4\lambda_3 g(x)G^3(x), \tag{13}$$

respectively, where

$-3 \leq \lambda_1 + \lambda_2 + \lambda_3 \leq 1,\ \lambda_1 \in [-1,1],\ \lambda_2 \in [-1,1],$.

$\lambda_3 \in [-1,1]$. Thus, the cdf and pdf of the QTE distributions are obtained by using (12) and (13) to obtain;

$$F_{QTE}(x) = (1+\lambda_1)(1-e^{-\theta x}) + (\lambda_2 - \lambda_1)\left[1-(1-e^{-\theta x})\right]^2 + (\lambda_3 - \lambda_2)\left[1-(1-e^{-\theta x})\right]^2 - \lambda_3\left[1-(1-e^{-\theta x})\right] \tag{14}$$

and

$$f_{QTE}(x) = \theta e^{-\theta x}\left[1+\lambda_1 + 2(\lambda_2 - \lambda_1)(1-e^{-\theta x})\right] + 3(\lambda_3 - \lambda_2)\theta e^{-\theta x}(1-e^{-\theta x})^2 - 4\lambda_3\theta e^{-\theta x}(1-e^{-\theta x})^3, \tag{15}$$

where $-3 \leq \lambda_1 + \lambda_2 + \lambda_3 \leq 1$, $\lambda_1 \in [-1,1]$, $\lambda_2 \in [-1,1], \theta > 0, x > 0$.

3. Maximum Likelihood Estimation

In this section, we provide the maximum likelihood estimators of parameters for examined the transmuted distributions.

3.1. Maximum likelihood estimation for transmuted exponential distribution

Let $X_1, X_2, \dots, X_n$ be a random sample from $TE(\theta, \lambda_1)$ distribution. The log-likelihood function is given as follows:

$$l(\theta, \lambda_1, | x) = n\log(\theta) - \theta\sum_{i=1}^{n} x_i + \sum_{i=1}^{n}\log(1-\lambda_1 + 2\lambda_1 e^{-\theta x_i}), \tag{16}$$

where $x = (x_1, x_2, \dots, x_n)$. The MLEs of unknown parameters of $TE(\theta, \lambda_1)$ distribution can be obtained by maximizing the log-likelihood function given in (16).

3.2. Maximum likelihood estimation for cubic transmuted exponential distribution

Let $X_1, X_2, \dots, X_n$ be a random sample from $CTE(\theta, \lambda_1, \lambda_2)$ distribution. The log-likelihood function is given as follows:

$$\ell(\theta, \lambda_1, \lambda_2 | x) = n\log(\theta) - \theta\sum_{i=1}^{n} x_i + \sum_{i=1}^{n}\log\left[\kappa(\theta, \lambda_1, \lambda_2)\right], \tag{17}$$

where $\boldsymbol{x} = (x_1, x_2, \dots, x_n)$ and $\kappa(\theta, \lambda_1, \lambda_2)$is as follows: $\kappa(\theta, \lambda_1, \lambda_2) = 1 - \lambda_1 - \lambda_2 + 2(3 + \lambda_1 - \lambda_2 - 3\lambda_1 e^{-\theta x_i})e^{-\theta x_i}$. The MLEs of unknown parameters of $CTE(\theta, \lambda_1, \lambda_2)$ distribution can be derived by maximizing the log-likelihood function given in (17).

3.3. Maximum likelihood estimation for quartic transmuted exponential distribution

Let $X_1, X_2, \dots, X_n$ be a random sample from $QTE(\theta, \lambda_1, \lambda_2, \lambda_3)$ distribution. The log-likelihood function is given as follows:

$$\ell(\theta, \lambda_1, \lambda_2 | x) = n\log(\theta) - \theta\sum_{i=1}^{n} x_i + \sum_{i=1}^{n}\log\left[\omega(\theta, \lambda_1, \lambda_2, \lambda_3) + \tau(\theta, \lambda_1, \lambda_2, \lambda_3)\right] \tag{18}$$

where $x = (x_1, x_2, \dots, x_n)$. $\omega(\theta, \lambda_1, \lambda_2, \lambda_3) = 1 - \lambda_1 - \lambda_2 - \lambda_3 + (4\lambda_2 + 2\lambda_1 + 6\lambda_3)e^{-\theta x_i}\tau(\theta, \lambda_1, \lambda_2, \lambda_3) = 4\lambda_3 e^{-3\theta x_i} - (3\lambda_2 + 9)e^{-2\theta x_i}$. The MLEs of unknown parameters of $QTE(\theta, \lambda_1, \lambda_2, \lambda_3)$ distribution can be derived by maximizing the log-likelihood function given in (18).

All maximization problem can be solved by some numerical methods such as Nelder-Mead, BFGS, or CG. These methods can be easily conducted by optim function in R software.

4. Simulation Study

In this section, we perform a Monte Carlo simulation study to evaluate classification in the determination of the true model for the transmuted family. The simulations are conducted based on 100 repetitions due to slow progression. We select sizes of samples as n=100, 200, 300 and 500. The considered initial values are given in Table 1. Tables 2-6 illustrate the results of the simulation study. An algorithm is considered to conduct the simulation study and determine the a in (3) weighting coefficient is given below.

Algorithm:

1. Generate random samples from TE, CTE, and QTE distributions.

2. The simulated samples were fitted with the TE, CTE and QTE distributions according to the AIC.

3. Then the 3×3 matrix was created. This matrix shows the correct classification rate according to AIC.

4. Then, we determine the coefficient a in (3), which maximizes the sum of the diagonal elements of defined matrix in Step 3. The coefficient a is determined by scanning in 0.1 increments.

5. After determining the value of a, MAIC values are calculated. Then, the correct classification matrix was created according to the MAIC.

A summary of the classifications is obtained in Tables 2-5. The total percentage of classifications of AIC and MAIC are given in Table 6. Table 6 provides the total correct classification percentages (CCP). It is calculated by the formula as follows:

$$CCP = \frac{\sum_{i=1}^{100} p_{TE_i} + p_{CTE_i} + p_{QTE_i}}{\sum_{j=1}^{3} r_j}$$

where p_i denotes the number of success ($p_i = 1$) or failure ($p_i = 0$) in selection of real model, and r_j denotes the total number of repetitions. Since 100 repetitions are considered separately for each model in this problem, it is computed $\sum_{j=1}^{3} r_j = 300$.

Table 1: The initial values of parameters in simulation study

Case	Distribution	θ	λ_1	λ_2	λ_3
	TE	1.5	0.8		
1	CTE	2	0.5	0.2	
	QTE	1.3	0.5	0.2	0.2
	TE	1.2	0.5		
2	CTE	1.5	0.4	0.1	
	QTE	1.8	0.38	0.3	0.3
	TE	0.9	-0.5		
3	CTE	0.9	-0.5	0.3	
	QTE	0.9	-0.5	0.3	0.2
	TE	2	0.4		
4	CTE	2	0.4	0.2	
	QTE	2	-0.4	0.2	0.1

Table 2: Classification percentage table for AIC and MAIC: Case 1

		Predicted Model					
		AIC			MAIC		
		TE	CTE	QTE	TE	CTE	QTE
	n=100			***a=0.5***			
Real Model	TE	92	8	0	92	8	0
	CTE	73	27	0	50	50	0
	QTE	97	3	0	86	14	0
	n=200			***a=0.5***			
	TE	94	6	0	91	9	0
	CTE	49	51	0	27	73	0
	QTE	93	7	0	80	20	0
	n=300			***a=0.6***			
	TE	93	7	0	92	8	0
	CTE	30	70	0	13	85	2
	QTE	85	15	0	77	23	0
	n=500			***a=1***			
	TE	95	5	0	95	5	0
	CTE	5	95	0	5	95	0
	QTE	75	25	0	75	25	0

Table 3: Classification percentage table for AIC and MAIC: Case 2

		Predicted Model					
		AIC			MAIC		
		TE	CTE	QTE	TE	CTE	QTE
	n=100			***a=0.3***			
Real Model	TE	95	5	0	76	24	0
	CTE	89	11	0	50	50	0
	QTE	96	4	0	77	22	1
	n=200			***a=0.5***			
	TE	86	14	0	77	23	0
	CTE	65	35	0	48	52	0
	QTE	73	22	5	77	15	8
	n=300			***a=0.3***			
	TE	90	10	0	66	24	10
	CTE	59	41	0	24	68	8
	QTE	81	8	11	52	25	23
	n=500			***a=0.3***			
	TE	87	12	1	62	21	17
	CTE	82	18	0	35	44	21
	QTE	77	22	1	38	38	24

Table 4: Classification percentage table for AIC and MAIC: Case 3

		Predicted Model					
		AIC			MAIC		
		TE	CTE	QTE	TE	CTE	QTE
	n=100			***a=0.2***			
Real Model	TE	99	1	0	75	25	0
	CTE	1	0	0	73	27	0
	QTE	97	3	0	73	25	2
	n=200			***a=0.2***			
	TE	93	7	0	57	27	16
	CTE	95	5	0	48	36	16
	QTE	94	6	0	40	41	19
	n=300			***a=0.4***			
	TE	91	9	0	73	23	4
	CTE	93	7	0	74	25	1
	QTE	94	6	0	62	35	3
	n=500			***a=0.3***			
	TE	85	15	0	53	20	27
	CTE	80	20	0	38	43	19
	QTE	75	25	0	27	51	22

Table 5: Classification percentage table for AIC and MAIC: Case 4

		Predicted Model					
		AIC			MAIC		
		TE	CTE	QTE	TE	CTE	QTE
	n=100			***a=0.6***			
Real Model	TE	98	2	0	92	8	0
	CTE	96	3	1	88	12	0
	QTE	1	0	0	98	2	0
	n=200			***a=0.2***			
	TE	93	7	0	56	33	11
	CTE	96	4	0	52	34	14
	QTE	96	4	0	43	41	16
	n=300			***a=0.1***			
	TE	89	11	0	36	24	40
	CTE	96	4	0	29	33	38
	QTE	94	6	0	15	44	41
	n=500			***a=0.4***			
	TE	94	6	0	61	21	18
	CTE	89	10	1	58	28	14
	QTE	80	20	0	35	47	18

Table 6: The percentage classification of AIC and MAIC

Case	n	a	MAIC	AIC
	100	0.5	47	39
1	200	0.5	54	48
	300	0.6	59	54
	500	1.0	63	63
	100	0.3	42	35
2	200	0.5	45	40
	300	0.3	52	43
	500	0.3	43	35
	100	0.2	34	33
3	200	0.2	37	32
	300	0.4	33	32
	500	0.3	39	35
	100	0.6	34	33
4	200	0.2	35	32
	300	0.1	36	31
	500	0.4	35	34

It can be seen from Tables 2-6 that MAIC is generally more successful in classification for the transmuted family of distribution than the AIC. According to Table 6, the MAIC appears to give better results than AIC in all parameter cases and sample sizes. It was observed that TE is the best model in the selection performance of MAIC, while QTE is the worst model in the selection performance of MAIC.

5. Real Data Applications

In this section, we perform a real data application to evaluate the fits of some transmuted distributions such as TE, CTE, and QTE. It considered AIC, MAIC, BIC, KS, p-values to compare the fits of these models. We consider the real dataset consists of 213 observations about the number of successive failures of the air conditioning system. These data were derived by [9]. These data were also analyzed by many authors such as [10-13]. The data set is given in Table 7.

Table 7: Successive failures of 213 items

194	413	90	74	55	23	97	50
50	130	487	102	15	14	10	57
261	51	44	9	254	493	18	209
58	60	48	56	87	11	102	12
100	14	29	37	186	29	104	7
72	270	283	7	57	33	100	61
220	120	141	22	603	35	98	54
65	49	12	239	14	18	39	3
5	32	9	14	70	47	62	142
104	85	67	169	24	21	246	47
15	2	91	59	447	56	29	176
77	197	438	43	134	184	20	386
71	80	188	230	152	36	79	59
246	1	79	3	27	201	84	27
16	88	130	14	118	44	15	42
46	230	59	153	104	20	206	5
34	29	26	35	5	82	5	61
118	326	12	54	36	34	18	25
31	22	18	156	11	216	139	67
3	46	210	57	76	14	111	97
26	71	39	30	7	44	11	63
22	23	14	18	13	34	62	11
14	16	18	130	90	163	208	1
70	16	101	52	208	95	359	320
41	5	4	502	181	12	3	68
225	182	33	21	106	66	31	120
310	62	23	191	24			

In real data analysis, one of the crucial points is the selection of coefficient a in the MAIC formula. Therefore, the parameter estimates based on real data analysis and the size of the data should be examined and the coefficient a should be decided according to the appropriate parameter values and appropriate data size. In this regard, Case 3 in the simulation study is the closest to the parameter estimates in Table 8, and considering that the sample size is 213, it is determined as a=0.2 according to Table 6. Table 9 provides the selection criteria statistics for successive failure data. Figs. 1-2 illustrates fitted cdfs and pdfs for successive failures data set respectively.

Table 8: Parameter estimates (standard errors) for successive failures data

Distribution	$\hat{\theta}$	$\hat{\lambda}_i$
TE	0.0081 (0.0013)	0.4998(0.2274)
CTE	0.0106(0.0013)	0.3771(0.2705) -0.6261(0.3153)
QTE	0.0088(0.0016)	0.2252(0.5273) 0.89009(1.3312) -0.84355(1.2006)

Table 9: Selection criterion for successive failures data

Distribution	-2log	AIC	MAIC	BIC	KS	p-value
TE	2352.5	**2356.5**	2353.3	**2363.2**	0.0428	0.8289
CTE	2356.8	2362.8	2358.0	2372.9	0.0726	0.2102
QTE	**2350.6**	2358.6	**2352.2**	2372.0	**0.0398**	**0.8874**

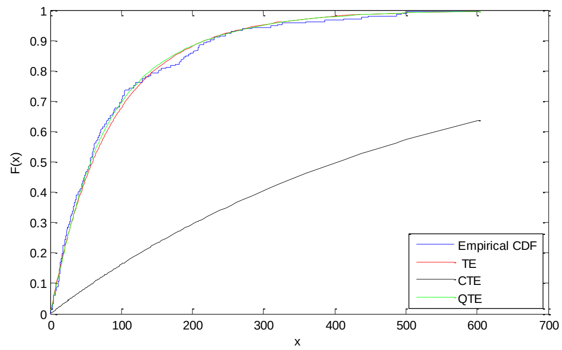

Figure 1. Fitted cdfs for successive failures data

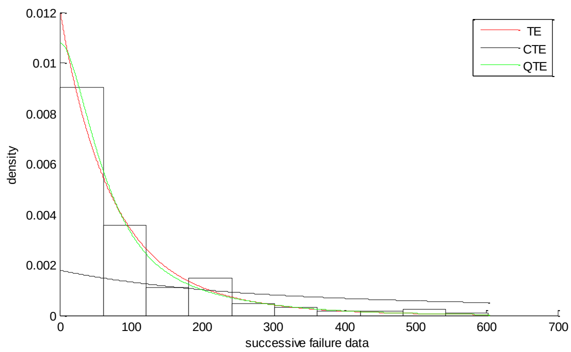

Figure 2. Fitted pdfs for successive failures data

From Table 9, TE distribution is best fitted model in real data analysis according to AIC and BIC while QTE is best fitted distribution in modelling successive failures data according to -2log, MAIC, K-S and its p-value. Thus, it can be concluded that -2log, K-S and its p-value support MAIC for successive failure data. However, AIC has a different results of selection. When it is examined Figs 1-2, it is seen that the fits of QTE and TE distributions to the data is quite close. However, Kolmogorov-Smirnov test statistics and p value give a very important support to the decision of MAIC for fitting successive failures data. In this case, it can be said that the real data analysis results are supported by the results of the simulation study.

6. Conclusion

We propose a new selection criterion as an alternative AIC. This new criterion is derived by modifying AIC. A simulation study is carried out to examine the performances of the correct classification and to compare the performances of AIC and MAIC for the transmuted family of distributions. It is clearly seen that MAIC is better than AIC in correct classification for the transmuted family of distributions based on the exponential distributions (TE, CTE, QTE) according to Tables 2-6. Although AIC is a well-known selection criterion, it gives bad results in the simulation study for general transmuted families based on exponential distribution. According to the results of the real data application, the decision of MAIC differed from AIC for successive failures data. It is confirmed that the MAIC made the right decision by Kolmogorov-Smirnov test statistics and its p value. Thus, simulation and real data results support each other. We recommend using MAIC in the studies about the transmuted family.

7. References

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