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Some Fixed Point Theorems for Cyclic Mapping in a Complete B-metric-like Space

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Abstract

In recent years, the research of fixed point theorem is a hot topic all the time. In this paper, we proposed the notion of a new mapping, that is, a general LW-type Lipschitz cyclic mapping, in complete b-metric-like spaces. Then, we obtained the existence and uniqueness theorems of its fixed point. Moreover, we give some examples to illustrate the main results in this paper.

Keywords: b-metric-like space, fixed point theorem, cyclic mapping.
2010 MSC: 47H09, 47H10, 47J25.

1. Introduction

Fixed point theory play an important role in the field of functional analysis, even in the whole mathematical field. With the development of generation of metric space ([1, 2, 3, 11, 12, 13]), the fixed point theory is further extended.

Next, we introduce the concept of a metric space as follows.

Definition 1.1. Let X be a nonempty set, $d : X \times X \to [0, \infty)$ be a function such that for all $x, y \in X$ the following three conditions satisfy:

$(d1)$ $d(x, y) \geq 0,\ d(x, y) = 0 \Longleftrightarrow x = y$;

$(d2)$ $d(x, y) = d(y, x)$;

$(d3)$ $d(x, y) \leq d(x, z) + d(z, y)$.

Then, the pair (X, d) is called a metric space.

Example 1 Usually, we define the metric by the absolute value, that is, $d(x, y) = |x - y|$. It is easy to verify that a metric defined like this satisfies the conditions $d1 - d3$.

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The concept of b-metric-like space was introduced in[4] as follows.

Definition 1.2. A b-metric-like on a nonempty set X is a function $D : X \times X \to [0, +\infty)$ such that for all $x, y, z \in X$ and a constant $K \geq 1$ the following three conditions hold true:

$(D1)$ if $D(x, y) = 0 \Rightarrow x = y$;

$(D2)$ $D(x, y) = D(y, x)$;

$(D3)$ $D(x, y) \leq K(D(x, z) + D(z, y))$.

The pair (X, D) is called a b-metric-like space.

Example 2 Let $X = [0, \infty)$ and let $r : X \times X \longrightarrow [0, \infty 1)$ be defined by $r(x, y) = (x + y)^2$. It is easy to know that $s = 2$ and $(X, r, 2)$ is a complete b-metric-like space.

Example 3 Let $X = C([0, T])$ be the set of real continuous functions on [0,T]. We endow X with the b-metric-like

$$R_\infty(u, v) = \sup_{t\in[0,T]} (|u(t)| + |v(t)|)^2,\ for\ all\ u, v \in X.$$

Clearly, $(X, R_\infty, 2)$ is a complete b-metric-like space.

In a metric space, Banach contractions principle is a classical result in fixed point theory. Here, we state the concept of contractive mapping as follows.

Definition 1.3. Let X be a metric space, $T : X \to X$ be a mapping, if there exists a constant $\alpha \in (0, 1)$ such that for all $x, y \in X$, the following condition is satisfied:

$$d(Tx, Ty) \leq \alpha d(x, y).$$

Then, T is called a contractive mapping.

It is natural that Banach contractions principle[5] is extended in kinds of metric space, for example, partial metric space, b-metric space, b-metric-like space and so on.

In 2017, Lei and Wu [6] proposed the concept of LW-type cyclic mapping in a complete b-metric-like space (especially, the meaning of b-metric-like is as Definition 1.2) as follows.

Definition 1.4. Let (X, r) be a b-metric-like space, $G_1,\ G_2$ be nonempty closed sets in (X, r). If (B, S) is a pair semi-cyclic mapping in $G_1 \times G_2$ and exists some nonnegative real constants γ, δ, t such that for all $x \in G_1, y \in G_2$ satisfy the following condition:

$$\beta r(x, Bx) + \delta r(y, Sy) + tr(Bx, Sy) \leq r(x, y).$$

Then, (B, S) is called as a LW-type cyclic mapping.

They obtained the one main fixed point theorem as follows.

Theorem 1.1 Suppose that (X, r) is a complete b-metric-like space. Let (B, S) be a general $LW-$type cyclic mapping, and $G_1,\ G_2$ be nonempty closed sets in (X, r). If $\beta + \delta + t > s,\ t > 1$, then there exists a unique $x^* \in G_1 \bigcap G_2$ such that $Bx^* = x^* = Sx^*$, that is, B and S have a unique common fixed point.

In 2018, S.Q.Weng, Y.Zhang and D.P. Wu [7] proposed the concept of $LW-$type cyclic mapping in a complete b-metric-like space as follows.

Definition 1.5. Let (X, r) be a b-metric-like space, $G_1,\ G_2$ be nonempty closed sets in (X, r). If (B, S) is a pair semi-cyclic mapping in $G_1 \times G_2$ and exists some nonnegative real constants γ, δ, t, L and $L < \gamma + \delta + t$ such that for all $x \in G_1, y \in G_2$ satisfy the following condition:

$$\beta r(x, Bx) + \delta r(y, Sy) + tr(Bx, Sy) \leq Lr(x, y).$$

Then, (B, S) is called as a LW-type cyclic mapping.

They obtained the one main fixed point theorem as follows.

Theorem 1.2 Suppose that (X, r) is a complete b-metric-like space. Let (B, S) be a general $LW-$type Lipschitz cyclic mapping, and $G_1,\ G_2$ be nonempty closed sets in (X, r). If $L \leq t, L = \max\{\beta, \delta\}, \beta \neq \delta$, and $s \in [1, \frac{L+t}{|\beta-\delta|})$, then there exists a unique $x^* \in G_1 \bigcap G_2$ such that $Bx^* = x^* = Sx^*$, that is, B and S have a unique common fixed point.

In 2019, S.Q.Weng [8] proposed the concept of a general LW -type cyclic mapping in a complete b-metric-like space as follows.

Based on this, we propose the notion of a mapping, and named it as a general LW-type cyclic mapping as follows.

Definition 1.6. Let (X, r) be a b-metric-like space, G_1, G_2 be nonempty closed sets in (X, r). If (B, S) is a pair semi-cyclic mapping in $G_1 \times G_2$ and exists some nonnegative functions γ, δ, t such that for all $x \in G_1, y \in G_2$ satisfy the following condition:

$$\beta(r(x,y))r(x,Bx) + \delta(r(x,y))r(y,Sy) + t(r(x,y))r(Bx,Sy) \leq r(x,y),$$

where $\gamma, \delta : [0, +\infty) \to [0, +\infty)$, $t : [0, +\infty) \to [1, +\infty)$. Then, (B, S) is called as a general LW-type cyclic mapping.

They obtained the one main fixed point theorem as follows.

Theorem 1.3 Suppose that (X, r) is a complete b-metric-like space. Let (B, S) be a general $LW-$type cyclic mapping, and G_1, G_2 be nonempty closed sets in (X, r). If it exists some λ such that the following conditions are satisfied: (1) $\max\{\frac{1-\beta(z)}{\delta(z)+t(z)}, \frac{1-\delta(z)}{\beta(z)+t(z)}, z \in [0, \infty)\} \leq \lambda < 1$, (2) $\lambda s < 1$. Then there exists a unique $x^* \in G_1 \bigcap G_2$ such that $Bx^* = x^* = Sx^*$, that is, B and S have a unique common fixed point.

Based on this, inspired by $Definition$ 1.5 and $Definition$ 1.6, we propose the notion of a mapping, and named it as a general $LW-$type cyclic mapping as follows.

Definition 1.7. Let (X, r) be a b-metric-like space, G_1, G_2 be nonempty closed sets in (X, r). If (B, S) is a pair semi-cyclic mapping in $G_1 \times G_2$ and exists some nonnegative functions γ, δ, t and a nonnegative real number L such that for all $x \in G_1, y \in G_2$ satisfy the following condition:

$$\beta(r(x,y))r(x,Bx) + \delta(r(x,y))r(y,Sy) + t(r(x,y))r(Bx,Sy) \leq Lr(x,y),$$

where $\gamma, \delta : [0, +\infty) \to [0, +\infty)$, $t : [0, +\infty) \to [1, +\infty)$. Then, (B, S) is called as a general LW-type Lipschitz cyclic mapping.

Later, we will show that the number L is constrainted by function $t(x)$ within a range.

In fact, it is easy to know that the general $LW-$type Lipschitz cyclic mapping is more general than the $LW-$type cyclic mapping. In fact, it is obvious that a general $LW-$type Lipschitz cyclic mapping is a general $LW-$type cyclic mapping when $L = 1$. Therefore, we extended the previous research work.

2. Preliminaries

We collect some necessary definitions, which will be used in next section.

Definition 2.1. [4] Let (X, r) be a b-metric-like space, and let $\{x_n\}$ be a sequence of points of X. A point $x \in X$ is said to be the limit of the sequence $\{x_n\}$ if $\lim_{n\to\infty} r(x, x_n) = r(x, x)$, and we say that the sequence $\{x_n\}$ is convergent to x and denote it by $x_n \to x$ as $n \to \infty$.

In the previous, we have given the definition of the quasi-B metric space, now we give the definition of cauchy sequence and convergence in the b-like-metric space.

Definition 2.2. [4] Let (X, r) be a b-metric-like space.

(i) A sequence $\{x_n\}$ is called Cauchy if and only if $\lim_{n\to\infty} r(x_n, x_m)$ exists and is finite.

(ii) A b-metric-like space (X, r) is said to be complete if and only if every Cauchy sequence $\{x_n\}$ in X converges to $x \in X$ so that

$$\lim_{m,n\to\infty} r(x_n, x_m) = r(x, x) = \lim_{n\to\infty} r(x_n, x).$$

Definition 2.3. [4] Let G_1, G_2 be nonempty sets of metric space, if $B(G_1) \subset G_2$, and $S(G_2) \subset G_1$, then the mapping $(B, S) : G_1 \times G_2 \to G_2 \times G_1$ is called as a pair semi-cyclic mapping, where B is said to be a lower semi-cyclic mapping, S is said to be a upper semi-cyclic mapping. If $B = S$, then B is said to be a cyclic mapping.

Definition 2.4. [9] If $\prec$ is a partially ordered in b-metric-like spaces (X, r), then $(X, r, \prec)$ is a partially ordered b-metric-like space.

3. Main results

Next, we give main results in this section.

Theorem 3.1 Suppose that (X,r) is a complete b-metric-like space. Let (B,S) be a general LW-type cyclic mapping, and G_1, G_2 be nonempty closed sets in (X,r) and $G_1\bigcap G_2$. If it exists some λ such that the following conditions are satisfied:

(1) $\max\{\frac{L-\beta(z)}{\delta(z)+t(z)}, \frac{L-\delta(z)}{\beta(z)+t(z)}, z\in[0,+\infty)\}\le\lambda<1$,

(2) $\lambda s<1$,

(3) $L\in[1,t(z)]$.

Then there exists a unique $x^*\in G_1\cap G_2$ such that $Bx^*=x^*=Sx^*$, that is, B and S have a unique common fixed point.

Proof. The sequence $\{x_n\}$ is defined as follows:

$$x_0\in G_1,\ x_1=Bx_0,\ x_2=Sx_1,\ x_3=Bx_2,\ x_4=Sx_3,...,\ n\ge 0.$$

Step 1 Prove that the sequence $\{x_n\}$ is a cauchy sequence. Because (B,S) be a general LW-Type cyclic mapping, thus we have

$$\begin{aligned}Lr(x_0,x_1)&\ge\beta(r(x_0,x_1))r(x_0,Bx_0)+\delta(r(x_0,x_1))r(x_1,Sx_1)+t(r(x_0,x_1))r(Bx_0,Sx_1)\\&=\beta(r(x_0,x_1))r(x_0,x_1)+\delta(r(x_0,x_1))r(x_1,x_2)+t(r(x_0,x_1))r(x_1,x_2),\end{aligned}$$

so we obtain

$$r(x_1,x_2)\le\frac{L-\beta(r(x_0,x_1))}{\delta(r(x_0,x_1))+t(r(x_0,x_1))}r(x_0,x_1). \tag{3.1}$$

Similarly we have

$$\begin{aligned}Lr(x_2,x_1)&\ge\beta(r(x_2,x_1))r(x_2,Bx_2)+\delta(r(x_2,x_1))r(x_1,Sx_1)+t(r(x_2,x_1))r(Bx_2,Sx_1)\\&=\beta(r(x_2,x_1))r(x_2,x_3)+\delta(r(x_2,x_1))r(x_1,x_2)+t(r(x_2,x_1))r(x_3,x_2)\end{aligned}$$

and we have

$$\begin{aligned}r(x_2,x_3)&\le\frac{L-\delta(r(x_2,x_1))}{\beta(r(x_2,x_1))+t(r(x_2,x_1))}r(x_1,x_2)\\&\le\frac{L-\delta(r(x_2,x_1))}{\beta(r(x_2,x_1))+t(r(x_2,x_1))}\cdot\frac{L-\beta(r(x_2,x_1))}{\delta(r(x_2,x_1))+t(r(x_2,x_1))}r(x_0,x_1)\end{aligned} \tag{3.2}$$

Because of $max\{\frac{1-\beta(z)}{\delta(z)+t(z)}, \frac{1-\delta(z)}{\beta(z)+t(z)}, z\in[0,+\infty)\}\le\lambda<1$, then

$$max\{\frac{L-\beta(r(x_i,x_{i-1}))}{\delta(r(x_i,x_{i-1}))+t(r(x_i,x_{i-1}))}, \frac{L-\delta(r(x_i,x_{i-1}))}{\beta(r(x_i,x_{i-1}))+t(r(x_i,x_{i-1}))}, x_i\in X, i\in N\}\le\lambda<1. \tag{3.3}$$

By (3.2) and (3.3), let $r(x_0,x_1)=C$, we have

$$r(x_2,x_3)\le\lambda^2C. \tag{3.4}$$

Let $m>n$, $\forall m,n\in N$, by $\lambda s<1$, this shows

$$\begin{aligned}r(x_n,x_m)&\le sr(x_n,x_{n+1})+s^2r(x_{n+1},x_{n+2})+...+s^{m-n}r(x_{m-1},x_m)\\&\le sL^nr(x_0,z_1)+s^2\lambda^{n+1}r(x_0,x_1)+...+s^{m-n}\lambda^{m-1}r(x_0,z_1)\\&=[s\lambda^n+s^2\lambda^{n+1}+...+s^{m-n}\lambda^{m-1}]C\\&=[(s\lambda)+(s\lambda)^2+...+(s\lambda)^{m-n}]\lambda^{n-1}C\\&=\frac{1-(s\lambda)^{m-n}}{1-(s\lambda)}s\lambda^nC\\&\le\frac{1}{1-(s\lambda)}\lambda^{n-1}C.\end{aligned} \tag{3.5}$$

From (3.4) (3.5), let $n \to \infty$, we get that

$$\lim_{n\to\infty} r(x_n, x_m) = 0. \tag{3.6}$$

This implies from (3.6) that the sequence $\{x_n\}$ is a Cauchy sequence. Due to, (X, r) is a complete b-metric-like space, then there exists a sequence $x \in X$ such that

$$x_n \to x \ (n \to \infty).$$

Therefore,

$$x_{2n} \to x; \ x_{2n+1} \to x \ (n \to \infty).$$

Because $\{x_{2n}\} \subset G_1$, $\{x_{2n+1}\} \subset G_2$, and G_1, G_2 is closed, then

$$x \in G_1 \cap G_2.$$

Step 2 Prove that x is a fixed point of the mapping B and S, that is, $Bx = x = Sx$. Because (B, S) be a general LW-type cyclic mapping, then we get that

$$\beta(r(x,x))r(x,Bx) + \delta(r(x,x))r(x,Sx) + t(r(x,x))r(Bx,Sx) \le Lr(x,x),$$

Due to $r(x,x) = \lim_{n,m\to\infty} r(x_m, x_n)$. Then, by (3.6) we have

$$r(x,Bx) = 0, \ r(x,Sx) = 0. \tag{3.7}$$

It implies that

$$Bx = x = Sx. \tag{3.8}$$

Step 3 Prove that the mappings B and S have a unique common fixed point. Now, let $x, x^* \in X$ are the common fixed points of mappings B and S in X. Then, by Definition 1.8 and (3.8), we have

$$\begin{aligned} r(x,x^*) &\ge \beta(r(x,x^*))r(x,Bx) + \delta(r(x,x^*))r(x^*,Sx^*) + t(r(x,x^*))r(Bx,Sx^*) \\ &\ge \beta(r(x,x^*))r(x,x) + \delta(r(x,x^*))r(x^*,x^*) + t(r(x,x^*))r(x,x^*) \\ &= t(r(x,x^*))r(x,x^*). \end{aligned}$$

That is,

$$Lr(x,x^*) \ge t(r(x,x^*))r(x,x^*).$$

Due to, $L \in [1, t(x)]$,that is $L < t(x)$, this is a contradiction. Thus, we obtain that

$$r(x,x^*) = 0, \ that \ is, \ x = x^*.$$

This completes the proof. □

Corollary 3.2 Suppose that (X, r) is a complete partially ordered b-metric-like space. Let (B, S) be a general LW-type Lipschitz cyclic mapping, and G_1, G_2 be nonempty closed sets in (X, r) and $G_1 \bigcap G_2$. If the following conditions are satisfied:

(1) $max\{\frac{1-\beta(z)}{\delta(z)+t(z)}, \frac{1-\delta(z)}{\beta(z)+t(z)}, z \in [0, +\infty)\} \le \lambda < 1$,

(2) $\lambda s < 1$,

(3) $L \in [1, t(z)]$.

Then there exists a unique $x^* \in G_1 \cap G_2$ such that $Bx^* = x^* = Sx^*$, that is, B and S have a unique common fixed point.

4. Illustration of Results

In this section, we will give a concrete example to illustrate the effectiveness of a general LW-type cyclic mapping and show the rationality of the obtained theorems.

Example 4.1 Consider $X = \{0,1,2\}, G_1 = \{0,1\}, G_2 = \{1,2\}$ and let $r : X \times X \to [0,+\infty)$ be defined by

$$r(0,0) = 0,\ r(1,1) = \frac{1}{2},\ r(2,2) = \frac{15}{4},\ r(0,1) = r(1,0) = \frac{3}{4},$$
$$r(0,2) = r(2,0) = \frac{3}{2},\ r(1,2) = r(2,1) = 3.$$

It is clear that (X,r) is a complete b-metric-like space with constant $s = \frac{4}{3}$. This can see the reference[10].

Moreover, the mapping B and S be defined as $B(0) = 2, B(1) = 1, S(1) = 1, S(2) = 1$.

According to the definition of a general LW-type cyclic mapping, we will choose proper functions β, δ, t to satisfy the conditions in Theorem 3.1.

We recall the definition of a general LW-type cyclic mapping, that is,

$$\beta(r(x,y))r(x,Bx) + \delta(r(x,y))r(y,Sy) + t(r(x,y))r(Bx,Sy) \le Lr(x,y),$$

and let

$$\beta(x) = \frac{x}{10+x},\ \delta(x) = \frac{x^2}{10+x^2},\ t(x) = 1+x.$$

In fact, we need to find a proper number λ to satisfy the conditions of Theorem 3.1 as follows:

(1) $max\{\frac{1-\beta(z)}{\delta(z)+t(z)}, \frac{1-\delta(z)}{\beta(z)+t(z)}, z \in [0,+\infty)\} \le \lambda < 1$,

(2) $\lambda s < 1$.

(3) $L \in [1, t(z)]$.

Taking L as $1 + \frac{t(z)-1}{2}$, that is $L = 1 + \frac{t(z)-1}{2}$, next, we will find λ by some concrete calculation, we will find λ by some concrete calculation. Recall that the mapping B and S be defined as $B(0) = 2, B(1) = 1, S(1) = 1, S(2) = 1$, so we obtain that it should take z as $r(0,1), r(0,2), r(1,1), r(1,2)$.

If we choose $z = r(1,1) = \frac{1}{2}$, then

$$\beta(\frac{1}{2}) = \frac{3}{11},\ \delta(\frac{1}{2}) = \frac{5}{11},\ t(\frac{1}{2}) = \frac{3}{2},$$

by $L \in [1, \frac{3}{2}]$ and let $L = \frac{5}{4}$, thus,

$$\max\{\frac{1}{2}, \frac{35}{74}\} = \frac{1}{2} \le \lambda < 1.$$

If we choose $z = r(0,1) = \frac{3}{4}$, then

$$\beta(\frac{3}{4}) = \frac{7}{43},\ \delta(\frac{3}{4}) = \frac{25}{169},\ t(\frac{3}{4}) = \frac{7}{4},$$

by $L \in [1, \frac{7}{4}]$ and let $L = \frac{11}{8}$, thus,

$$\max\{\frac{413}{638}, \frac{1659}{2566}\} = \frac{417}{638} \le \lambda < 1.$$

If we choose $z = r(0,2) = \frac{3}{2}$, then

$$\beta(\frac{3}{2}) = \frac{5}{23},\ \delta(\frac{3}{2}) = \frac{13}{49},\ t(\frac{3}{2}) = \frac{5}{2},$$

by $L \in [1, \frac{5}{2}]$ and let $L = \frac{7}{4}$, thus,

$$\max\{\frac{141}{250}, \frac{291}{542}\} = \frac{291}{542} \le \lambda < 1.$$

If we choose $z = r(1, 2) = 3$, then

$$\beta(3) = \frac{4}{13},\ \delta(3) = \frac{10}{19},\ t(3) = 4,$$

by $L \in [1, 4]$ and let $L = \frac{5}{2}$, thus,

$$\max\{\frac{57}{112}, \frac{75}{172}\} = \frac{57}{112} \leq \lambda < 1.$$

From above, it shows that we can choose $\lambda \in [\frac{291}{542}, \frac{3}{4})$, by virtue of $s = \frac{4}{3}$, and we obtain that $\lambda s < 1$. This imply that the conditions given in Theorems 3.1 are feasible and meaningful. At the same time, we can know that the mappings B and S have a unique common fixed point under that conditions.

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