Hacettepe Journal of Mathematics & Statistics

Hacet. J. Math. Stat.
Volume 52 (2) (2023), 487 – 498
DOI : 10.15672/hujms.831074

Research Article

Significantly improved dominance relation for no-wait flowshop scheduling problems with uncertain setup times

Muberra Allahverdi

Department of Mathematics, Ankara Yıldırım Beyazıt University, Ankara, Turkey

Abstract

The problem of minimizing total completion time (TCT) in an uncertain environment is a crucial problem in production engineering. Minimizing the TCT of a two-machine no-wait scheduling problem with uncertain and bounded setup times is known to be very difficult, and is very likely to have no optimal solution. Such problems are known as Non-deterministic Polynomial-time hard. Scheduling literature provides a mathematical dominance relation for the problem. In this article, a substantially more effective mathematical dominance relation is established. In fact, computational methods reveal that the average percentage improvement comparing the established one in this article to the one in the literature is 1407.80%. Furthermore, statistical hypothesis testing is conducted to compare the means of the established dominance relation to that given in the literature, with p-values of (almost) 0 for every case, meaning that the mean of the established dominance relation is substantially larger than the one given in the literature. Additionally, confidence intervals are constructed for each mean of the randomly generated cases for the proposed dominance relation to confirm the accuracy of the means.

Mathematics Subject Classification (2020). 90B30, 90B36

Keywords. Mathematical dominance relation, statistical inference, no-wait flowshop, total completion time, uncertain setup times

1. Introduction

A manufacturing model with two machines where jobs flow from the first machine to the next is known as a two-machine flowshop model. Specifically, each job has two operations where the first operation is performed on the first machine and the second one is performed on the second machine. In some environments, however, certain production settings require that jobs be processed without any delay. That is, a job must continuously be processed from beginning to end with no idle time in between the processes. This is especially crucial with manufacturing settings relating to heat, where pausing may lead to cooling the material and possibly result in complications [11]. Such a manufacturing flowshop with no breaks is known as a *no-wait flowshop* and is used in many industries, including chemical, plastic, and pharmaceutical and many scheduling problems such as

Email addresses: m.allahverdi.abdulhafiz@aybu.edu.tr (M. Allahverdi)
Received: 28.11.2020; Accepted: 19.09.2022

bakery production and aircraft landing [5, 14]. As indicated in the survey papers of [14] and [5], research relating to the no-wait flowshop problem has been increasing with various performance measures such as the makespan by [25] and total flow time by [19].

Since unpredictability is a common part of manufacturing settings (see [22, 23]), it is crucial to consider cases when job descriptors, such as processing times,setup times, and due dates are not known in advance. The study of stochastic scheduling problems aims to give solutions when job descriptors are uncertain and prone to change. Generally, in such problems the aim is to find a solution to the *expected* job descriptors as opposed to the actual ones. Some papers in this area, include [21], which studies normally distributed processing times to minimize the expected number of tardy jobs, Pinedo [20] which considers exponentially distributed processing times with the objective of minimizing the expected weighted number of tardy jobs, Cunningham and Dutta [12] and Ku and Niu [18] which study exponentially distributed processing times, and Kalczynski and Kamburowski [15] which considers job processing times with the Weibull distribution, and so on.

The time it takes to set up a machine for a new job is called the *setup time* of that job on that particular machine. In some manufacturing cases setup times are too small to make any significant difference in production and are therefore neglected in the vast majority of scheduling literature. Nonetheless, many other manufacturing settings require longer setup times, Kopanos et al. [17] making a considerable difference in production. Neglecting setup times in such cases results in poor efficiency. Allahverdi [4] states that setup times need to be considered separately from processing times for increased productivity, better resource utilization, eliminating waste, and meeting deadlines.

Research in scheduling problems generally assumes setup times to be deterministic, which becomes problematic in settings where they change stochastically. This could result due to equipment shortage, crew skills, or the breakdown of tools [16]. Gonzalez-Neira et al. [13] and Wang and Choi [24] discuss the large number of uncertainties in manufacturing. In an effort to resolve these types of problems, Aydilek et al. [10] suggests setting lower and upper bounds for setup times. That is, for an uncertain setup time of job i on machine k, denoted $s_{i,k}$, there is a lower bound $Ls_{i,k}$ and an upper bound $Us_{i,k}$ with $Ls_{i,k} \leq s_{i,k} \leq Us_{i,k}$.

The motivation for the study comes from the reasons stated in the above paragraphs. The fact that it is a no-wait flowshop combined with the unpredictability of its setup times, given only the lower and upper bounds, is essential for these reasons.

Research regarding uncertain setup times has primarily focused on flowshop scheduling. These, include [6], which establishes dominance relations for a two-machine flowshop scheduling problem, denoted $F2$, with respect to the makespan, i.e. completion time of the last job (C_{max}) and total completion time ($\sum C_j$), stated mathematically as ($F2|L_{s_{i,k}} \leq s_{i,k} \leq U_{s_{i,k}}|C_{max}, \sum C_j$). Other papers, include [1–3], which consider the same problem with the performance measure of C_{max} (makespan), $\sum C_j$ (total completion time), and L_{max} (maximum lateness), respectively.

Finding an optimal solution to minimizing the TCT of a two-machine no-wait problem is known to be very difficult and likely impossible. Such problems are classified as NP (Non-deterministic Polynomial-time)-hard, meaning that in terms of difficulty, they are either equivalent to or more difficult than a class of problems called NP-complete problems. No solutions to NP-complete problems have been found and the conjecture is that no polynomial algorithm exists. Furthermore, if one NP-complete problem has a polynomial algorithm, then so do the rest, [11].

The two machine no-wait flowshop problem with a performance measure of L_{max}, that is $F2|no-wait,\ Ls_{i,k} \leq s_{i,k} \leq Us_{i,k}|L_{max}$, was addressed by [7] and a dominance relation was established. Furthermore, the problem $F2|no-wait,\ Ls_{i,k} \leq s_{i,k} \leq Us_{i,k}|\sum C_j$ was addressed in [9]. The paper provided a dominance relation to minimize $\sum C_j$. In this article, the same problem is addressed and a new dominance relation is proposed. It is shown that the new dominance relation is significantly better than the current one (around a 1500% improvement) in the literature. This is further confirmed using a statistical test of hypothesis with a significance level $\alpha = 0.01$.

The remainder of the paper is organized as follows: Section 2 discusses the proposed dominance relation. Section 3 discusses the results of computational experiments to compare the proposed dominance relation to the best existing one in literature. Computational experiments are carried out using the programming language Python and the results are confirmed using a test of hypothesis. Finally, Section 4 is the conclusion, where the computational experiments and the hypothesis test are summarized.

2. Proposed dominance relation

Suppose we have n jobs. Let γ_1 be a sequence of these jobs and let γ_2 be another sequence obtained from γ_1 by permuting the rth element g and $r+1$st element h. So, if we let σ_1 denote the first $r-1$st elements of γ_1 and σ_2 the elements in positions $r+2$ to n, we have $\gamma_1 = (\sigma_1, g, h, \sigma_2)$ and $\gamma_2 = (\sigma_1, h, g, \sigma_2)$.

The following lemma is from [7].

Lemma 2.1. *The completion time of the job in position j on machine 2 is computed as follows:*

$$C_{[j]} = \sum_{z=1}^{j} \max\left(s_{[z,2]} + t_{[z-1,2]}, s_{[z,1]} + t_{[z,1]}\right) + t_{[j,2]}.$$

Theorem 2.2. *Consider a two machine no-wait flow-shop scheduling problem with uncertain setup times within some bounds. For two adjacent jobs g and h, job h should precede job g if the following conditions hold, in order to minimise the performance measure of total completion time, denoted,* TCT*. That is,* $\mathrm{TCT}(\gamma_2) \leq \mathrm{TCT}(\gamma_1)$ *when the conditions below are satisfied.*

(1) $t_{h,2} \leq t_{g,2}$,
(2) $Us_{h,2} + \max\{t_{[k,2]}\}_{k=r-1,r,r+1} \leq Ls_{h,1} + t_{h,1}$,
(3) $Us_{g,2} + \max\{t_{[k,2]}\}_{k=r-1,r,r+1} \leq Ls_{g,1} + t_{g,1}$,
(4) $Us_{g,1} + t_{g,1} \leq Ls_{h,1} + t_{h,1}$,

where $\mathrm{TCT}(\gamma)$ *is the total completion time of a sequence* γ.

Proof. We know from Lemma 2.1 that

$$C_{[j]} = \sum_{z=1}^{j} \max\left\{s_{[z,2]} + t_{[z-1,2]}, s_{[z,1]} + t_{[z,1]}\right\} + t_{[j,2]}$$

and that $\mathrm{TCT} = \sum_{i=1}^{n} C_{[i]}$.
We need to prove that $\mathrm{TCT}(\gamma_2) \leq \mathrm{TCT}(\gamma_1)$.

Case 1: $j = 1, \cdots, r-1$

It is trivial that $C_{[j]}(\gamma_2) = C_{[j]}(\gamma_1)$ for $j = 1, \cdots, r-1$ since γ_1 and γ_2 are the exact same sequence up to the rth element and therefore have the same setup and processing times. Hence, $\sum_{j=1}^{r-1} C_{[j]}(\gamma_2) = \sum_{j=1}^{r-1} C_{[j]}(\gamma_1)$.

Case 2: $j = r, r+1, r+2$

$$\begin{aligned} C_{[r]}(\gamma_1) &= \sum_{z=1}^{r-1} \max\left(s_{[z,2]} + t_{[z-1,2]}, s_{[z,1]} + t_{[z,1]}\right) + \max\left(s_{[r,2]} + t_{[r-1,2]}, s_{[r,1]} + t_{[r,1]}\right) + t_{[r,2]} \\ &= \sum_{z=1}^{r-1} \max\left(s_{[z,2]} + t_{[z-1,2]}, s_{[z,1]} + t_{[z,1]}\right) + \max\left(s_{g,2} + t_{[r-1,2]}, s_{g,1} + t_{g,1}\right) + t_{g,2} \end{aligned}$$

$$C_{[r]}(\gamma_2) = \sum_{z=1}^{r-1} \max\left(s_{[z,2]} + t_{[z-1,2]}, s_{[z,1]} + t_{[z,1]}\right) + \max\left(s_{h,2} + t_{[r-1,2]}, s_{h,1} + t_{h,1}\right) + t_{h,2}$$

$$\begin{aligned} C_{[r+1]}(\gamma_1) = &\sum_{z=1}^{r-1} \max\left(s_{[z,2]} + t_{[z-1,2]}, s_{[z,1]} + t_{[z,1]}\right) + \max\left(s_{g,2} + t_{[r-1,2]}, s_{g,1} + t_{g,1}\right) \\ &+ \max\left(s_{h,2} + t_{[r,2]}, s_{h,1} + t_{h,1}\right) + t_{h,2} \end{aligned}$$

$$\begin{aligned} C_{[r+1]}(\gamma_2) = &\sum_{z=1}^{r-1} \max\left(s_{[z,2]} + t_{[z-1,2]}, s_{[z,1]} + t_{[z,1]}\right) + \max\left(s_{h,2} + t_{[r-1,2]}, s_{h,1} + t_{h,1}\right) \\ &+ \max\left(s_{g,2} + t_{[r,2]}, s_{g,1} + t_{g,1}\right) + t_{g,2} \end{aligned}$$

$$\begin{aligned} C_{[r+2]}(\gamma_1) = &\sum_{z=1}^{r-1} \max\left(s_{[z,2]} + t_{[z-1,2]}, s_{[z,1]} + t_{[z,1]}\right) + \max\left(s_{g,2} + t_{[r-1,2]}, s_{g,1} + t_{g,1}\right) \\ &+ \max\left(s_{h,2} + t_{[r,2]}, s_{h,1} + t_{h,1}\right) + \max\left(s_{[r+2,2]} + t_{h,2}, s_{[r+2,1]} + t_{[r+2,1]}\right) + t_{[r+2,2]} \end{aligned}$$

$$\begin{aligned} C_{[r+2]}(\gamma_2) = &\sum_{z=1}^{r-1} \max\left(s_{[z,2]} + t_{[z-1,2]}, s_{[z,1]} + t_{[z,1]}\right) + \max\left(s_{h,2} + t_{[r-1,2]}, s_{h,1} + t_{h,1}\right) \\ &+ \max\left(s_{g,2} + t_{[r,2]}, s_{g,1} + t_{g,1}\right) + \max\left(s_{[r+2,2]} + t_{g,2}, s_{[r+2,1]} + t_{[r+2,1]}\right) + t_{[r+2,2]} \end{aligned}$$

Hence,

$$\begin{aligned} \sum_{z=r}^{r+2} C_{[z]}(\gamma_1) - \sum_{z=r}^{r+2} C_{[z]}(\gamma_2) = &\, 3\max\left(s_{g,2} + t_{[r-1,2]}, s_{g,1} + t_{g,1}\right) + 2\max\left(s_{h,2} + t_{[r,2]}, s_{h,1} + t_{h,1}\right) \\ &+ \max\left(s_{[r+2,2]} + t_{h,2}, s_{[r+2,1]} + t_{[r+2,1]}\right) + t_{[r+2,2]} + t_{h,2} + t_{g,2} \\ &- 3\max\left(s_{h,2} + t_{[r-1,2]}, s_{h,1} + t_{h,1}\right) - 2\max\left(s_{g,2} + t_{[r,2]}, s_{g,1} + t_{g,1}\right) \\ &- \max\left(s_{[r+2,2]} + t_{g,2}, s_{[r+2,1]} + t_{[r+2,1]}\right) - t_{[r+2,2]} - t_{g,2} - t_{h,2}. \end{aligned}$$

By conditions 2 and 3, we have

$$\begin{aligned} &= 3(s_{g,1} + t_{g,1}) + 2(s_{h,1} + t_{h,1}) + \max\left(s_{[r+2,2]} + t_{h,2}, s_{[r+2,1]} + t_{[r+2,1]}\right) \\ &\quad - 3(s_{h,1} + t_{h,1}) - 2(s_{g,1} + t_{g,1}) - \max\left(s_{[r+2,2]} + t_{g,2}, s_{[r+2,1]} + t_{[r+2,1]}\right) \\ &= (s_{g,1} + t_{g,1}) + \max\left(s_{[r+2,2]} + t_{h,2}, s_{[r+2,1]} + t_{[r+2,1]}\right) \\ &\quad - (s_{h,1} + t_{h,1}) - \max\left(s_{[r+2,2]} + t_{g,2}, s_{[r+2,1]} + t_{[r+2,1]}\right) \\ &\le 0. \end{aligned}$$

Case 3: $j = r + 3, \cdots, n$

As in Case 1, we show that $C_{[j]}(\gamma_2) = C_{[j]}(\gamma_1)$ for $j = r + 3, \cdots, n$.

$$\begin{aligned} C_{[j]}(\gamma_1) = &\sum_{z=1}^{r-1} \max\left(s_{[z,2]} + t_{[z-1,2]}, s_{[z,1]} + t_{[z,1]}\right) + \max\left(s_{g,2} + t_{[r-1,2]}, s_{g,1} + t_{g,1}\right) \\ &+ \max\left(s_{h,2} + t_{[r,2]}, s_{h,1} + t_{h,1}\right) + \max\left(s_{[r+2,2]} + t_{[r+1,2]}, s_{[r+2,1]} + t_{[r+2,1]}\right) \\ &+ \sum_{z=r+3}^{n} \max\left(s_{[z,2]} + t_{[z-1,2]}, s_{[z,1]} + t_{[z,1]}\right) + t_{[j,2]} \end{aligned}$$

$$C_{[j]}(\gamma_2) = \sum_{z=1}^{r-1} \max\left(s_{[z,2]} + t_{[z-1,2]}, s_{[z,1]} + t_{[z,1]}\right) + \max\left(s_{h,2} + t_{[r-1,2]}, s_{h,1} + t_{h,1}\right)$$
$$+ \max\left(s_{g,2} + t_{[r,2]}, s_{g,1} + t_{g,1}\right) + \max\left(s_{[r+2,2]} + t_{[r+1,2]}, s_{[r+2,1]} + t_{[r+2,1]}\right)$$
$$+ \sum_{z=r+3}^{n} \max\left(s_{[z,2]} + t_{[z-1,2]}, s_{[z,1]} + t_{[z,1]}\right) + t_{[j,2]}$$

We have

$$C_{[j]}(\gamma_1) - C_{[j]}(\gamma_2) = \max\left(s_{g,2} + t_{[r-1,2]}, s_{g,1} + t_{g,1}\right) + \max\left(s_{h,2} + t_{[r,2]}, s_{h,1} + t_{h,1}\right)$$
$$+ \max\left(s_{[r+2,2]} + t_{[r+1,2]}, s_{[r+2,1]} + t_{[r+2,1]}\right)$$
$$- \max\left(s_{h,2} + t_{[r-1,2]}, s_{h,1} + t_{h,1}\right) - \max\left(s_{g,2} + t_{[r,2]}, s_{g,1} + t_{g,1}\right)$$
$$- \max\left(s_{[r+2,2]} + t_{[r+1,2]}, s_{[r+2,1]} + t_{[r+2,1]}\right).$$

By conditions 2 and 3,

$$= s_{g,1} + t_{g,1} + s_{h,1} + t_{h,1} - s_{h,1} - t_{h,1} - s_{g,1} - t_{g,1}$$
$$= 0.$$

It follows from Cases 1, 2, and 3 that $\mathrm{TCT}(\gamma_2) \leq \mathrm{TCT}(\gamma_1)$ as desired. □

3. Computational experiments

Computational experiments are conducted to compare the older and newer versions of the dominance relations in an effort to confirm that the newer version is more efficient. "Efficiency" here refers to how often randomly generated data satisfy conditions of the newer version compared to that of the older version. How often these dominance relations are satisfied translates to how effective they are in real-life situations. It is shown in this article that the newer version is significantly more effective.

To conduct experiments it is necessary to generate data for the processing times and lower and upper bounds of the setup times. This is done in the following way: similar to [8] and [10], different values of n are used (number of jobs) from 50 to 500 with an increment of 50. So 10 different values of n are considered. A value H that determines how to choose the lower bound of setup times based on the upper bounds is defined as shown below. The variable H is set to be equal to eight different values $H = 5, 10, 15, 20, 25, 30, 35, 40$ and the following is done for each combination of n and H.

- Generate $t_{j,k}$ from $U(1, 100)$.
- Generate $Us_{j,k}$ from $U(1, 100)$,
- Generate $Ls_{j,k}$ from $U(\max(1, Us_{j,k} - H), Us_{j,k})$,

for $j = 1, \cdots, n$ and $k = 1, 2$.

In total, $(10)(8) = 80$ combinations of n and H are considered.

3.1. Computing examples with Python

The programming language Python is utilized to compare the older and newer versions of the dominance relation, stated from now on as Theorem-old and Theorem-new, respectively. First, a function called "counter-satisfy" is created which, given a certain sequence of jobs and a theorem (either Theorem-old or Theorem-new), counts the number of times the given theorem is satisfied for every two adjacent jobs in the given sequence.

For each combination of H and n, 300 instances are generated for the processing times and lower and upper bounds of setup times. Then, for each of the 300 instances, the sequence $1-2-3-\cdots-n$ is shuffled and entered into the "counter-satisfy" function along with each of Theorem-old and Theorem-new to keep track of how often the theorems are satisfied.

Finally, the average of the 300 instances corresponding to the different shuffles is taken, so that there is one average estimate for each instance of processing times and lower and upper bounds of setup times. As a result, there are 300 averages in total. Lastly, the average and standard deviation of those 300 averages are taken and the result is recorded for that particular H and n combination in Table 1. Since there are a total of 8 values for H, 10 values of n, and $300 \times 300 = 90,000$ cases for each H and n combination; the total number of instances considered is $8 \times 10 \times 300 \times 300 = 7,200,000$.

Table 1 summarizes the computational results for all combinations of n and H. The first two columns list the various values for n and H and the third and fourth columns list the mean and standard deviation of how often the randomly generated data satisfy conditions of Theorem-old. Similarly, the fifth and sixth values list the mean and standard deviation of how often the data satisfy conditions of Theorem-new. Finally the last column lists the percentage improvement which shows the improvement of Theorem-new compared to Theorem-old. For a given combination of H and n, the percentage improvement is calculated by dividing the difference between the means of Theorem-old and Theorem-new by the mean of Theorem-old.

Table 1. Computational results.

H	n	Thm-old mean	Thm-old std	Thm-new mean	Thm-new std	Per. Imp.
5	50	0.022	0.03	2.785	0.984	125.59
5	100	0.046	0.043	5.598	1.453	120.7
5	150	0.074	0.058	8.541	1.871	114.42
5	200	0.09	0.056	11.774	2.181	129.82
5	250	0.115	0.059	14.543	2.399	125.46
5	300	0.137	0.071	17.605	2.54	127.5
5	350	0.158	0.075	20.427	2.802	128.28
5	400	0.181	0.084	23.315	3.053	127.81
5	450	0.215	0.088	26.117	3.064	120.47
5	500	0.231	0.088	28.892	3.024	124.07
10	50	0.007	0.017	2.554	0.972	363.86
10	100	0.015	0.02	5.227	1.459	347.47
10	150	0.021	0.023	7.98	1.562	379
10	200	0.027	0.023	10.622	1.948	392.41
10	250	0.035	0.03	13.213	2.208	376.51
10	300	0.042	0.03	15.871	2.396	376.88
10	350	0.046	0.031	18.546	2.734	402.17
10	400	0.051	0.033	21.606	2.966	422.65
10	450	0.061	0.032	23.76	2.989	388.51
10	500	0.068	0.036	26.717	3.173	391.9
15	50	0.003	0.009	2.233	0.911	743.33
15	100	0.008	0.015	4.81	1.326	600.25
15	150	0.01	0.015	7.226	1.607	721.6
15	200	0.013	0.014	9.769	1.856	750.46
15	250	0.017	0.016	12.142	2.021	713.24
15	300	0.018	0.018	14.703	2.38	815.83
15	350	0.021	0.018	16.943	2.369	805.81
15	400	0.025	0.019	19.901	2.663	795.04
15	450	0.027	0.02	22.237	2.821	822.59
15	500	0.032	0.023	24.745	3.014	772.28
20	50	0.002	0.007	2.13	0.899	1064
20	100	0.003	0.006	4.47	1.313	1489
20	150	0.006	0.011	6.758	1.696	1125.33
20	200	0.008	0.011	8.964	1.847	1119.5
20	250	0.01	0.012	11.161	1.982	1115.1
20	300	0.013	0.013	13.267	2.118	1019.54
20	350	0.012	0.012	15.521	2.295	1292.42
20	400	0.016	0.014	17.859	2.526	1115.19
20	450	0.017	0.015	20.053	2.562	1178.59
20	500	0.019	0.015	22.404	2.82	1178.16

25	50	0.001	0.005	1.973	0.787	1972
25	100	0.003	0.008	4.051	1.192	1349.33
25	150	0.003	0.007	6.158	1.46	2051.67
25	200	0.005	0.009	8.15	1.555	1629
25	250	0.007	0.009	10.104	1.74	1442.43
25	300	0.009	0.01	12.389	1.945	1375.56
25	350	0.009	0.011	14.601	2.246	1621.33
25	400	0.01	0.01	16.597	2.373	1658.7
25	450	0.012	0.012	18.657	2.758	1553.75
25	500	0.012	0.011	20.869	2.643	1738.08
30	50	0.001	0.007	1.757	0.769	1756
30	100	0.002	0.007	3.7	1.068	1849
30	150	0.003	0.006	5.54	1.269	1845.67
30	200	0.004	0.008	7.563	1.596	1889.75
30	250	0.005	0.008	9.486	1.846	1896.2
30	300	0.006	0.008	11.328	1.965	1887
30	350	0.006	0.009	13.216	1.983	2201.67
30	400	0.007	0.009	15.093	2.24	2155.14
30	450	0.008	0.008	17.156	2.324	2143.5
30	500	0.01	0.01	19.173	2.269	1916.3
35	50	0.001	0.004	1.695	0.681	1694
35	100	0.001	0.005	3.514	1.084	3513
35	150	0.002	0.005	5.17	1.208	2584
35	200	0.003	0.006	7.196	1.654	2397.67
35	250	0.003	0.005	8.862	1.829	2953
35	300	0.004	0.007	10.609	1.856	2651.25
35	350	0.005	0.007	12.387	1.989	2476.4
35	400	0.006	0.007	14.402	2.127	2399.33
35	450	0.007	0.009	15.705	2.248	2242.57
35	500	0.008	0.009	17.622	2.443	2201.75
40	50	0.001	0.004	1.559	0.681	1558
40	100	0.001	0.005	3.309	1.036	3308
40	150	0.001	0.004	4.789	1.327	4788
40	200	0.003	0.006	6.452	1.35	2149.67
40	250	0.003	0.007	8.197	1.463	2731.33
40	300	0.004	0.006	9.83	1.638	2456.5
40	350	0.004	0.006	11.384	1.926	2845
40	400	0.006	0.007	12.979	1.911	2162.17
40	450	0.005	0.007	14.608	2.13	2920.6
40	500	0.007	0.008	16.367	2.341	2337.14

As can be seen in Table 1, the newer version of the theorem is considerably better than the older version. The average percentage improvement is 1407.80.% Furthermore, the median is $\frac{1349.33+1375.56}{2} = 1362.44\%$. Since the median is close to the average, it indicates that the average is an accurate representation of the numbers. Likewise, the standard deviations of both Thm-old and Thm-new are reasonably small, which further indicate that the means give an accurate representation of the data. Finally, the percentage improvement is higher for greater values of H and n as seen in Figures 1 and 2 below. The fact that the percentage improvement rises with greater values of n and H is an advantage, since it illustrates the effectiveness of the new dominance relation.

As can be seen in Figures 1 and 2, there are points where the graph decreases. Nonetheless, the general direction of the graph is upward as we move to the right.

3.2. Hypothesis testing

Hypothesis testing is used to confirm that the new dominance relation is much more effective than the earlier one. For a given H and n, the aim is to compare the average number of times the earlier dominance relation, denoted μ_0, was satisfied versus the newer dominance relation, denoted μ_1 and confirm that $\mu_1 > \mu_0$.

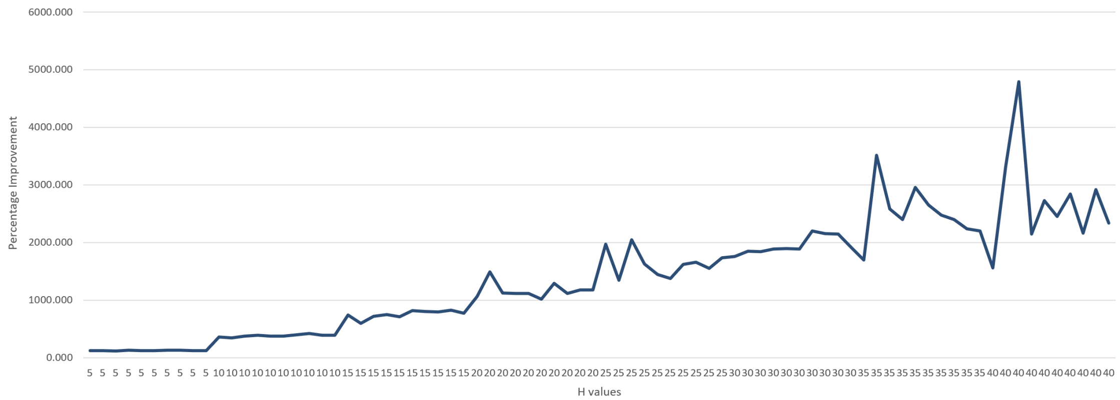

Figure 1. Percentage improvement vs H.

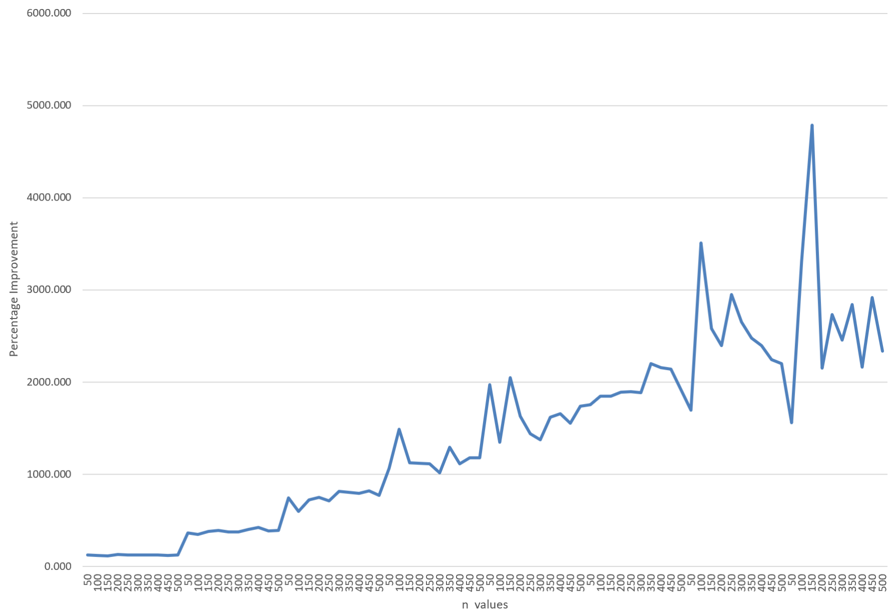

Figure 2. Percentage improvement vs n.

The the null and alternative hypotheses are as follows:

- $H_0 : \mu_1 - \mu_0 = 0$
- $H_1 : \mu_1 - \mu_0 > 0$

We take the significance level of $\alpha = 0.01$. Given n and H, we compute the Z-scores for a difference of means using the test statistic formula

$$Z = \frac{\bar{X}_1 - \bar{X}_0}{\sqrt{\frac{S_0^2}{90000} + \frac{S_1^2}{90000}}},$$

where $\bar{X}_0, \bar{X}_1$, S_0, and S_1 are the sample means and standard deviations of the earlier and newer dominance relations, respectively.

Since $P(Z \leq 2.33) = 0.99$, the critical Z-value for the 99th percentile of a standard normal distribution $N(0,1)$ is 2.33. Hence, if the Z-score is greater than 2.33, we reject the null hypothesis and accept the alternative hypothesis that the newer dominance relation is in fact better. Otherwise, we fail to reject the null hypothesis. The results are listed in Table 2.

Table 2. Z-scores and 95% and 99% confidence intervals for Thm-old and Thm-new.

n	H	95% CI-old	95% CI-new	99% CI-old	99% CI-new	Z-score
50	5	(0.02, 0.03)	(2.67, 2.9)	(0.02, 0.03)	(2.64, 2.93)	841.987
100	5	(0.04, 0.05)	(5.43, 5.76)	(0.04, 0.05)	(5.38, 5.81)	1145.816
150	5	(0.07, 0.08)	(8.33, 8.75)	(0.07, 0.08)	(8.26, 8.82)	1356.964
200	5	(0.08, 0.1)	(11.53, 12.02)	(0.08, 0.1)	(11.45, 12.1)	1606.623
250	5	(0.11, 0.12)	(14.27, 14.81)	(0.11, 0.12)	(14.19, 14.9)	1803.706
300	5	(0.13, 0.15)	(17.32, 17.89)	(0.13, 0.15)	(17.23, 17.98)	2062.344
350	5	(0.15, 0.17)	(20.11, 20.74)	(0.15, 0.17)	(20.01, 20.84)	2169.352
400	5	(0.17, 0.19)	(22.97, 23.66)	(0.17, 0.19)	(22.86, 23.77)	2272.379
450	5	(0.21, 0.22)	(25.77, 26.46)	(0.2, 0.23)	(25.66, 26.57)	2535.051
500	5	(0.22, 0.24)	(28.55, 29.23)	(0.22, 0.24)	(28.44, 29.34)	2842.15
50	10	(0.01, 0.01)	(2.44, 2.66)	(0, 0.01)	(2.41, 2.7)	785.991
100	10	(0.01, 0.02)	(5.06, 5.39)	(0.01, 0.02)	(5.01, 5.44)	1071.592
150	10	(0.02, 0.02)	(7.8, 8.16)	(0.02, 0.02)	(7.75, 8.21)	1528.451
200	10	(0.02, 0.03)	(10.4, 10.84)	(0.02, 0.03)	(10.33, 10.91)	1631.56
250	10	(0.03, 0.04)	(12.96, 13.46)	(0.03, 0.04)	(12.88, 13.54)	1790.324
300	10	(0.04, 0.05)	(15.6, 16.14)	(0.04, 0.05)	(15.51, 16.23)	1981.773
350	10	(0.04, 0.05)	(18.24, 18.86)	(0.04, 0.05)	(18.14, 18.95)	2029.862
400	10	(0.05, 0.05)	(21.27, 21.94)	(0.05, 0.06)	(21.16, 22.05)	2180.074
450	10	(0.06, 0.06)	(23.42, 24.1)	(0.06, 0.07)	(23.32, 24.21)	2378.485
500	10	(0.06, 0.07)	(26.36, 27.08)	(0.06, 0.07)	(26.24, 27.19)	2519.441
50	15	(0, 0)	(2.13, 2.34)	(0, 0)	(2.1, 2.37)	734.322
100	15	(0.01, 0.01)	(4.66, 4.96)	(0.01, 0.01)	(4.61, 5.01)	1086.356
150	15	(0.01, 0.01)	(7.04, 7.41)	(0.01, 0.01)	(6.99, 7.47)	1347.048
200	15	(0.01, 0.01)	(9.56, 9.98)	(0.01, 0.02)	(9.49, 10.05)	1576.895
250	15	(0.02, 0.02)	(11.91, 12.37)	(0.01, 0.02)	(11.84, 12.44)	1799.795
300	15	(0.02, 0.02)	(14.43, 14.97)	(0.02, 0.02)	(14.35, 15.06)	1850.997
350	15	(0.02, 0.02)	(16.67, 17.21)	(0.02, 0.02)	(16.59, 17.3)	2142.868
400	15	(0.02, 0.03)	(19.6, 20.2)	(0.02, 0.03)	(19.5, 20.3)	2239.072
450	15	(0.02, 0.03)	(21.92, 22.56)	(0.02, 0.03)	(21.82, 22.66)	2361.869
500	15	(0.03, 0.03)	(24.4, 25.09)	(0.03, 0.04)	(24.3, 25.19)	2459.749
50	20	(0, 0)	(2.03, 2.23)	(0, 0)	(2, 2.26)	710.101
100	20	(0, 0)	(4.32, 4.62)	(0, 0)	(4.27, 4.67)	1020.629
150	20	(0, 0.01)	(6.57, 6.95)	(0, 0.01)	(6.51, 7.01)	1194.315
200	20	(0.01, 0.01)	(8.76, 9.17)	(0.01, 0.01)	(8.69, 9.24)	1454.657
250	20	(0.01, 0.01)	(10.94, 11.39)	(0.01, 0.01)	(10.87, 11.46)	1687.81
300	20	(0.01, 0.01)	(13.03, 13.51)	(0.01, 0.01)	(12.95, 13.58)	1877.302
350	20	(0.01, 0.01)	(15.26, 15.78)	(0.01, 0.01)	(15.18, 15.86)	2027.293
400	20	(0.01, 0.02)	(17.57, 18.14)	(0.01, 0.02)	(17.48, 18.23)	2119.089
450	20	(0.02, 0.02)	(19.76, 20.34)	(0.01, 0.02)	(19.67, 20.44)	2346.096
500	20	(0.02, 0.02)	(22.09, 22.72)	(0.02, 0.02)	(21.98, 22.82)	2381.349
50	25	(0, 0)	(1.88, 2.06)	(0, 0)	(1.86, 2.09)	751.7
100	25	(0, 0)	(3.92, 4.19)	(0, 0)	(3.87, 4.23)	1018.769
150	25	(0, 0)	(5.99, 6.32)	(0, 0)	(5.94, 6.38)	1264.711
200	25	(0, 0.01)	(7.97, 8.33)	(0, 0.01)	(7.92, 8.38)	1571.356
250	25	(0.01, 0.01)	(9.91, 10.3)	(0.01, 0.01)	(9.85, 10.36)	1740.839
300	25	(0.01, 0.01)	(12.17, 12.61)	(0.01, 0.01)	(12.1, 12.68)	1909.486
350	25	(0.01, 0.01)	(14.35, 14.86)	(0.01, 0.01)	(14.27, 14.94)	1949.042
400	25	(0.01, 0.01)	(16.33, 16.87)	(0.01, 0.01)	(16.24, 16.95)	2096.947
450	25	(0.01, 0.01)	(18.34, 18.97)	(0.01, 0.01)	(18.25, 19.07)	2028.081
500	25	(0.01, 0.01)	(20.57, 21.17)	(0.01, 0.01)	(20.48, 21.26)	2367.403
50	30	(0, 0)	(1.67, 1.84)	(0, 0)	(1.64, 1.87)	685.017
100	30	(0, 0)	(3.58, 3.82)	(0, 0)	(3.54, 3.86)	1038.742
150	30	(0, 0)	(5.4, 5.68)	(0, 0)	(5.35, 5.73)	1308.969
200	30	(0, 0)	(7.38, 7.74)	(0, 0.01)	(7.32, 7.8)	1420.847
250	30	(0, 0.01)	(9.28, 9.7)	(0, 0.01)	(9.21, 9.76)	1540.776
300	30	(0.01, 0.01)	(11.11, 11.55)	(0, 0.01)	(11.04, 11.62)	1728.535
350	30	(0, 0.01)	(12.99, 13.44)	(0, 0.01)	(12.92, 13.51)	1998.467
400	30	(0.01, 0.01)	(14.84, 15.35)	(0.01, 0.01)	(14.76, 15.43)	2020.43
450	30	(0.01, 0.01)	(16.89, 17.42)	(0.01, 0.01)	(16.81, 17.5)	2213.584
500	30	(0.01, 0.01)	(18.92, 19.43)	(0.01, 0.01)	(18.84, 19.51)	2533.647

50	35	(0, 0)	(1.62, 1.77)	(0, 0)	(1.59, 1.8)	746.243
100	35	(0, 0)	(3.39, 3.64)	(0, 0)	(3.35, 3.68)	972.222
150	35	(0, 0)	(5.03, 5.31)	(0, 0)	(4.99, 5.35)	1283.433
200	35	(0, 0)	(7.01, 7.38)	(0, 0)	(6.95, 7.44)	1304.647
250	35	(0, 0)	(8.65, 9.07)	(0, 0)	(8.59, 9.13)	1453.084
300	35	(0, 0)	(10.4, 10.82)	(0, 0.01)	(10.33, 10.89)	1714.158
350	35	(0, 0.01)	(12.16, 12.61)	(0, 0.01)	(12.09, 12.68)	1867.56
400	35	(0.01, 0.01)	(14.16, 14.64)	(0, 0.01)	(14.09, 14.72)	2030.454
450	35	(0.01, 0.01)	(15.45, 15.96)	(0.01, 0.01)	(15.37, 16.04)	2094.912
500	35	(0.01, 0.01)	(17.35, 17.9)	(0.01, 0.01)	(17.26, 17.99)	2162.982
50	40	(0, 0)	(1.48, 1.64)	(0, 0)	(1.46, 1.66)	686.332
100	40	(0, 0)	(3.19, 3.43)	(0, 0)	(3.15, 3.46)	957.904
150	40	(0, 0)	(4.64, 4.94)	(0, 0)	(4.59, 4.99)	1082.437
200	40	(0, 0)	(6.3, 6.6)	(0, 0)	(6.25, 6.65)	1433.097
250	40	(0, 0)	(8.03, 8.36)	(0, 0)	(7.98, 8.41)	1680.227
300	40	(0, 0)	(9.65, 10.02)	(0, 0)	(9.59, 10.07)	1799.622
350	40	(0, 0)	(11.17, 11.6)	(0, 0)	(11.1, 11.67)	1772.577
400	40	(0.01, 0.01)	(12.76, 13.2)	(0, 0.01)	(12.69, 13.26)	2036.564
450	40	(0, 0.01)	(14.37, 14.85)	(0, 0.01)	(14.29, 14.93)	2056.749
500	40	(0.01, 0.01)	(16.1, 16.63)	(0.01, 0.01)	(16.02, 16.72)	2096.528

As seen in Table 2, the z-scores are much larger than 2.33, rejecting the null hypothesis that the earlier theorem is at least as good as the newer one. The minimum z-score in the table is 685 and the p-value is $1 - \Phi(685)$, which is practically 0.

Furthermore, 95% and 99% confidence intervals are constructed for the means of the earlier and proposed dominance relations. The average 95% and 99% confidence intervals for Thm-old are $(0.0244, 0.0286)$ and $(0.0237, 0.0293)$, respectively. Those of Thm-new are $(11.87, 12.31)$ and $(11.80, 12.38)$, respectively. Clearly, there is no overlap between these average confidence intervals. In fact, not only is there no overlap in the averages, there is also no overlap in any individual case in the table. This further confirms the conclusion of the hypothesis test that the proposed dominance relation is significantly better than the earlier one. The 95% and 99% confidence intervals were calculated using the following formulas, respectively.

$$x \pm (1.96)(\frac{s}{300})$$

and

$$x \pm (2.58)(\frac{s}{300}),$$

where x is the sample mean and s is the sample standard deviation.

4. Conclusion

A dominance relation was proposed by [9] to solve the problem of a two-machine no-wait flowshop scheduling problem with uncertain and bounded setup times. In this article, a new dominance relation is proposed which is substantially more effective than the one proposed earlier. In other words, conditions for the new dominance relation are satisfied much more frequently than the previous dominance relation.

The difference between the two dominance relations is confirmed with randomly generated data of $7,200,000$ instances. Computational Results are presented with tables and graphs. Hypothesis testing with a significance level of $\alpha = 0.01$ along with confidence intervals are also constructed to further verify the new dominance relation's effectiveness.

For future research, it would be interesting to see what kind of dominance relation would solve the problem of total tardiness as opposed to total completion time.

References

[1] A. Allahverdi, *Two-machine flowshop scheduling problem to minimize makespan with bounded setup and processing times*, IJAM **8**, 145-153, 2005.

[2] A. Allahverdi, *Two-machine flowshop scheduling problem to minimize total completion time with bounded setup and processing times*, Int. J. Prod. Econ. **103** (1), 386-400, 2006.

[3] A. Allahverdi, *Two-machine flowshop scheduling problem to minimize maximum lateness with bounded setup and processing times*, Kuwait J. Sci. Eng. **33** (2), 233-251, 2006.

[4] A. Allahverdi, *The third comprehensive survey on scheduling problems with setup times/costs*, Eur. J. Oper. Res. **246** (2), 345-378, 2015.

[5] A. Allahverdi, *A survey of scheduling problems with no-wait in process*, Eur. J. Oper. Res. **255** (3), 665-686, 2016.

[6] A. Allahverdi, T. Aldowaisa and Y. Sotskov, *Two-machine flowshop scheduling problem to minimize makespan or total completion time with random and bounded setup times*, Int. J. Math. Math. Sci. **39**, 2475-2486, 2003.

[7] A. Allahverdi and M. Allahverdi, *Two-machine no-wait flowshop scheduling problem with uncertain setup times to minimize maximum lateness*, Comput. Appl. Math. **37** (5), 6774-6794, 2018.

[8] A. Allahverdi and H. Aydilek, *Heuristics for two-machine flowshop scheduling problem to minimize maximum lateness with bounded processing times*, Comput. Math. with Appl. **60** (5), 1374-1384, 2010.

[9] M. Allahverdi and A. Allahverdi, *Minimizing total completion time in a two-machine no-wait flowshop with uncertain and bounded setup times*, J. Ind. Manag. Optim. **16** (5), 2439-2457, 2020.

[10] A. Aydilek, H. Aydilek and A. Allahverdi, *Increasing the profitability and competitivess in a production environment with random and bounded setup times*, Int. J. Prod. Res. **51** (1), 106-117, 2013.

[11] K.R. Baker and D. Trietsch, *Principles of Sequencing and Scheduling*, John Wiley and Sons, New jersy, 2009.

[12] A.A. Cunningham and S.K. Dutta, *Scheduling jobs with exponentially distributed processing times on two machines of a flow shop*, Nav. Res. Logist. Q. **20** (1), 69-81, 1973.

[13] E.M. Gonzalez-Neira, D. Ferone, S. Hatami and A.A. Juan, *A biased-randomized simheuristic for the distributed assembly permutation flowshop problem with stochastic processing times*, Simulat. Model. Pract. Theor. **79**, 23-36, 2017.

[14] N.G. Hall and C. Sriskandarajah, *A survey of machine scheduling problems with blocking and no-wait in process*, Oper. Res. **44** (3), 510-525, 1996.

[15] P.J. Kalczynski and J. Kamburowski, *A heuristic for minimizing the expected makespan in two-machine flowshops with consistent coefficients of variation*, Eur. J. Oper. Res. **169** (3), 742-750, 2006.

[16] S.C. Kim and P.M. Bobrowski, *Scheduling jobs with uncertain setup times and sequence dependency*, Omega **25** (4), 437-447, 1997.

[17] G.M. Kopanos, J.M. Lainez and J. Puigjaner, *An efficient mixed-integer linear programming scheduling framework for addressing sequence-dependent setup issues in batch plants*, Ind. Eng. Chem. Res. **48** (13), 6346-6357, 2009.

[18] P.S. Ku and S.C. Niu, *On Johnson's two-machine flow shop with random processing times*, Oper. Res. **34** (1), 130-136, 1986.

[19] X. Li, Z. Yang, R. Ruiz, T. Chen, and S. Sui, *An iterated greedy heuristic for no-wait flow shops with sequence dependent setup times, learning and forgetting effects*, Inf. Sci. **453**, 408-425, 2018.

[20] M. Pinedo, *Stochastic scheduling with release dates and due dates*, Oper. Res. **31** (3), 559-572, 1983.

[21] D.K. Seo, C.M., Klein and W. Jang, *Single machine stochastic scheduling to minimize the expected number of tardy jobs using mathematical programming models*, Comput. Ind. Eng. **48** (2), 153-161, 2005.

[22] H.M. Soroush, *Sequencing and due-date determination in the stochastic single machine problem with earliness and tardiness costs*, Eur. J. Oper. Res. **113** (2), 450-468, 1999.

[23] H.M. Soroush, *Minimizing the weighted number of early and tardy jobs in a stochastic single machine scheduling problem*, Eur. J. Oper. Res. **181** (1), 266-287, 2007.

[24] K. Wang and S.H. Choi, *A decomposition-based approach to flexible flow shop scheduling under machine breakdown*, Int. J. Prod. Res. **50** (1), 215-234, 2012.

[25] K.C. Ying and S.W. Lin, *Minimizing makespan for no-wait flowshop scheduling problems with setup times*, Comput. Ind. Eng. **121**, 73-81, 2018.