# alphanumeric journal 

The Journal of Operations Research, Statistics, Econometrics and Management Information Systems

## Pharmacy Duty Scheduling Problem: Gümüşhane Case

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#### Abstract

In the Turkish Healthcare System, pharmacies are an important echelon of the drug supply chain. The pharmacy on duty practice ensures that the demand for drugs and similar medical supplies is met 24/7. The duty pharmacy scheduling problem aims to satisfy the customers and the pharmacists without violating the regulations while determining the duty sequence of the pharmacies in a certain planning period. This study handles the duty pharmacy scheduling problem of Gümüşhane City Center where 13 pharmacies operate. In the current system, the representative pharmacist prepares the duty schedule manually. The planning period is divided into subsets, and pharmacies are assigned to the duties in each subset respectively in such the number of duties of each pharmacy in the subsets is as equal as possible. The managerial problem of the case is that the duty frequency of each pharmacy is variable in the manual schedule and pharmacists perceive this as an unfairness. Because pharmacists expect to be on duty 1 in 13 days. We propose a mathematical model that adopts the fixed duty frequency and the inheritance of duty distribution. We test the model according to the 2020 calendar and compare model results with the real duty schedule belong to 2020 using the 2019 distribution of pharmacies. The comparison and test results show that the duties are distributed as fairly as possible and the turn of each pharmacy is repeated once every 13 days. The contribution of this study is that it introduces innovative components to the pharmacy duty scheduling problem, offers practical policy recommendations, and provides a casespecific example to limited PDS literature.


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## 1. Introduction

The drug supply chain is a critical issue for any healthcare system. Unrestricted access to drugs can lead to improper and unnecessary use, thereby deteriorating public health. On the other hand, strict control or prohibition may fail treatments and high demand in healthcare facilities. Policymakers try to control access to drugs through various regulations. In the Turkish Healthcare System, only pharmacies are authorized for a drug sale. Besides selling drugs, the role of a health consultant makes pharmacies an important component of the system. Pharmacies are subject to the Regional Chamber of Pharmacists to which they are affiliated. Ensuring continuity of the drug supply $24 / 7$ is among the tasks of the Chamber. For this purpose, it regulates the working hours of pharmacies and makes sure that a sufficient number of pharmacies are on duty to cover the region in the off-hour period. The pharmacy on duty continues to serve from the end to the beginning of the working hours on weekdays and Saturdays, and also all day on days such as Sundays and Bank Holidays, based on a schedule.

Although regulations exist, the process of identifying the pharmacy on duty remains a black box, as it is not defined how to combine stakeholder-based factors such as accessibility, capacity, and proximity to each other of pharmacies. The decision on the pharmacy on duty should satisfy the two key stakeholders of the system: the pharmacist and the customer. The pharmacist, who operates the pharmacy, expects the schedule to be as fair as possible and the maximum distance between the duty pharmacies, since being on duty is economically more profitable. The customer, who is a patient or someone who needs a medical item, wants to reach the pharmacy on duty with the least effort to meet the urgent need for medical supplies as soon as possible. The schedule is widely prepared by representative pharmacists manually. The handicap of manual scheduling is that the quality of the schedule varies depending on the judgment and experience of the representative.

Operations Research (OR) supports decision-makers from different disciplines to make impersonal decisions using analytical methods. The OR literature provides effective solutions to healthcare management challenges such as planning (Swisher, Jacobson, Jun, \& Balci, 2001), performance measurement (Varela, de Martins, \& Fávero, 2010), location (Kanuganti, Sarkar, Singh, \& Arkatkar, 2015), patient flow (Jiang \& Giachetti, 2008), and scheduling (Adan, Bekkers, Dellaert, Vissers, \& Yu, 2009) with techniques such as integer programming (Brunner, Bard, \& Kolisch, 2009), simulation (Abo-Hamad \& Arisha, 2013) and multi-criteria decision making (Dursun, Karsak, \& Karadayi, 2011). Kahraman \& Topcu (2018) collect OR techniques used in healthcare management and Tuzun \& Topcu (2018) present a detailed taxonomy. The representative's challenge is handled as Pharmacy Duty Scheduling (PDS) in the scope of OR. PDS focuses on satisfying well-defined constraints of both stakeholders and regulations, and also optimizing the objectives of decision-maker when determining the duty sequence of pharmacies within a certain planning period.

The PDS has scarce literature with two branches: solution-oriented and case-specific. Solution-oriented studies focus on computing acceptable solutions with heuristic methods in cases where exact solution methods to large-scale PDS problems are inoperative. Variable Neighborhood Search Algorithm (Kocatürk \& Özpeynirci, 2014),

Tabu Search Algorithm (Özpeynirci \& Ağlamaz, 2016), Branch and Price Algorithm (Ceyhan \& Özpeynirci, 2016) are recommended methods in the literature for largescale PDS problems.

The case-specific PDS problems address real-life challenges and include soft and hard constraints depending on the attribute of the case. In a feasible solution, hard constraints must be satisfied, and soft constraints should be fulfilled as much as possible. For example, the assignment of at least one pharmacy on duty is a hard constraint, whereas the distribution of duties to pharmacies as equally as possible is a soft constraint. Ozgur et al. (2009) developed a web-based application for the PDS problem of Ankara Pharmacist Association (APA). The application utilizes a stochastic optimization approach that uses a Simulated Annealing (SA) algorithm and satisfies the requirements of the APA. Remarkable requirements are the maximum distance between pharmacies on duty and the fair schedule. Uncu et al. (2018) proposed a distance restricted maximal covering location model for the PDS problem of Adana Chamber of Pharmacists. The objective of the study is fair scheduling, subject to a minimum distance between pharmacies on duty. Fair scheduling and maximizing the distance between duty pharmacies, which are commonly considered, are major characteristics of the PDS problem.

This study deals with the PDS problem of Gümüşhane City Center. The special situation of the case mentioned in Section 2 reduces the PDS only to a fair schedule. The critical notes from the interview with pharmacists are as follows; (i) The duty schedule is prepared annually by the representative pharmacist manually and published monthly, (ii) Pharmacies' duty sequences are followed in 3 subsets: Weekdays, Sundays, and Bank Holidays, (iii) The goal is to assign an equal number of duties to each pharmacy in each subset, (iv) The variable frequency of duties is the biggest complaint of pharmacists. To overcome mentioned complaint, we propose a mathematical model that adopts the fixed duty frequency and also the heredity of duty distribution to provide fair distribution. Then we analyze the impact of the subset configuration on the schedule fairness. We also present a comparison model results with the real duty list belong to 2020 using the 2019 distribution of pharmacies.

The remainder of the study is organized as follows. Section 2 introduces the case and identifies the problem. Mathematical formulations and analyses, respectively, are in Sections 3 and 4. Section 5 contains the conclusions and policy recommendations.

## 2. Gümüşhane Case

In this section, we give information about the case and highlight the problem. Gümüşhane is a small city with only 13 pharmacies in the city center. While 10 of 13 are located on the same street in a side by side and/or opposite position, 3 of 13 are all together at a distance of approximately 1 km from others. Figure 1 shows the location of the pharmacies. The maximum distance between pharmacies (E5 and E12 pharmacies) is about 1.5 km . As the pharmacies are few and too close to each other, the city center is considered as a single region and only one pharmacy is on duty each day. From a different viewpoint, the special situation of the city can be emphasized as follows. Let's imagine that two pharmacies are on duty every day in the Gümüşhane. In this setting, the distance between pharmacies on duty is too short to
cause idle service delivery, and also the maximum distance is even less than the minimum distance ( 1.6 km ) accepted in Uncu et al. (2018).

In cities with many high population zones, more than one pharmacy is on duty to spread and satisfy demand, and to meet the needs of customers with the least effort, taking account of their distance from each other. However, Gümüşhane city center does not have a population density that would require more than one pharmacy on duty. These details minify the influence of the customers on the decision. Fair scheduling remains the only goal of the PDS of the case.


Figure 1. Pharmacy locations
The planning period is one year and consists of three subsets according to regulations: Weekdays (W), Sundays (S), and Bank Holidays (BH). For instance, Saturday, October 31, 2020 is in W, Sunday, November 1, 2020 is in S, Thursday, 29 October 2020 (Republic Day in Turkey) is in BH. In subset W, the pharmacy on duty runs approximately 16 hours from the end of working hours to the beginning, and 24 hours in the subsets S and BH. According to the 2020 calendar data, the distribution of the duties to the subset is as follows: $\mathrm{W}: 301, \mathrm{~s}: 49, \mathrm{BH}: 16$.

In current practice, the representative pharmacist follows a turn-based approach for the fair distribution of days in each subset to pharmacies in such a way that the same pharmacy will not be on duty on consecutive days. Although the calendar day order of the days in the subsets W and S are in a regular pattern, the days in the BH are irregular. This situation leads to a fluctuation in the duty frequency in the preparation of a fair schedule. Due to the economic worth of the duty days, the variability of duty frequency creates conflict between pharmacies. While pharmacists expect to be on duty 1 in 13 days, with a manual schedule the duty frequency may decrease to 1 in 9 10 days. Mainly the underlying cause of this is the effort to distribute the duties equally, but the fair distribution may not be possible within a single planning period, only by taking into account multiple planning periods. Pharmacists can accept the allocation of fair duty distribution in the long term because calendar days can not be changed, whereas the variable duty frequency is not acceptable as it causes a difference in earnings in the short term. Therefore, we handle the fixed duty frequency as a hard constraint.

The fixed frequency approach implies that each pharmacy will be on duty in a locked turn during the planning period. This approach causes pharmacies to have a different number of duties in the same subset due to the irregularity of the days in the BH
subset. Since the 24 -hour duty is more profitable than the 16 -hour duty, the unbalanced sharing causes conflict between pharmacies. The decision-maker can compensate for this in the next period and allocate balanced sharing with a holistic perspective. This opportunity eliminates obstacles to the fixed duty frequency approach, as long as the inheritance of the duty distribution is taken into account. The inheritance of the duty distribution is realized by including the duty distribution of each subset from the previous periods to the current planning. Also, it facilitates rescheduling in events that interrupt the planning period, such as opening a new pharmacy or closing an existing pharmacy.

When the variability in duties frequencies is interpreted as a symptom, it can be seen that the main source of the problem is subsets. The current subset configuration is based on the attribute of the calendar day but the duration of the duties is the main attribute that distinguishes the duty days. The duties in the $S$ and $B H$ subsets can be combined as they last for a full day. Two subsets based on duty duration are more valid. Decision-makers may explain more easily that two different duty sequences are followed for both subsets as the reason for the variability in duty frequencies.

## 3. Problem Formulation

In this section, we present the mathematical formulation of the problem.
Sets and Indices
$E:\left\{E_{1}, E_{2}, \ldots, E_{13}\right\}: \quad$ Set of Pharmacies, $e \in E$
$G:\{1,2, \ldots, 365\}: \quad$ Set of Planning Period, $g \in G$
$T:\{W, S, B H\}: \quad$ Set of Subsets of Planning Period, $t \in T$

## Parameters

$T_{g}$ : Indicates in which subset day $g$ is.
$d_{e, t}: \quad$ Number of duties in the subset $t$ of pharmacy $e$ in the previous planning period.
Decision Variables
$x_{e, g}:\left\{\begin{array}{l}1, \text { If the pharmacy } e \text { is on duty on day } g \\ 0, \text { Otherwise }\end{array}\right.$
$y_{e, t}: \quad$ Number of duties under subset $t$ of pharmacy $e$
$y_{t}^{\text {max }}: \quad$ The maximum number of duties in a pharmacy's subset $t$
Objective Function

$$
\begin{equation*}
\operatorname{Min} Z=\sum_{t \in T} y_{t}^{\max } \tag{1}
\end{equation*}
$$

Constraints

$$
\begin{equation*}
\sum_{e \in E} x_{e, g}=1 \quad \forall g \in G \tag{2}
\end{equation*}
$$

$$
\begin{array}{cc}
\sum_{a \in G: T_{n}=t} x_{e, g}+d_{e, t}=y_{e, t} & \forall e \in E, t \in T \\
y_{e, t} \leq y_{t}^{\max } & \forall e \in E, t \in T \\
\sum_{a^{\prime}=1}^{|E|} x_{e, g+g^{\prime}}=1 & \forall e \in E, g \in\{1, \ldots,|G|-|E|\} \tag{5}
\end{array}
$$

The objective function (1) ensures that minimize the maximum number of duties of a pharmacy in each subset. Constraint (2) allows only one pharmacy to be on duty in a day. (3) calculates the number of duties in each subset of each pharmacy by including the previous planning period. The inheritance of the duty distribution is provided by this constraint. (4) limits the maximum number of duties of a pharmacy in each subset. (5) guarantees that each pharmacy is on duty in a cycle and once per cycle. The pharmacy number determines the cycle length. This constraint also prevents a pharmacy from being on duty on consecutive days. The combination of (1) and (4) reflects a min-max approach, ensuring the distribution of duties as equally as possible.

Note that the solution can be sought with the Goal Programming approach or Pareto Optimal solution with the multi-objective formulation. However, the decision problem discussed is more suitable for the described biobjective modeling.

## 4. Analysis

This section aims to test the behavior of the model according to the 2020 calendar data and to observe the effect of changing in subset configurations. We perform all tests in the Python/Gurobi modeling environment with a 3.5 GHz i5-8250U processor and 12 GB RAM. We measure the quality of the schedule with the standard deviation of the duty distribution, and we call it the fairness score. The fairness score represents the difference in pharmacy duty numbers and it is desirable to be close to zero.

We use a test platform consisting of two stages illustrated in Figure 2. The first stage acts as the previous period and the second stage represents the current period. In the first stage, the model runs with parameter $d$ that is null. The inheritance of the duty distribution is provided by the parameter $d$. In the second stage, the ability of the model is tested according to parameter $d$ that is full.

Expectations in the results are as follows: i) The low fairness score, ii) The fixed duty frequency, iii) Decrease in the fairness score in a holistic view.


Figure 2. Schema of the testing platform

| Pharmacy | First Stage |  |  |  |  | Second Stage |  |  |  |  | Cumulative |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | W | 5 | BH | Total | Turn | W | S | BH | Total | Turn | W | 5 | BH | Total |
| E1 | 23 | 4 | 1 | 28 | 8 | 24 | 3 | 1 | 28 | 7 | 47 | 7 | 2 | 56 |
| E2 | 23 | 4 | 1 | 28 | 11 | 24 | 3 | 2 | 29 | 2 | 47 | 7 | 3 | 57 |
| E3 | 24 | 4 | 0 | 28 | 13 | 23 | 4 | 1 | 28 | 3 | 47 | 8 | 1 | 56 |
| E4 | 24 | 3 | 2 | 29 | 2 | 23 | 4 | 1 | 28 | 11 | 47 | 7 | 3 | 57 |
| E5 | 22 | 4 | 2 | 28 | 5 | 24 | 4 | 0 | 28 | 12 | 46 | 8 | 2 | 56 |
| E6 | 24 | 4 | 1 | 29 | 1 | 22 | 4 | 2 | 28 | 4 | 46 | 8 | 3 | 57 |
| E7 | 22 | 4 | 2 | 28 | 6 | 24 | 4 | 0 | 28 | 13 | 46 | 8 | 2 | 56 |
| E8 | 24 | 3 | 1 | 28 | 9 | 22 | 4 | 2 | 28 | 5 | 46 | 7 | 3 | 56 |
| E9 | 24 | 3 | 1 | 28 | 7 | 22 | 4 | 2 | 28 | 6 | 46 | 7 | 3 | 56 |
| E10 | 24 | 4 | 0 | 28 | 12 | 22 | 4 | 2 | 28 | 10 | 46 | 8 | 2 | 56 |
| E11 | 22 | 4 | 2 | 28 | 4 | 23 | 4 | 1 | 28 | 8 | 45 | 8 | 3 | 56 |
| E12 | 23 | 4 | 1 | 28 | 3 | 24 | 3 | 1 | 28 | 9 | 47 | 7 | 2 | 56 |
| E13 | 22 | 4 | 2 | 28 | 10 | 24 | 4 | 1 | 29 | 1 | 46 | 8 | 3 | 57 |
| Fairness Score | 0.86 | 0.42 | 0.70 | 0.36 |  | 0.86 | 0.42 | 0.70 | 0.36 |  | 0.61 | 0.50 | 0.63 | 0.46 |
| Average | 0.66 |  |  |  |  | 0.66 |  |  |  |  | 0.58 |  |  |  |
| Values in the columns W, S, BH, and Total show the number of duties. |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| The turn column shows the fixed duty sequence of the pharmacy. |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Fairness Score represents the standard deviation of the relevant column. |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Table 1. Test Results of the Model
The annual duty schedule obtained at the end of the second stage is in the appendix ${ }^{1}$. Table 1 shows the number of duties in each subset and the duty sequence. When the duty schedule is examined, it is seen that the duty sequence (Turn column in Table 1) in the first stage is followed, the model has implemented the fixed duty frequency. The results in Table 1 have a low fairness score. The reason for the score not being 0 is that the number of days in the subsets is not proportional to the number of pharmacies and the pharmacies have the fixed on duty sequence throughout the planning period. The average fairness score of the subsets is 0.66 in the first stage, 0.66 in the second stage, and 0.58 in the cumulative results. The low score in the cumulative results indicates that the model is successful in balancing. Test results show that the model meets expectations. We can claim that the fixed duty frequency and the inheritance of duty distribution approaches eliminate the main problem of the case and automated scheduling can replace manual scheduling. Also, we analyze the effect of subset settings on the schedule. Table 2 displays the test subset configurations. S1 is the basic subset configuration of the problem. In S2, S and BH subsets are combined and the number of subsets is reduced to 2 . Underlying the idea of reducing subsets lies the similarities in the duration of duties. On the days in subset W, the duties last about 16 hours from the end of working hours to the beginning (18:00 $\rightarrow$ 09:00), whereas the days in the $S$ and BH subsets last for a full day (09:00 $\rightarrow 09: 00,24$ hours). In S2, W covers 16-hour duty days and 5 includes 24-hour duty days.

|  | W | S | BH |
| :--- | :--- | :--- | :--- |
| S1 | 301 | 49 | 16 |
| S2 | 301 | 65 | - |

The values show the number of duties in each subset.
Table 2. Subsets Configurations

[^1]Since the results of S1 are the same as Table 1, Table 3 shows the number of duties in each subset and the duty sequence, according to S 2 . The model is consistent to obtain a fair schedule with the low fairness score and to balance the number of duties in subsets in both configurations by changing the duty sequence, see E2, E3, E4 as examples. The duty distribution of S1 and S2 indicates that the model is not sensitive to data, stable, and also can handle different subset configurations.

|  | First Stage |  |  |  | Second Stage |  |  |  | Cumulative |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Pharmacy | W | S | Total | Turn | W | S | Total | Turn | W | 5 | Total |
| E1 | 24 | 5 | 29 | 2 | 23 | 5 | 28 | 8 | 47 | 10 | 57 |
| E2 | 23 | 5 | 28 | 8 | 24 | 5 | 29 | 1 | 47 | 10 | 57 |
| E3 | 23 | 5 | 28 | 11 | 23 | 5 | 28 | 11 | 46 | 10 | 56 |
| E4 | 22 | 6 | 28 | 5 | 24 | 4 | 28 | 7 | 46 | 10 | 56 |
| E5 | 24 | 5 | 29 | 1 | 23 | 5 | 28 | 3 | 47 | 10 | 57 |
| E6 | 24 | 4 | 28 | 9 | 22 | 6 | 28 | 6 | 46 | 10 | 56 |
| E7 | 24 | 4 | 28 | 7 | 22 | 6 | 28 | 4 | 46 | 10 | 56 |
| E8 | 24 | 4 | 28 | 12 | 22 | 6 | 28 | 5 | 46 | 10 | 56 |
| E9 | 22 | 6 | 28 | 4 | 24 | 4 | 28 | 9 | 46 | 10 | 56 |
| E10 | 24 | 4 | 28 | 13 | 22 | 6 | 28 | 10 | 46 | 10 | 56 |
| E11 | 23 | 5 | 28 | 3 | 24 | 5 | 29 | 2 | 47 | 10 | 57 |
| E12 | 22 | 6 | 28 | 10 | 24 | 4 | 28 | 12 | 46 | 10 | 56 |
| E13 | 22 | 6 | 28 | 6 | 24 | 4 | 28 | 13 | 46 | 10 | 56 |
| Fairness Score | 0.86 | 0.78 | 0.36 |  | 0.86 | 0.78 | 0.36 |  | 0.46 | 0.00 | 0.46 |
| Average | 0.82 |  |  |  | 0.82 |  |  |  | 0.23 |  |  |
| Values in the colu The turn column s Fairness Score rep | ns W, ows th esents | and T pharm he stan | al show cy's fixe ard dev | the num duty 5 tion of | er of d quence. he relev | ties. |  |  |  |  |  |

Table 3. Test Results of S2
In the comparison of the cumulative fairness scores of S1 and S2, it is seen that the results of S2 reflect fairness better than S1 and the confusing minor differences in S1 are not in S 2 .

If manual scheduling continues, the S 2 configuration is a more practical option to a fair schedule. In manual scheduling based on S2, even if the frequency of duty varies, believing that this is due to two different sequences for the two subsets and making sure that they do not have a financial loss due to the total number of duties in both subsets is the same makes the situation more acceptable to pharmacists.

### 4.1. Comparison of Model Results with the Real Duty List

In this part of the study, we compare the model results for 2020 with the real duty list of 2020 by using the 2019 duty distributions of pharmacies. Before the comparison, we displayed the variability of the duty frequency, which constitutes the motivation for the problem, in Table 4. The links to real duty lists are in footnotes².

[^2]|  | 2019 Duty List |  |  |  | 2020 Duty List |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Pharmacy | Max | Min | Average | Std. Dev. | Max | Min | Average | Std. Dev. |
| Derman | 25 | 5 | 13,58 | 5,82 | 24 | 5 | 12,71 | 4,16 |
| Elif | 29 | 5 | 13,33 | 5,52 | 28 | 7 | 13,73 | 4,92 |
| Gümüşhane | 27 | 7 | 13,73 | 5,10 | 21 | 6 | 12,86 | 3,12 |
| Hayat | 23 | 6 | 13,19 | 4,33 | 21 | 5 | 12,36 | 3,88 |
| Işık | 24 | 4 | 13,50 | 4,81 | 27 | 5 | 12,79 | 5,16 |
| Karaca | 25 | 5 | 12,50 | 5,47 | 28 | 6 | 13,56 | 5,25 |
| Merkez | 24 | 5 | 12,11 | 5,17 | 31 | 4 | 13,81 | 5,54 |
| Özkan | 29 | 6 | 13,00 | 5,06 | 24 | 6 | 13,15 | 4,20 |
| Sağlık | 23 | 6 | 13,15 | 4,60 | 19 | 5 | 12,86 | 3,34 |
| Sinem | 26 | 6 | 12,71 | 4,63 | 21 | 6 | 12,70 | 3,99 |
| Şifa | 28 | 6 | 12,68 | 5,40 | 24 | 4 | 12,96 | 4,76 |
| Yalçın | 37 | 6 | 13,07 | 6,01 | 24 | 4 | 12,29 | 5,09 |
| Yüce | 22 | 5 | 12,39 | 4,39 | 27 | 8 | 13,26 | 4,31 |
| Average | 26,31 | 5,54 | 13,00 | 5,10 | 24,54 | 5,46 | 13,00 | 4,44 |
|  | Max: The pharmacy's maximum number of days between dutiesMin: The pharmacy's minimum number of days between dutiesAverage: Average number of days between the pharmacy's dutiesStd. Dev.: The standard deviation of the number of days between the pharmacy's duties |  |  |  |  |  |  |  |

Table 4. The duty frequencies of pharmacies for 2019 and 2020.
The difference between the maximum and minimum duty frequencies justifies the complaints of pharmacists. For example, the Yalçın Pharmacy in 2019, after its duty on 14 July 2019, its next duty took place 37 days later on 20 August 2019. In 2020, Şifa Pharmacy is an opposite example. After its duty on 31 May 2020, its next duty took place 4 days later on 4 June 2020. Although the average duty frequency of pharmacies is around 1 in 13 days, standard deviations greater than 0 reveals variability. The current duty distribution policy does not meet the expectations as stated in the motivation of the problem.

The pharmacy duty distributions in 2019 and 2020, which were prepared manually by the representative pharmacist, are shown in Table 5. The fairness scores in the 2020 duty distribution show an improvement in the $S$ subset but a worsening in the W and BH subsets. The cumulative distribution demonstrates that the unfairness in 2019 was not tolerated in 2020.

When the 2020 duty distributions of pharmacies are evaluated keeping in mind the duty variability shown in Table 4, it is visible that manual scheduling is far from satisfying the pharmacies' expectations.

|  | Real-2019 |  |  |  | Real-2020 |  |  |  | Cumulative |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Pharmacy | W | 5 | BH | Total | W | 5 | BH | Total | W | 5 | BH | Total |
| Derman | 23 | 4 | 0 | 27 | 24 | 4 | 1 | 29 | 47 | 8 | 1 | 56 |
| Elif | 23 | 4 | 1 | 28 | 21 | 4 | 2 | 27 | 44 | 8 | 3 | 55 |
| Gümüşhane | 22 | 4 | 1 | 27 | 24 | 4 | 1 | 29 | 46 | 8 | 2 | 56 |
| Hayat | 23 | 3 | 1 | 27 | 24 | 4 | 1 | 29 | 47 | 7 | 2 | 56 |
| Işık | 22 | 4 | 1 | 27 | 24 | 4 | 1 | 29 | 46 | 8 | 2 | 56 |
| Karaca | 24 | 3 | 2 | 29 | 22 | 3 | 1 | 26 | 46 | 6 | 3 | 55 |
| Merkez | 23 | 5 | 1 | 29 | 22 | 4 | 1 | 27 | 45 | 9 | 2 | 56 |
| Özkan | 23 | 4 | 1 | 28 | 24 | 4 | 0 | 28 | 47 | 8 | 1 | 56 |
| Sağlık | 23 | 4 | 1 | 28 | 24 | 4 | 1 | 29 | 47 | 8 | 2 | 57 |
| Sinem | 24 | 4 | 1 | 29 | 23 | 3 | 2 | 28 | 47 | 7 | 3 | 57 |
| Şifa | 23 | 4 | 2 | 29 | 24 | 4 | 0 | 28 | 47 | 8 | 2 | 57 |
| Yalçın | 24 | 3 | 1 | 28 | 23 | 3 | 3 | 29 | 47 | 6 | 4 | 57 |
| Yüce | 24 | 4 | 1 | 29 | 22 | 4 | 2 | 28 | 46 | 8 | 3 | 57 |
| Fairness Score | 0,66 | 0,53 | 0,47 | 0,83 | 1,03 | 0,42 | 0,80 | 0,95 | 0,91 | 0,84 | 0,82 | 0,70 |
| Average | 0,56 |  |  |  | 0,75 |  |  |  | 0,86 |  |  |  |

Table 5. The distribution of the pharmacies in the 2019 and 2020

While the proposed model enables pharmacies to be on duty at a fixed frequency, it tries to balance cumulative duty distribution. For this, it needs the distribution of duties from the previous period. Using the 2019 duty distribution as the parameter d, the model result of 2020 allows us to measure model performance with the real duty list of 2020. Table 6 shows the duty distribution of the model for 2020 using the 2019 duty distribution and the duty frequency of pharmacies in this distribution.

|  | Model-2020 |  |  |  | Cumulative <br> (Real-2019 + Model-2020) |  |  |  | The duty frequencies of pharmacies for Model-2020 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Pharmacy | W | 5 | BH | Total | W | 5 | BH | Total | Max | Min | Average | Std. Dev. |
| Derman | 23 | 4 | 1 | 28 | 46 | 8 | 1 | 55 | 13 | 13 | 13 | 0 |
| Elif | 24 | 3 | 1 | 28 | 47 | 7 | 2 | 56 | 13 | 13 | 13 | 0 |
| Gümüşhane | 24 | 4 | 0 | 28 | 46 | 8 | 1 | 55 | 13 | 13 | 13 | 0 |
| Hayat | 24 | 4 | 0 | 28 | 47 | 7 | 1 | 55 | 13 | 13 | 13 | 0 |
| Işık | 23 | 4 | 1 | 28 | 45 | 8 | 2 | 55 | 13 | 13 | 13 | 0 |
| Karaca | 23 | 4 | 1 | 28 | 47 | 7 | 3 | 57 | 13 | 13 | 13 | 0 |
| Merkez | 24 | 3 | 1 | 28 | 47 | 8 | 2 | 57 | 13 | 13 | 13 | 0 |
| Özkan | 24 | 3 | 2 | 29 | 47 | 7 | 3 | 57 | 13 | 13 | 13 | 0 |
| Sağlık | 22 | 4 | 2 | 28 | 45 | 8 | 3 | 56 | 13 | 13 | 13 | 0 |
| Sinem | 22 | 4 | 2 | 28 | 46 | 8 | 3 | 57 | 13 | 13 | 13 | 0 |
| Şifa | 24 | 4 | 1 | 29 | 47 | 8 | 3 | 58 | 13 | 13 | 13 | 0 |
| Yalçın | 22 | 4 | 2 | 28 | 46 | 7 | 3 | 56 | 13 | 13 | 13 | 0 |
| Yüce | 22 | 4 | 2 | 28 | 46 | 8 | 3 | 57 | 13 | 13 | 13 | 0 |
| Fairness Score | 0,86 | 0,42 | 0,70 | 0,36 | 0,72 | 0,49 | 0,82 | 0,97 |  |  |  |  |
| Average | 0,66 |  |  |  | 0,68 |  |  |  |  |  |  |  |

Table 6. Model results and the duty frequencies for 2020
When the Real-2020 results in Table 5 and the Model- 2020 results in Table 6 are compared, it is seen that the model provides a fairer distribution. In addition, the cumulative distribution shows that unfairness in 2019 can be tolerated in 2020, the average fairness score has increased considerably. The improvement in the fairness score and the stability in duty frequency shows that the model meets the expectations of the pharmacies.

## 5. Conclusion and Policy Recommendation

In this study, we handle the duty scheduling problem of pharmacies in Gümüşhane City Center. While the PDS literature focuses on combining the conflicting interests of the customers and pharmacist stakeholders, this study focused only on the satisfaction of pharmacists due to the special situation of the Gümüşhane. This specialty causes a single pharmacy to be on duty every day and reduces the problem to only ensuring a fair distribution of duties. The factor that affects the satisfaction of pharmacists is that the frequency of the duty is variable. While the decision-maker manually assigned the duties in each subset to the pharmacies to create a fair schedule, it cannot prevent the duty day from converging.

The proposed model can distribute the duties as fairly as possible, taking into account the previous duty distribution, provided that it ensures a fixed duty sequence. The results of the analysis verify the behavior of the model. Automatic scheduling allows the decision-maker to create schedules that are quick and do not require accuracy control.

In the current practice, the planning period is divided into three subsets according to the attribute of the days. However, the days of duties for pharmacists differ according
to the duration of the duty. From this point of view, it is reasonable to divide the planning period into two subsets, which last for 16 hours and 24 hours. When the distribution of the duties was examined using the two subset configurations, it was found that a fairer schedule was obtained. Accordingly, a two-subset approach based on duty duration can be offered to policymakers as a recommendation.

The contributions of the study are that it provides a case-specific example of the limited PDS literature. Also, the factors considered in the study have the potential to be a solution for the problems faced by future researchers and decision-makers.

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## Appendix

## A-1 January-April Duty Schedule

| January | Duty | February | Duty | March | Duty | April | Duty |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Wednesday, January 1, 2020 | E6 | Saturday, February 1, 2020 | E7 | Sunday, March 1, 2020 | E8 | Wednesday, April 1, 2020 | E6 |
| Thursday, January 2, 2020 | E4 | Sunday, February 2, 2020 | E9 | Monday, March 2, 2020 | E13 | Thursday, April 2, 2020 | E4 |
| Friday, January 3, 2020 | E12 | Monday, February 3, 2020 | E1 | Tuesday, March 3, 2020 | E2 | Friday, April 3, 2020 | E12 |
| Saturday, January 4, 2020 | E11 | Tuesday, February 4, 2020 | E8 | Wednesday, March 4, 2020 | E10 | Saturday, April 4, 2020 | E11 |
| Sunday, January 5, 2020 | E5 | Wednesday, February 5, 2020 | E13 | Thursday, March 5, 2020 | E3 | Sunday, April 5, 2020 | E5 |
| Monday, January 6, 2020 | E7 | Thursday, February 6, 2020 | E2 | Friday, March 6, 2020 | E6 | Monday, April 6, 2020 | E7 |
| Tuesday, January 7, 2020 | E9 | Friday, February 7, 2020 | E10 | Saturday, March 7, 2020 | E4 | Tuesday, April 7, 2020 | E9 |
| Wednesday, January 8, 2020 | E1 | Saturday, February 8, 2020 | E3 | Sunday, March 8, 2020 | E12 | Wednesday, April 8, 2020 | E1 |
| Thursday, January 9, 2020 | E8 | Sunday, February 9, 2020 | E6 | Monday, March 9, 2020 | E11 | Thursday, April 9, 2020 | E8 |
| Friday, January 10, 2020 | E13 | Monday, February 10, 2020 | E4 | Tuesday, March 10, 2020 | E5 | Friday, April 10, 2020 | E13 |
| Saturday, January 11, 2020 | E2 | Tuesday, February 11, 2020 | E12 | Wednesday, March 11, 2020 | E7 | Saturday, April 11, 2020 | E2 |
| Sunday, January 12, 2020 | E10 | Wednesday, February 12, 2020 | E11 | Thursday, March 12, 2020 | E9 | Sunday, April 12, 2020 | E10 |
| Monday, January 13, 2020 | E3 | Thursday, February 13, 2020 | E5 | Friday, March 13, 2020 | E1 | Monday, April 13, 2020 | E3 |
| Tuesday, January 14, 2020 | E6 | Friday, February 14, 2020 | E7 | Saturday, March 14, 2020 | E8 | Tuesday, April 14, 2020 | E6 |
| Wednesday, January 15, 2020 | E4 | Saturday, February 15, 2020 | E9 | Sunday, March 15, 2020 | E13 | Wednesday, April 15, 2020 | E4 |
| Thursday, January 16, 2020 | E12 | Sunday, February 16, 2020 | E1 | Monday, March 16, 2020 | E2 | Thursday, April 16, 2020 | E12 |
| Friday, January 17, 2020 | E11 | Monday, February 17, 2020 | E8 | Tuesday, March 17, 2020 | E10 | Friday, April 17, 2020 | E11 |
| Saturday, January 18, 2020 | E5 | Tuesday, February 18, 2020 | E13 | Wednesday, March 18, 2020 | E3 | Saturday, April 18, 2020 | E5 |
| Sunday, January 19, 2020 | E7 | Wednesday, February 19, 2020 | E2 | Thursday, March 19, 2020 | E6 | Sunday, April 19, 2020 | E7 |
| Monday, January 20, 2020 | E9 | Thursday, February 20, 2020 | E10 | Friday, March 20, 2020 | E4 | Monday, April 20, 2020 | E9 |
| Tuesday, January 21, 2020 | E1 | Friday, February 21, 2020 | E3 | Saturday, March 21, 2020 | E12 | Tuesday, April 21, 2020 | E1 |
| Wednesday, January 22, 2020 | E8 | Saturday, February 22, 2020 | E6 | Sunday, March 22, 2020 | E11 | Wednesday, April 22, 2020 | E8 |
| Thursday, January 23, 2020 | E13 | Sunday, February 23, 2020 | E4 | Monday, March 23, 2020 | E5 | Thursday, April 23, 2020 | E13 |
| Friday, January 24, 2020 | E2 | Monday, February 24, 2020 | E12 | Tuesday, March 24, 2020 | E7 | Friday, April 24, 2020 | E2 |
| Saturday, January 25, 2020 | E10 | Tuesday, February 25, 2020 | E11 | Wednesday, March 25, 2020 | E9 | Saturday, April 25, 2020 | E10 |
| Sunday, January 26, 2020 | E3 | Wednesday, February 26, 2020 | E5 | Thursday, March 26, 2020 | E1 | Sunday, April 26, 2020 | E3 |
| Monday, January 27, 2020 | E6 | Thursday, February 27, 2020 | E7 | Friday, March 27, 2020 | E8 | Monday, April 27, 2020 | E6 |
| Tuesday, January 28, 2020 | E4 | Friday, February 28, 2020 | E9 | Saturday, March 28, 2020 | E13 | Tuesday, April 28, 2020 | E4 |
| Wednesday, January 29, 2020 | E12 | Saturday, February 29, 2020 | E1 | Sunday, March 29, 2020 | E2 | Wednesday, April 29, 2020 | E12 |
| Thursday, January 30, 2020 | E11 |  |  | Monday, March 30, 2020 | E10 | Thursday, April 30, 2020 | E11 |
| Friday, January 31, 2020 | E5 |  |  | Tuesday, March 31, 2020 | E3 |  |  |

A-2 May-August Duty Schedule

| May | Duty | June | Duty | July | Duty | August | Duty |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Friday, May 1, 2020 | E5 | Monday, June 1, 2020 | E13 | Wednesday, July 1, 2020 | E6 | Saturday, August 1, 2020 | E7 |
| Saturday, May 2, 2020 | E7 | Tuesday, June 2, 2020 | E2 | Thursday, July 2, 2020 | E4 | Sunday, August 2, 2020 | E9 |
| Sunday, May 3, 2020 | E9 | Wednesday, June 3, 2020 | E10 | Friday, July 3, 2020 | E12 | Monday, August 3, 2020 | E1 |
| Monday, May 4, 2020 | E1 | Thursday, June 4, 2020 | E3 | Saturday, July 4, 2020 | E11 | Tuesday, August 4, 2020 | E8 |
| Tuesday, May 5, 2020 | E8 | Friday, June 5, 2020 | E6 | Sunday, July 5, 2020 | E5 | Wednesday, August 5, 2020 | E13 |
| Wednesday, May 6, 2020 | E13 | Saturday, June 6, 2020 | E4 | Monday, July 6, 2020 | E7 | Thursday, August 6, 2020 | E2 |
| Thursday, May 7, 2020 | E2 | Sunday, June 7, 2020 | E12 | Tuesday, July 7, 2020 | E9 | Friday, August 7, 2020 | E10 |
| Friday, May 8, 2020 | E10 | Monday, June 8, 2020 | E11 | Wednesday, July 8, 2020 | E1 | Saturday, August 8, 2020 | E3 |
| Saturday, May 9, 2020 | E3 | Tuesday, June 9, 2020 | E5 | Thursday, July 9, 2020 | E8 | Sunday, August 9, 2020 | E6 |
| Sunday, May 10, 2020 | E6 | Wednesday, June 10, 2020 | E7 | Friday, July 10, 2020 | E13 | Monday, August 10, 2020 | E4 |
| Monday, May 11, 2020 | E4 | Thursday, June 11, 2020 | E9 | Saturday, July 11, 2020 | E2 | Tuesday, August 11, 2020 | E12 |
| Tuesday, May 12, 2020 | E12 | Friday, June 12, 2020 | E1 | Sunday, July 12, 2020 | E10 | Wednesday, August 12, 2020 | E11 |
| Wednesday, May 13, 2020 | E11 | Saturday, June 13, 2020 | E8 | Monday, July 13, 2020 | E3 | Thursday, August 13, 2020 | E5 |
| Thursday, May 14, 2020 | E5 | Sunday, June 14, 2020 | E13 | Tuesday, July 14, 2020 | E6 | Friday, August 14, 2020 | E7 |
| Friday, May 15, 2020 | E7 | Monday, June 15, 2020 | E2 | Wednesday, July 15, 2020 | E4 | Saturday, August 15, 2020 | E9 |
| Saturday, May 16, 2020 | E9 | Tuesday, June 16, 2020 | E10 | Thursday, July 16, 2020 | E12 | Sunday, August 16, 2020 | E1 |
| Sunday, May 17, 2020 | E1 | Wednesday, June 17, 2020 | E3 | Friday, July 17, 2020 | E11 | Monday, August 17, 2020 | E8 |
| Monday, May 18, 2020 | E8 | Thursday, June 18, 2020 | E6 | Saturday, July 18, 2020 | E5 | Tuesday, August 18, 2020 | E13 |
| Tuesday, May 19, 2020 | E13 | Friday, June 19, 2020 | E4 | Sunday, July 19, 2020 | E7 | Wednesday, August 19, 2020 | E2 |
| Wednesday, May 20, 2020 | E2 | Saturday, June 20, 2020 | E12 | Monday, July 20, 2020 | E9 | Thursday, August 20, 2020 | E10 |
| Thursday, May 21, 2020 | E10 | Sunday, June 21, 2020 | E11 | Tuesday, July 21, 2020 | E1 | Friday, August 21, 2020 | E3 |
| Friday, May 22, 2020 | E3 | Monday, June 22, 2020 | E5 | Wednesday, July 22, 2020 | E8 | Saturday, August 22, 2020 | E6 |
| Saturday, May 23, 2020 | E6 | Tuesday, June 23, 2020 | E7 | Thursday, July 23, 2020 | E13 | Sunday, August 23, 2020 | E4 |
| Sunday, May 24, 2020 | E4 | Wednesday, June 24, 2020 | E9 | Friday, July 24, 2020 | E2 | Monday, August 24, 2020 | E12 |
| Monday, May 25, 2020 | E12 | Thursday, June 25, 2020 | E1 | Saturday, July 25, 2020 | E10 | Tuesday, August 25, 2020 | E11 |
| Tuesday, May 26, 2020 | E11 | Friday, June 26, 2020 | E8 | Sunday, July 26, 2020 | E3 | Wednesday, August 26, 2020 | E5 |
| Wednesday, May 27, 2020 | E5 | Saturday, June 27, 2020 | E13 | Monday, July 27, 2020 | E6 | Thursday, August 27, 2020 | E7 |
| Thursday, May 28, 2020 | E7 | Sunday, June 28, 2020 | E2 | Tuesday, July 28, 2020 | E4 | Friday, August 28, 2020 | E9 |
| Friday, May 29, 2020 | E9 | Monday, June 29, 2020 | E10 | Wednesday, July 29, 2020 | E12 | Saturday, August 29, 2020 | E1 |
| Saturday, May 30, 2020 | E1 | Tuesday, June 30, 2020 | E3 | Thursday, July 30, 2020 | E11 | Sunday, August 30, 2020 | E8 |
| Sunday, May 31, 2020 | E8 |  |  | Friday, July 31, 2020 | E5 | Monday, August 31, 2020 | E13 |

## A-3 September-December Duty Schedule

| September | Duty | October | Duty | November | Duty | December | Duty |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Tuesday, September 1, 2020 | E2 | Thursday, October 1, 2020 | E4 | Sunday, November 1, 2020 | E9 | Tuesday, December 1, 2020 | E2 |
| Wednesday, September 2, 2020 | E10 | Friday, October 2, 2020 | E12 | Monday, November 2, 2020 | E1 | Wednesday, December 2, 2020 | E10 |
| Thursday, September 3, 2020 | E3 | Saturday, October 3, 2020 | E11 | Tuesday, November 3, 2020 | E8 | Thursday, December 3, 2020 | E3 |
| Friday, September 4, 2020 | E6 | Sunday, October 4, 2020 | E5 | Wednesday, November 4, 2020 | E13 | Friday, December 4, 2020 | E6 |
| Saturday, September 5, 2020 | E4 | Monday, October 5, 2020 | E7 | Thursday, November 5, 2020 | E2 | Saturday, December 5, 2020 | E4 |
| Sunday, September 6, 2020 | E12 | Tuesday, October 6, 2020 | E9 | Friday, November 6, 2020 | E10 | Sunday, December 6, 2020 | E12 |
| Monday, September 7, 2020 | E11 | Wednesday, October 7, 2020 | E1 | Saturday, November 7, 2020 | E3 | Monday, December 7, 2020 | E11 |
| Tuesday, September 8, 2020 | E5 | Thursday, October 8, 2020 | E8 | Sunday, November 8, 2020 | E6 | Tuesday, December 8, 2020 | E5 |
| Wednesday, September 9, 2020 | E7 | Friday, October 9, 2020 | E13 | Monday, November 9, 2020 | E4 | Wednesday, December 9, 2020 | E7 |
| Thursday, September 10, 2020 | E9 | Saturday, October 10, 2020 | E2 | Tuesday, November 10, 2020 | E12 | Thursday, December 10, 2020 | E9 |
| Friday, September 11, 2020 | E1 | Sunday, October 11, 2020 | E10 | Wednesday, November 11, 2020 | E11 | Friday, December 11, 2020 | E1 |
| Saturday, September 12, 2020 | E8 | Monday, October 12, 2020 | E3 | Thursday, November 12, 2020 | E5 | Saturday, December 12, 2020 | E8 |
| Sunday, September 13, 2020 | E13 | Tuesday, October 13, 2020 | E6 | Friday, November 13, 2020 | E7 | Sunday, December 13, 2020 | E13 |
| Monday, September 14, 2020 | E2 | Wednesday, October 14, 2020 | E4 | Saturday, November 14, 2020 | E9 | Monday, December 14, 2020 | E2 |
| Tuesday, September 15, 2020 | E10 | Thursday, October 15, 2020 | E12 | Sunday, November 15, 2020 | E1 | Tuesday, December 15, 2020 | E10 |
| Wednesday, September 16, 2020 | E3 | Friday, October 16, 2020 | E11 | Monday, November 16, 2020 | E8 | Wednesday, December 16, 2020 | E3 |
| Thursday, September 17, 2020 | E6 | Saturday, October 17, 2020 | E5 | Tuesday, November 17, 2020 | E13 | Thursday, December 17, 2020 | E6 |
| Friday, September 18, 2020 | E4 | Sunday, October 18, 2020 | E7 | Wednesday, November 18, 2020 | E2 | Friday, December 18, 2020 | E4 |
| Saturday, September 19, 2020 | E12 | Monday, October 19, 2020 | E9 | Thursday, November 19, 2020 | E10 | Saturday, December 19, 2020 | E12 |
| Sunday, September 20, 2020 | E11 | Tuesday, October 20, 2020 | E1 | Friday, November 20, 2020 | E3 | Sunday, December 20, 2020 | E11 |
| Monday, September 21, 2020 | E5 | Wednesday, October 21, 2020 | E8 | Saturday, November 21, 2020 | E6 | Monday, December 21, 2020 | E5 |
| Tuesday, September 22, 2020 | E7 | Thursday, October 22, 2020 | E13 | Sunday, November 22, 2020 | E4 | Tuesday, December 22, 2020 | E7 |
| Wednesday, September 23, 2020 | E9 | Friday, October 23, 2020 | E2 | Monday, November 23, 2020 | E12 | Wednesday, December 23, 2020 | E9 |
| Thursday, September 24, 2020 | E1 | Saturday, October 24, 2020 | E10 | Tuesday, November 24, 2020 | E11 | Thursday, December 24, 2020 | E1 |
| Friday, September 25, 2020 | E8 | Sunday, October 25, 2020 | E3 | Wednesday, November 25, 2020 | E5 | Friday, December 25, 2020 | E8 |
| Saturday, September 26, 2020 | E13 | Monday, October 26, 2020 | E6 | Thursday, November 26, 2020 | E7 | Saturday, December 26, 2020 | E13 |
| Sunday, September 27, 2020 | E2 | Tuesday, October 27, 2020 | E4 | Friday, November 27, 2020 | E9 | Sunday, December 27, 2020 | E2 |
| Monday, September 28, 2020 | E10 | Wednesday, October 28, 2020 | E12 | Saturday, November 28, 2020 | E1 | Monday, December 28, 2020 | E10 |
| Tuesday, September 29, 2020 | E3 | Thursday, October 29, 2020 | E11 | Sunday, November 29, 2020 | E8 | Tuesday, December 29, 2020 | E3 |
| Wednesday, September 30, 2020 | E6 | Friday, October 30, 2020 | E5 | Monday, November 30, 2020 | E13 | Wednesday, December 30, 2020 | E6 |
|  |  | Saturday, October 31, 2020 | E7 |  |  | Thursday, December 31, 2020 | E4 |


[^0]:    Keywords:
    Pharmacy Duty Scheduling, Integer Programming, The Fixed Duty Frequency, The Inheritance of Duty Distribution

[^1]:    ${ }^{1}$ For the relevant readers, the average calculation time of the two stages is approximately 2 seconds.

[^2]:    ${ }^{2}$ Duty Pharmacy Lists
    2019 January, February, March: https://www.trabzoneczaciodasi.org.tr/duyuru/31122018-gumushane-ocak-subat-mart-nobet-listeleri-4559
    2019 April, May, June:
    https://www.trabzoneczaciodasi.org.tr/duyuru/gumushane-ili-nisan-mayis-haziran-2019-aylarina-ait-nobet-listesi-4647
    2019 July, August, September: https://www.trabzoneczaciodasi.org.tr/duyuru/28062019-gumushane-merkez-temmuz-agustos-eylul-2019-nobet-listesi-4705 2019 October, November, December: https://www.trabzoneczaciodasi.org.tr/duyuru/30092019-gumushane-merkez-ekim-kasim-aralik-2019-nobet-listeleri-4749 2020 January, February, March: https://www.trabzoneczaciodasi.org.tr/duyuru/27122019-gumushane-merkez-ocak-subat-mart-2020-nobet-listesi-4847
    https://www.trabzoneczaciodasi.org.tr/duyuru/26032020-gumushane-ili-nisan-mayis-haziran-2020-nobetleri-4984 2020 October, November, December: https://www.trabzoneczaciodasi.org.tr/duyuru/28092020-gumushane-ili-ekim-kasim-aralik-2020-nobetleri-5241

