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Mathematical Modeling of the Effect of CO2 Laser Parameter on Shape and Geometry of Polymer Plate

Timur Canel[a], **İrem Bağlan**[b]

[a] *Department of Physics, Faculty of Science and Arts, Kocaeli University, Kocaeli, Turkey.*
[b] *Department of Mathematics, Faculty of Science and Arts, Kocaeli University, Kocaeli, Turkey.*

Abstract. In recent years, the use of polymer-based materials is in almost every aspect of daily life [1]. PMMA can be used in many areas from aircraft to the medical industry with their good chemical stability, high strength, high corrosion and aging resistance, insulation performance, and smooth surface [2]. In this study, grooves were formed on Polymethyl Methacrylate (PMMA) Plates with different scanning speeds with CO2 laser. Since the scan speed of the laser is increased, the interaction time between the laser beam and the material decreases then the amount of energy transferred to the material also decreases. Measurements were made from high-resolution optical microscope images of the grooves created on PMMA. In this study, the distribution of heat energy transferred to the material was modeled mathematically. The change to groove size depending on the laser scan speed is modeled. To validate the mathematical model, the surfaces of the PMMA plate were ablated with different scan speed at constant power. The CO2 laser that has 10600 nm wavelengths and 130 Watts maximum power was used in the ablation.

1. Introduction

Polymeric materials can be divided into two groups; Thermoplastics and thermosets. The main difference between the two is their reaction to heating. Thermoplastics can be reheated, coated and cooled as required. No chemical treatment is required during this process. Thermosets, on the other hand, cannot be reshaped after being heated and shaped. It becomes very strong and durable in the first forming. PMMA is classified as thermoplastic. PMMA has various performance benefits such as high strength, shrink-resistance, and easy flexibility. Polymer materials are frequently preferred in the industry as they can be processed easily. Although it can be processed by mechanical and chemical methods, laser processing of polymer materials has superior properties compared to other methods. Due to the difficulty of controlling chemical reactions and their negative effects on the environment, the application area of the chemical method is very limited. Although mechanical processing is one of the frequently used methods, it has disadvantages such as abrasion of the abrasive elements used and the inability to obtain a product with the same precision.

The tribology, wettability, adhesion and hydrophobization properties have been improving by surface texturing. Many different methods have been developed for texturing the surfaces of polymers with

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Corresponding author: TC, mail address: tcanel@kocaeli.edu.tr ORCID:0000-0002-4282-1806, IB ORCID:0000-0002-2877-9791

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different specialties [3]. Many laser parameters such as wavelength, frequency, power and spot size can be selected in accordance with the material and the desired surface structure. In addition to these features, lasers are preferred in many areas today because they are compact and do not require additional systems other than ambient gas.

Although the ablation mechanism in laser material processing is strictly dependent on material properties and process parameters, it is very difficult to obtain a surface structure with the desired precision. The effective thermophysical properties in the ablation mechanism are thermal conduction, absorption coefficient and specific heat. Besides the laser properties such as the wavelength, frequency and power of the laser used, process parameters such as scan speed, overlap rate, number of pulses and beam size determine the ablation and therefore the quality of the processed material.

Regular textures such as micro-sized cavities and grooves created on the polymeric material surface improve the friction and adhesion behavior of the materials. The geometries, density and orientation of the microstructures created on the surface play an important role in increasing the surface performance. [4,5]. For these reasons, many optimization studies have been carried out in order to obtain the desired texture on the surface of many kinds of materials. [6,7,8]. In addition to optimization studies, mathematical modeling of the heat distribution in the material can be obtained from data about the geometry of the cavities to be obtained by laser. [9,10,11]. In this study, the mathematical modeling of the heat distribution for the width of the grooves created by laser on the PMMA plate was made. In the mathematical model, the Fourier method with a homogenous approach was used. To obtain a numerical model, the effects of the laser scan speed on the groove size of PMMA sheet were investigated and a simple mathematical model of the heat distribution on surface is proposed.

The heat distribution equation on surface can be written as below;

$$\frac{\partial T(x,t)}{\partial t} = \alpha^2 \frac{\partial^2 T(x,t)}{\partial x^2}, \tag{1}$$

where T is the temperature as a function of time t and distance x, α is the thermal diffusivity of the investigate material.

$$\alpha^2 = \frac{\lambda}{c\rho}$$

where, λ denotes the thermal conductivity, c specific heat ρ density.

Let $t_p > 0$ be a fixed number and denote by $D = \{(x.t) : 0 < x < l, 0 < t < t_p\}$, where t_p is the pulse duration.

The initial condition can be written as;

$$T(x,0) = T_0, \quad 0 < x < l$$

where T_0 is the initial temperature of the material. It was assumed that all the energy absorbed by the surface was transmitted to the material. Thus, the boundary condition ($x = 0$) on the surface can be written as follows:

$$\frac{\partial T(0,t)}{\partial t} = 0, \quad \frac{\partial T(l,t)}{\partial t} = 0$$

This problem is called a parabolic problem. Classical solution of the problem (1)-(3) is $T(x,t) \in C^{2,1}(D) \cap C^{1,0}(D)$. The heat source problem has been investigated with parabolic equation in many studies. Then the following solution is obtained using Fourier method.

$$T(x,t) = \sum_{k=1}^{\infty} (T_{ck}(t) \cos \frac{2\pi\alpha k}{l} x + T_{sk}(t) \sin \frac{2\pi\alpha k}{l} x) e^{-(\frac{2\pi\alpha k}{l})^2 t} \tag{2}$$

The laser intensity within the material can be found using the Beer-Lambert's Law:
$\frac{dI(x)}{dx} = -aI$

Where $I(x)$ is the laser intensity as a function of distance from laser spot and α is the absorption coefficient of the material respectively. Although absorption coefficient is changed within the material but it was taken as constant in our study. Laser intensity as a function of distance within material can be written as;

$I = I_0 e^{-\int_b^z a dx}$

Actually most of the beam intensities have Gaussian distribution. We made one more assumption that our laser beam is top-hat beam that means intensity is homogeneously distributed in spot area.

The heat generation from the laser beam absorbed by the material is defined as,

$S = -dI/dx$

Using Leibniz rule yields, the heat source can be written as;

$S = I_0 e^{-\int_b^z a dx}$.

The temperature distribution as a function was obtained as given below;

$$\begin{aligned} T(x,t) \quad = \quad & \sum_{k=1}^{\infty}\left(\varphi_{ck} e^{-\left(\frac{2\pi\alpha k}{l}\right)^2 t} + \int_0^t \int_0^l S(x,t) \cos\frac{2\pi k}{l} x e^{-\left(\frac{2\pi\alpha k}{l}\right)^2 (t-\tau)} dx d\tau\right) \cos\frac{2\pi k}{l} x \\ & + \sum_{k=1}^{\infty}\left(\varphi_{sk} e^{-\left(\frac{2\pi\alpha k}{l}\right)^2 t} + \int_0^t \int_0^l S(x,t) \sin\frac{2\pi k}{l} x e^{-\left(\frac{2\pi\alpha k}{l}\right)^2 (t-\tau)} dx d\tau\right) \sin\frac{2\pi k}{l} x - \frac{xH}{l\lambda} \end{aligned} \tag{3}$$

2. Material and Experimental Setup

The surfaces of 10 mm thick PMMA sheets to be used were polished before ablation to cleaning and increase the transparency of the surfaces. Some physical and thermal properties of PMMA sheet which were used in ablation and mathematical modeling have been listed in Table 1. In the ablation process commercial 130 W CO_2 laser was used with different scan speeds at constant power. Laser spot diameter is 160 μm the laser beam intensity $6.5 \times 10^9 W/m^2$.

Table 1 Some physical and thermal properties of PMMA

Properties	Value	Unit
Density	1180	kg/m^3
Coefficient of Thermal Expansion	75	$(.10^{-6} K^{-1})$
Melting point	130	$°C$
Heat Deflection Temperature	95	$°C$
Specific heat	69	$J.K^{-1}kg^{-1}$
Thermal Conductivity	0.18	$W.m^{-1}.K^{-1}$

3. Results and Discussion

In this study, mathematical model has been proposed for the groove formation on PMMA sheet with various scan speeds and constant power. Groove sizes were measured from optical microscope images of ablated surfaces of PMMA sheets.

The Heat Deflection Zone boundary and molten zone boundary distances were calculated as 2059 μm and 1733 μm respectively. Temperatures at Heat Deflection boundary and molten zone boundary are 368 K and 403 K respectively. Fourier coefficients in the mathematical model were obtained using these boundary temperatures.

The coefficients in the temperature distribution equation φ_c and φ_s were calculated as 321.45 and -201.15 respectively. These coefficients depend on the thermo physical properties of PMMA. Then, in order to verify the validity of mathematical model, new grooves were obtained using 100, 150, 200, 250, 300, 350 mm/s scan speeds. To verify the mathematical model, these coefficients were used to calculate the melting and heat deflection temperatures for the same material and different scan speeds. The calculated temperatures for boundaries (melting and heat deflection region) are given in Table 3.

Table 2 Laser scan speeds and groove widths measured from images.

Scan Speed mm/s	Molten Zone width (μm)	Heat Deflection Zone width (μm)
50	1733	2059
100	1707	2027
150	1677	1991
200	1642	1949
250	1677	1897
300	1707	1830
350	1733	1735

Table 3 The calculated melting and heat deflection temperatures temperatures for boundaries.

Scan Speed mm/s		T(x,t) (K)	T(x,t) (K) (Calculated)	error
100	Melting	403	416.69	3.40
100	Heat Deflection	368	377.89	2.69
150	Melting	403	423.73	5.14
150	Heat Deflection	368	382.84	4.03
200	Melting	403	429.12	6.48
200	Heat Deflection	368	389.68	5.89
250	Melting	403	438.25	8.75
250	Heat Deflection	368	396.47	7.74
300	Melting	403	445.54	10.45
300	Heat Deflection	368	406.17	10.37
350	Melting	403	453.59	12.55
350	Heat Deflection	368	413.71	12.42

4. Conclusion

It can be used for different purposes such as improving the mechanical properties of the materials by laser processing the surfaces of polymer materials, as well as using them in electronic devices. It is very important for the quality of the product to control the dimensions of the geometries to be obtained by laser on the material. By modeling the heat dissipation mechanism in material processing with laser, the dimensions of the shape to be obtained on the material can be controlled. Applicable mathematical modeling plays an important role in explaining this mechanism. In accordance with the purpose of the study, applicable mathematical modeling has been created and the applicability of this model has been proven.

In this study, grooves were formed on Polymethyl Methacrylate (PMMA) Plates with different scanning speeds with CO2 laser. Since the scan speed of the laser is increased, the interaction time between the laser beam and the material decreases then the amount of energy transferred to the material also decreases. Measurements were made from high-resolution optical microscope images of the grooves created on PMMA. In this study, the distribution of heat energy transferred to the material was modeled mathematically. The change to groove size depending on the laser scan speed is modeled. The heat distribution that causes the formation of grooves is modeled with the Fourier method. First, material-specific coefficients were calculated with the proposed mathematical model. In order to prove the validity of these coefficients, 7 different grooves obtained with 7 different scanning speeds were examined. The results obtained show that the proposed mathematical model is reliable.

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