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Evaluation of SP anomalies caused by two dimensional sheet like structures with different inversion techniques

İki boyutlu levha biçimli yapıların neden olduğu SP anomalilerinin farklı ters çözüm teknikleriyle değerlendirilmesi

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Abstract

Usage of the least squares and inversion methods are commonly applied to the geophysical data analysis. Solution of the theoretical anomalies of inclined sheet like bodies for the self-potential method were compared by writing a Fortran based computer program which is using simple iterative methods with damped least squares (Marquardt-Levenberg) algorithm. As a result of theoretical model studies, model parameters have been reached with very little number of iterations at the small error limits. Applied Marquardt-Levenberg method damping factor has been carried out automatically in the program depending on converging and non-converging conditions. Depth, horizontal length and starting point (X_0) parameters of the inclined sheet model were obtained within low error limits compared with the iteration methods for model and real field data.

Keywords: Self-Potential; Inclined sheet model; Least Squares; Marquardt-Levenberg.

Öz

En küçük kareler ve ters çözüm yöntemlerinin kullanımı jeofizik veri analizinde yaygın olarak uygulanmaktadır. Eğilimli tabaka benzeri cisimlerin kuramsal anomalilerinin doğal potansiyel yöntemi için çözümü, basit yinelemeli yöntemler ve sönümlü en küçük kareler (Marquardt-Levenberg) yöntemi kullanılarak Fortran tabanlı bir bilgisayar programı yazılarak karşılaştırılmıştır. Kuramsal model çalışmaları sonucunda, küçük hata limitlerinde çok az sayıda yineleme ile model parametrelere ulaşılmıştır. Uygulanan Marquardt-Levenberg yöntemi sönümleme faktörü programda, yakınsak ve yakınsak olmayan koşullara bağlı olarak otomatik olarak gerçekleştirilmiştir. Eğimli levha modelinin derinlik, yatay uzunluk ve başlangıç noktası (X_0) parametreleri, model ve gerçek alan verileri için iterasyon yöntemleri ile karşılaştırıldığında düşük hata sınırları içinde elde edilmiştir.

Anahtar kelimeler: Doğal-Potansiyel, Eğimli levha modeli, En küçük kareler, Marquardt-Levenberg.

1 Introduction

The self-potential (SP) method is based on measurement of naturally occured potential differences generated mainly by electrokinetic, electrochemical, and thermoelectric sources. The self-potential method is employed wide range of applications in Engineering and Geothecnical investigations [1],[2], in the exploration of metallic sulfides [3] and graphite deposits [4], Geothermal explorations [5]-[8], shallow flow of ground water in sinkholes [9]-[11], and cavity detection [12],[13].

There are quantitative methods used to determine the parameters of a polarized structure assuming a model with simple geometry. There are available various graphical and numerical methods developed to interpret SP anomalies, including curve matching [4],[14],[15], characteristic points [16]-[18], least squares [19]-[21], derivative and gradient analysis [22], [23], [24], nonlinear modeling [25]-[27], simple iterative [28], singular value decomposition [29], neural networks [30], genetic algorithm [31], particle swarm optimization [32],[33], Whale optimization [34], and Fourier analysis techniques [35],[36].

Since the ore bodies of metallic sulfides and graphites which are found in nature as veins, they could be approximated to two dimensional simple geometric models, they may be considered as two dimensional sheets (Figure 1). Oppositely polarized two ends of sheet model were thought to be in the x-y plane.

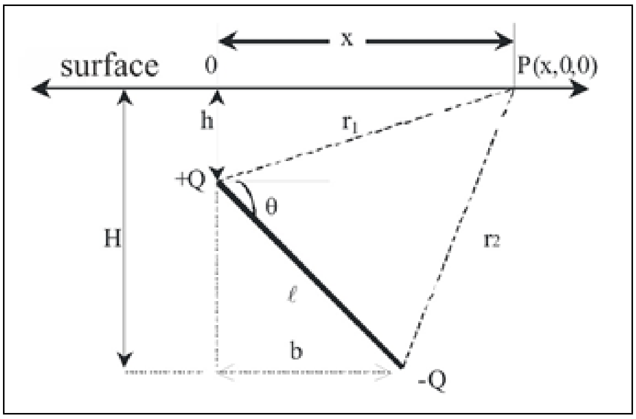

Figure 1. A sketch view of a inclined sheet body.

- P : Measurement point,
- 0 : Surface imprint of the upper end of sheeted ore body,
- l : Length of the ore body,
- r_1, r_2 : Distances to the point P of the upper and lower ends of the ore body,
- q : Inclination of the ore body (Polarization angle),
- x : Distance between the point 0 and the measurement point P(x, 0, 0),
- h, H : Vertical distances of the upper and lower ends of the ore body (Depths).

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2 Interpretation with a simple iteration method

In order to determine the parameters of inclined sheets, a digital approach which was proposed by [37], was used. This method contains the calculations of body parameters iteratively depending on depths to top and bottom of the body.

SP anomaly at the point P of arbitrarily polarized two-dimensional sheet was given by [38];

$$V(x) = K \cdot ln\frac{x^2 + h^2}{(x-b)^2 + H^2} \tag{1}$$

$$b = \frac{H - h}{\tan\theta} \tag{2}$$

where b is a stationary parameter depending on the depths of h and H which are belong to of the inclined sheet upper and lower ends respectively and θ is the slope of sheet. The below equations can be derived from the characteristic properties of potential equation given by [38] and [39]. Parameter K can obtained from the potential value $V(0)$ at the centre of model $(x = 0)$.

$$K = \frac{V(0)}{ln\frac{h^2}{(b^2 + H^2)}} \tag{3}$$

$$K = \frac{V_{Mm}}{\ln(h^2/H^2)} \tag{4}$$

The X_0 value would be at the condition of $V(x) = 0$;

$$X_0 = \frac{b^2 + H^2 - h^2}{2b} \tag{5}$$

In order to find x distances giving maximum and minimum values of $V(x)$ potentials, the derivative of the potential should be analyzed;

$$\frac{\partial V(x)}{\partial x} = 0 \tag{6}$$

These are as given below.

$$x_{max} = x_0 + \sqrt{x_0^2 + h^2} \tag{7}$$

$$x_{min} = x_0 - \sqrt{x_0^2 + h^2} \tag{8}$$

Parameters of h and H can be reached from x_{max} equation and from x_0 axis value at the situation of $V(x) = 0$, respectively.

$$h = \sqrt{x_{max}^2 - 2x_0 x_{max}} \tag{9}$$

$$H = \sqrt{\square^2 - b^2 + 2bx_0} = \sqrt{x_{Max}^2 - 2x_0 x_{Max} - b^2 + 2bx_0} \tag{10}$$

The equation depending on the depth parameters were found from the ratios of total potentials at the roots of x_{max} and x_{min} for $V(0)$ potential value at $x = 0$.

$$V_{max} + V_{min} = V_{Mm} = K.\ell n\frac{h^2}{H^2} \tag{11}$$

$$V(0) = K.ln\frac{h^2}{b^2 + H^2} \tag{12}$$

$$S = \frac{V(0)}{V_{Mm}} \tag{13}$$

$$S.\ell n\frac{h^2}{H^2} = ln\frac{h^2}{b^2 + H^2} \tag{14}$$

$$\left[\frac{h^2}{H^2}\right]^S = \frac{h^2}{b^2 + H^2} \tag{15}$$

Parameter b can be expressed as below by putting the equations of H and h to the appropriate places.

$$b = \frac{(2bx_0 - b^2 + x_{max}^2 - 2x_0 x_{max})^s}{2x_0(x_{max}^2 - 2x_0 x_{max})^{s-1}} \tag{16}$$

This equation which is non-linear, was tried to be solved by a simple iterative inverse method developed for the solution of non-linear equations [40]. The iteration form of this equation is described as;

$$b_j = f(b_i) \tag{17}$$

The initial value is b_i and the assigned value is b_j. Afterwards b_j is utilized as b_i for the next iteration. Iteration is finalized when $|b_i - b_j| \le \varepsilon$ is less the predetermined small value ε (error amount) or when it is being reached the maximum number of iterations. As it could be seen in the equation, the method is effective for the determination of "b" parameter of sheet depending on two parameters (X_0, X_{max} points) of theoretical or measured anomaly. This is the reason to reach to the solution with large number of iterations in this method.

2.1 Dampened least squares (Marquardt-Levenberg) method

Coefficients of non-linear equations could be solved iteratively with the Marquardt-Levenberg method [41] where it is basis depending on the Newton method.

$$F = (F_{X_i}^{obs} - F_{X_i}^{cal}) \tag{18}$$

$$E = \sum_{i=1}^{N} F^2 \ge \text{Minimum} \tag{19}$$

The purpose is to make sum of squares of errors minimum. This could be expressed as below.

$$F = (F_{X_i}^{obs} - F_{X_i}^{cal}) = \begin{bmatrix} \frac{\partial F_{X_1}^{cal}}{\partial \beta_1} & \frac{\partial F_{X_1}^{cal}}{\partial \beta_2} \cdots & \cdots \frac{\partial F_{X_1}^{cal}}{\partial \beta_N} \\ \frac{\partial F_{X_2}^{cal}}{\partial \beta_1} & \frac{\partial F_{X_2}^{cal}}{\partial \beta_2} \cdots & \cdots \frac{\partial F_{X_2}^{cal}}{\partial \beta_N} \\ \vdots & \vdots & \vdots \\ \frac{\partial F_{X_N}^{cal}}{\partial \beta_1} & \frac{\partial F_{X_N}^{cal}}{\partial \beta_2} \cdots & \cdots \frac{\partial F_{X_N}^{cal}}{\partial \beta_N} \end{bmatrix} * \begin{bmatrix} \delta P_1 \\ \delta P_2 \\ \vdots \\ \delta P_N \end{bmatrix} = A * \delta P_i \tag{20}$$

Matrix F is not a square matrix; therefore the both sides are multiplied with the transpose of the matrix.

$$A^T.F = (A^T.A).\delta P_i \tag{21}$$

Singularity problem of $(A^T A)$ matrix could be solved as given below.

$$A^T.F = (A^T.A + \lambda I).\delta P_i \tag{22}$$

$$G = (B + \lambda I).\delta P_i \tag{23}$$

The initial value of λ should be taken as 0.01, for the situation of convergence $\lambda = \lambda/10$ and for the situation of divergence $\lambda = 10 \times \lambda$ should be considered [41]. Model equation is a non-linear function in our equation. Matrix of derivatives is made up of the derivatives analytical model equation with respect to the coefficients.

The model equation could be expressed in two forms as depending on the parameters of (K, h, b, H) and (K, h, b, X_0) in the model equation. Accordingly two different derivative matrices can be used for the same approximation. This situation can be considered as the insurance for the result being reached. Here, the results would be given for the solutions being obtained with two different derivative matrices.

2.2 Calculations of differential derivative equations

$$V(x) = K.ln\left(\frac{x^2 + h^2}{(x-b)^2 + H^2}\right) \tag{24}$$

According to the inclined sheet model potential equation, the coefficients are described as $B(1) = K, B(2) = h, B(3) = b, B(4) = H$. Derivatives of the equation according to the coefficients and the derivative matrix are written.

$$\frac{\partial V(x)}{\partial K} = ln\left(\frac{x^2 + h^2}{(x-b)^2 + H^2}\right) \tag{25}$$

$$\frac{\partial V(x)}{\partial h} = K.\left(\frac{2.h}{x^2 + H^2}\right) \tag{26}$$

$$\frac{\partial V(x)}{\partial b} = K.\left(\frac{2.(x-b)}{(x-b)^2 + H^2}\right) \tag{27}$$

$$\frac{\partial V(x)}{\partial H} = K.\left(\frac{-2.H}{(x-b)^2 + H^2}\right) \tag{28}$$

$$F = \left(F_{X_i}^{obs} - F_{X_i}^{cal}\right)$$
$$= \begin{bmatrix} \frac{\partial F_{X_1}^{cal}}{\partial K} & \frac{\partial F_{X_1}^{cal}}{\partial h} & \frac{\partial F_{X_1}^{cal}}{\partial b} & \frac{\partial F_{X_1}^{cal}}{\partial H} \\ \frac{\partial F_{X_2}^{cal}}{\partial K} & \frac{\partial F_{X_2}^{cal}}{\partial h} & \frac{\partial F_{X_2}^{cal}}{\partial b} & \frac{\partial F_{X_2}^{cal}}{\partial H} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial F_{X_N}^{cal}}{\partial K} & \frac{\partial F_{X_N}^{cal}}{\partial h} & \frac{\partial F_{X_N}^{cal}}{\partial b} & \frac{\partial F_{X_N}^{cal}}{\partial H} \end{bmatrix} * \begin{bmatrix} \delta K \\ \delta h \\ \delta b \\ \delta H \end{bmatrix} \tag{29}$$

If we rewrite the potential equation by putting the equation given before instead of H, the coefficients would become as $B(1) = K, B(2) = h, B(3) = b, B(4) = X_0$.

$$V(x) = K\,ln\left(\frac{x^2 + h^2}{x^2 - 2xb + h^2 + 2b.x_0}\right) \tag{30}$$

$$\frac{\partial V(x)}{\partial K} = ln\left(\frac{x^2 + h^2}{x^2 - 2xb + h^2 + 2b.x_0}\right) \tag{31}$$

$$\frac{\partial V(x)}{\partial \square} = K.\left(\frac{4\square b(x_0 - x)}{(x^2 - 2xb + \square^2 + 2bx_0).(x^2 + \square^2)}\right) \tag{32}$$

$$\frac{\partial V(x)}{\partial b} = K.\left(\frac{2.(x - x_0)}{(x^2 - 2xb + h^2 + 2bx_0)}\right) \tag{33}$$

$$\frac{\partial V(x)}{\partial x_0} = K.\left(\frac{-2b}{(x^2 - 2xb + \square^2 + 2bx_0)}\right) \tag{34}$$

Derivative matrix is formed with derivatives according to coefficients.

$$F = \left(F_{X_i}^{obs} - F_{X_i}^{cal}\right)$$
$$= \begin{bmatrix} \frac{\partial F_{X_1}^{cal}}{\partial K} & \frac{\partial F_{X_1}^{cal}}{\partial h} & \frac{\partial F_{X_1}^{cal}}{\partial b} & \frac{\partial F_{X_1}^{cal}}{\partial Xo} \\ \frac{\partial F_{X_2}^{cal}}{\partial K} & \frac{\partial F_{X_2}^{cal}}{\partial h} & \frac{\partial F_{X_2}^{cal}}{\partial b} & \frac{\partial F_{X_2}^{cal}}{\partial Xo} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial F_{X_N}^{cal}}{\partial K} & \frac{\partial F_{X_N}^{cal}}{\partial h} & \frac{\partial F_{X_N}^{cal}}{\partial b} & \frac{\partial F_{X_N}^{cal}}{\partial Xo} \end{bmatrix} * \begin{bmatrix} \delta K \\ \delta h \\ \delta b \\ \delta Xo \end{bmatrix} \tag{35}$$

2.3 Theoretical model solutions

The parameter values obtained from nomogram and graphical methods were used as initial model parameters in inverse solution process. In the theoretical models, the solution results of $H1 = 2$ unit, $H2 = 5$ unit, $K = 100$ mV and $\theta = 30^o, 45^o$ and 60^o would be given in order to compare with the other methods for the parameters given in the literature. As an extra to the results of nomogram and graphical method, the results of the simple iteration and Marquardt-Levenberg algorithm would be given in Table 1, 2 and 3.

In order to escape complexity of curves in the graphics, only the model curves and the results of simple iteration and Marquardt-Levenberg algorithm were shown. The graphics of the other methods were not included in the comparison. The same parameter values were observed to be reached for the Marquardt solution although two different initial values were used depending on two different parameters (H and X_0).

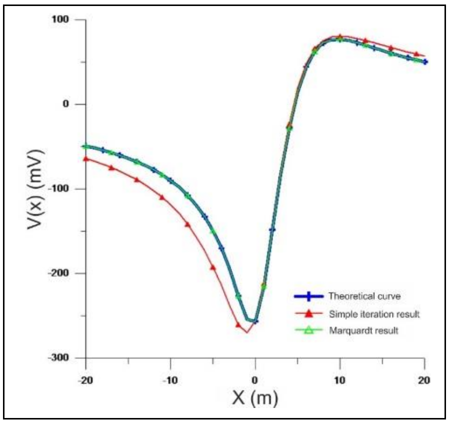

Figure 2. A graph for the theoretical SP anomaly of Model I and inversion results. Model parameters are given as $h = 2$ *units* $H = 5$ *units*, $K = 100$ *mV and* $\theta = 30^o$.

Table 1. Results obtained with different evaluation techniques.

	Model I	Nomogram	Graphics	Simple Iteration	Marquardt Method
K	100		111.18	174.48	99.99
θ	30	32.7	30	31.6	29.99
h	2	2.83	2.34	3.16	1.99
H	5	5.26	5.48	5.44	4.99
b	5.19	3.79	5.44	3.71	5.196

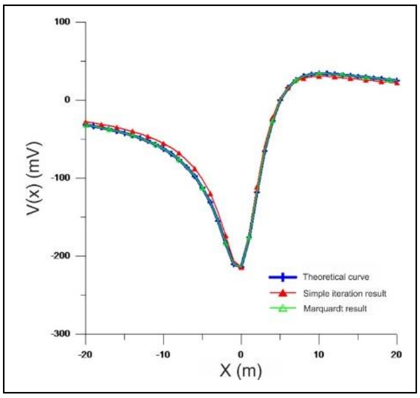

Figure 3. A graph for the theoretical SP anomaly of Model II and inversion results. Model parameters are given as as $h = 2$ $units$ $H = 5$ $units$, $K = 100$ mV and $\theta = 45^o$.

Table 2. Results obtained with different evaluation techniques.

	Model II	Nomogram	Graphics	Simple Iteration	Marquardt Method
K	100		64.38	92.04	100.0
θ	45	50	45	46.8	44.999
h	2	2.78	1.13	1.74	2.0
H	5	3.98	4.87	4.78	5.0
b	3	1.07	3.74	2.85	3.0

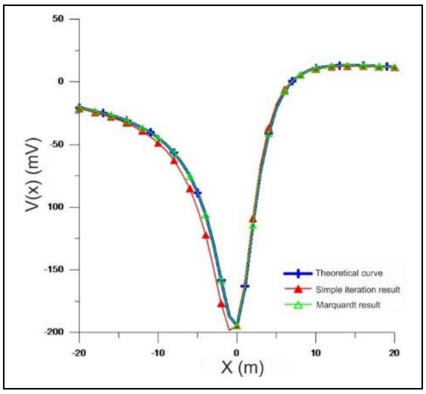

Figure 4. A graph for the theoretical SP anomaly of Model III and inversion results. Model parameters are given as as $h = 2$ $units$ $H = 5$ $units$, $K = 100$ mV and $\theta = 60^o$.

Table 3. Results obtained with different evaluation techniques.

	Model III	Nomogram	Graphics	Simple Iteration	Marquardt Method
K	100		50	301.92	99.998
θ	60	59.5	60	62.1	59.999
h	2	1.8	0.8	2.92	2.0
H	5	4.74	5.35	3.99	5.0
b	1.73	1.73	2.63	0.57	1.73

2.4 Field data applications

SP anomaly over the graphite ore body in the Bavaria forest area in the southern Germany was used as the real field data. SP measurements were made by [4] and he interpreted it with the least squares and he described the anomaly as polarized sheet model.

The profile of 520.5 meter in length was sampled as $dx = 10.41$ meter. By adding the results of the simple iteration and Marquardt methods the sheet parameters obtained with the nomogram and graphics methods are given in Table 4.

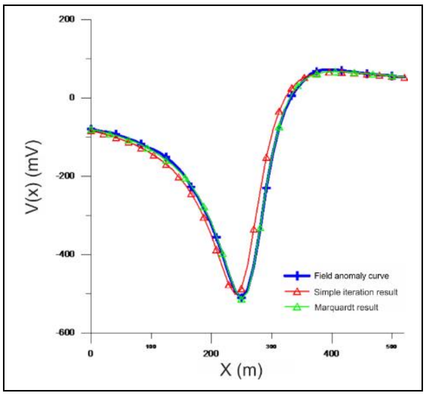

Figure 5. A graph for the field SP anomaly of the Bavaria forest area in the southern Germany [4] and its inversion results.

Table 4. Results obtained with different evaluation techniques.

	Model	Nomogram	Graphics	Simple Iteration	Marquardt Method
K	190.13		363	355.3	414.59
θ	48	48	49.8	49.5	49.60
h	23.81	32.5	33.8	35.82	36.62
H	74.26	70.68	64.3	66.29	62.74
b	45,43	34,38	25,77	26,03	22,23

3 Results

Interpretations of theoretical anomalies of the sheet or rod like bodies having the slopes of $30^o, 45^o$ and 60^o were carried out, but the better results were obtained with the simple iteration and Marquardt methods.

It is well known case that the interpretation sensitivity of nomogram and graphical methods varies with respect to the feelings of interpreter. Since the methods in our study comprised of the whole data, accordingly they gave way to the reliable results being independent of the errors and failures of the interpreter. Compared with the other workers in this subject, the error levels were observed to be reduced the level of $1E^{-6}$.

It was only approached to the model curve more than 20 iterations in the simple iteration method. However it was generally reached to the result less than 10 iterations with the Marquardt method algorithm.

The same precision was also observed for the interpretations on the field data. Parameters obtained from the Least Squares, Simple Iteration and Marquardt methods as the digital techniques were observed to be very close to the body parameters of field data.

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Appendix A

Flow chart of iterative inversion method is presented below.