



INFLUENCE OF UNCONVENTIONAL CURRENT-PHASE RELATION ON CHAOTIC DYNAMICS OF JOSEPHSON JUNCTIONS

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ABSTRACT

In this paper, we study the influence of unconventional current-phase relation (CPR) of Josephson junction on chaotic dynamics. The influence of the second term of CPR on an externally shunted Josephson junction with nonzero inductance has been studied. Using the circuit model, the time-dependent simulations are carried out for a variety of control parameters. It is shown that the presence of the second term on CPR leads to a change in the boundary of the chaotic region in the bifurcation diagram.

Keywords: Josephson junction, Current-phase relation, Chaotic dynamics

1. INTRODUCTION

Josephson junction devices have been used in many applications such as ultrahigh sensitive detectors and superconducting quantum interference devices [1,2]. It is well known that many simple nonlinear systems, including Josephson circuits can exhibit chaotic dynamics. In this manner, Josephson junction devices could be useful for ultrahigh-speed chaotic generators for applications of code generation in spread-spectrum communications [3] and true random number generation in secure communication and encryption [4, 5]. From this point of view, the dynamics of Josephson junctions are of great importance in contemporary superconducting electronics [6] so that junction devices can be used as high-frequency chaotic signal generators.

It is well known that, in the study dynamics of Josephson junction up to now, the harmonic CPR $I = I_c \sin \phi$ was used [7]. The corresponding equation of the resistively shunted junction model [7, 8] is given by the equation

$$\beta \ddot{\phi} + \dot{\phi} + \sin \phi = i_e, \quad (1)$$

where i_e is normalized external current via Josephson junction in units of critical current I_c , dots over ϕ corresponds to the derivative with respect to dimensionless time $\frac{\Phi_0}{2\pi I_c R_N}$, Φ_0 is the magnetic flux quantum, R_N is the normal resistance of junction. In Eq. (1) notation β is the McCumber parameter of Josephson junction $\beta = \frac{2e}{\hbar} I_c R_N^2 C$. Harmonic CPR in JJ is based on low temperatures

superconductors experimentally demonstrated many years ago [7,8]. A very recent study shows that the in junctions on new topological superconductors, the CPR include additional fractional term [9, 10],

$$I = I_c f_m(\phi) = I_{c0}(\sin\phi + m \sin(\phi/2)) \quad (2a)$$

The second term in Eq. (2a) related to Majorana quasi-particles and dynamical detection of these particles seems very intriguing condensed matter physics. Some dynamical properties of Josephson junction with anharmonic CPR of the type

$$I = I_{c0} f_\alpha(\phi) = (\sin\phi + \alpha \sin(2\phi)) \quad (2b)$$

was investigated in Ref. [11-13]. Such type of CPR was observed experimentally in Josephson junctions based on high temperature and many band superconductors [8]. In the present study, we conduct the influence of unconventional CPR (2a,2b) on the chaotic behavior of the shunted Josephson junction circuits for various parameters. Our investigation is based on numerical simulations using an externally shunted Josephson junction circuit model with nonzero inductance similar to the one given in [13] with nonlinear resistance.

2. BASIC EQUATIONS and RESULTS

The dynamics of shunted Josephson junction for the case of general CPR (2a,b) is given by the equation of resistive model [7, 8] (Figure 1)

$$\dot{\phi} = v, \quad (3a)$$

$$\dot{v} = \frac{1}{\beta} (i_e - g(v)v - f_{m,\alpha}(\phi) - i_s), \quad (3b)$$

$$i_s = \frac{1}{\beta_L} (v - i_s). \quad (3c)$$

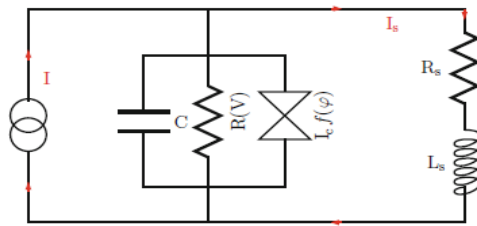


Figure 1. Equivalent circuit of externally shunted Josephson junction with unconventional CPR

$g(v) = R_s/R(v)$ is the normalized tunnel junction conductance; $i_s = I_s/I_c$ is dimensionless shunt current; $i_{dc} = I_{dc}/I_c$ is dimensionless external direct bias current; $\beta_L = \frac{2e}{\hbar} I_c L_s$ is the dimensionless inductance; $\tau = \omega_c t$ is the normalized time in which $\omega_c = \frac{2e}{\hbar} V_c$ is the characteristic frequency and $V_c = I_c R_s$ is the characteristic voltage.

The quantitative criterion for a time series to be chaotic is the positivity of one of its Lyapunov exponents [8,14]. We have three independent variables in the resistively shunted models [7] and we compute only the largest Lyapunov exponent, which is sufficient to distinguish a chaotic signal from an aperiodic signal. Lyapunov exponents are obtained check whether the time series to be chaotic by looking at the positivity of one of its Lyapunov exponents. A bifurcation diagram is useful tool for assessing the characteristics of the steady-state solutions of a system over the range of control parameters. Figure 2 shows bifurcation diagram obtained using the local maxima method. As shown in Figure 2, the presence of the second term in CPR leads to a change in the bifurcation diagrams with changing the control parameter β_L . Such shifting behavior of the bifurcation diagram also observed for different values of the amplitude of second term (m and α).

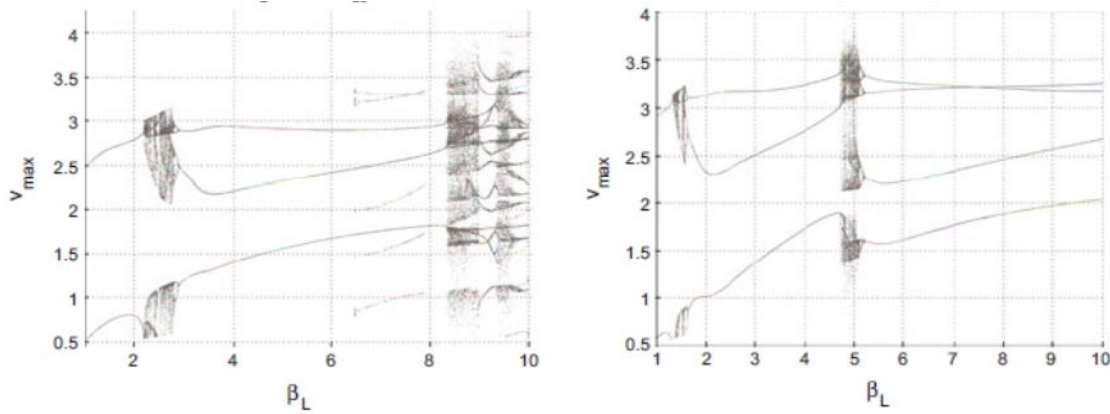


Figure 2. Bifurcation diagram of Josephson junction with conventional ($m = \alpha = 0$). (left) and unconventional (right) CPR $\alpha = 0.5$, $i_e = 1.55$, $\beta = 0.70$.

Determining appropriate chaotic regions from bifurcation diagrams in Figure 2 we can obtain time series representations in Figure 3.

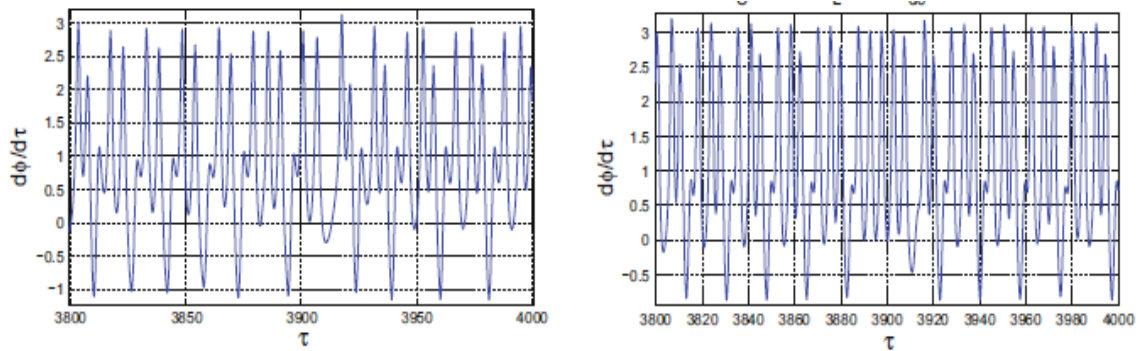


Figure 3. Time series for Josephson junction with harmonic ($m = \alpha = 0$). (left) and unconventional (right) CPR $\alpha = 0.5$, $i_e = 1.55$, $\beta = 0.70$ (left) and anharmonics (right) CPR

For the interpretation of the influence of the second term in unconventional CPR leads to an increase in the normalized critical current: I_c/I_{c0} where I_{c0} is the critical current at $\alpha = m = 0$. Therefore, it causes a decrease in the inductance of the JJ

$$L_J(\alpha, m) = \frac{\hbar}{2eI_{c0}} \frac{1}{\max(f_{m,\alpha})} \quad , \quad (4)$$

where explicit expression $\max(f_{\alpha,m})$ presented in Ref. [15]:

$$f_{m,\alpha} = \begin{cases} 1 + 2\alpha^2, \text{anharmonic term} \\ 1 + \frac{m}{\sqrt{2}} + \frac{7}{64}m^2, \text{Majorana term} \end{cases} \quad (5)$$

It is well known that the influence of shunted inductance L_s on the dynamics of Josephson junction is important if it is comparable with Josephson inductance (i.e., $L_s \approx L_J$). Consequently, the values of the shunted inductance L_s together with L_J has shifted to lower values (Figure 2) and it leads to the above-mentioned changes in bifurcation diagrams of the junctions with unconventional CPR. Decreasing of normalized inductance β_L , which corresponds to the first bifurcation point as a function of the amplitude of second term (2a,2b) presented in Figure 4. In calculation, we also use results of Ref. [15] renormalization of the critical current of the junction with CPR (2a) and (2b). It means that in both cases of CPR (2a) and (2b), the boundary of bifurcation region shifted to lower values of normalized inductance β_L . Also clear that in the case of CPR with Majorana term (2a) such influence becomes more effective than in case (2b).

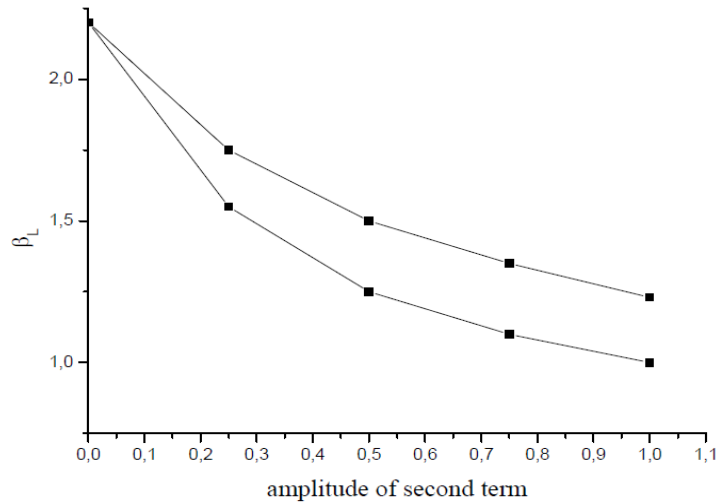


Figure 4. The boundary of the chaotic region in the bifurcation diagram versus amplitude of second term (upper line correspond to CPR (2b), the lower line to CPR (2a))

Thus, the influence of unconventional CPR of Josephson junction on chaotic dynamics was analyzed. Using the circuit model it is shown that the presence of the second term on CPR leads to a decreasing of the boundary value of the chaotic region in the bifurcation diagram.

I thank N.Kartlı, H.B.Yildirim for the discussion. This study was supported by TÜBİTAK grant 118F093.

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