

## Determination of Moment Capacity $M_p$ for Rectangular Reinforced Concrete Columns<sup>†</sup>

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### ABSTRACT

*Turkish Seismic Code stipulates capacity design procedure for the calculation of design shear force of RC beams and columns. Therefore, the plastic hinge moment capacity ( $M_p$ ) at the ends of columns must be determined as accurately as possible. Turkish Seismic Design Code recommends an increase of 40% in ultimate moment capacity ( $M_r$ ) to obtain the plastic hinge moment capacity ( $M_p$ ), unless a detailed calculation is carried out. However, this recommendation remains insufficient in attaining plastic hinge moment capacity for high levels of axial load.*

*In this study, new analytical equations have been derived to obtain the plastic hinge moment capacity of rectangular RC columns in an accurate and practical way. Plastic hinge moment capacity of columns calculated with the proposed equations is compared to the capacity obtained from experimental results. It is shown that the proposed equations yield accurate results.*

**Keywords:** Column moment capacity, M-N interaction diagrams, strain hardening, confined concrete

### 1. INTRODUCTION

In line with the requirement for capacity design procedure in the Turkish Seismic Design Code, the design shear force of R/C columns should be calculated based on the flexural moment capacities at the ends of columns (Figure 1). Therefore, the plastic hinge moment capacity ( $M_p$ ) at the end of the column must be determined as accurately as possible.

The moment capacity of a R/C member under pure bending or combined bending and axial force can be calculated taking into consideration different assumptions. Three different definitions of moment capacities are summarized below.

#### **$M_r$ , Ultimate Moment Capacity (TS500) [1]**

Ultimate moment capacity is obtained using design strengths of materials ( $f_{cd} = f_{ck} / \gamma_c$ ,  $f_{yd} = f_{yk} / \gamma_s$ ) with the assumptions given by TS500 [1]. The assumptions for

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calculation of ultimate strength can be summarized as: Plane sections remain plane after bending, tensile strength of concrete is neglected, steel is an elasto-plastic material, stress distribution at the compression zone is expressed as an equivalent rectangular stress block and maximum strain in the extreme fiber of concrete in compression is 0.003.

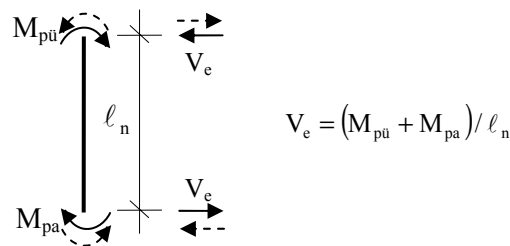
#### **$M_n$ , Nominal Moment Capacity (ACI-318) [2]**

Nominal moment capacity given by ACI-318 [2] is obtained with characteristic material strengths. In design process nominal moment capacity is reduced with a strength reduction factor ( $\phi$ ).

#### **$M_p$ , Flexural Moment Capacity**

For a flexure dominant structural element, flexural moment capacity calculated based on code assumptions ( $M_r$ ,  $M_n$ ), is always less than the actual capacity ( $M_p$ ) since strain hardening of reinforcing steel is neglected and confinement effect of concrete is overlooked and material strengths are reduced with partial safety factors.

The maximum flexural moment capacity ( $M_p$ ) can be defined as the maximum moment obtained from moment-curvature analyses considering strain hardening of steel, crushing of cover concrete, tensile strength of concrete and stress-strain relationship of confined concrete. However; calculation of  $M_p$  through moment-curvature analysis during the design process is not always practical and useful.



*Figure 1: Bending moment values of columns for based on capacity design procedure in the Turkish Seismic Design Code [3, 4]*

Moment curvature relations of a sample column section, under various axial load levels, are shown in Fig. 2. In Fig. 3,  $M_p$  moment capacities of sample column sections with corresponding axial load levels are given as an interaction diagram (PEMKED) [5].

Modified Kent-Park model for confined and unconfined concrete under compression, a strain hardening material model for reinforcement steel, a material model considering tensile strength of concrete are used to obtain moment curvature relations as well as a fiber model that is used for geometrical definition of the section [5]. Maximum stress, strain corresponding to maximum stress and effective maximum strain for unconfined concrete are assumed to be  $0.85 f_{ck}$ , 0.002 and 0.004, respectively [6].

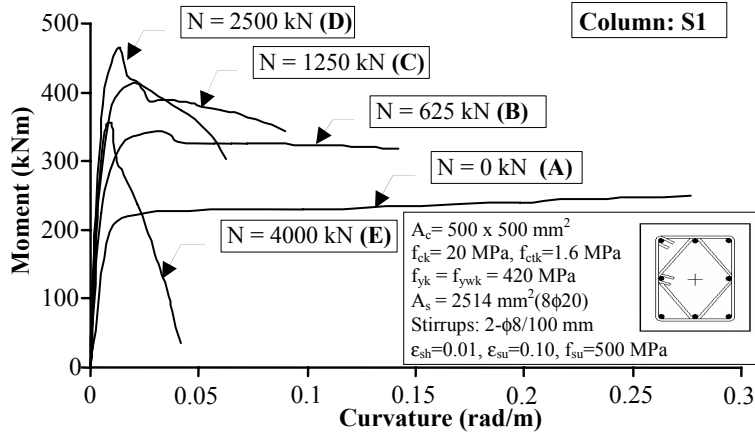


Figure 2: Moment curvature relations for various axial load levels

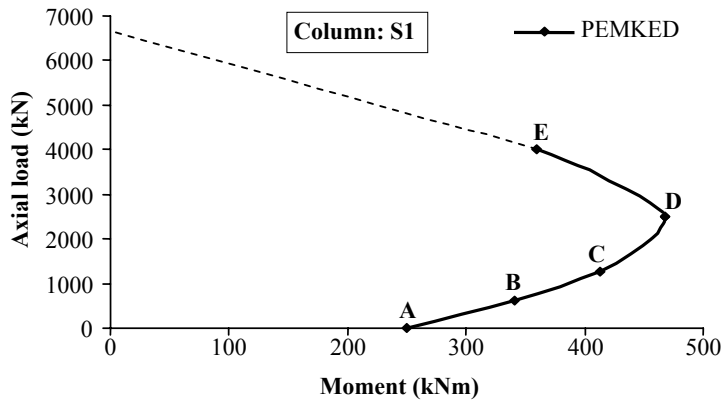


Figure 3:  $M_p$  moment capacities with corresponding axial load levels [5]

In Turkish Seismic Design Code, it is designated that maximum bending moment capacity  $M_p$  can be taken as  $1.4M_r$  [3, 4], unless detailed calculations are performed. Although, the value of  $M_p/M_r$  ratio is given as 1.4, regardless of column axial load level, this ratio is influenced by many design variables primarily the column axial load level [5, 6]. Due to this fact,  $M_p/M_r \approx 1.4$  assumption is sometimes unrealistic with increasing axial load [5, 6].

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Ersoy has carried out analytic studies on obtaining column moment capacities  $M_p$ , with a more realistic approach than  $1.4M_r$  approach of TSDC [6]. Ersoy showed that confinement effect in concrete is more effective than strain hardening of reinforcement steel on column moment capacities  $M_p$ , and recommended use of  $f_{yk}$  and  $f_{cc}$  ( $f_{cc}=1.15f_{ck}$ ) instead of  $f_{yd}$  and  $f_{cd}$ . This approach yields more accurate results for increasing levels of axial load, however; it underestimates the moment capacity for lower levels of axial load since the effect of strain hardening on moment capacity can not be considered in this approach [5].

In the current study, more realistic and practical approaches to obtain column moment capacities  $M_p$ , are proposed along with comparisons with experimental results.

## 2. EXPERIMENTAL COMPARISONS

In this section; test results of columns subjected to constant axial load and cyclic flexure simulating earthquake loading, found in the literature, have been considered.  $M_p$  moment capacities obtained from test elements ( $M_{p,experimental}=V_{max}L$ ), are compared with the moment capacities calculated based on the approximate methods mentioned above.

Schematical views of test mechanisms, typical column sections and general properties of test elements used in comparisons are summarized in Fig. 4, Table 1 and Table 2, respectively.

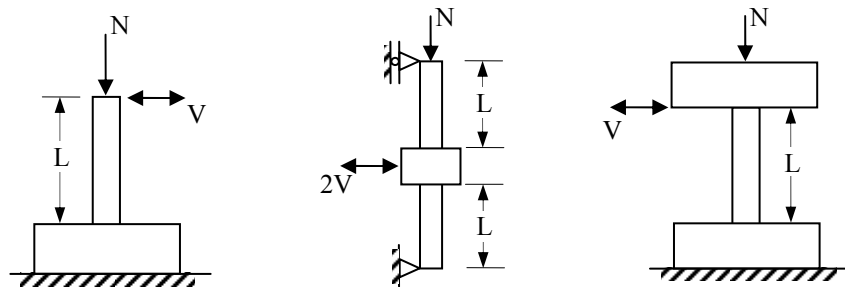


Figure 4: Schematical view of test mechanisms

Table 1: Typical column sections of test columns

Typical column section						
1	2	3	4	5	6	7

*Table 2: General properties of test columns*

<b>No</b>	<b>Reference/ Element</b>	<b>Type</b>	<b>b/h (mm/mm)</b>	<b><math>\rho_t</math></b>	<b><math>f_{ck} / f_{yk} / f_{su}</math> (MPa)</b>	<b><math>f_{ywk}</math> (MPa)</b>	<b>Stirrups <math>\emptyset/s</math>(mm)</b>	<b>n</b>
1	[7]/No1	1	550/550	0.0179	23.1/375/635.6	297	$\emptyset 10/80$	0.26
2	[7]/No2	1	550/550	0.0179	41.4/375/635.6	316	$\emptyset 12/75$	0.214
3	[7]/No3	2	550/550	0.0179	21.4/375/635.6	297	$\emptyset 10/75$	0.42
4	[7]/No4	2	550/550	0.0179	23.5/375/635.6	294	$\emptyset 12/62$	0.60
5	[8]/No3	2	400/400	0.0151	23.6/427/670	320	$\emptyset 12/100$	0.38
6	[8]/No4	2	400/400	0.0151	25/427/670	280	$\emptyset 10/90$	0.21
7	[9]/No1	1	400/400	0.0151	46.5/446/702	364	$\emptyset 7/85$	0.10
8	[9]/No2	1	400/400	0.0151	44/446/702	360	$\emptyset 8/78$	0.30
9	[9]/No3	1	400/400	0.0151	44/446/702	364	$\emptyset 7/91$	0.30
10	[9]/No4	1	400/400	0.0151	40/446/702	255	$\emptyset 6/95$	0.30
11	[10]/No9	3	400/600	0.0188	26.9/432*	305	$\emptyset 12/80$	0.10
12	[11]/No2	4	400/400	0.0157	25.6/474/721	333	$\emptyset 12/80$	0.20
13	[11]/No4	4	400/400	0.0157	25.6/474/721	333	$\emptyset 12/80$	0.20
14	[11]/No5	2	550/550	0.0125	32/511/675	325	$\emptyset 12/110$	0.10
15	[11]/No6	2	550/550	0.0125	32/511/675	325	$\emptyset 12/110$	0.10
16	[12]/85STC-1	5	250/250	0.0162	27.9/374/494	506	$\emptyset 5.5/50$	0.106
17	[12]/85STC-2	5	250/250	0.0162	27.9/374/494	506	$\emptyset 5.5/50$	0.106
18	[12]/85STC-3	5	250/250	0.0162	27.9/374/494	506	$\emptyset 5.5/50$	0.106
19	[12]/85PDC-1	5	250/250	0.0162	24.8/374/494	352	$\emptyset 5.5/50$	0.106

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*Table 2: General properties of test columns (continued)*

No	Reference/ Element	Type	b/h (mm/mm)	$\rho_t$	$f_{ck} / f_{yk} / f_{su}$ (MPa)	$f_{ywk}$ (MPa)	Stirrups $\phi/s$ (mm)	n
20	[12]/85PDC-2	5	250/250	0.0162	27.9/374/494	506	$\phi 5.5/50$	0.106
21	[12]/85PDC-3	5	250/250	0.0162	27.9/374/494	506	$\phi 5.5/50$	0.106
22	[13]/No. 1S1	6	305/305	0.0163	29.1/367/578	363	$\phi 9.5/76$	0.099
23	[13]/No. 2S1	6	305/305	0.0163	30.7/367/578	363	$\phi 9.5/127$	0.093
24	[13]/No. 5S1	6	305/305	0.0163	29.4/429/657	392	$\phi 9.5/76$	0.195
25	[13]/No. 6S1	6	305/305	0.0163	31.8/429/657	392	$\phi 9.5/127$	0.181
26	[14]/U-1	5	350/350	0.0321	43.6/430*	470	$\phi 10/150$	0.0
27	[14]/U-4	5	350/350	0.0321	32/438*	470	$\phi 10/50$	0.153
28	[14]/U-6	4	350/350	0.0321	37.3/437*	425	$\phi 6.4/65$	0.131
29	[14]/U-7	4	350/350	0.0321	39/437*	425	$\phi 6.4/65$	0.126
30	[15]/C1-1	2	400/400	0.0214	24.9/497/592	459.5	$\phi 6.3/50$	0.113
31	[15]/C1-2	2	400/400	0.0214	26.7/497/592	459.5	$\phi 6.3/50$	0.158
32	[15]/C1-3	2	400/400	0.0214	26.1/497/592	459.5	$\phi 6.3/50$	0.216
33	[15]/C2-1	2	400/400	0.0214	25.3/497/592	459.5	$\phi 6.3/52$	0.111
34	[15]/C2-2	2	400/400	0.0214	27.1/497/592	459.5	$\phi 6.3/52$	0.156
35	[15]/C2-3	2	400/400	0.0214	26.8/497/592	459.5	$\phi 6.3/52$	0.21
36	[15]/C3-1	2	400/400	0.0214	26.4/497/592	459.5	$\phi 6.3/54$	0.107
37	[15]/C3-2	2	400/400	0.0214	27.5/497/592	459.5	$\phi 6.3/54$	0.153

\* Ultimate strength of steel is not given.

Table 2: General properties of test columns (continued)

No	Reference/ Element	Type	b/h (mm/mm)	$\rho_t$	$f_{ck} / f_{yk} / f_{su}$ (MPa)	$f_{ywk}$ (MPa)	Stirrups $\emptyset/s$ (mm)	n
38	[15]/C3-3	2	400/400	0.0214	26.9/497/592	459.5	$\emptyset 6.3/54$	0.209
39	[16]/L1	7	400/400	0.0142	24.8/362/*	325	$\emptyset 9/100$	0.032
40	[16]/L2	7	400/400	0.0142	24.8/362/*	325	$\emptyset 9/100$	0.032
41	[17]/No7	2	400/400	0.0151	28.3/440/674	466	$\emptyset 10/117$	0.223
42	[17]/No8	2	400/400	0.0151	40.1/440/674	466	$\emptyset 10/92$	0.39
43	[18]/No-5	1	400/400	0.0151	41/474/633.3	372	$\emptyset 8/81$	0.50
44	[18]/No-6	1	400/400	0.0151	40/474/633.3	388	$\emptyset 7/96$	0.50
45	[19]/BG-1	4	350/350	0.0195	34/455.6/660	570	$\emptyset 9.5/152$	0.428
46	[19]/BG-2	4	350/350	0.0195	34/455.6/660	570	$\emptyset 9.5/76$	0.428
47	[19]/BG-3	4	350/350	0.0195	34/455.6/660	570	$\emptyset 9.5/76$	0.20
48	[19]/BG-4	2	350/350	0.0293	34/455.6/660	570	$\emptyset 9.5/152$	0.462
49	[20]/c5-40N	6	203/203	0.0193	38.1/572/729.1	513.7	$\emptyset 9.5/76$	0.362
50	[21]/D1N60	2	250/250	0.0243	37.6/461/634.3	485	$\emptyset 4/40$	0.60

\* Ultimate strength of steel is not given.

The ratios of the moment capacity ( $M_{p,experimental}=V_{max}L$ ), obtained from maximum base shear occurring at the ends of test columns with properties given in Table 2 to moment capacity obtained with  $1.4M_r$  assumption given by TSDC and their variations with column axial load level are shown in Fig. 5.

It can be seen from Fig. 5 that, the ratios of  $M_{p,experimental}/1.4M_r$  – in other words – the ratios of maximum shear forces occurring at the ends of elements to shear forces obtained with  $1.4M_r$  assumption ( $V_{max}/V_e$ ), vary between 0.81~2.1 and  $M_{p,experimental}/1.4M_r$  ratio increases for increasing dimensionless axial load level. It is a normal tendency because of ultimate moment ( $M_r$ ) becomes less than the actual capacity and decreases from a lower axial load level (balanced axial load), for increasing dimensionless axial load levels. The increase in  $M_{p,experimental}/1.4M_r$  ratios can be much more than that given above for axial load levels higher than the maximum value considered above. In Fig. 6,  $M_{p,experimental}$  moment

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capacity (actual capacity) is marked for axial load level of  $\approx 0.7$ , on the interaction diagram obtained with  $1.4M_r$  assumption for a column section to explain this behaviour briefly. The ratio of  $M_{p,experimental}/1.4M_r$  is nearly 66 for axial load level  $n \approx 0.7$ . This result is very interesting even it is unusual in practice.

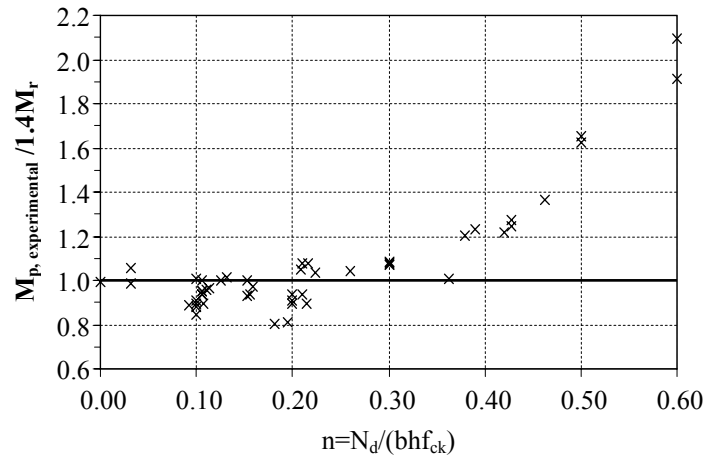


Figure 5: Variation of  $M_{p,experimental}/1.4M_r$  ratio with column axial load level

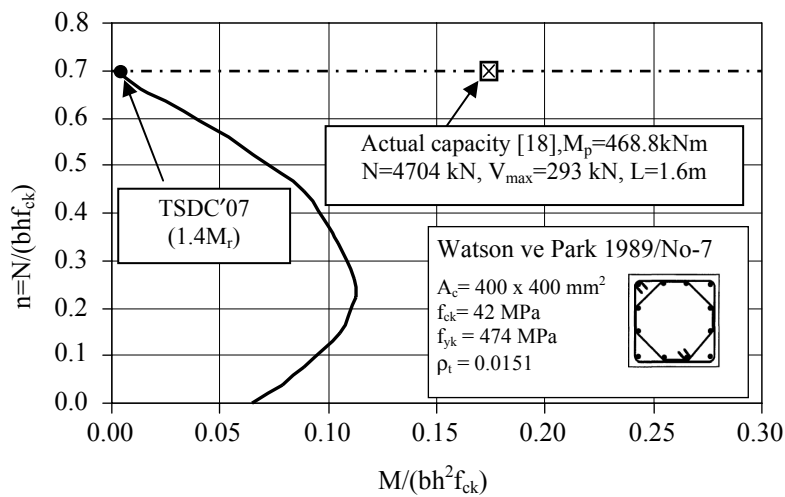


Figure 6:  $M_{p,experimental}/1.4M_r$  ratio for higher axial loads



The ratio of  $M_{p,experimental}/1.4M_r$  vary between 0.81~1.1 for axial load levels lower than 0.3 as can be seen in Fig.5. Mean ratio is 0.97 and the error range is about  $\pm 10\%$  for these axial load levels.

The comparisons of moment capacities obtained with the method proposed by Ersoy ( $f_{cc}=1.15f_{ck}$ ) with the experimental moment capacities ( $M_{p,experimental}=V_{max}L$ ) are shown in Fig. 7.

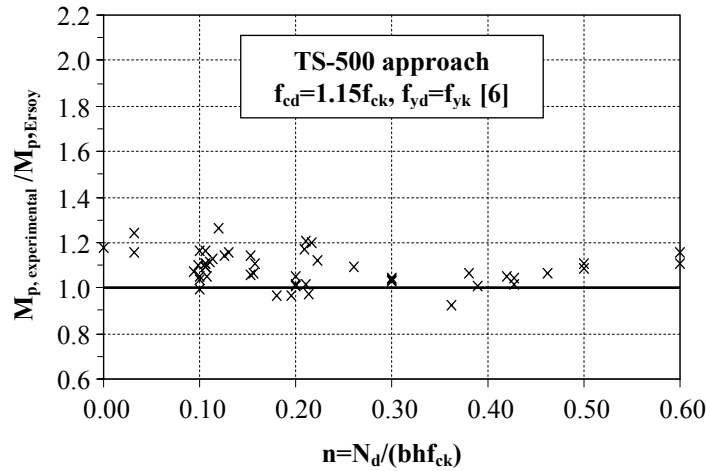


Figure 7: Variation of  $M_{p,experimental}/M_{Ersoy}$  ratio with column axial load level

It can be seen from Fig. 7 that  $M_{p,experimental}/M_{Ersoy}$  ratios vary between 0.93~1.26 and the error rate increases for decreasing axial load level. The authors find it beneficial to investigate the reasons of this increase. Ersoy has emphasized that, confined concrete strength can not be less than  $1.15f_{ck}$  and moment capacities calculated with this assumption are less than the real capacities for columns having minimum confinement given in TSDC. Besides, Ersoy has highlighted that, moment capacities calculated for confined concrete strength equal to  $1.15f_{ck}$ , can be considerably less than the real capacity for columns having confinement more than the minimum value [6]. As a matter of fact, there are some columns having confined concrete strength greater than  $1.15f_{ck}$  in Fig. 7. The effect of increase in concrete strength on column moment capacity is limited for relatively lower axial load levels. The test elements of 26, 27 and 28 (U-1, U-4, U-6) in Table 2 can be given as an example. Concrete strengths of these elements should be increased 6.2, 2.5 and 3.6 times, respectively, to reach experimental moment capacities with the TS-500 approach mentioned above. These increased values are not realistic for confined concrete strengths. The increase in concrete strength begin to show a decrease tendency and get close to confined concrete strength for increasing axial load levels. In this context, not taking into consideration the stress increase because of strain hardening effect will lead to underestimation of moment capacities for low axial load levels. The moment capacities

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obtained by Ersoy's method are considerably close to real moment capacities for axial load levels higher than 0.25 and error rates are less than 10%.

### **3. PROPOSED METHODS TO OBTAIN COLUMN MOMENT CAPACITIES, $M_p$**

In this section, two simple procedures to obtain column moment capacities are proposed. In the first method, column moment capacity is obtained based on the ultimate moment, whereas increased concrete and steel strengths are used in the second method.

The investigations made in the previous section show that,  $1.4M_r$  approach and Ersoy's method provide good results for specific axial load levels (Fig. 5 and 7). An equation can be derived to obtain column moment capacity due to axial load level based on the relationship between ultimate moment capacity and moment capacities found by Ersoy's method.

With this purpose, equations proposed by Çakıroğlu and Özer as functions of interaction curves for symmetrically reinforced rectangular R/C columns based on ultimate design will be used as below [22].

- **For material design strengths ( $f_{cd}=f_{ck}/1.5$  ;  $f_{yd}=f_{yk}/1.15$ ) ;**

$$n = \frac{N_d}{b \cdot h \cdot f_{ck}} \quad , \quad m = \frac{M_d}{b \cdot h^2 \cdot f_{ck}} \quad (1)$$

$$\omega_t = \frac{A_{st} \cdot f_{yk}}{k_1 \cdot k_2 \cdot b \cdot h \cdot f_{ck}} \quad (2)$$

for  $n < 0.2$ ;

$$\omega_t = 2.86 \cdot m + 2.92 \cdot n^2 - 1.48 \cdot n \quad (3a)$$

for  $0.2 < n \leq 0.3$ ;

$$\omega_t = 2.92 \cdot m - 0.192 \quad (3b)$$

for  $n > 0.3$ ;

$$\omega_t = 2.92 \cdot m + 0.62 \cdot n^2 + 0.254 \cdot n - 0.32 \quad (3c)$$

- **For any given value of material coefficients;**

$$\bar{n} = \frac{N_d}{0.85 \cdot b \cdot h \cdot f_c} \quad , \quad \bar{m} = \frac{M_d}{0.85 \cdot b \cdot h^2 \cdot f_c} \quad (4)$$

$$\bar{\omega}_t = \frac{A_{st} \cdot f_y}{k_1 \cdot k_2 \cdot 0.85 \cdot b \cdot h \cdot f_c} \quad (5)$$

for  $\bar{n} < 0.35$ ;

$$\bar{\omega}_t = 2.49 \cdot \bar{m} + 1.44 \cdot \bar{n}^2 - 1.29 \cdot \bar{n} \quad (6a)$$

for  $0.35 \leq \bar{n} \leq 0.50$ ;

$$\bar{\omega}_t = 2.54 \cdot \bar{m} - 0.295 \quad (6b)$$

for  $\bar{n} > 0.50$ ;

$$\bar{\omega}_t = 2.54 \cdot \bar{m} + 0.305 \cdot \bar{n}^2 + 0.22 \cdot \bar{n} - 0.49 \quad (6c)$$

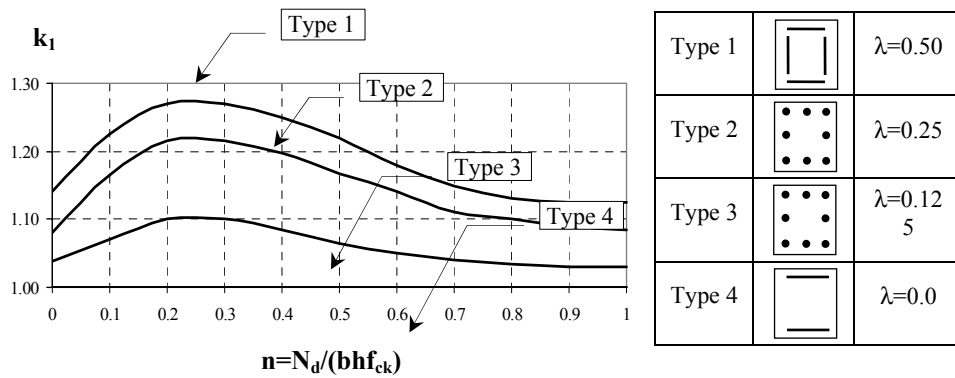


Figure 8:  $k_1$  coefficients for different longitudinal reinforcement configurations [22]

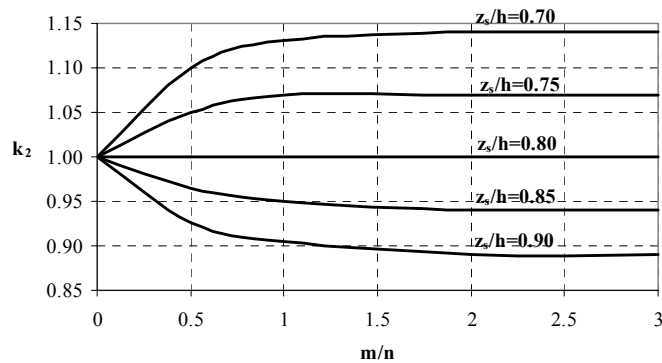


Figure 9:  $k_2$  coefficients for different  $z_s/h$  ratios [22]

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In (1-6) equations;  $A_{st}$  is the total longitudinal reinforcement area,  $\omega_t$  is the mechanical reinforcement index,  $k_1$  is the coefficient to represent different longitudinal reinforcement configurations, (Fig. 8),  $k_2$  is a coefficient to represent different  $z_s/h$  ratios (Fig. 9),  $f_c$  is concrete compression strength in case of any given material coefficient,  $f_y$  is steel strength in any given material coefficient.

The steps to obtain a unique  $M_{p,Ersoy}/M_r$  valid for all R/C rectangular columns are summarized below.

Material design strengths ( $f_{cd}$  and  $f_{yd}$ ) are used to obtain ultimate moment capacity ( $M_r$ ) using Eq.(1-3) for any axial load level, whereas  $f_c=1.15f_{ck}$  and  $f_y=f_{yk}$  [6] are used to obtain the moment capacity ( $M_p$ ) in Ersoy's method using Eq.(4-6).

Equation (7) is derived from Equations (2) and (5) by rearranging two equations according to total longitudinal reinforcement area ( $A_{st}$ ), and equalizing to each other and simplifying, respectively. (In equation (5)  $f_c=1.15f_{ck}$ ,  $f_y=f_{yk}$ ).

$$\omega_t = \bar{\omega}_t \cdot 0.85 \cdot 1.15 \quad (7)$$

For the terms  $\omega_t$  and  $\bar{\omega}_t$  in equation (7), functions defined for corresponding axial load levels (equations (3a, b, c) and (6a, b, c)) are used and  $\bar{n} = 1.023 \cdot n$  transformation is adopted, so  $M_p/M_r$  ratio can be obtained with the equations given below.

$$M_p = \bar{m} \cdot 0.85 \cdot b \cdot h^2 \cdot 1.15 \cdot f_{ck} \quad (8)$$

$$n = \frac{N_d}{b \cdot h \cdot f_{ck}} \quad , \quad m = \frac{M_r}{b \cdot h^2 \cdot f_{ck}} \quad (9)$$

$$\frac{M_p}{M_r} = \beta_1 + \frac{\beta_2}{m} \quad (10)$$

In Eq.(10);  $\beta_1$  ve  $\beta_2$  are coefficients depending on axial load level,  $M_r$ , that is column ultimate moment capacity,  $m$  is a dimensionless column ultimate moment capacity, respectively. Variations of  $\beta_1$  and  $\beta_2$  coefficients for different levels of axial load are given in Table 3.

The  $M_{p,Ersoy}/M_r$  ratios, varying against dimensionless axial load level and dimensionless column ultimate moment capacity are calculated from Eq.(10) for all symmetrically reinforced R/C rectangular column sections for an axial load level range of  $0.15bhf_{ck} \sim 0.5bhf_{ck}$  regardless of column longitudinal reinforcement configuration and  $z_s/h$  ratio can be seen in Fig. 10.

Table 3: Variations of  $\beta_1$  and  $\beta_2$  coefficients against axial load levels

Axial load	$\beta_1$	$\beta_2$
$n < 0.2$	1.15	$0.58n^2 - 0.076n$
$0.2 \leq n \leq 0.3$	1.173	$0.52n - 0.59n^2 - 0.08$
$0.3 < n < 0.34$	1.173	$0.62n - 0.34n^2 - 0.13$
$0.34 \leq n \leq 0.5$	1.16	$0.10n + 0.24n^2 - 0.013$

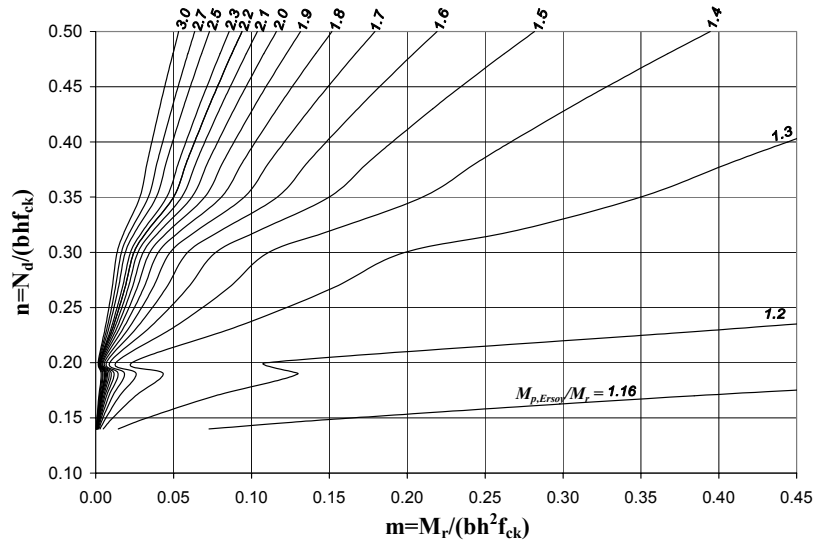


Figure 10: Distributions of  $M_{p,Ersoy}/M_r$  ratios obtained with Eq.(10) for rectangular R/C columns

As it can be seen in Fig. 5,  $1.4M_r$  approach provides good results for low axial load levels. The moment capacities obtained with this approach are less than real moment capacities of columns for increasing axial load levels. Moment capacities obtained with the method proposed by Ersoy provide very close results to actual capacity for these axial load level ranges. As a result, it is meaningful to use  $1.4M_r$  approach for axial load levels lower than 0.25, whereas Ersoy's method can be used for axial load levels higher or equal to 0.25.

The coefficients used in Eq.(10) vary according to the axial load level. Besides, these coefficients can be defined as a single equation for axial load level higher or equal to 0.25 in a similar form of Eq.(10). For this purpose, Eq.(11) is derived by Statistica program [23].

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$$\frac{M_p}{M_r} = 1.16 + \frac{0.34 \cdot n - 0.07}{m} \quad (11)$$

The comparisons for  $M_{p,Ersoy}/M_r$  ratios obtained from Eq.(11) with Eq.(10) are shown in Fig. 11 for axial load levels higher or equal to 0.25. The coefficient of correlation of Eq.(11) is 0.998.

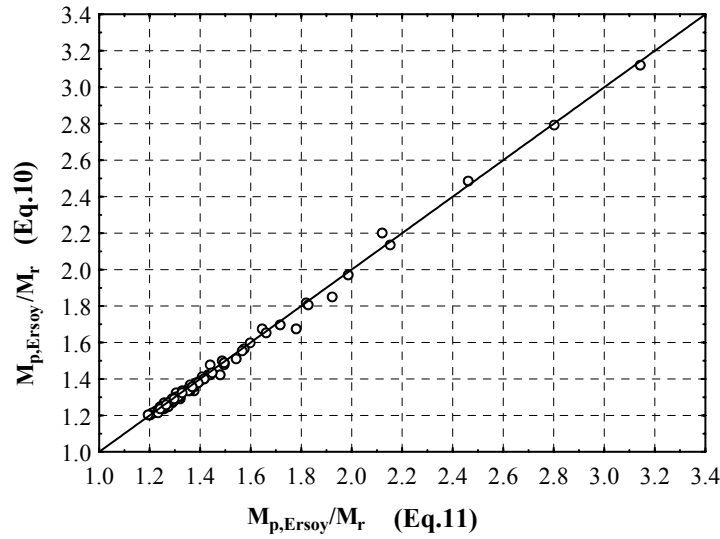


Figure 11: Comparisons of  $M_{p,Ersoy}/M_r$  ratios obtained with Eq. (10) and Eq.(11) for  $n \geq 0.25$

From this point of view, column moment capacities  $M_p$  can be obtained with equation (12). Values of  $\beta$  coefficient in Eq.(12) for corresponding axial load level are given in Table 4.

$$\frac{M_p}{M_r} = 1.4 + \frac{\beta}{m} \quad (12)$$

Table 4: Variations of  $\beta$  coefficient against axial load level

Axial load	$\beta$
$n < 0.25$	0
$0.25 \leq n \leq 0.5$	$(0.34n - 0.24m - 0.07) \geq 0$

Comparisons for experimental moment capacities ( $M_{p,experimental}=V_{max}L$ ) with proposed moment capacities by Eq.(12) of test columns with given properties in Table 2 are shown in Fig. 12.

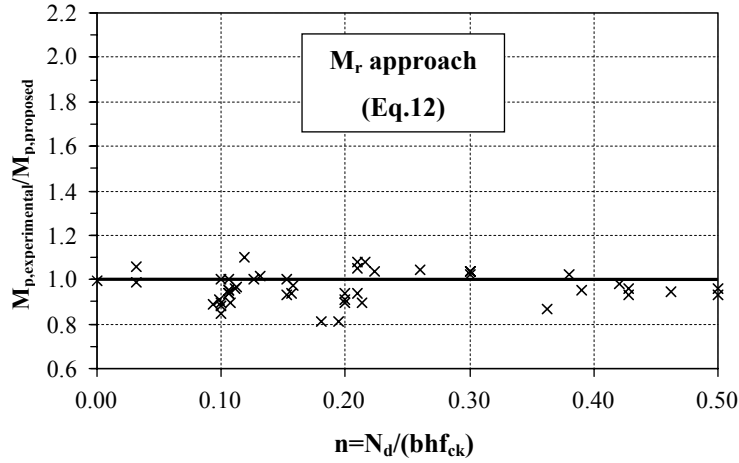


Figure 12: Comparisons for experimental moment capacities with proposed moment capacities by Eq.(12)

In the second proposed method to obtain column moment capacity, a similar approach to Ersoy's method is used. The effect of strain hardening on increase in moment capacities should be taken into consideration for decreasing axial load level as can be seen in Fig.7. In this context, strain hardening effect is thought to be taken into consideration by increasing yield strength and experimental moment capacities are used with this purpose. Confined concrete strength can be taken as  $1.15f_{ck}$  as proposed by Ersoy's method.

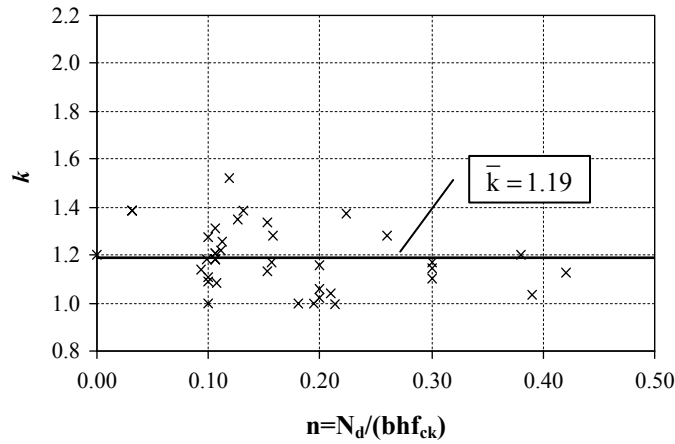


Figure 13: Variations of coefficients  $k$  against axial load levels

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In Fig.13, variations of coefficient  $k$  which represents the increase in yield strength of steel against the axial load level can be seen. The mean value of the ratio of ultimate strength to yield strength ( $f_{su}/f_{yk}$ ) is 1.42 for test columns given in Table 2. In this proposed method, the increase in the amount of yield strength ( $k$ ) is equal to 1.19 which is the mean value of results shown in Fig.13. It is well known that the effect of strain hardening on moment capacity shows a decreasing tendency for increasing axial load level. The similar tendency can be observed for coefficients  $k$  in Fig.13.

The comparison of the experimental moment capacities with moment capacities calculated by using increased material strengths ( $f_{cd}=1.15f_{ck}$  and  $f_{yd}=1.19f_{yk}$ ) for columns given in Table 2 are shown in Fig.14 with an error band of 10%. It can be seen in Fig. 14 that the results of the mentioned approach provide similar results to experimental capacities for all axial load levels and error is usually less than 10%.

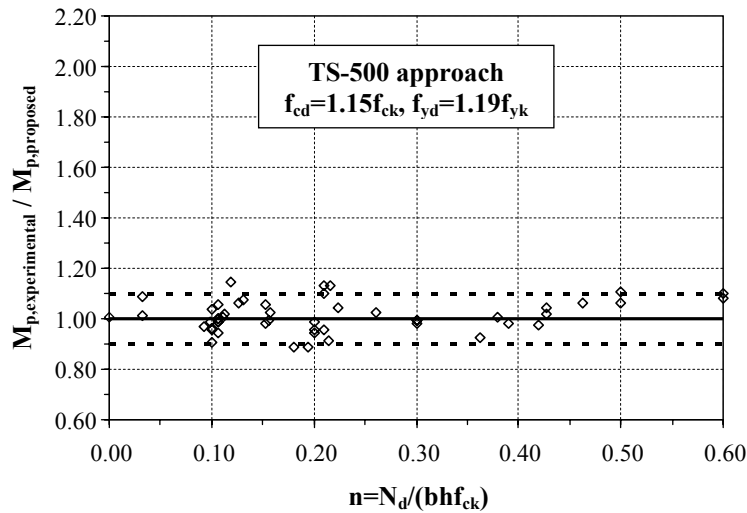


Figure 14: The comparisons of the experimental moment capacities with proposed moment capacities ( $f_{cd}=1.15f_{ck}$  and  $f_{yd}=1.19f_{yk}$ )

## 4. CONCLUSIONS

In this study, two methods are proposed to calculate  $M_p$  moment capacity of R/C rectangular columns for various axial load levels in an accurate and practical way. In the first method, column moment capacity is calculated as a function of the ultimate moment and axial load level. In the second method, column moment capacity is calculated using increased material design strengths with the assumptions given in TS-500.

Results based on comparison of existing and proposed methods with experimental results can be summarized as below.



- $M_{p,experimental}/M_r$  ratio varies between 1.13 ~ 2.93 for axial load level range of  $0 \sim 0.6bhf_{ck}$ . For higher axial load levels, these ratios increase considerably. Thus, shear forces at column ends obtained with  $1.4M_r$  assumption are considerably lower than the shear strength required for ductile behaviour especially for high axial load levels.
- $M_{p,experimental}/M_{p,Ersoy}$  ratio varies between 0.93 ~ 1.26 for axial load levels given above. Especially, axial load levels lower than 0.25, error rates increase and yield lower results than actual capacity in the range of 26%.
- Column moment capacity  $M_p$ , can be expressed as a function of ultimate moment  $M_r$  and axial load level with the help of analytical expressions of M-N interaction diagrams depending on increased design strengths. Moment capacities obtained with Eq.(12) derived based on the approach mentioned above, provide similar results to actual moment capacities.
- The effect of strain hardening on increase of moment capacities should be taken into consideration in columns for decreasing axial load levels. The effect of strain hardening on moment capacity shows a decreasing tendency for increasing axial load levels.
- Column moment capacity  $M_p$  can be calculated using increased strengths ( $f_{cd}=1.15f_{ck}$  ve  $f_{yd}=1.19f_{yk}$ ) instead of material design strengths given in TS500. It is also found that moment capacities obtained with this approach provide similar results to actual moment capacities for all axial load levels.
- Characteristic material strengths considered for concrete and steel represent the strength with a probability range falling under of 10%. In other words, in-place material strength will be higher than the characteristic strength with a probability of 90%. From this point of view, statistical investigations should be carried out to take the influence of physical and modelling uncertainties on moment capacity of R/C columns into consideration.

### **Symbols**

- $b$  : Width of column section  
 $f_{cc}$  : Confined concrete compressive strength  
 $f_{cd}$  : Design compressive strength of concrete  
 $f_{ck}$  : Characteristic compressive cylinder strength of concrete  
 $f_{ctk}$  : Characteristic tensile strength of concrete  
 $f_{yd}$  : Design yield strength of longitudinal reinforcement  
 $f_{yk}$  : Characteristic yield strength of longitudinal reinforcement  
 $f_{ywk}$  : Characteristic yield strength of transverse reinforcement  
 $f_{su}$  : Ultimate strength of steel  
 $h$  : Height of column section  
 $k$  : Coefficient k which represents the increase in the amount of yield strength of steel

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- $k_1$  : Coefficient to represent different longitudinal reinforcement configurations
- $k_2$  : Coefficients for different  $z_s/h$  ratios
- $L$  : Shear length
- $\ell_n$  : Clear height of column
- $m$  : Dimensionless ultimate moment capacity
- $M_n$  : Nominal moment capacity
- $M_p$  : Moment capacity
- $M_{pa}$  : Ultimate moment capacity calculated at the bottom of column clear height
- $M_{p\bar{u}}$  : Ultimate moment capacity calculated at the top of column clear height
- $M_{p,Ersoy}$  : Moment capacity based on factored material strength
- $M_r$  : Ultimate moment capacity
- $n$  : Dimensionless axial load level
- $N$  : Axial load
- $N_d$  : Factored axial force calculated under simultaneous action of vertical loads and seismic loads
- $s$  : Spacing of transverse reinforcement
- $V_e$  : Shear force taken into account for the calculation of transverse reinforcement of column
- $V_{maks}$  : Maximum base shear
- $\epsilon_{cu}$  : Ultimate compressive strain of unconfined concrete
- $\epsilon_{sh}$  : The strain at which strain-hardening of steel begins
- $\epsilon_{su}$  : The ultimate strain of steel
- $\phi$  : Curvature
- $\emptyset$  : Bar diameter
- $\gamma_c$  : Partial safety factors for concrete
- $\gamma_s$  : Partial safety factors for reinforcement steel
- $\lambda$  : The ratio between the reinforcing steel area located away from either tension or compression side of the section and the total reinforcement area of the column
- $\rho_t$  : Longitudinal column reinforcement ratio
- $\omega_t$  : Mechanical reinforcement index

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