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Mathematical Modeling of the Effect of CO2 Laser Power on Texture Size on Polyoxmethylene (POM) Sheet

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Abstract. Variation of the groove size depending on the laser power has been modeled in this proposed mathematical model. It is obtained by polymerization of Polyoxmethylene formaldehyde, a semi-crystalline polymer, and is among the hardest and strongest thermoplastics. Polyoxmethylene can be used in slip-friction pairs without lubrication. Polyoxmethylene materials are widely used for materials in tribological applications. They also show good friction properties.

1. Introduction

The tribologic properties wettability, hydrophobization and adhesion properties surface can be improved by surface texturing. The polymer surfaces have been modified with many commercial methods [9]. Chemical and physical modification can be applied to the polymer surfaces. There are some disadvantages in the chemical processing of polymer materials. Since chemical processes are very difficult to control in chemical surface treatment, the application areas of this method are also very limited. In addition, the measures to be taken to prevent environmental pollution by chemical methods are costly and increase the number of processes. One of the foremost disadvantages in the processing of polymer surfaces by mechanical methods is the wear of the tools used during the process. In addition to increasing the cost of wear, it also decreases the sensitivity of the process as the processing time increases.

Most of the materials can be easily processed with a laser. Many polymer materials can be processed precisely with the appropriate laser selection. Material processing precision is continuous and does not change over time. The selection of suitable parameters is very important in laser material processing. For each material and the desired product, the effects of many parameters such as laser power, frequency, the wavelength should be investigated and the most appropriate parameter selection should be made. The material processing time is short since high energy can be transferred very precisely to a small area by laser in a very short time.

In laser material processing, when the laser beam hits the surface, the material surface heats first. When the laser application time increases, if the energy is high enough, melting, evaporation or burning occurs respectively. The ablation mechanism in laser material processing has not been fully explained. In addition to the process parameters of the laser used in material processing with laser, the thermophysical properties

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of the material such as specific heat, absorption coefficient and thermal conduction significantly affect the quality of the processed material.

The effects of laser parameters on the surface texture have been investigated in many studies in order to obtain surface textures such as grooves and small cavities in the desired shape and size [10, 11]. Many studies have been carried out to obtain suitable laser and parameters for many different materials in order to obtain the desired surface properties [8, 12]. In addition to optimization studies to determine the material properties to be obtained by selecting the appropriate parameters, mathematical modeling studies are also carried out for the product to be obtained The applicability of mathematical modeling in laser material processing of polymer has also been proven by experimental results [12–15].

In this study, mathematical modeling of the width of micro-sized grooves created with laser on a Polyoxmethylene (POM) sheet has been made. Fourier method was used in the mathematical modeling of the heat distribution on the surface of the Polyoxymethylene. To obtain a mathematical model, the effects of the laser power on the groove width of Polyoxymethylene sheet were investigated. A mathematical model has been obtained by using the thermophysical properties of Polyoxymethylene and laser parameters.

The following problem of parabolic equations with various boundary conditions was studied [1–7].

The heat distribution equation on surface can be written as below;

$$\frac{\partial T(x,t)}{\partial t} = \alpha^2 \frac{\partial^2 T(x,t)}{\partial x^2} \tag{1}$$

where, T is the temperature as a function of time "t" and distance "x", α is the thermal diffusivity of the investigate material.

$$\alpha^2 = \frac{\lambda}{c\rho},$$

where λ denotes the thermal conductivity, c specific heat, ρ density.

Let $t_p > 0$ be a fixed number and denote by $D = \{(x.t) : 0 < x < l, 0 < t < t_p\}$, where t_p is the pulse duration.

The initial condition can be written as;

$$T(x,0) = T_0, \quad 0 < x < l$$

where T_0 is the initial temperature of the material. It was assumed that all the energy absorbed by the surface was transmitted to the material. Thus, the boundary condition ($x = 0$) on the surface can be written as follows:

$$\frac{\partial T(0,t)}{\partial t} = 0, \quad \frac{\partial T(l,t)}{\partial t} = 0.$$

This problem is called a parabolic problem. Classical solution of the problem (1)-(3) is $T(x,t) \in C^{2,1}(D) \cap C^{1,0}(D)$. The heat source problem has been investigated with parabolic equation in many studies. Then the following solution is obtained using Fourier method.

$$T(x,t) = \sum_{k=1}^{\infty} (T_{ck}(t) \cos \frac{2\pi\alpha k}{l} x + T_{sk}(t) \sin \frac{2\pi\alpha k}{l} x) e^{-(\frac{2\pi\alpha k}{l})^2 t} \tag{2}$$

The laser intensity within the material can be found using the Beer-Lambert's Law:

$\frac{dI(x)}{dx} = -aI$

Where $I(x)$ is the laser intensity as a function of distance from laser spot and α is the absorption coefficient of the material respectively. Although absorption coefficient is changed within the material but it was taken as constant in our study. Laser intensity as a function of distance within material can be written as;

$I = I_0 e^{-\int_b^z a dx}$

Actually most of the beam intensities have Gaussian distribution. We made one more assumption that our laser beam is top-hat beam that means intensity is homogeneously distributed in spot area.

The heat generation from the laser beam absorbed by the material is defined as,
$S = -dI/dx$
Using Leibniz rule yields, the heat source can be written as;
$S = I_0 e^{-\int_b^z a dx}$.
The temperature distribution as a function was obtained as given below;

$$T(x,t) = \sum_{k=1}^{\infty} \left(\varphi_{ck} e^{-(\frac{2\pi\alpha k}{l})^2 t} + \int_0^t \int_0^\pi S(x,t) \cos \frac{2\pi\alpha k}{l} x e^{-(\frac{2\pi\alpha k}{l})^2 (t-\tau)} dx d\tau \right) \cos \frac{2\pi\alpha k}{l} x + \sum_{k=1}^{\infty} \left(\varphi_{sk} e^{-(\frac{2\pi\alpha k}{l})^2 t} + \int_0^t \int_0^\pi S(x,t) \sin \frac{2\pi\alpha k}{l} x e^{-(\frac{2\pi\alpha k}{l})^2 (t-\tau)} dx d\tau \right) \sin \frac{2\pi\alpha k}{l} x \quad (3)$$

2. Material and Experimental Setup

The surfaces of 5 mm thick Polyoxymethylene sheets were used to ablation process. Some physical and thermal properties of Polyoxymethylene which were used in mathematical modeling have been listed in Table 1.

In the ablation process commercial 130 W CO2 laser was used with different power at constant scan speed. Laser spot diameter is 160 μm.

Table 1 Some physical and thermal properties of Polyoxymethylene

Properties	Value	Unit
Density	1410	kg/m^3
Thermal Capacity	1.5	$kJ/kg.K$
Melting point	165	$°C$
Heat Deflection Temperature	110	$°C$
Tensile module of elasticity	2800	MPa
Thermal Conductivity	0.31	W/mK

3. Results and Discussion

In this study, mathematical model has been proposed for the groove formation on Polyoxymethylene sheet with various power and constant scan speed. Groove widths were measured from optical microscope images of ablated surfaces of Polyoxymethylene sheets.

For 26 Watts of laser power, from the optical microscope images, the Heat Deflection Zone boundary and molten zone boundary distances were measured as 1434 μm and 1252 μm respectively. Temperatures at Heat Deflection boundary and molten zone boundary are 383 K and 438 K respectively. These values are used in temperature distribution equation obtain the Fourier coefficients which are depends on the material properties. The coefficients in the temperature distribution equation (2) were calculated as φ_{ck} (=451,32) and φ_{sk} (-205.15). These are the coefficients depend on the thermal properties of Polyoxymethylene. Then, in order to verify the validity of mathematical model, new grooves were obtained using 39, 52, 65, 78, 91 and 104 Watts. These coefficients were used to calculate temperature distribution for the Polyoxymethylene and variour laser beam powers.

Table 2 Laser Powers and groove widths measured from images.

Lazer Powerwatt	Molten Zone width (μm)	Heat Deflection Zone width (μm)
26	1252	1434
39	1319	1513
52	1367	1568
65	1404	1611
78	1434	1646
91	1459	1676
104	1482	1702

Each laser powers and the coefficients obtained previously were used in the temperature distribution equation to calculate the temperatures for each speed of laser beam. The calculated temperatures for boundaries are given in Table 3.

Table 3 Melting and Heat Deflection Temperatures calculated with mathematical model, real values and percent error between them.

Powerwatt		T(x,t) (K)	T(x,t) (K) (Calculated)	error
39	Melting	438	441.316	0.76
39	Heat Deflection	383	385.021	0.53
52	Melting	438	442.256	0.97
52	Heat Deflection	383	385.895	0.76
65	Melting	438	443.462	1.25
65	Heat Deflection	383	387.021	1.05
78	Melting	438	444.891	1.57
78	Heat Deflection	383	388.105	1.33
91	Melting	438	446.114	1.85
91	Heat Deflection	383	389.206	1.62
104	Melting	438	447.365	2.14
104	Heat Deflection	383	390.170	1.86

4. Conclusion

By texturing the surfaces, the mechanical properties of my surfaces can be changed. The properties of material surfaces can be improved by many methods such as mechanical and chemical methods. While texturing surfaces with a laser have many advantages, it can require complex processes to be controlled. Mathematical modeling of the heat distribution of the surface to be obtained with laser texture can be known in advance the properties of the product to be obtained. This saves time and material.

Grooves were formed on the Polyoxymethylene material surface with seven different laser beam power. Measurements were made from the images of the groove obtained by using a 26-watt laser beam. The measurement results were applied to the proposed mathematical model and the φ_{ck} and φ_{sk} coefficients in the mathematical model were calculated. These coefficients are coefficients depending on the properties of Polyoxymethylene. These coefficients were applied for grooves obtained using 39, 52, 65, 78, 91 and 104 W laser beams. Heat deflection and melting point values obtained in the mathematical model are quite compatible with the actual values. As the laser power increased, the error rate increased acceptably.

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