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RESEARCH ARTICLE

ON ESTIMATING PARAMETERS OF LINDLEY-GEOMETRIC DISTRIBUTION

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ABSTRACT

Lindley-geometric (LG) distribution is a mixture of Lindley and geometric distribution. We tackle the problem of estimation parameters for the LG distribution. For this purpose maximum likelihood, least-squares, weighted least-squares, Anderson-Darling, and Crámer–von-Mises methods are used to estimate the two parameters of the LG distribution. We also consider an extensive Monte Carlo simulation study to evaluate these methods according to the biases and mean-squared errors (MSEs). Finally, eight real data applications are presented.

Keywords: Point estimation, Geometric distribution, Lindley distribution, Lindley-geometric distribution, Monte Carlo simulation

1. INTRODUCTION

Statistical distributions are crucial in various lifetime modeling. The parameter(s) of the distribution provides information about the shape, skewness, or kurtosis of the distribution. It is necessary to estimate the parameter as accurately as possible to obtain efficient information about the distribution. The maximum likelihood method is generally preferred for parameter estimation. However, some methods of estimation are alternative to maximum likelihood method have been used. Many authors or researchers have studied the comparison of different estimation methods for various distributions. It is well-known that it is very important to support these studies with simulations and real data applications. Some of such studies in the literature can be listed as follows: Mazucheli et al. [1] compared different methods of parameters for the weighted Lindley distribution through Monte Carlo simulations. Gupta and Singh [2] discussed the estimation of the parameter of Lindley distribution under hybrid censoring. Singh et al. [3] considered Bayes estimation of parameters for Lindley distribution. Al-Zahrani and Gindwan [4] studied about two parameters Lindley distribution under hybrid censoring. Santo and Mazucheli [5] discussed the comparison of methods of estimation for Marshall-Olkin extended Lindley distribution. They considered six different methods to estimate the parameters of Marshall-Olkin extended Lindley distribution such as maximum likelihood, maximum product of spacing method, least-squares, weighted least-squares, Cramér–von Mises, and Anderson–Darling methods in this study. Generalized Lindley distribution which is a generalization of Lindley distribution was examined by Gui and Chen [6] in terms of joint confidence regions of its parameters.

The main purpose of this paper is to compare five methods (maximum likelihood, least-squares, weighted least-squares, Anderson-Darling, and Crámer–von-Mises) of estimation for Lindley-geometric distribution which is a compound a Lindley distribution and geometric distribution. The rest of this paper is organized as follows: In Section 2, Lindley-geometric distribution and a relevant literature review are given. Five methods of estimation are described in Section 3. A Monte Carlo simulation study is conducted to assess the performances of these estimators according to bias and MSEs criteria in Section 4. Section 5 provides eight real data applications. Lastly, conclusions are presented in Section 6.

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2. LINDLEY-GEOMETRIC (LG) DISTRIBUTION

LG distribution is suggested by Zakerzadeh and Mahmoudi [7]. The probability density function (pdf) and cumulative distribution function (cdf) are given by

$$f(x)=\frac{\theta^2}{\theta+1}(1-p)(1+x)e^{-\theta x}\left[1-p\left(1+\frac{\theta x}{\theta+1}\right)e^{-\theta x}\right]^{-2} \tag{1}$$

where $\theta>0, p\in(0,1)$ and $x>0$. The LG distribution reduces the Lindley distribution when $p=0$ in (1). Some authors have generated new models based on modifications of the LG distribution. Liyanage and Pararai [8] investigated the Lindley power series class of distributions. They described special cases of Lindley power series including LG, Lindley binomial, Lindley Poisson, and Lindley logarithmic (LL) distributions. They examined some properties of these models. Besides, they performed simulations and applications based on LL, including also their properties in [8]. Merovci and Elbatal [9] suggested a new distribution called transmuted Lindley geometric distribution. Diab and Muhammed [10] produced a new extension of LG distribution called Quasi Lindley geometric distribution. Gui and Zhang [11] proposed the complementary Lindley geometric distribution as an alternative LG and Lindley distribution. Elbatal and Khalil [12] introduced a new three parameters LG distribution using pdf (1).

3. METHODS OF ESTIMATION

In this section, five estimation methods are examined for estimating the unknown parameters of LG distribution. We study the maximum likelihood, least-squares, weighted least squares, Anderson-Darling, and Crámer–von-Mises methods of estimation.

Let $X_1, X_2, \ldots, X_n$ be a random sample from the LG distribution. $X_{(1)}<X_{(2)}<\ldots<X_{(n)}$ symbolize the corresponding order statistics. Also, $x_{(i)}$ indicate the observed value of $X_{(i)}$ for $i=1,2,\ldots,n$. Then, the likelihood and log-likelihood function of the LG distribution are given, respectively, by

$$L(\boldsymbol{\Theta})=\prod_{i=1}^{n}\frac{\theta^2}{\theta+1}(1-p)(1+x_i)\exp(-\theta x_i)\left\{1-p\left(1+\frac{\theta x_i}{\theta+1}\right)\exp(-\theta x_i)\right\}^{-2} \tag{2}$$

and

$$\ell(\boldsymbol{\Theta})=2n\log(\theta)-n\log(\theta+1)+n\log(1-p)\sum_{i=1}^{n}\log(1-x_i)-\theta\sum_{i=1}^{n}x_i-2\sum_{i=1}^{n}\log\left\{1-p\left(1+\frac{\theta x_i}{\theta+1}\right)\exp(-\theta x_i)\right\} \tag{3}$$

, where $\boldsymbol{\Theta}=(\theta,p)$.

Then, maximum likelihood estimator (MLE) of $\boldsymbol{\Theta}$ is given by

$$\boldsymbol{\Theta}_1=\underset{\boldsymbol{\Theta}}{\arg\max}\{\ell(\boldsymbol{\Theta})\}. \tag{4}$$

Let us define the following functions which are used to obtain the least-squares, weighted least squares, Anderson-Darling and Crámer–von-Mises estimators:

$$Q_{LS}(\boldsymbol{\Theta})=\sum_{i=1}^{n}\left(\left\{\frac{1-\left(1+\frac{\theta x_{(i)}}{\theta+1}\right)\exp\left(-\theta x_{(i)}\right)}{1-p\left(1+\frac{\theta x_{(i)}}{\theta+1}\right)\exp\left(-\theta x_{(i)}\right)}\right\}-\frac{i}{n+1}\right)^{2}$$

$$Q_{WLS}(\boldsymbol{\Theta})=\sum_{i=1}^{n}\frac{(n+2)(n+1)^{2}}{i(n-i+1)}\left(\left\{\frac{1-\left(1+\frac{\theta x_{(i)}}{\theta+1}\right)\exp\left(-\theta x_{(i)}\right)}{1-p\left(1+\frac{\theta x_{(i)}}{\theta+1}\right)\exp\left(-\theta x_{(i)}\right)}\right\}-\frac{i}{n+1}\right)^{2}$$

$$Q_{AD}(\theta)=-n-\frac{1}{n}\sum_{i=1}^{n}(2i-1)\times\left\{\log\left(\left\{\frac{1-\left(1+\frac{\theta x_{(i)}}{\theta+1}\right)\exp\left(-\theta x_{(i)}\right)}{1-p\left(1+\frac{\theta x_{(i)}}{\theta+1}\right)\exp\left(-\theta x_{(i)}\right)}\right\}\right)+\log\left(1-\left\{\frac{1-\left(1+\frac{\theta x_{(i)}}{\theta+1}\right)\exp\left(-\theta x_{(i)}\right)}{1-p\left(1+\frac{\theta x_{(i)}}{\theta+1}\right)\exp\left(-\theta x_{(i)}\right)}\right\}\right)\right\}$$

and

$$Q_{CvM}(\boldsymbol{\Theta})=\frac{1}{12n}+\sum_{i=1}^{n}\left(\left\{\frac{1-\left(1+\frac{\theta x_{(i)}}{\theta+1}\right)\exp\left(-\theta x_{(i)}\right)}{1-p\left(1+\frac{\theta x_{(i)}}{\theta+1}\right)\exp\left(-\theta x_{(i)}\right)}\right\}-\frac{2i-1}{2n}\right)^{2}.$$

Then, least square estimator (LSE), weighted least square estimator (WLSE), Anderson-Darling estimator (ADE) and Crámer–von-Mises estimator (CvME) of the parameter vector $\boldsymbol{\Theta}$ are given, respectively by

$$\hat{\boldsymbol{\Theta}}_2=\underset{\boldsymbol{\Theta}}{\arg\min}\left\{Q_{LS}(\boldsymbol{\Theta})\right\}, \quad (5)$$

$$\hat{\boldsymbol{\Theta}}_3=\underset{\boldsymbol{\Theta}}{\arg\min}\left\{Q_{WLS}(\boldsymbol{\Theta})\right\}, \quad (6)$$

$$\hat{\boldsymbol{\Theta}}_4=\underset{\boldsymbol{\Theta}}{\arg\min}\left\{Q_{AD}(\boldsymbol{\Theta})\right\}, \quad (7)$$

$$\hat{\boldsymbol{\Theta}}_5=\underset{\boldsymbol{\Theta}}{\arg\min}\left\{Q_{CVM}(\boldsymbol{\Theta})\right\}, \quad (8)$$

Five estimates given in (4)-(8) can be obtained by optim function included in the stats package in R with Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm which is known as a quasi-Newton method. The BFGS algorithm was firstly studied by Fletcher [13].

4. SIMULATION STUDY

In this section, 5000 trials are conducted to estimate the biases and MSEs of all estimators. Four true parameter settings are considered as follows: $\boldsymbol{\Theta}=(0.5,0.3),(0.75,0.9),(1.25,0.5),(1.5,0.7)$. The sample sizes are selected as $n=(25,50,100,150,200,250)$. The acceptance-rejection algorithm is used to generate data from $\mathrm{LG}(\boldsymbol{\Theta})$ distribution. The BFGS algorithm which is available in R is used to obtain five estimates given in (4)-(8). In Tables 1-2, biases and MSEs of MLE, LSE, WLSE, ADE, and CVME are reported. Tables 1-2 indicate that the biases and MSEs of five estimators are close to zero when the sample size increases as expected. From the simulation results, if one will use LG distribution in real data modeling, one can choose any estimation methods.

Table 1. Average biases of all estimators

		$\boldsymbol{\Theta}_1$		$\boldsymbol{\Theta}_2$		$\boldsymbol{\Theta}_3$		$\boldsymbol{\Theta}_4$		$\boldsymbol{\Theta}_5$	
$\boldsymbol{\Theta}$	n	θ	p	θ	p	θ	p	θ	p	θ	p
(0.5,0.3)	25	-0.0291	0.1437	-0.1396	0.2557	-0.1276	0.2486	-0.0963	0.2499	-0.0416	0.0206
	50	-0.0140	0.0853	-0.0733	0.1630	-0.0565	0.1483	-0.0469	0.1433	-0.0231	0.0421
	100	-0.0084	0.0504	-0.0377	0.0917	-0.0259	0.0786	-0.0242	0.0800	-0.0129	0.0290
	150	-0.0027	0.0234	-0.0223	0.0539	-0.0134	0.0424	-0.0130	0.0442	-0.0061	0.0112
	200	-0.0001	0.0129	-0.0124	0.0304	-0.0061	0.0220	-0.0065	0.0252	-0.0004	-0.0020
	250	0.0023	0.0020	-0.0094	0.0237	-0.0035	0.0132	-0.0038	0.01526	-0.0002	-0.0022
(0.75,0.9)	25	0.4227	-0.1371	0.2556	-0.1312	0.1970	-0.1116	0.1902	-0.0817	0.5950	-0.2602
	50	0.2563	-0.0827	0.1998	-0.0879	0.1584	-0.0751	0.1430	-0.0606	0.3692	-0.1407
	100	0.1387	-0.0431	0.1347	-0.0541	0.0978	-0.0426	0.0886	-0.0362	0.2259	-0.0790
	150	0.0984	-0.0297	0.0947	-0.0369	0.0682	-0.0281	0.0611	-0.0243	0.1577	-0.0531
	200	0.0688	-0.0212	0.0678	-0.0276	0.0449	-0.0199	0.0361	-0.0164	0.1153	-0.0394
	250	0.0507	-0.0158	0.0564	-0.0231	0.0368	-0.0163	0.0293	-0.0135	0.0961	-0.0328
(1.25,0.5)	25	0.0055	0.0062	-0.2593	0.0853	-0.2341	0.0856	-0.1759	0.0952	0.0182	-0.1122
	50	0.0218	-0.0185	-0.1403	0.0490	-0.0950	0.0359	-0.0753	0.0332	0.0008	-0.0472
	100	0.0338	-0.0330	-0.0403	-0.0033	-0.0086	-0.0141	-0.0079	-0.0105	0.0302	-0.0531
	150	0.0304	-0.0329	-0.0180	-0.0130	0.0039	-0.0206	0.0020	-0.0174	0.0286	-0.0462
	200	0.0313	-0.0306	-0.0035	-0.0162	0.0136	-0.0225	0.0117	-0.0199	0.0313	-0.0410
	250	0.0257	-0.0272	0.0020	-0.0183	0.0149	-0.0230	0.0126	-0.0205	0.0297	-0.0381
(1.5,0.7)	25	0.2146	-0.0911	-0.1559	-0.0256	-0.1399	-0.0210	-0.0751	-0.0109	0.2602	-0.1875
	50	0.1564	-0.0774	-0.0449	-0.0404	-0.0071	-0.0428	0.0055	-0.0370	0.1715	-0.1189
	100	0.1039	-0.0509	-0.0136	-0.0282	0.0280	-0.0343	0.0283	-0.0312	0.0985	-0.0661
	150	0.0682	-0.0340	-0.0061	-0.0205	0.0269	-0.0253	0.0228	-0.0227	0.0693	-0.0455
	200	0.0604	-0.0286	-0.0072	-0.0151	0.0241	-0.0205	0.0188	-0.0178	0.0494	-0.0337
	250	0.0531	-0.0248	0.0003	-0.0147	0.0283	-0.0199	0.0233	-0.0176	0.0456	-0.0296

Table 2. Average MSEs of all estimators

		Θ_1		Θ_2		Θ_3		Θ_4		Θ_5	
Θ	n	θ	p	θ	p	θ	p	θ	p	θ	p
(0.5,0.3)	25	0.0155	0.0782	0.0693	0.4962	0.0578	0.6842	0.0340	0.1709	0.0668	0.7935
	50	0.0092	0.0548	0.0344	0.2103	0.0233	0.1539	0.0179	0.1041	0.0321	0.2488
	100	0.0056	0.0392	0.0174	0.1140	0.0109	0.0779	0.0096	0.0646	0.0166	0.1219
	150	0.0043	0.0314	0.0116	0.0822	0.0073	0.0544	0.0067	0.0480	0.0112	0.0867
	200	0.0033	0.0253	0.0085	0.0629	0.0053	0.0420	0.0050	0.0381	0.0084	0.0661
	250	0.0029	0.0243	0.0068	0.0518	0.0044	0.0359	0.0042	0.0340	0.0068	0.0538
(0.75,0.9)	25	0.6821	0.0655	1.2410	0.2586	1.0630	0.2213	0.6216	0.0615	2.1034	0.5247
	50	0.3292	0.0307	0.6448	0.0757	0.5444	0.0618	0.3737	0.0348	0.8815	0.1133
	100	0.1419	0.0111	0.3149	0.0287	0.2479	0.0215	0.1934	0.0152	0.3876	0.0374
	150	0.0926	0.0064	0.2079	0.0164	0.1554	0.0114	0.1283	0.0088	0.2430	0.0201
	200	0.0637	0.0041	0.1618	0.0115	0.1153	0.0077	0.0969	0.0061	0.1828	0.0136
	250	0.0481	0.0030	0.1313	0.0090	0.0906	0.0058	0.0779	0.0048	0.1456	0.0104
(1.25,0.5)	25	0.1580	0.0601	0.5947	0.4159	0.4908	0.3695	0.2725	0.1010	0.6709	0.7187
	50	0.0983	0.0489	0.2899	0.1361	0.2032	0.1003	0.1609	0.0721	0.2962	0.1763
	100	0.0650	0.0378	0.1561	0.0834	0.1054	0.0604	0.0927	0.0502	0.1592	0.0970
	150	0.0456	0.0295	0.1080	0.0597	0.0710	0.0432	0.0659	0.0389	0.1100	0.0665
	200	0.0379	0.0246	0.0835	0.0468	0.0560	0.0343	0.0528	0.0317	0.0852	0.0509
	250	0.0311	0.0200	0.0658	0.0374	0.0445	0.0274	0.0426	0.0258	0.0671	0.0402
(1.5,0.7)	25	0.5394	0.0649	1.4047	0.2946	1.2132	0.2475	0.6856	0.0751	1.9377	0.5644
	50	0.3362	0.0505	0.7723	0.1148	0.5975	0.0897	0.4576	0.0607	0.9083	0.1643
	100	0.1847	0.0280	0.4098	0.0547	0.2885	0.0413	0.2544	0.0347	0.4374	0.0655
	150	0.1186	0.0179	0.2813	0.0348	0.1861	0.0254	0.1717	0.0229	0.2924	0.0394
	200	0.0908	0.0131	0.2161	0.0272	0.1403	0.0193	0.1307	0.0173	0.2212	0.0297
	250	0.0717	0.0100	0.1761	0.0212	0.1128	0.0149	0.1073	0.0139	0.1798	0.0229

5. REAL DATA APPLICATIONS

In this section, eight real data applications for the LG distribution are applied. LG distribution is fitted to the real data sets estimating the parameter using five methods of estimation. The MLE, LSE, WLSE, ADE, and CVME of the parameters of LG distribution are also obtained by the BFGS algorithm and reported in Table 4. The detailed information on the real data is given in Appendix, also descriptive information about the data is presented in Table 3.

Table 3. Descriptive statistics and their references based on the eight real data sets

	Number of observation	Mean	Standard deviation	Minimum	Maximum	References
Data 1	55	259.1636	258.8003	1	1146	Mudholkar et al. [14]
Data 2	101	1.0248	1.1193	0.01	7.89	Andrews and Herzberg [15]
Data 3	40	4.0125	5.1652	0.5	24.5	Jorgensen [16])
Data 4	15	27.5466	20.7633	1.4	66.2	Lawless [17]
Data 5	50	7.8210	9.2063	0.013	48.105	Murthy et al. [18])
Data 6	34	1.8794	1.9525	0.1	8	Bhaumik [19]
Data 7	116	42.1293	32.9878	1	168	Nadarajah [20], Leiva et al. [21]
Data 8	45	1.3414	1.2466	0.047	4.033	Bekker et al. [22]

Table 4. The parameter estimates of the parameters of LG distribution for all data sets

	Θ_1		Θ_2		Θ_3		Θ_4		Θ_5	
Data set	θ	p	θ	p	θ	p	θ	p	θ	p
1	0.0039	0.8133	0.0038	0.8116	0.0038	0.8112	0.0036	0.8482	0.0038	0.8116
2	1.0886	0.4375	1.2537	0.2068	1.2343	0.2688	1.1035	0.4229	1.3065	0.1407
3	0.1224	0.9390	0.1030	0.9572	0.0604	0.9825	0.0760	0.9749	0.0968	0.9618
4	0.0661	0.1590	0.0308	0.8117	0.0362	0.7525	0.0450	0.6271	0.0420	0.6603
5	0.1026	0.8764	0.0927	0.8837	0.0792	0.9147	0.0919	0.8996	0.1007	0.8649
6	0.5455	0.6348	0.3010	0.8781	0.3670	0.8222	0.4774	0.7149	0.4402	0.7587
7	0.0365	0.4662	0.0252	0.7474	0.0306	0.6254	0.0305	0.6298	0.0264	0.7238
8	0.9136	0.3791	0.4691	0.8196	0.7253	0.6022	0.6858	0.6378	0.5668	0.7476

6. CONCLUSION

In this paper, LG distribution is studied in terms of some point estimations. LG distribution is proposed by Zakerzadeh and Mahmoudi [7]. They only examined the maximum likelihood for point estimation of LG distribution. Unlike their study, we provide five estimators including MLE, LSE, WLSE, ADE, and CvME of parameters of LG distribution. Thus, a new extension is provided for the estimation of the parameters for LG distribution. Monte Carlo simulation studies are performed for different parameter values and different sample sizes. It is concluded that as the sizes of samples increase, the MSEs and biases of all estimators decrease and close to zero. According to the simulation study results, it is seen that any of the five methods of estimation examined in this paper can be used in modeling real data. Also, we analyze eight real data set to illustrate the usefulness of LG distribution. The parameter estimates of LG distribution are also obtained using five different estimation methods for these practical data sets.

CONFLICT OF INTEREST

The authors stated that there are no conflicts of interest regarding the publication of this article.

REFERENCES

[1] Mazucheli J, Louzada F, Ghitany, ME. Comparison of estimation methods for the parameters of the weighted Lindley distribution. Applied Mathematics and Computation 2013; 220: 463-471.

[2] Gupta PK, Singh, B. Parameter estimation of Lindley distribution with hybrid censored data. International Journal of System Assurance Engineering and Management 2013; 4: 378-385.

[3] Singh SK, Singh U, Sharma VK. Bayesian estimation and prediction for the generalized Lindley distribution under asymmetric loss function. Hacettepe Journal of Mathematics and Statistics 2014; 43: 661-678.

[4] Al-Zahrani B, Gindwan M. Parameter estimation of a two-parameter Lindley distribution under hybrid censoring. International Journal of System Assurance Engineering and Management 2014; 5: 628-636.

[5] Santo APJ, Mazucheli J. Comparison of estimation methods for the Marshall–Olkin extended Lindley distribution. Journal of Statistical Computation and Simulation 2015; 85: 3437-3450.

[6] Gui W, Chen M. Parameter estimation and joint confidence regions for the parameters of the generalized Lindley distribution. Mathematical Problems in Engineering 2016; Avaible at http://dx.doi.org/10.1155/2016/7946828.

[7] Zakerzadeh H, Mahmoudi E. A new two parameter lifetime distribution: model and properties. 2012; arXiv preprint arXiv:1204.4248.

[8] Warahena-Liyanage G, Pararai M. The Lindley power series class of distributions: Model, properties and applications. Journal of Computations and Modelling 2015; 5: 35-80.

[9] Merovci F. Elbatal I. Transmuted Lindley-geometric distribution and its applications. Journal of Statistics Applications and Probability 2013; 3: 77-91.

[10] Diab LS, Muhammed HZ. Quasi Lindley Geometric Distribution. International Journal of Computer Applications 2014; 95: 9-16.

[11] Gui W, Zhang H, Guo L. The Complementary Lindley-Geometric Distribution and Its Application in Lifetime Analysis. Sankhya B 2017; 79: 316-335.

[12] Elbatal I, Khalil MG. A New Extension of Lindley Geometric Distribution and its Applications. Pakistan Journal of Statistics and Operation Research 2019; 15: 249-263.

[13] Fletcher R. Practical methods of optimization. John and Sons, Chichester 1987.

[14] Mudholkar GS, Srivastava DK, Freimer M. The exponentiated Weibull family: A reanalysis of the bus-motor-failure data. Technometrics 1995; 37: 436-445.

[15] Andrews DF, Herzberg AM. Data: A Collection of Problems from Many Fields for the Student and Research Worker 1985; New York: Springer Series in Statistics.

[16] Jorgensen B. Statistical Properties of the Generalized Inverse Gaussian distribution. 2012; 9: Springer Science and Business Media.

[17] Lawless JF. Statistical Models and Methods for Lifetime Data. 2003; Wiley: New York.

[18] Murthy D, Xie M, Jiang R. Weibull models. John Wiley and Sons 2004; 505: 180.

[19] Bhaumik DK, Kapur KG, Gibbons RD. Testing parameters of a gamma distribution for small samples. Technometrics 2009; 51: 326-334.

[20] Nadarajah S. A truncated inverted beta distribution with application to air pollution data. Stochastic Environmental Research and Risk 2008; 22: 285–289.

[21] Leiva V, Vilca F, Balakrishnan N, Sanhueza A. A skewed sinh-normal distribution and its properties and application to air pollution. Communications in Statistics-Theory and Methods 2010; 39: 426–443.

[22] Bekker A, Roux JJJ, Mosteit PJ. A generalization of the compound Rayleigh distribution: using a Bayesian method on cancer survival times. Communications in Statistics-Theory and Methods 2000; 29: 1419-1433.

APPENDIX

The eight real data sets used in the real data applications are given in detail below:

Data Set 1: 7, 34, 42, 63, 64, 74, 83, 84, 91, 108, 112, 129, 133, 133, 139, 140, 140, 146, 149, 154, 157, 160, 160, 165, 173, 176, 185, 218, 225, 241, 248, 273, 277, 279, 297, 319, 405, 417, 420, 440, 523, 523, 583, 594, 1101, 1116, 1146, 1, 226, 1, 349, 1, 412,1, 417.

Data Set 2: 0.01, 0.01, 0.02, 0.02, 0.02, 0.03, 0.03, 0.04, 0.05, 0.06, 0.07, 0.07, 0.08, 0.09, 0.09, 0.10, 0.10, 0.11, 0.11, 0.12, 0.13, 0.18, 0.19, 0.20, 0.23, 0.24, 0.24, 0.29, 0.34, 0.35, 0.36, 0.38, 0.40, 0.42, 0.43, 0.52, 0.54, 0.56, 0.60, 0.60, 0.63, 0.65, 0.67, 0.68, 0.72, 0.72, 0.72, 0.73, 0.79, 0.79, 0.80, 0.80, 0.83, 0.85, 0.90, 0.92, 0.95, 0.99, 1.00, 1.01, 1.02, 1.03, 1.05, 1.10, 1.10, 1.11, 1.15, 1.18, 1.20, 1.29, 1.31, 1.33, 1.34, 1.40, 1.43, 1.45, 1.50, 1.51, 1.52, 1.53, 1.54, 1.54, 1.55, 1.58, 1.60, 1.63, 1.64, 1.80, 1.80, 1.81, 2.02, 2.05, 2.14, 2.17, 2.33, 3.03, 3.03, 3.34, 4.20, 4.69, 7.89.

Data Set 3: 0.50, 0.60, 0.60, 0.70, 0.70, 0.70, 0.80, 0.80, 1.00, 1.00, 1.00, 1.00, 1.10, 1.30, 1.50, 1.50, 1.50, 1.50, 2.00, 2.00, 2.20, 2.50, 2.70, 3.00, 3.00, 3.30, 4.00, 4.00, 4.50, 4.70, 5.00, 5.40, 5.40, 7.00, 7.50, 8.80, 9.00, 10.20, 22.00, 24.50.

Data Set 4: 1.4, 5.1, 6.3, 10.8, 12.1, 18.5, 19.7, 22.2, 23, 30.6, 37.3, 46.3, 53.9, 59.8, 66.2.

Data Set 5: 0.013, 0.065, 0.111, 0.111, 0.163, 0.309, 0.426, 0.535, 0.684, 0.747, 0.997, 1.284, 1.304, 1.647, 1.829, 2.336, 2.838, 3.269, 3.977, 3.981, 4.520, 4.789, 4.849, 5.202, 5.291, 5.349, 5.911, 6.018, 6.427, 6.456, 6.572, 7.023, 7.087, 7.291, 7.787, 8.596, 9.388, 10.261, 10.713, 11.658, 13.006, 13.388, 13.842, 17.152, 17.283, 19.418, 23.471, 24.777, 32.795, 48.105.

Data Set 6: 5.1, 1.2, 1.3, 0.6, 0.5, 2.4, 0.5, 1.1, 8.0, 0.8, 0.4, 0.6, 0.9, 0.4, 2.0, 0.5, 5.3, 3.2, 2.7, 2.9, 2.5, 2.3, 1.0, 0.2, 0.1, 0.1, 1.8, 0.9, 2.0, 4.0, 6.8, 1.2, 0.4,0.2.

Data Set 7: 41, 36, 12, 18, 28, 23, 19, 8, 7, 16, 11, 14, 18, 14, 34, 6, 30, 11, 1, 11, 4, 32, 23, 45, 115, 37, 29, 71, 39, 23, 21, 37, 20, 12, 13, 135, 49, 32, 64, 40, 77, 97, 97, 85, 10, 27, 7, 48, 35, 61, 79, 63, 16, 80, 108, 20, 52, 82, 50, 64, 59, 39, 9, 16, 78, 35, 66, 122, 89, 110, 44, 28, 65, 22, 59, 23, 31, 44, 21, 9, 45, 168, 73, 76, 118, 84, 85, 96, 78, 73, 91, 47, 32, 20, 23, 21, 24, 44, 21, 28, 9, 13, 46, 18, 13, 24, 16, 13, 23, 36, 7, 14, 30, 14, 18, 20.

Data Set 8: 0.047, 0.115, 0.121, 0.132, 0.164, 0.197, 0.203, 0.260, 0.282, 0.296, 0.334, 0.395, 0.458, 0.466, 0.501, 0.507, 0.529, 0.534, 0.540, 0.641, 0.644, 0.696, 0.841, 0.863, 1.099, 1.219, 1.271, 1.326, 1.447, 1.485, 1.553, 1.581, 1.589, 2.178, 2.343, 2.416, 2.444, 2.825, 2.830, 3.578, 3.658, 3.743, 3.978, 4.003, 4.033.