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Spin Three-Body Problem of Classical Electrodynamics with Radiation Terms – (I) Derivation of Spin Equations

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Abstract

In the present paper spin equations for three-body problem of classical electrodynamics are introduced. They should be considered jointly with 3-body equations of motion derived in a previous paper of the author. The system of spin equations is an overdetermined one. It is shown that the independent spin equations are nine in number as many as the components of the unknown spin functions. The system obtained will be solved by the fixed-point method in a next paper.

Keywords: Spin three-body problem, Classical electrodynamics, Spin equations.
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1. Introduction

The present paper is one of the series devoted to study of the three-body problem of classical electrodynamics. In [5] we have formulated 3-dimensional three-body problem with radiation terms and in [6] we have proved the existence-uniqueness of periodic solution based on the previous results for two-body problem [2], [3]. The system of equations of motion is of neutral type with delays depending on unknown trajectories. We have derived spin equations for two-body problem in [4] using the Corben's considerations of classical spinning particles [10], [11]. The problem for spinning particles in many papers has been considered (cf. for instance [7], [14], [16]-[20]). Here in a similar way we extend the results from [4] applying the interaction presentation for N-body problem from [1] and derive spin equations for three charged particles. We show that to every particle correspond six equations but three of them are consequences from the first three ones

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in the following sense: every such equation differs from the first one by a summand which is a cross product of spin vector and arbitrary constant vector perpendicular to the spin vector. So, we obtain nine equations for nine unknown spin functions, namely the component of the three spin vectors. We note the papers D.-A. Deckert et al. [8], [9], [12], [13], [15] based on the Wheeler- Feynman approach. The paper consists of five sections, Appendix and References. In Section 2 preliminary results for 3-body problem equations of motion with radiation terms and their relations to spin equations are considered. In Section 3 the spin equations are derived and the elements of the spin tensor are defined. In Section 4 a vector form of the Lorentz part and radiation parts of the first three spin equations is presented. Lorentz parts we call the summands in the right-hand sides of the spin equations which take into account for a given particle the influence of the rest ones. The terms which take into account self-interaction of every particle we call radiation terms. In Section 5 we show that the last three equations are consequences of the first three ones. In this way we obtain nine equations for nine unknown spin functions, or we get as many equations as spin functions are. The existence-uniqueness of a periodic solution of the general system in the second part of the present paper will be proved.

2. Preliminary Results for3-Body Problem Equations of Motion with Radiation Terms and their Relations to Spin Equations

First, we recall some denotations from [2] - [6]. The investigations are in the Minkowski space. Roman suffixes run over 1, 2, 3, 4, while Greek – over 1, 2, 3 with Einstein summation convention. By $\langle .,. \rangle_4$ we denote the scalar product in the Minkowski space, while by $\langle .,. \rangle$ – the scalar product in three-dimensional Euclidean subspace.

Following the approach from [4] and [10] we introduce spin equations jointly with equations of motion with radiation terms derived in [5] for three-body problem:

$$\begin{aligned}
\frac{d\lambda_r^{(1)}}{ds_1} &= \frac{e_1}{m_1 c^2}\left(\left(F_{rs}^{(12)}+F_{rs}^{(13)}\right)\lambda_s^{(1)}+F_{rs}^{(1)rad}\lambda_s^{(1)}\right);\\
\frac{d\lambda_r^{(2)}}{ds_2} &= \frac{e_2}{m_2 c^2}\left(\left(F_{rs}^{(21)}+F_{rs}^{(23)}\right)\lambda_s^{(2)}+F_{rs}^{(2)rad}\lambda_s^{(2)}\right);\\
\frac{d\lambda_r^{(3)}}{ds_3} &= \frac{e_3}{m_3 c^2}\left(\left(F_{rs}^{(31)}+F_{rs}^{(32)}\right)\lambda_s^{(3)}+F_{rs}^{(3)rad}\lambda_s^{(3)}\right);
\end{aligned}\tag{2.1}$$

$$\begin{aligned}
\frac{d\sigma_{ij}^{(1)}}{ds_1} &= \frac{e_1}{m_1 c^2}\left(\left(F_{im}^{(12)}+F_{im}^{(13)}+F_{im}^{(1)rad}\right)\sigma_{mj}^{(1)}-\sigma_{im}^{(1)}\left(F_{mj}^{(12)}+F_{mj}^{(13)}+F_{mj}^{(1)rad}\right)\right);\\
\frac{d\sigma_{ij}^{(2)}}{ds_2} &= \frac{e_2}{m_2 c^2}\left(\left(F_{im}^{(21)}+F_{im}^{(23)}+F_{im}^{(2)rad}\right)\sigma_{mj}^{(2)}-\sigma_{im}^{(2)}\left(F_{mj}^{(21)}+F_{mj}^{(23)}+F_{mj}^{(2)rad}\right)\right);\\
\frac{d\sigma_{ij}^{(3)}}{ds_3} &= \frac{e_3}{m_3 c^2}\left(\left(F_{im}^{(31)}+F_{im}^{(32)}+F_{im}^{(3)rad}\right)\sigma_{mj}^{(3)}-\sigma_{im}^{(3)}\left(F_{mj}^{(31)}+F_{mj}^{(32)}+F_{mj}^{(3)rad}\right)\right).
\end{aligned}\tag{2.2}$$

The electromagnetic tensors $F_{rs}^{(kn)}$ in the right-hand sides we call Lorentz parts. For each particle, they take into account the influence of other particles. The quantities relating to the particles are: $\left(x_1^{(k)}(t), x_2^{(k)}(t), x_3^{(k)}(t), x_4^{(k)}=ict\right) \equiv \left(\vec{x}^{(k)}, ict\right)$, $(k=1,2,3)$ – space-time coordinates of the moving particles; c – the vacuum speed of light; L_k – the world lines; m_k – proper masses; e_k – charges $(k=1,2,3)$. The elements of the electromagnetic tensors $F_{rl}^{(kn)} = \frac{\partial\Phi_l^{(n)}}{\partial x_r^{(k)}} - \frac{\partial\Phi_r^{(n)}}{\partial x_l^{(k)}}$ can be calculated by the retarded

Lienard-Wiechert potentials $\Phi_r^{(n)} = -\frac{e_n\lambda_r^{(n)}}{\left\langle\lambda^{(n)},\xi^{(kn)}\right\rangle_4}$, where

$$\begin{aligned}\xi^{(kn)} &= \left(\xi_1^{(kn)},\xi_2^{(kn)},\xi_3^{(kn)},\xi_4^{(kn)}\right)\\ &= \left(x_1^{(k)}(t)-x_1^{(n)}(t-\tau_{kn}),x_2^{(k)}(t)-x_2^{(n)}(t-\tau_{kn}),x_3^{(k)}(t)-x_3^{(n)}(t-\tau_{kn}),ic\tau_{kn}(t)\right);\\ \lambda^{(k)} &= \left(\lambda_1^{(k)},\lambda_2^{(k)},\lambda_3^{(k)},\lambda_4^{(k)}\right) = \left(\vec{\lambda}^{(k)},\lambda_4^{(k)}\right) = \left(\frac{u_1^{(k)}}{\Delta_k},\frac{u_2^{(k)}}{\Delta_k},\frac{u_3^{(k)}}{\Delta_k},\frac{ic}{\Delta_k}\right) = \left(\frac{\vec{u}^{(k)}}{\Delta_k},\frac{ic}{\Delta_k}\right);\end{aligned}$$

$$\Delta_k = \sqrt{c^2-\left\langle\vec{u}^{(k)}(t),\vec{u}^{(k)}(t)\right\rangle};\Delta_n = \sqrt{c^2-\left\langle\vec{u}^{(n)}(t-\tau_{kn}),\vec{u}^{(n)}(t-\tau_{kn})\right\rangle}.$$

Since $\xi^{(kn)}$ are isotropic 4-vectors, i.e. $\left\langle\xi^{(kn)},\xi^{(kn)}\right\rangle_4=0$, the retarded functions $\tau_{kn}(t)$ can be defined as solutions of the functional equations,

$$\tau_{kn}(t) = \frac{1}{c}\sqrt{\left\langle\vec{\xi}^{(kn)},\vec{\xi}^{(kn)}\right\rangle} \equiv \frac{1}{c}\sqrt{\sum_{\alpha=1}^{3}\left[x_\alpha^{(k)}(t)-x_\alpha^{(n)}(t-\tau_{kn}(t))\right]^2}, \tag{2.3}$$

where$(k\,n)=(1\,2),(1\,3),(2\,1),(2\,3),(3\,1),(3\,2)$.

Following denotations from [5] we obtain

$$\frac{d\lambda_r^{(k)}}{ds_k} = \frac{e_k}{m_kc^2}\sum_{n=1,n\neq k}^{3}F_{rm}^{(kn)}\lambda_m^{(k)};F_{rm}^{(kn)} = \frac{\partial\Phi_m^{(n)}}{\partial x_r^{(k)}}-\frac{\partial\Phi_r^{(n)}}{\partial x_m^{(k)}} = e_n\left(P_r^{(kn)}\xi_m^{(kn)}-P_m^{(kn)}\xi_r^{(kn)}\right);$$

$$\frac{d\lambda_r^{(k)}}{ds_k} = \frac{e_k}{m_kc^2}\sum_{n=1,n\neq k}^{3}e_n\left(P_r^{(kn)}\xi_m^{(kn)}-P_m^{(kn)}\xi_r^{(kn)}\right)\lambda_m^{(k)};$$

$$\frac{d\vec{\lambda}^{(k)}}{ds_k} = -\frac{e_k^2}{m_kc^2}\sum_{n=1,n\neq k}^{3}e_n\left[\vec{P}^{(kn)}\left\langle\lambda^{(k)},\xi^{(kn)}\right\rangle_4-\vec{\xi}^{(kn)}\left\langle\lambda^{(k)},P^{(kn)}\right\rangle_4\right];$$

$$\frac{d\vec{\lambda}^{(k)}}{ds_k} = \frac{1}{\Delta_k^2}\frac{d\vec{u}^{(k)}(t)}{dt}+\frac{\vec{u}^{(k)}(t)}{\Delta_k^4}\left\langle\vec{u}^{(k)}(t),\frac{d\vec{u}^{(k)}(t)}{dt}\right\rangle;\frac{d\lambda_4^{(k)}}{ds_k} = \frac{ic}{\Delta_k^4}\left\langle\vec{u}^{(k)}(t),\frac{d\vec{u}^{(k)}(t)}{dt}\right\rangle;$$

$$D_{kn} = \frac{c^2\tau_{kn}-\left\langle\vec{\xi}^{(kn)},\vec{u}^{(n)}\right\rangle}{c^2\tau_{kn}-\left\langle\vec{\xi}^{(kn)},\vec{u}^{(k)}\right\rangle};\frac{d\vec{\lambda}^{(n)}}{ds_n} = \frac{D_{kn}}{\Delta_n^2}\left(\frac{d\vec{u}^{(n)}}{dt}+\frac{\vec{u}^{(n)}}{\Delta_{kn}^2}\left\langle\vec{u}^{(n)},\frac{d\vec{u}^{(n)}}{dt}\right\rangle\right);$$

$$\frac{d\lambda_4^{(n)}}{ds_n} = \frac{icD_{kn}}{\Delta_{kn}^4}\left\langle\vec{u}^{(n)},\frac{d\vec{u}^{(n)}}{dt}\right\rangle;P_r^{(kn)} = -\frac{\lambda_r^{(n)}}{\left\langle\lambda^{(n)},\xi^{(kn)}\right\rangle_4^3}\left[1+\left\langle\xi^{(kn)},\frac{d\lambda^{(n)}}{ds_n}\right\rangle_4\right]+\frac{1}{\left\langle\lambda^{(n)},\xi^{(kn)}\right\rangle_4^{\,2}}\frac{d\lambda_r^{(n)}}{ds_n};$$

$$\vec{P}^{(kn)} = -\frac{\vec{\lambda}^{(n)}M_{kn}}{\left\langle\lambda^{(n)},\xi^{(kn)}\right\rangle_4^3}+\frac{1}{\left\langle\lambda^{(n)},\xi^{(kn)}\right\rangle_4^{\,2}}\frac{d\vec{\lambda}^{(n)}}{ds_n}\;;\;M_{kn}=1+\left\langle\xi^{(kn)},\frac{d\lambda^{(n)}}{ds_n}\right\rangle_4;$$

$$\begin{aligned}P_4^{(kn)} &= -\frac{\lambda_4^{(n)}M_{kn}}{\left\langle\lambda^{(n)},\xi^{(kn)}\right\rangle_4^3}+\frac{1}{\left\langle\lambda^{(n)},\xi^{(kn)}\right\rangle_4^{\,2}}\frac{d\lambda_4^{(n)}}{ds_n} = -ic\left(\frac{M_{kn}}{\Delta_n\left\langle\lambda^{(n)},\xi^{(kn)}\right\rangle_4^3}-\frac{D_{kn}\left\langle\vec{u}^{(n)},\dot{\vec{u}}^{(n)}\right\rangle}{\left\langle\lambda^{(n)},\xi^{(kn)}\right\rangle_4^{\,2}\Delta_n^4}\right)\equiv -icL_{kn};\\ \frac{i}{c}P_4^{(kn)} &= L_{kn}\,(k=1,2,3\;;\;r=1,2,3,4).\end{aligned}$$

The radiation terms are defined as a half of the difference of retarded and advanced potentials in accordance with the Dirac assumption:

$$F_{ml}^{(k)rad} = \frac{1}{2}\left[\left(\frac{\partial\Phi_l^{(k)ret}}{\partial x_m^{(k)ret}}-\frac{\partial\Phi_m^{(n)ret}}{\partial x_l^{(k)ret}}\right)-\left(\frac{\partial\Phi_l^{(k)adv}}{\partial x_m^{(k)adv}}-\frac{\partial\Phi_m^{(n)adv}}{\partial x_l^{(k)adv}}\right)\right],$$

where $\Phi_r^{(k)ret} = -\frac{e_k\lambda_r^{(k)ret}}{\left\langle\lambda^{(k)ret},\xi^{(k)ret}\right\rangle_4}$, $\Phi_r^{(k)adv} = -\frac{e_k\lambda_r^{(k)adv}}{\left\langle\lambda^{(k)adv},\xi^{(k)adv}\right\rangle_4}$ and

$$F_{rs}^{(k)ret} = \frac{\partial\Phi_s^{(k)ret}}{\partial x_r^{(k)}} - \frac{\partial\Phi_r^{(k)ret}}{\partial x_s^{(k)}} = e_k\left(P_r^{(k)ret}\xi_s^{(k)ret} - P_s^{(k)ret}\xi_r^{(k)ret}\right);$$

$$F_{rs}^{(k)adv} = \frac{\partial\Phi_s^{(k)adv}}{\partial x_r^{(k)}} - \frac{\partial\Phi_r^{(k)adv}}{\partial x_s^{(k)}} = e_k\left(P_r^{(k)adv}\xi_s^{(k)adv} - P_s^{(k)adv}\xi_r^{(k)adv}\right),$$

where

$$P_r^{(k)ret} = -\frac{\lambda_r^{(k)ret}}{\left\langle\lambda^{(k)ret},\xi^{(k)ret}\right\rangle_4^3}\left[1+\left\langle\xi^{(k)ret},\frac{d\lambda^{(k)ret}}{ds_k}\right\rangle_4\right] + \frac{1}{\left\langle\lambda^{(k)ret},\xi^{(k)ret}\right\rangle_4{}^2}\frac{d\lambda_r^{(k)ret}}{ds_k};$$

$$P_r^{(k)adv} = -\frac{\lambda_r^{(k)adv}}{\left\langle\lambda^{(k)adv},\xi^{(k)adv}\right\rangle_4^3}\left[1+\left\langle\xi^{(k)adv},\frac{d\lambda^{(k)adv}}{ds_k}\right\rangle_4\right] + \frac{1}{\left\langle\lambda^{(k)adv},\xi^{(k)adv}\right\rangle_4{}^2}\frac{d\lambda_r^{(k)adv}}{ds_k};$$

$$F_{rs}^{(k)rad} = \frac{F_{rs}^{(k)ret} - F_{rs}^{(k)adv}}{2} = e_k\frac{P_r^{(k)ret}\xi_s^{(k)ret} - P_s^{(k)ret}\xi_r^{(k)ret} - \left(P_r^{(k)adv}\xi_s^{(k)adv} - P_s^{(k)adv}\xi_r^{(k)adv}\right)}{2}.$$

In the previous paper [5] it is proved that every fourth equation of (2.1) is a consequence of the first three ones. So under the assumption $\sqrt{\left\langle\vec{u}^{(k)}(t),\vec{u}^{(k)}(t)\right\rangle} \leq \bar{c} < c$ $(k = 1,2,3)$ the equations of motion (2.1) are 12 in number but it is proved that three of them are consequence of the rest ones so we obtain nine in number independent equations. In [6] an existence-uniqueness of a periodic solution of (2.1) is proved.

3. Derivation of the Spin Equations

Recalling the summation in repeated Latin indices we introduce an explicit form of the spin equations:

$$\frac{d\sigma_{ij}^{(1)}}{ds_1} = \frac{e_1}{m_1c^2}\left[\left(F_{im}^{(12)} + F_{im}^{(13)} + F_{im}^{(1)rad}\right)\sigma_{mj}^{(1)} - \sigma_{im}^{(1)}\left(F_{mj}^{(12)} + F_{mj}^{(13)} + F_{mj}^{(1)rad}\right)\right];$$

$$\frac{d\sigma_{ij}^{(2)}}{ds_2} = \frac{e_2}{m_2c^2}\left[\left(F_{im}^{(21)} + F_{im}^{(23)} + F_{im}^{(2)rad}\right)\sigma_{mj}^{(2)} - \sigma_{im}^{(2)}\left(F_{mj}^{(21)} + F_{mj}^{(23)} + F_{mj}^{(2)rad}\right)\right];$$

$$\frac{d\sigma_{ij}^{(3)}}{ds_3} = \frac{e_3}{m_3c^2}\left[\left(F_{im}^{(31)} + F_{im}^{(32)} + F_{im}^{(3)rad}\right)\sigma_{mj}^{(3)} - \sigma_{im}^{(3)}\left(F_{mj}^{(31)} + F_{mj}^{(32)} + F_{mj}^{(3)rad}\right)\right]$$

or in abbreviated form

$$\frac{d\sigma_{ij}^{(k)}}{ds_k} = \frac{e_1}{m_1c^2}\left[\left(\sum_{n=1,n\neq k}^{3} F_{im}^{(kn)} + F_{im}^{(k)rad}\right)\sigma_{mj}^{(1)} - \sigma_{im}^{(1)}\left(\sum_{n=1,n\neq k}^{3} F_{mj}^{(kn)} + F_{mj}^{(1)rad}\right)\right].$$

Introduce denotations following [4]:

$$\vec{\theta}^{(k)} = \left(\theta_1^{(k)}(t),\theta_2^{(k)}(t),\theta_3^{(k)}(t)\right); \vec{\sigma}^{(k)} = \left(\sigma_1^{(k)},\sigma_2^{(k)},\sigma_3^{(k)}\right);$$

$$\vec{\theta}^{(k)} = \frac{1}{c}\left(\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)}\right) = \frac{1}{c}\begin{vmatrix}\vec{e}_1 & \vec{e}_2 & \vec{e}_3\\ \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)}\\ \sigma_1^{(k)} & \sigma_2^{(k)} & \sigma_3^{(k)}\end{vmatrix} = \frac{1}{c}\begin{vmatrix}\lambda_2^{(k)} & \lambda_3^{(k)}\\ \sigma_2^{(k)} & \sigma_3^{(k)}\end{vmatrix}\vec{e}_1 - \frac{1}{c}\begin{vmatrix}\lambda_1^{(k)} & \lambda_3^{(k)}\\ \sigma_1^{(k)} & \sigma_3^{(k)}\end{vmatrix}\vec{e}_2 + \frac{1}{c}\begin{vmatrix}\lambda_1^{(k)} & \lambda_2^{(k)}\\ \sigma_1^{(k)} & \sigma_2^{(k)}\end{vmatrix}\vec{e}_3$$

that is,

$$\vec{\theta}^{(k)} = \left(\frac{1}{c}\begin{vmatrix}\lambda_2^{(k)} & \lambda_3^{(k)}\\ \sigma_2^{(k)} & \sigma_3^{(k)}\end{vmatrix}; -\frac{1}{c}\begin{vmatrix}\lambda_1^{(k)} & \lambda_3^{(k)}\\ \sigma_1^{(k)} & \sigma_3^{(k)}\end{vmatrix}, \frac{1}{c}\begin{vmatrix}\lambda_1^{(k)} & \lambda_2^{(k)}\\ \sigma_1^{(k)} & \sigma_2^{(k)}\end{vmatrix}\right)$$

we define the spin tensor

$$\sigma^{(k)}_{\mu\nu}=\begin{pmatrix} 0 & \sigma^{(k)}_3 & -\sigma^{(k)}_2 & i\,\theta^{(k)}_1\\ -\sigma^{(k)}_3 & 0 & \sigma^{(k)}_1 & i\,\theta^{(k)}_2\\ \sigma^{(k)}_2 & -\sigma^{(k)}_1 & 0 & i\,\theta^{(k)}_3\\ -i\,\theta^{(k)}_1 & -i\,\theta^{(k)}_2 & -i\,\theta^{(k)}_3 & 0\end{pmatrix}$$

$$=\begin{pmatrix} 0 & \sigma^{(k)}_3 & -\sigma^{(k)}_2 & \frac{i}{c}\begin{vmatrix}\lambda^{(k)}_2 & \lambda^{(k)}_3\\ \sigma^{(k)}_2 & \sigma^{(k)}_3\end{vmatrix}\\ -\sigma^{(k)}_3 & 0 & \sigma^{(k)}_1 & -\frac{i}{c}\begin{vmatrix}\lambda^{(k)}_1 & \lambda^{(k)}_3\\ \sigma^{(k)}_1 & \sigma^{(k)}_3\end{vmatrix}\\ \sigma^{(k)}_2 & -\sigma^{(k)}_1 & 0 & \frac{i}{c}\begin{vmatrix}\lambda^{(k)}_1 & \lambda^{(k)}_2\\ \sigma^{(k)}_1 & \sigma^{(k)}_2\end{vmatrix}\\ -\frac{i}{c}\begin{vmatrix}\lambda^{(k)}_2 & \lambda^{(k)}_3\\ \sigma^{(k)}_2 & \sigma^{(k)}_3\end{vmatrix} & \frac{i}{c}\begin{vmatrix}\lambda^{(k)}_1 & \lambda^{(k)}_3\\ \sigma^{(k)}_1 & \sigma^{(k)}_3\end{vmatrix} & -\frac{i}{c}\begin{vmatrix}\lambda^{(k)}_1 & \lambda^{(k)}_2\\ \sigma^{(k)}_1 & \sigma^{(k)}_2\end{vmatrix} & 0\end{pmatrix}.$$

Thus, we introduce the following equations describing the spin of three charged particles:

$$\frac{d\sigma^{(k)}_{12}}{ds_k}=\frac{e_k}{m_kc^2}\left[\left(\sum_{n=1,n\neq k}^{3}F^{(kn)}_{1\,m}+F^{(k)rad}_{1m}\right)\sigma^{(k)}_{m2}-\sigma^{(k)}_{1\,m}\left(\sum_{n=1,n\neq k}^{3}F^{(kn)}_{m2}+F^{(k)rad}_{m2}\right)\right];$$

$$\frac{d\sigma^{(k)}_{13}}{ds_k}=\frac{e_k}{m_kc^2}\left[\left(\sum_{n=1,n\neq k}^{3}F^{(kn)}_{1\,m}+F^{(k)rad}_{1\,m}\right)\sigma^{(k)}_{m3}-\sigma^{(k)}_{1m}\left(\sum_{n=1,n\neq k}^{3}F^{(kn)}_{m3}+F^{(k)rad}_{m3}\right)\right];$$

$$\frac{d\sigma^{(k)}_{23}}{ds_k}=\frac{e_k}{m_kc^2}\left[\left(\sum_{n=1,n\neq k}^{3}F^{(kn)}_{2\,m}+F^{(k)rad}_{2\,m}\right)\sigma^{(k)}_{m3}-\sigma^{(k)}_{2m}\left(\sum_{n=1,n\neq k}^{3}F^{(kn)}_{m3}+F^{(k)rad}_{m3}\right)\right];$$

$$\frac{d\sigma^{(k)}_{14}}{ds_k}=\frac{e_k}{m_kc^2}\left[\left(\sum_{n=1,n\neq k}^{3}F^{(kn)}_{1\,m}+F^{(k)rad}_{1\,m}\right)\sigma^{(k)}_{m4}-\sigma^{(k)}_{1m}\left(\sum_{n=1,n\neq k}^{3}F^{(kn)}_{m4}+F^{(k)rad}_{m4}\right)\right];$$

$$\frac{d\sigma^{(k)}_{24}}{ds_k}=\frac{e_k}{m_kc^2}\left[\left(\sum_{n=1,n\neq k}^{3}F^{(kn)}_{2m}+F^{(k)rad}_{2\,m}\right)\sigma^{(k)}_{m4}-\sigma^{(k)}_{2m}\left(\sum_{n=1,n\neq k}^{3}F^{(kn)}_{m4}+F^{(k)rad}_{m4}\right)\right];$$

$$\frac{d\sigma^{(k)}_{34}}{ds_k}=\frac{e_k}{m_kc^2}\left[\left(\sum_{n=1,n\neq k}^{3}F^{(kn)}_{3m}+F^{(k)rad}_{3m}\right)\sigma^{(k)}_{m4}-\sigma^{(k)}_{3m}\left(\sum_{n=1,n\neq k}^{3}F^{(kn)}_{m4}+F^{(k)rad}_{m4}\right)\right]$$

or

$$\frac{d\sigma^{(k)}_{12}}{ds_k}=\frac{e_k}{m_kc^2}\sum_{n=1,n\neq k}^{3}\left(F^{(kn)}_{1\,m}\sigma^{(k)}_{m2}-F^{(kn)}_{m2}\sigma^{(k)}_{1\,m}\right)+\frac{e_k}{m_kc^2}\left(F^{(k)rad}_{1m}\sigma^{(k)}_{m2}-F^{(k)rad}_{m2}\sigma^{(k)}_{1\,m}\right); \tag{3.1}$$

$$\frac{d\sigma^{(k)}_{13}}{ds_k}=\frac{e_k}{m_kc^2}\sum_{n=1,n\neq k}^{3}\left(F^{(kn)}_{1\,m}\sigma^{(k)}_{m3}-F^{(kn)}_{m3}\sigma^{(k)}_{1m}\right)+\frac{e_k}{m_kc^2}\left(F^{(k)rad}_{1\,m}\sigma^{(k)}_{m3}-F^{(k)rad}_{m3}\sigma^{(k)}_{1m}\right); \tag{3.2}$$

$$\frac{d\sigma^{(k)}_{23}}{ds_k}=\frac{e_k}{m_kc^2}\sum_{n=1,n\neq k}^{3}\left(F^{(kn)}_{2\,m}\sigma^{(k)}_{m3}-F^{(kn)}_{m3}\sigma^{(k)}_{2m}\right)+\frac{e_k}{m_kc^2}\left(F^{(k)rad}_{2\,m}\sigma^{(k)}_{m3}-F^{(k)rad}_{m3}\sigma^{(k)}_{2m}\right); \tag{3.3}$$

$$\frac{d\sigma^{(k)}_{14}}{ds_k}=\frac{e_k}{m_kc^2}\sum_{n=1,n\neq k}^{3}\left(F^{(kn)}_{1\,m}\sigma^{(k)}_{m4}-F^{(kn)}_{m4}\sigma^{(k)}_{1m}\right)+\frac{e_k}{m_kc^2}\left(F^{(k)rad}_{1\,m}\sigma^{(k)}_{m4}-F^{(k)rad}_{m4}\sigma^{(k)}_{1m}\right); \tag{3.4}$$

$$\frac{d\sigma_{24}^{(k)}}{ds_k}=\frac{e_k}{m_kc^2}\sum_{n=1,n\neq k}^{3}\left(F_{2m}^{(kn)}\sigma_{m4}^{(k)}-F_{m4}^{(kn)}\sigma_{2m}^{(k)}\right)+\frac{e_k}{m_kc^2}\left(F_{2\,m}^{(k)rad}\sigma_{m4}^{(k)}-F_{m4}^{(k)rad}\sigma_{2m}^{(k)}\right); \tag{3.5}$$

$$\frac{d\sigma_{34}^{(k)}}{ds_k}=\frac{e_k}{m_kc^2}\sum_{n=1,n\neq k}^{3}\left(F_{3m}^{(kn)}\sigma_{m4}^{(k)}-F_{m4}^{(kn)}\sigma_{3m}^{(k)}\right)+\frac{e_k}{m_kc^2}\left(F_{3m}^{(k)rad}\sigma_{m4}^{(k)}-F_{m4}^{(k)rad}\sigma_{3m}^{(k)}\right). \tag{3.6}$$

4. Vector Form of the Lorentz and Radiation Parts of the First Three Spin Equations

The primary goal of the present section is to transform equations (3.1) - (3.6) and to obtain a vector form of the spin equations. Recall that $F_{i\,m}^{(kn)}$ are the Lorentz parts while $F_{i\,m}^{(k)rad}$ – the radiation parts.

Let us consider the cross products with respect to some base $\vec{e}_1$, $\vec{e}_2$, $\vec{e}_3$:

$$\vec{\sigma}^{(k)}\times\vec{\xi}^{(k)}=\left(\begin{vmatrix}\sigma_2^{(k)} & \sigma_3^{(k)}\\ \xi_2^{(kn)} & \xi_3^{(kn)}\end{vmatrix},\ -\begin{vmatrix}\sigma_1^{(k)} & \sigma_3^{(k)}\\ \xi_1^{(kn)} & \xi_3^{(kn)}\end{vmatrix},\ \begin{vmatrix}\sigma_1^{(k)} & \sigma_2^{(k)}\\ \xi_1^{(kn)} & \xi_2^{(kn)}\end{vmatrix}\right);$$

$$\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)}=\left(\begin{vmatrix}\lambda_2^{(k)} & \lambda_3^{(k)}\\ \sigma_2^{(k)} & \sigma_3^{(k)}\end{vmatrix},\ -\begin{vmatrix}\lambda_1^{(k)} & \lambda_3^{(k)}\\ \sigma_1^{(k)} & \sigma_3^{(k)}\end{vmatrix},\ \begin{vmatrix}\lambda_1^{(k)} & \lambda_2^{(k)}\\ \sigma_1^{(k)} & \sigma_2^{(k)}\end{vmatrix}\right);$$

$$\vec{\sigma}^{(k)}\times\vec{P}^{(kn)}=\left(\begin{vmatrix}\sigma_2^{(k)} & \sigma_3^{(k)}\\ P_2^{(kn)} & P_3^{(kn)}\end{vmatrix},\ -\begin{vmatrix}\sigma_1^{(k)} & \sigma_3^{(k)}\\ P_1^{(kn)} & P_3^{(kn)}\end{vmatrix},\ \begin{vmatrix}\sigma_1^{(k)} & \sigma_2^{(k)}\\ P_1^{(kn)} & P_2^{(kn)}\end{vmatrix}\right)$$

and

$$\vec{P}^{(kn)}\times\left(\vec{\sigma}^{(k)}\times\vec{\xi}^{(kn)}\right)=\begin{vmatrix}\vec{e}_1 & \vec{e}_2 & \vec{e}_3\\ P_1{}^{(kn)} & P_2{}^{(kn)} & P_3{}^{(kn)}\\ \begin{vmatrix}\sigma_2^{(k)} & \sigma_3^{(k)}\\ \xi_2^{(kn)} & \xi_3^{(kn)}\end{vmatrix} & -\begin{vmatrix}\sigma_1^{(k)} & \sigma_3^{(k)}\\ \xi_1^{(kn)} & \xi_3^{(kn)}\end{vmatrix} & \begin{vmatrix}\sigma_1^{(k)} & \sigma_2^{(k)}\\ \xi_1^{(kn)} & \xi_2^{(kn)}\end{vmatrix}\end{vmatrix}=$$

$$\begin{aligned}&=\left(P_2{}^{(kn)}\begin{vmatrix}\sigma_1^{(k)} & \sigma_2^{(k)}\\ \xi_1^{(kn)} & \xi_2^{(kn)}\end{vmatrix}+P_3{}^{(kn)}\begin{vmatrix}\sigma_1^{(k)} & \sigma_3^{(k)}\\ \xi_1^{(kn)} & \xi_3^{(kn)}\end{vmatrix}\right)\vec{e}_1-\\ &-\left(P_1{}^{(kn)}\begin{vmatrix}\sigma_1^{(k)} & \sigma_2^{(k)}\\ \xi_1^{(kn)} & \xi_2^{(kn)}\end{vmatrix}-P_3{}^{(kn)}\begin{vmatrix}\sigma_2^{(k)} & \sigma_3^{(k)}\\ \xi_2^{(kn)} & \xi_3^{(kn)}\end{vmatrix}\right)\vec{e}_2+\\ &+\left(-P_1{}^{(kn)}\begin{vmatrix}\sigma_1^{(k)} & \sigma_3^{(k)}\\ \xi_1^{(kn)} & \xi_3^{(kn)}\end{vmatrix}-P_2{}^{(kn)}\begin{vmatrix}\sigma_2^{(k)} & \sigma_3^{(k)}\\ \xi_2^{(kn)} & \xi_3^{(kn)}\end{vmatrix}\right)\vec{e}_3\,.\end{aligned}$$

We infer

$$P_2{}^{(kn)}\begin{vmatrix}\sigma_1^{(k)} & \sigma_2^{(k)}\\ \xi_1^{(kn)} & \xi_2^{(kn)}\end{vmatrix}+P_3{}^{(kn)}\begin{vmatrix}\sigma_1^{(k)} & \sigma_3^{(k)}\\ \xi_1^{(kn)} & \xi_3^{(kn)}\end{vmatrix}=\left[\vec{P}^{(kn)}\times\left(\vec{\sigma}^{(k)}\times\vec{\xi}^{(k)}\right)\right]_1;$$

$$-P_1{}^{(kn)}\begin{vmatrix}\sigma_1^{(k)} & \sigma_2^{(k)}\\ \xi_1^{(kn)} & \xi_2^{(kn)}\end{vmatrix}+P_3{}^{(kn)}\begin{vmatrix}\sigma_2^{(k)} & \sigma_3^{(k)}\\ \xi_2^{(kn)} & \xi_3^{(kn)}\end{vmatrix}=\left[\vec{P}^{(kn)}\times\left(\vec{\sigma}^{(k)}\times\vec{\xi}^{(k)}\right)\right]_2;$$

$$-P_1^{(kn)}\begin{vmatrix}\sigma_1^{(k)} & \sigma_3^{(k)}\\ \xi_1^{(kn)} & \xi_3^{(kn)}\end{vmatrix}-P_2^{(kn)}\begin{vmatrix}\sigma_2^{(k)} & \sigma_3^{(k)}\\ \xi_2^{(kn)} & \xi_3^{(kn)}\end{vmatrix}=\left[\vec{P}^{(kn)}\times\left(\vec{\sigma}^{(k)}\times\vec{\xi}^{(k)}\right)\right]_3$$

where the right subscripts mean the number of the coordinate. In the same way

$$\vec{P}^{(kn)}\times\left(\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)}\right)=\begin{vmatrix}\vec{e}_1 & \vec{e}_2 & \vec{e}_3\\ P_1^{(kn)} & P_2^{(kn)} & P_3^{(kn)}\\ \begin{vmatrix}\lambda_2^{(k)} & \lambda_3^{(k)}\\ \sigma_2^{(k)} & \sigma_3^{(k)}\end{vmatrix} & -\begin{vmatrix}\lambda_1^{(k)} & \lambda_3^{(k)}\\ \sigma_1^{(k)} & \sigma_3^{(k)}\end{vmatrix} & \begin{vmatrix}\lambda_1^{(k)} & \lambda_2^{(k)}\\ \sigma_1^{(k)} & \sigma_2^{(k)}\end{vmatrix}\end{vmatrix}=$$

$$= \left(P_2^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix} + P_3^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{vmatrix} \right) \vec{e}_1 - \left(P_1^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix} - P_3^{(kn)} \begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} \right) \vec{e}_2 +$$
$$+ \left(-P_1^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{vmatrix} - P_2^{(kn)} \begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} \right) \vec{e}_3$$

that means

$$P_2^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix} + P_3^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{vmatrix} = \left[\vec{P}^{(kn)} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \right]_1 ;$$

$$-P_1^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix} + P_3^{(kn)} \begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} = \left[\vec{P}^{(kn)} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \right]_2 ;$$

$$-P_1^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{vmatrix} - P_2^{(kn)} \begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} = \left[\vec{P}^{(kn)} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \right]_3 .$$

For the next term

$$\vec{\xi}^{(kn)} \times \left(\vec{\sigma}^{(k)} \times \vec{P}^{(kn)} \right) = \begin{vmatrix} \vec{e}_1 & \vec{e}_2 & \vec{e}_3 \\ \xi_1{}^{(kn)} & \xi_2{}^{(kn)} & \xi_3{}^{(kn)} \\ \begin{vmatrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ P_2^{(kn)} & P_3^{(kn)} \end{vmatrix} & -\begin{vmatrix} \sigma_1^{(k)} & \sigma_3^{(k)} \\ P_1^{(kn)} & P_3^{(kn)} \end{vmatrix} & \begin{vmatrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ P_1^{(kn)} & P_2^{(kn)} \end{vmatrix} \end{vmatrix} =$$

$$= \left(\xi_2{}^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ P_1^{(kn)} & P_2^{(kn)} \end{vmatrix} + \xi_3{}^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_3^{(k)} \\ P_1^{(kn)} & P_3^{(kn)} \end{vmatrix} \right) \vec{e}_1 -$$
$$- \left(\xi_1{}^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ P_1^{(kn)} & P_2^{(kn)} \end{vmatrix} - \xi_3{}^{(kn)} \begin{vmatrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ P_2^{(kn)} & P_3^{(kn)} \end{vmatrix} \right) \vec{e}_2 +$$
$$+ \left(-\xi_1{}^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_3^{(k)} \\ P_1^{(kn)} & P_3^{(kn)} \end{vmatrix} - \xi_2{}^{(kn)} \begin{vmatrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ P_2^{(kn)} & P_3^{(kn)} \end{vmatrix} \right) \vec{e}_3 .$$

Therefore

$$\xi_2{}^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ P_1^{(kn)} & P_2^{(kn)} \end{vmatrix} + \xi_3{}^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_3^{(k)} \\ P_1^{(kn)} & P_3^{(kn)} \end{vmatrix} = \left[\vec{\xi}^{(kn)} \times \left(\vec{\sigma}^{(k)} \times \vec{P}^{(k)} \right) \right]_1 ;$$

$$-\xi_1{}^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ P_1^{(kn)} & P_2^{(kn)} \end{vmatrix} + \xi_3{}^{(kn)} \begin{vmatrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ P_2^{(kn)} & P_3^{(kn)} \end{vmatrix} = \left[\vec{\xi}^{(kn)} \times \left(\vec{\sigma}^{(k)} \times \vec{P}^{(k)} \right) \right]_2 ;$$

$$-\xi_1^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_3^{(k)} \\ P_1^{(kn)} & P_3^{(kn)} \end{vmatrix} - \xi_2^{(kn)} \begin{vmatrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ P_2^{(kn)} & P_3^{(kn)} \end{vmatrix} = \left[\vec{\xi}^{(kn)} \times \left(\vec{\sigma}^{(k)} \times \vec{P}^{(k)} \right) \right]_3$$

and finally

$$\vec{\xi}^{(kn)} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) = \begin{vmatrix} \vec{e}_1 & \vec{e}_2 & \vec{e}_3 \\ \xi_1{}^{(kn)} & \xi_2{}^{(kn)} & \xi_3{}^{(kn)} \\ \begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} & -\begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{vmatrix} & \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix} \end{vmatrix} =$$

$$= \left(\xi_2{}^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix} + \xi_3{}^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{vmatrix} \right) \vec{e}_1 - \left(\xi_1{}^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix} - \xi_3{}^{(kn)} \begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} \right) \vec{e}_2 +$$
$$+ \left(-\xi_1{}^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{vmatrix} - \xi_2{}^{(kn)} \begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} \right) \vec{e}_3$$

that is,

$$\xi_2^{(kn)}\begin{vmatrix}\lambda_1^{(k)} & \lambda_2^{(k)}\\ \sigma_1^{(k)} & \sigma_2^{(k)}\end{vmatrix}+\xi_3^{(kn)}\begin{vmatrix}\lambda_1^{(k)} & \lambda_3^{(k)}\\ \sigma_1^{(k)} & \sigma_3^{(k)}\end{vmatrix}=\left[\vec{\xi}^{(kn)}\times\left(\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)}\right)\right]_1;$$

$$-\xi_1^{(kn)}\begin{vmatrix}\lambda_1^{(k)} & \lambda_2^{(k)}\\ \sigma_1^{(k)} & \sigma_2^{(k)}\end{vmatrix}+\xi_3^{(kn)}\begin{vmatrix}\lambda_2^{(k)} & \lambda_3^{(k)}\\ \sigma_2^{(k)} & \sigma_3^{(k)}\end{vmatrix}=\left[\vec{\xi}^{(kn)}\times\left(\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)}\right)\right]_2;$$

$$-\xi_1^{(kn)}\begin{vmatrix}\lambda_1^{(k)} & \lambda_3^{(k)}\\ \sigma_1^{(k)} & \sigma_3^{(k)}\end{vmatrix}-\xi_2^{(kn)}\begin{vmatrix}\lambda_2^{(k)} & \lambda_3^{(k)}\\ \sigma_2^{(k)} & \sigma_3^{(k)}\end{vmatrix}=\left[\vec{\xi}^{(kn)}\times\left(\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)}\right)\right]_3.$$

Transformation of the Lorentz Part of Equation (3.1)

First, we transform the Lorentz part, because the radiation parts can be transformed in the same way. So instead of

$$\frac{d\sigma_{12}^{(k)}}{ds_k}=\frac{e_k}{m_kc^2}\sum_{n=1,n\neq k}^{3}\left(F_{1\,m}^{(kn)}\sigma_{m2}^{(k)}-\sigma_{1\,m}^{(k)}F_{m2}^{(kn)}\right)+\frac{e_k}{m_kc^2}\left(F_{1\,m}^{(k)rad}\sigma_{m2}^{(k)}-\sigma_{1\,m}^{(k)}F_{m2}^{(k)rad}\right)$$

we consider only

$$\frac{d\sigma_{12}^{(k)}}{ds_k}=\sum_{n=1,n\neq k}^{3}\frac{e_ke_n}{m_kc^2}\left[\sigma_{m2}^{(k)}\left(P_1^{(kn)}\xi_m^{(kn)}-P_m^{(kn)}\xi_1^{(kn)}\right)-\sigma_{1\,m}^{(k)}\left(P_m^{(kn)}\xi_2^{(kn)}-P_2^{(kn)}\xi_m^{(kn)}\right)\right]=$$

$$=\sum_{n=1,n\neq k}^{3}\frac{e_ke_n}{m_kc^2}\left(P_1^{(kn)}\sigma_{m2}^{(k)}\xi_m^{(kn)}-\xi_1^{(kn)}\sigma_{m2}^{(k)}P_m^{(kn)}-\xi_2^{(kn)}\sigma_{1\,m}^{(k)}P_m^{(kn)}+P_2^{(kn)}\sigma_{1\,m}^{(k)}\xi_m^{(kn)}\right).$$

But

$$P_1^{(kn)}\sigma_{m2}^{(k)}\xi_m^{(kn)}=P_1^{(kn)}\left(\sigma_{12}^{(k)}\xi_1^{(kn)}+\sigma_{32}^{(k)}\xi_3^{(kn)}+\sigma_{42}^{(k)}\xi_4^{(kn)}\right)=P_1^{(kn)}\left(\sigma_3^{(k)}\xi_1^{(kn)}-\sigma_1^{(k)}\xi_3^{(kn)}+\sigma_{42}^{(k)}\xi_4^{(kn)}\right)=$$
$$=-P_1^{(kn)}\begin{vmatrix}\sigma_1^{(k)} & \sigma_3^{(k)}\\ \xi_1^{(kn)} & \xi_3^{(kn)}\end{vmatrix}-P_1^{(kn)}\tau_{kn}\begin{vmatrix}\lambda_1^{(k)} & \lambda_3^{(k)}\\ \sigma_1^{(k)} & \sigma_3^{(k)}\end{vmatrix};$$

$$-\xi_1^{(kn)}\sigma_{m2}^{(k)}P_m^{(kn)}=-\xi_1^{(kn)}\left(\sigma_{1\,2}^{(k)}P_1^{(kn)}+\sigma_{32}^{(k)}P_3^{(kn)}+\sigma_{42}^{(k)}P_4^{(kn)}\right)$$
$$=-\xi_1^{(kn)}\left(\sigma_3^{(k)}P_1^{(kn)}-\sigma_1^{(k)}P_3^{(kn)}+\sigma_{42}^{(k)}P_4^{(kn)}\right)=\xi_1^{(kn)}\begin{vmatrix}\sigma_1^{(k)} & \sigma_3^{(k)}\\ P_1^{(kn)} & P_3^{(kn)}\end{vmatrix}-\xi_1^{(kn)}L_{kn}\begin{vmatrix}\lambda_1^{(k)} & \lambda_3^{(k)}\\ \sigma_1^{(k)} & \sigma_3^{(k)}\end{vmatrix};$$

$$-\xi_2^{(kn)}\sigma_{1m}^{(k)}P_m^{(kn)}=-\xi_2^{(kn)}\left(\sigma_{1\,2}^{(k)}P_2^{(kn)}+\sigma_{13}^{(k)}P_3^{(kn)}+\sigma_{14}^{(k)}P_4^{(kn)}\right)=$$
$$=-\xi_2^{(kn)}\left(\sigma_3^{(k)}P_2^{(kn)}-\sigma_2^{(k)}P_3^{(kn)}+\sigma_{14}^{(k)}P_4^{(kn)}\right)=\xi_2^{(kn)}\begin{vmatrix}\sigma_2^{(k)} & \sigma_3^{(k)}\\ P_2^{(kn)} & P_3^{(kn)}\end{vmatrix}-\xi_2^{(kn)}L_{kn}\begin{vmatrix}\lambda_2^{(k)} & \lambda_3^{(k)}\\ \sigma_2^{(k)} & \sigma_3^{(k)}\end{vmatrix};$$

$$P_2^{(kn)}\sigma_{1m}^{(k)}\xi_m^{(kn)}=P_2^{(kn)}\left(\sigma_{12}^{(k)}\xi_2^{(kn)}+\sigma_{13}^{(k)}\xi_3^{(kn)}+\sigma_{14}^{(k)}\xi_4^{(kn)}\right)=P_2^{(kn)}\left(\sigma_3^{(k)}\xi_2^{(kn)}-\sigma_2^{(k)}\xi_3^{(kn)}+\sigma_{14}^{(k)}\xi_4^{(kn)}\right)=$$
$$=-P_2^{(kn)}\begin{vmatrix}\sigma_2^{(k)} & \sigma_3^{(k)}\\ \xi_2^{(kn)} & \xi_3^{(kn)}\end{vmatrix}-\tau_{kn}P_2^{(kn)}\begin{vmatrix}\lambda_2^{(k)} & \lambda_3^{(k)}\\ \sigma_2^{(k)} & \sigma_3^{(k)}\end{vmatrix}$$

and then in view of $\frac{d\sigma_{12}^{(k)}}{ds_k}=\frac{d\sigma_3^{(k)}}{ds_k}$ we obtain

$$\frac{d\sigma_3^{(k)}}{ds_k}=\sum_{n=1,n\neq k}^{3}\frac{e_ke_n}{m_kc^2}\left(-P_1^{(kn)}\begin{vmatrix}\sigma_1^{(k)} & \sigma_3^{(k)}\\ \xi_1^{(kn)} & \xi_3^{(kn)}\end{vmatrix}-\tau_{kn}P_1^{(kn)}\begin{vmatrix}\lambda_1^{(k)} & \lambda_3^{(k)}\\ \sigma_1^{(k)} & \sigma_3^{(k)}\end{vmatrix}+\xi_1^{(kn)}\begin{vmatrix}\sigma_1^{(k)} & \sigma_3^{(k)}\\ P_1^{(kn)} & P_3^{(kn)}\end{vmatrix}-\right.$$
$$-L_{kn}\xi_1^{(kn)}\begin{vmatrix}\lambda_1^{(k)} & \lambda_3^{(k)}\\ \sigma_1^{(k)} & \sigma_3^{(k)}\end{vmatrix}+\xi_2^{(kn)}\begin{vmatrix}\sigma_2^{(k)} & \sigma_3^{(k)}\\ P_2^{(kn)} & P_3^{(kn)}\end{vmatrix}-L_{kn}\xi_2^{(kn)}\begin{vmatrix}\lambda_2^{(k)} & \lambda_3^{(k)}\\ \sigma_2^{(k)} & \sigma_3^{(k)}\end{vmatrix}-$$
$$\left.-P_2^{(kn)}\begin{vmatrix}\sigma_2^{(k)} & \sigma_3^{(k)}\\ \xi_2^{(kn)} & \xi_3^{(kn)}\end{vmatrix}-\tau_{kn}P_2^{(kn)}\begin{vmatrix}\lambda_2^{(k)} & \lambda_3^{(k)}\\ \sigma_2^{(k)} & \sigma_3^{(k)}\end{vmatrix}\right)$$

or

$$\frac{d\sigma_3^{(k)}}{ds_k}=\sum_{n=1,n\neq k}^{3}\frac{e_ke_n}{m_kc^2}\left(-P_1^{(kn)}\begin{vmatrix}\sigma_1^{(k)} & \sigma_3^{(k)}\\ \xi_1^{(kn)} & \xi_3^{(kn)}\end{vmatrix}-P_2^{(kn)}\begin{vmatrix}\sigma_2^{(k)} & \sigma_3^{(k)}\\ \xi_2^{(kn)} & \xi_3^{(kn)}\end{vmatrix}-\tau_{kn}P_1^{(kn)}\begin{vmatrix}\lambda_1^{(k)} & \lambda_3^{(k)}\\ \sigma_1^{(k)} & \sigma_3^{(k)}\end{vmatrix}-\right.$$
$$-\tau_{kn}P_2^{(kn)}\begin{vmatrix}\lambda_2^{(k)} & \lambda_3^{(k)}\\ \sigma_2^{(k)} & \sigma_3^{(k)}\end{vmatrix}+\xi_1^{(kn)}\begin{vmatrix}\sigma_1^{(k)} & \sigma_3^{(k)}\\ P_1^{(kn)} & P_3^{(kn)}\end{vmatrix}+\xi_2^{(kn)}\begin{vmatrix}\sigma_2^{(k)} & \sigma_3^{(k)}\\ P_2^{(kn)} & P_3^{(kn)}\end{vmatrix}-L_{kn}\xi_1^{(kn)}\begin{vmatrix}\lambda_1^{(k)} & \lambda_3^{(k)}\\ \sigma_1^{(k)} & \sigma_3^{(k)}\end{vmatrix}-$$
$$\left.-L_{kn}\xi_2^{(kn)}\begin{vmatrix}\lambda_2^{(k)} & \lambda_3^{(k)}\\ \sigma_2^{(k)} & \sigma_3^{(k)}\end{vmatrix}\right)$$

and consequently

$$\frac{d\sigma_3^{(k)}}{ds_k}=$$
$$\sum_{n=1,n\neq k}^{3}\frac{e_ke_n}{m_kc^2}\left\{\left[\vec{P}^{(kn)}\times\left(\vec{\sigma}^{(k)}\times\vec{\xi}^{(kn)}\right)\right]_3+\tau_{kn}\left[\vec{P}^{(kn)}\times\left(\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)}\right)\right]_3-\left[\vec{\xi}^{(kn)}\times\left(\vec{\sigma}^{(k)}\times\vec{P}^{(kn)}\right)\right]_3+\right.$$
$$\left.+L_{kn}\left[\vec{\xi}^{(kn)}\times\left(\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)}\right)\right]_3\right\}.$$

Transformation of the Lorentz Part of Equation (3.2)

Here

$$\frac{d\sigma_{13}^{(k)}}{ds_k}=\frac{e_k}{m_kc^2}\sum_{n=1,n\neq k}^{3}\left(F_{1\,m}^{(kn)}\sigma_{m3}^{(k)}-\sigma_{1m}^{(k)}F_{m3}^{(kn)}\right)+\frac{e_k}{m_kc^2}\left(F_{1\,m}^{(k)rad}\sigma_{m3}^{(k)}-\sigma_{1m}^{(k)}F_{m3}^{(k)rad}\right);$$

$$\begin{aligned}\frac{d\sigma_{13}^{(k)}}{ds_k}&=\frac{e_k}{m_kc^2}\sum_{n=1,n\neq k}^{3}\left(F_{1\,m}^{(kn)}\sigma_{m3}^{(k)}-\sigma_{1\,m}^{(k)}F_{m3}^{(kn)}\right)\\&=\sum_{n=1,n\neq k}^{3}\frac{e_ke_n}{m_kc^2}\left[\sigma_{m3}^{(k)}\left(P_1^{(kn)}\xi_m^{(kn)}-P_m^{(kn)}\xi_1^{(kn)}\right)-\sigma_{1\,m}^{(k)}\left(P_m^{(kn)}\xi_3^{(kn)}-P_3^{(kn)}\xi_m^{(kn)}\right)\right]\\&=\sum_{n=1,n\neq k}^{3}\frac{e_ke_n}{m_kc^2}\left(P_1^{(kn)}\sigma_{m3}^{(k)}\xi_m^{(kn)}-\sigma_{m3}^{(k)}P_m^{(kn)}\xi_1^{(kn)}-\sigma_{1\,m}^{(k)}P_m^{(kn)}\xi_3^{(kn)}+P_3^{(kn)}\sigma_{1\,m}^{(k)}\xi_m^{(kn)}\right);\end{aligned}$$

$$\begin{aligned}P_1^{(kn)}\sigma_{m3}^{(k)}\xi_m^{(kn)}&=P_1^{(kn)}\left(\sigma_{13}^{(k)}\xi_1^{(kn)}+\sigma_{23}^{(k)}\xi_2^{(kn)}+\sigma_{43}^{(k)}\xi_4^{(kn)}\right)=P_1^{(kn)}\left(-\sigma_2^{(k)}\xi_1^{(kn)}+\sigma_1^{(k)}\xi_2^{(kn)}+\sigma_{43}^{(k)}\xi_4^{(kn)}\right)\\&=P_1^{(kn)}\begin{vmatrix}\sigma_1^{(k)} & \sigma_2^{(k)}\\ \xi_1^{(kn)} & \xi_2^{(kn)}\end{vmatrix}+\tau_{kn}P_1^{(kn)}\begin{vmatrix}\lambda_1^{(k)} & \lambda_2^{(k)}\\ \sigma_1^{(k)} & \sigma_2^{(k)}\end{vmatrix};\end{aligned}$$

$$\begin{aligned}-\xi_1^{(kn)}\sigma_{m3}^{(k)}P_m^{(kn)}&=-\xi_1^{(kn)}\left(\sigma_{13}^{(k)}P_1^{(kn)}+\sigma_{23}^{(k)}P_2^{(kn)}+\sigma_{43}^{(k)}P_4^{(kn)}\right)=-\xi_1^{(kn)}\left(-\sigma_2^{(k)}P_1^{(kn)}+\sigma_1^{(k)}P_2^{(kn)}+\sigma_{43}^{(k)}P_4^{(kn)}\right)\\&=-\xi_1^{(kn)}\begin{vmatrix}\sigma_1^{(k)} & \sigma_2^{(k)}\\ P_1^{(kn)} & P_2^{(kn)}\end{vmatrix}+L_{kn}\xi_1^{(kn)}\begin{vmatrix}\lambda_1^{(k)} & \lambda_2^{(k)}\\ \sigma_1^{(k)} & \sigma_2^{(k)}\end{vmatrix};\end{aligned}$$

$$-\xi_3^{(kn)}\sigma_{1\,m}^{(k)}P_m^{(kn)}=-\xi_3^{(kn)}\left(\sigma_{1\,2}^{(k)}P_2^{(kn)}+\sigma_{1\,3}^{(k)}P_3^{(kn)}+\sigma_{1\,4}^{(k)}P_4^{(kn)}\right)=-\xi_3^{(kn)}\left(\sigma_3^{(k)}P_2^{(kn)}-\sigma_2^{(k)}P_3^{(kn)}+\sigma_{1\,4}^{(k)}P_4^{(kn)}\right)$$

$$= \xi_3^{(kn)} \begin{vmatrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ P_2^{(kn)} & P_3^{(kn)} \end{vmatrix} - L_{kn}\xi_3^{(kn)} \begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix};$$

$$P_3^{(kn)}\sigma_{1\,m}^{(k)}\xi_m^{(kn)} = P_3^{(kn)}\left(\sigma_{1\,2}^{(k)}\xi_2^{(kn)} + \sigma_{1\,3}^{(k)}\xi_3^{(kn)} + \sigma_{1\,4}^{(k)}\xi_4^{(kn)}\right) = P_3^{(kn)}\left(\sigma_3^{(k)}\xi_2^{(kn)} - \sigma_2^{(k)}\xi_3^{(kn)} + \sigma_{1\,4}^{(k)}\xi_4^{(kn)}\right) =$$

$$= -P_3^{(kn)} \begin{vmatrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ \xi_2^{(kn)} & \xi_3^{(kn)} \end{vmatrix} - \tau_{kn}P_3^{(kn)} \begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix}.$$

Then

$$\sum_{n=1,n\neq k}^{3}\left(F_{1\,m}^{(kn)}\sigma_{m3}^{(k)} - \sigma_{1\,m}^{(k)}F_{m3}^{(kn)}\right) =$$

$$= \sum_{n=1,n\neq k}^{3} e_n \left(P_1^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ \xi_1^{(kn)} & \xi_2^{(kn)} \end{vmatrix} + \tau_{kn}P_1^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix} - \xi_1^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ P_1^{(kn)} & P_2^{(kn)} \end{vmatrix} + \right.$$
$$+L_{kn}\xi_1^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix} + +\xi_3^{(kn)} \begin{vmatrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ P_2^{(kn)} & P_3^{(kn)} \end{vmatrix} - L_{kn}\xi_3^{(kn)} \begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} - P_3^{(kn)} \begin{vmatrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ \xi_2^{(kn)} & \xi_3^{(kn)} \end{vmatrix} -$$
$$\left. -\tau_{kn}P_3^{(kn)} \begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} \right).$$

In view of $\frac{d\sigma_{13}^{(k)}}{ds_k} = -\frac{d\sigma_2^{(k)}}{ds_k}$ we have

$$-\frac{d\sigma_2^{(k)}}{ds_k} = \sum_{n=1,n\neq k}^{3} \frac{e_k e_n}{m_k c^2}\left(P_1^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ \xi_1^{(kn)} & \xi_2^{(kn)} \end{vmatrix} - P_3^{(kn)} \begin{vmatrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ \xi_2^{(kn)} & \xi_3^{(kn)} \end{vmatrix} + \tau_{kn}P_1^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix} - \right.$$
$$-\tau_{kn}P_3^{(kn)} \begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} - \xi_1^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ P_1^{(kn)} & P_2^{(kn)} \end{vmatrix} + \xi_3^{(kn)} \begin{vmatrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ P_2^{(kn)} & P_3^{(kn)} \end{vmatrix} +$$
$$\left. +L_{kn}\xi_1^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix} - L_{kn}\xi_3^{(kn)} \begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} \right).$$

Therefore

$$P_1^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ \xi_1^{(kn)} & \xi_2^{(kn)} \end{vmatrix} - P_3^{(kn)} \begin{vmatrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ \xi_2^{(kn)} & \xi_3^{(kn)} \end{vmatrix} = -\left[\vec{P}^{(kn)} \times \left(\vec{\sigma}^{(k)} \times \vec{\xi}^{(k)}\right)\right]_2;$$

$$\tau_{kn}P_1^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix} - \tau_{kn}P_3^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix} = -\tau_{kn}\left[\vec{P}^{(kn)} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)}\right)\right]_2;$$

$$\xi_1^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ P_1^{(kn)} & P_2^{(kn)} \end{vmatrix} - \xi_3^{(kn)} \begin{vmatrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ P_2^{(kn)} & P_3^{(kn)} \end{vmatrix} = \left[\vec{\xi}^{(k)} \times \left(\vec{\sigma}^{(k)} \times \vec{P}^{(kn)}\right)\right]_2;$$

$$L_{kn}\xi_1^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix} - L_{kn}\xi_3^{(kn)} \begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} = -L_{kn}\left[\vec{\xi}^{(k)} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)}\right)\right]_2$$

that is,

$$-\frac{d\sigma_2^{(k)}}{ds_k} = \sum_{n=1,n\neq k}^{3} \frac{e_k e_n}{m_k c^2}\left\{-\left[\vec{P}^{(kn)} \times \left(\vec{\sigma}^{(k)} \times \vec{\xi}^{(k)}\right)\right]_2 - \tau_{kn}\left[\vec{P}^{(kn)} \times \left(\vec{\sigma}^{(k)} \times \vec{\lambda}^{(k)}\right)\right]_2 + \right.$$
$$\left. + \left[\vec{\xi}^{(k)} \times \left(\vec{\sigma}^{(k)} \times \vec{P}^{(kn)}\right)\right]_2 - L_{kn}L_{kn}\left[\vec{\xi}^{(k)} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)}\right)\right]_2 \right\}$$

or

$$\frac{d\sigma_2^{(k)}}{ds_k}=\sum_{n=1,n\neq k}^{3}\frac{e_ke_n}{m_kc^2}\left\{\left[\vec{P}^{(kn)}\times\left(\vec{\sigma}^{(k)}\times\vec{\xi}^{(kn)}\right)\right]_2+\tau_{kn}\tau_{kn}\left[\vec{P}^{(kn)}\times\left(\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)}\right)\right]_2-\right.$$
$$\left.-\left[\vec{\xi}^{(kn)}\times\left(\vec{\sigma}^{(k)}\times\vec{P}^{(kn)}\right)\right]_2+L_{kn}L_{kn}\left[\vec{\xi}^{(kn)}\times\left(\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)}\right)\right]_2\right\}.$$

Transformation of the Lorentz part of equation (3.3)

For the Lorentz part of the third equation

$$\frac{d\sigma_{23}^{(k)}}{ds_k}=\frac{e_k}{m_kc^2}\sum_{n=1,n\neq k}^{3}\left(F_{2\,m}^{(kn)}\sigma_{m3}^{(k)}-F_{m3}^{(kn)}\sigma_{2m}^{(k)}\right)+\frac{e_k}{m_kc^2}\left(F_{2\,m}^{(k)rad}\sigma_{m3}^{(k)}-F_{m3}^{(k)rad}\sigma_{2m}^{(k)}\right)$$

we have

$$\sum_{n=1,n\neq k}^{3}\left(F_{2\,m}^{(kn)}\sigma_{m3}^{(k)}-F_{m3}^{(kn)}\sigma_{2m}^{(k)}\right)=\sum_{n=1,n\neq k}^{3}e_n\left(\left(P_2^{(kn)}\xi_m^{(kn)}-P_m^{(kn)}\xi_2^{(kn)}\right)\sigma_{m3}^{(k)}-\right.$$
$$\left.-\left(P_m^{(kn)}\xi_3^{(kn)}-P_3^{(kn)}\xi_m^{(kn)}\right)\sigma_{2m}^{(k)}\right)$$
$$=\sum_{n=1,n\neq k}^{3}e_n\left(P_2^{(kn)}\sigma_{m3}^{(k)}\xi_m^{(kn)}-\sigma_{m3}^{(k)}P_m^{(kn)}\xi_2^{(kn)}-P_m^{(kn)}\sigma_{2m}^{(k)}\xi_3^{(kn)}+P_3^{(kn)}\sigma_{2m}^{(k)}\xi_m^{(kn)}\right);$$

$$P_2^{(kn)}\sigma_{m3}^{(k)}\xi_m^{(kn)}=P_2^{(kn)}\left(\sigma_{13}^{(k)}\xi_1^{(kn)}+\sigma_{23}^{(k)}\xi_2^{(kn)}+\sigma_{43}^{(k)}\xi_4^{(kn)}\right)=P_2^{(kn)}\left(-\sigma_2^{(k)}\xi_1^{(kn)}+\sigma_1^{(k)}\xi_2^{(kn)}+\sigma_{43}^{(k)}\xi_4^{(kn)}\right)$$
$$=P_2^{(kn)}\begin{vmatrix}\sigma_1^{(k)}&\sigma_2^{(k)}\\\xi_1^{(kn)}&\xi_2^{(kn)}\end{vmatrix}+\tau_{kn}P_2^{(kn)}\begin{vmatrix}\lambda_1^{(k)}&\lambda_2^{(k)}\\\sigma_1^{(k)}&\sigma_2^{(k)}\end{vmatrix};$$

$$-\xi_2^{(kn)}\sigma_{m3}^{(k)}P_m^{(kn)}=-\xi_2^{(kn)}\left(\sigma_{13}^{(k)}P_1^{(kn)}+\sigma_{23}^{(k)}P_2^{(kn)}+\sigma_{43}^{(k)}P_4^{(kn)}\right)=-\xi_2^{(kn)}\left(-\sigma_2^{(k)}P_1^{(kn)}+\sigma_1^{(k)}P_2^{(kn)}+\sigma_{43}^{(k)}P_4^{(kn)}\right)$$
$$=-\xi_2^{(kn)}\begin{vmatrix}\sigma_1^{(k)}&\sigma_2^{(k)}\\P_1^{(kn)}&P_2^{(kn)}\end{vmatrix}+L_{kn}\xi_2^{(kn)}\begin{vmatrix}\lambda_1^{(k)}&\lambda_2^{(k)}\\\sigma_1^{(k)}&\sigma_2^{(k)}\end{vmatrix};$$

$$-\xi_3^{(kn)}P_m^{(kn)}\sigma_{2m}^{(k)}=-\xi_3^{(kn)}\left(P_1^{(kn)}\sigma_{21}^{(k)}+P_3^{(kn)}\sigma_{23}^{(k)}+P_4^{(kn)}\sigma_{24}^{(k)}\right)=-\xi_3^{(kn)}\left(-P_1^{(kn)}\sigma_3^{(k)}+P_3^{(kn)}\sigma_1^{(k)}+P_4^{(kn)}\sigma_{24}^{(k)}\right)$$
$$=-\xi_3^{(kn)}\begin{vmatrix}\sigma_1^{(k)}&\sigma_3^{(k)}\\P_1^{(kn)}&P_3^{(kn)}\end{vmatrix}+L_{kn}\xi_3^{(kn)}\begin{vmatrix}\lambda_1^{(k)}&\lambda_3^{(k)}\\\sigma_1^{(k)}&\sigma_3^{(k)}\end{vmatrix};$$

$$P_3^{(kn)}\sigma_{2m}^{(k)}\xi_m^{(kn)}=P_3^{(kn)}\left(\sigma_{21}^{(k)}\xi_1^{(kn)}+\sigma_{23}^{(k)}\xi_3^{(kn)}+\sigma_{24}^{(k)}\xi_4^{(kn)}\right)=P_3^{(kn)}\left(-\sigma_3^{(k)}\xi_1^{(kn)}+\sigma_1^{(k)}\xi_3^{(kn)}+\sigma_{24}^{(k)}\xi_4^{(kn)}\right)$$
$$=P_3^{(kn)}\begin{vmatrix}\sigma_1^{(k)}&\sigma_3^{(k)}\\\xi_1^{(kn)}&\xi_3^{(kn)}\end{vmatrix}+\tau_{kn}P_3^{(kn)}\begin{vmatrix}\lambda_1^{(k)}&\lambda_3^{(k)}\\\sigma_1^{(k)}&\sigma_3^{(k)}\end{vmatrix}.$$

Further on we have

$$\sum_{n=1,n\neq k}^{3}\left(F_{2\,m}^{(kn)}\sigma_{m3}^{(k)}-F_{m3}^{(kn)}\sigma_{2m}^{(k)}\right)=e_n\left(P_2^{(kn)}\sigma_{m3}^{(k)}\xi_m^{(kn)}-\sigma_{m3}^{(k)}P_m^{(kn)}\xi_2^{(kn)}-P_m^{(kn)}\sigma_{2m}^{(k)}\xi_3^{(kn)}+P_3^{(kn)}\sigma_{2m}^{(k)}\xi_m^{(kn)}\right)=$$

$$=e_n\left(P_2^{(kn)}\begin{vmatrix}\sigma_1^{(k)}&\sigma_2^{(k)}\\\xi_1^{(kn)}&\xi_2^{(kn)}\end{vmatrix}+\tau_{kn}P_2^{(kn)}\begin{vmatrix}\lambda_1^{(k)}&\lambda_2^{(k)}\\\sigma_1^{(k)}&\sigma_2^{(k)}\end{vmatrix}-\xi_2^{(kn)}\begin{vmatrix}\sigma_1^{(k)}&\sigma_2^{(k)}\\P_1^{(kn)}&P_2^{(kn)}\end{vmatrix}+L_{kn}\xi_2^{(kn)}\begin{vmatrix}\lambda_1^{(k)}&\lambda_2^{(k)}\\\sigma_1^{(k)}&\sigma_2^{(k)}\end{vmatrix}-\right.$$
$$\left.-\xi_3^{(kn)}\begin{vmatrix}\sigma_1^{(k)}&\sigma_3^{(k)}\\P_1^{(kn)}&P_3^{(kn)}\end{vmatrix}+L_{kn}\xi_3^{(kn)}\begin{vmatrix}\lambda_1^{(k)}&\lambda_3^{(k)}\\\sigma_1^{(k)}&\sigma_3^{(k)}\end{vmatrix}+P_3^{(kn)}\begin{vmatrix}\sigma_1^{(k)}&\sigma_3^{(k)}\\\xi_1^{(kn)}&\xi_3^{(kn)}\end{vmatrix}-\tau_{kn}P_3^{(kn)}\begin{vmatrix}\lambda_1^{(k)}&\lambda_3^{(k)}\\\sigma_1^{(k)}&\sigma_3^{(k)}\end{vmatrix}\right).$$

Then

$$\frac{d\sigma_1^{(k)}}{ds_k}=\sum_{\substack{n=1,\\ n\neq k}}^{3}\frac{e_ke_n}{m_kc^2}\left(P_2^{(kn)}\begin{vmatrix}\sigma_1^{(k)} & \sigma_2^{(k)}\\ \xi_1^{(kn)} & \xi_2^{(kn)}\end{vmatrix}+P_3^{(kn)}\begin{vmatrix}\sigma_1^{(k)} & \sigma_3^{(k)}\\ \xi_1^{(kn)} & \xi_3^{(kn)}\end{vmatrix}+\tau_{kn}P_2^{(kn)}\begin{vmatrix}\lambda_1^{(k)} & \lambda_2^{(k)}\\ \sigma_1^{(k)} & \sigma_2^{(k)}\end{vmatrix}+\right.$$
$$+\tau_{kn}P_3^{(kn)}\begin{vmatrix}\lambda_1^{(k)} & \lambda_3^{(k)}\\ \sigma_1^{(k)} & \sigma_3^{(k)}\end{vmatrix}-\xi_2^{(kn)}\begin{vmatrix}\sigma_1^{(k)} & \sigma_2^{(k)}\\ P_1^{(kn)} & P_2^{(kn)}\end{vmatrix}-\xi_3^{(kn)}\begin{vmatrix}\sigma_1^{(k)} & \sigma_3^{(k)}\\ P_1^{(kn)} & P_3^{(kn)}\end{vmatrix}+$$
$$\left.+L_{kn}\xi_2^{(kn)}\begin{vmatrix}\lambda_1^{(k)} & \lambda_2^{(k)}\\ \sigma_1^{(k)} & \sigma_2^{(k)}\end{vmatrix}+L_{kn}\xi_3^{(kn)}\begin{vmatrix}\lambda_1^{(k)} & \lambda_3^{(k)}\\ \sigma_1^{(k)} & \sigma_3^{(k)}\end{vmatrix}\right).$$

But

$$P_2^{(kn)}\begin{vmatrix}\sigma_1^{(k)} & \sigma_2^{(k)}\\ \xi_1^{(kn)} & \xi_2^{(kn)}\end{vmatrix}+P_3^{(kn)}\begin{vmatrix}\sigma_1^{(k)} & \sigma_3^{(k)}\\ \xi_1^{(kn)} & \xi_3^{(kn)}\end{vmatrix}=\left[\vec{P}^{(kn)}\times(\vec{\sigma}^{(k)}\times\vec{\xi}^{(kn)})\right]_1;$$

$$\tau_{kn}P_2^{(kn)}\begin{vmatrix}\lambda_1^{(k)} & \lambda_2^{(k)}\\ \sigma_1^{(k)} & \sigma_2^{(k)}\end{vmatrix}+\tau_{kn}P_3^{(kn)}\begin{vmatrix}\lambda_1^{(k)} & \lambda_3^{(k)}\\ \sigma_1^{(k)} & \sigma_3^{(k)}\end{vmatrix}=\tau_{kn}\left[\vec{P}^{(kn)}\times(\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)})\right]_1;$$

$$-\xi_2^{(kn)}\begin{vmatrix}\sigma_1^{(k)} & \sigma_2^{(k)}\\ P_1^{(kn)} & P_2^{(kn)}\end{vmatrix}-\xi_3^{(kn)}\begin{vmatrix}\sigma_1^{(k)} & \sigma_3^{(k)}\\ P_1^{(kn)} & P_3^{(kn)}\end{vmatrix}=-\left[\vec{\xi}^{(kn)}\times(\vec{\sigma}^{(k)}\times\vec{P}^{(kn)})\right]_1;$$

$$L_{kn}\xi_2^{(kn)}\begin{vmatrix}\lambda_1^{(k)} & \lambda_2^{(k)}\\ \sigma_1^{(k)} & \sigma_2^{(k)}\end{vmatrix}+L_{kn}\xi_3^{(kn)}\begin{vmatrix}\lambda_1^{(k)} & \lambda_3^{(k)}\\ \sigma_1^{(k)} & \sigma_3^{(k)}\end{vmatrix}=L_{kn}\left[\vec{\xi}^{(kn)}\times(\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)})\right]_1$$

and then

$$\frac{d\sigma_1^{(k)}}{ds_k}=\sum_{n=1,n\neq k}^{3}\frac{e_ke_n}{m_kc^2}\left\{\left[\vec{P}^{(kn)}\times(\vec{\sigma}^{(k)}\times\vec{\xi}^{(kn)})\right]_1+\tau_{kn}\left[\vec{P}^{(kn)}\times(\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)})\right]_1-\right.$$
$$\left.-\left[\vec{\xi}^{(kn)}\times(\vec{\sigma}^{(k)}\times\vec{P}^{(kn)})\right]_1+L_{kn}\left[\vec{\xi}^{(kn)}\times(\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)})\right]_1\right\}.$$

Finally, we rewrite the Lorentz parts of the first three spin equation in a vector form

$$\frac{d\vec{\sigma}^{(k)}}{ds_k}=\sum_{n=1,n\neq k}^{3}\frac{e_ke_n}{m_kc^2}\left[\vec{P}^{(kn)}\times\left(\vec{\sigma}^{(k)}\times\vec{\xi}^{(kn)}\right)+\tau_{kn}\vec{P}^{(kn)}\times\left(\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)}\right)-\vec{\xi}^{(kn)}\times\left(\vec{\sigma}^{(k)}\times\vec{P}^{(kn)}\right)+\right.$$
$$\left.+L_{kn}\vec{\xi}^{(kn)}\times\left(\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)}\right)\right].$$

Having in mind that

$$\frac{e_k}{m_kc^2}\left(F_{1m}^{(k)rad}\sigma_{m2}^{(k)}-\sigma_{1m}^{(k)}F_{m2}^{(k)rad}\right)=\frac{{e_k}^2}{2m_kc^2}$$

$$\left\{\left[\vec{P}^{(k)ret}\times\left(\vec{\sigma}^{(k)}\times\vec{\xi}^{(k)ret}\right)\right]_3+{\tau_k}^{ret}\left[\vec{P}^{(k)ret}\times\left(\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)}\right)\right]_3-\left[\vec{\xi}^{(k)ret}\times\left(\vec{\sigma}^{(k)}\times\vec{P}^{(k)ret}\right)\right]_3+\right.$$
$$\left.+L_{k,ret}\left[\vec{\xi}^{(k)ret}\times\left(\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)}\right)\right]_3\right\}-$$
$$-\left\{\left[\vec{P}^{(k)adv}\times\left(\vec{\sigma}^{(k)}\times\vec{\xi}^{(k)adv}\right)\right]_3+{\tau_k}^{adv}\left[\vec{P}^{(k)adv}\times\left(\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)}\right)\right]_3-\left[\vec{\xi}^{(k)adv}\times\left(\vec{\sigma}^{(k)}\times\vec{P}^{(k)adv}\right)\right]_3+\right.$$
$$\left.+L_{k,adv}\left[\vec{\xi}^{(k)adv}\times\left(\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)}\right)\right]_3\right\};$$

$$\frac{e_k}{m_k c^2}\left(F_{1m}^{(k)rad}\sigma_{m3}^{(k)} - \sigma_{1m}^{(k)}F_{m3}^{(k)rad}\right) = \frac{{e_k}^2}{2m_k c^2}$$

$$\left\{\left[\vec{P}^{(k)ret}\times\left(\vec{\sigma}^{(k)}\times\vec{\xi}^{(k)ret}\right)\right]_2 + {\tau_k}^{ret}\left[\vec{P}^{(k)ret}\times\left(\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)}\right)\right]_2 - \left[\vec{\xi}^{(k)ret}\times\left(\vec{\sigma}^{(k)}\times\vec{P}^{(k)ret}\right)\right]_2 + \right.$$
$$\left. + L_{k,ret}\left[\vec{\xi}^{(k)ret}\times\left(\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)}\right)\right]_2\right\} - \left\{\left[\vec{P}^{(k)adv}\times\left(\vec{\sigma}^{(k)}\times\vec{\xi}^{(k)adv}\right)\right]_2 + {\tau_k}^{adv}\left[\vec{P}^{(k)adv}\times\left(\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)}\right)\right]_2 - \right.$$
$$\left. - \left[\vec{\xi}^{(k)adv}\times\left(\vec{\sigma}^{(k)}\times\vec{P}^{(k)adv}\right)\right]_2 + L_{k,adv}\left[\vec{\xi}^{(k)adv}\times\left(\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)}\right)\right]_2\right\};$$

$$\frac{e_k}{m_k c^2}\left(F_{2m}^{(k)rad}\sigma_{m3}^{(k)} - F_{m3}^{(k)rad}\sigma_{2m}^{(k)}\right) = \frac{{e_k}^2}{2m_k c^2}$$

$$\left\{\left[\vec{P}^{(k)ret}\times\left(\vec{\sigma}^{(k)}\times\vec{\xi}^{(k)ret}\right)\right]_1 + {\tau_k}^{ret}\left[\vec{P}^{(k)ret}\times\left(\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)}\right)\right]_1 - \left[\vec{\xi}^{(k)ret}\times\left(\vec{\sigma}^{(k)}\times\vec{P}^{(k)ret}\right)\right]_1 + \right.$$
$$\left. + L_{k,ret}\left[\vec{\xi}^{(k)ret}\times\left(\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)}\right)\right]_1\right\} -$$
$$-\left\{\left[\vec{P}^{(k)adv}\times\left(\vec{\sigma}^{(k)}\times\vec{\xi}^{(k)adv}\right)\right]_1 + {\tau_k}^{adv}\left[\vec{P}^{(k)adv}\times\left(\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)}\right)\right]_1 - \left[\vec{\xi}^{(k)adv}\times\left(\vec{\sigma}^{(k)}\times\vec{P}^{(k)adv}\right)\right]_1 + \right.$$
$$\left. + L_{k,adv}\left[\vec{\xi}^{(k)adv}\times\left(\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)}\right)\right]_1\right\}$$

we obtain

$$\frac{d\vec{\sigma}^{(k)}}{ds_k} = \sum_{n=1,n\neq k}^{3}\frac{e_k e_n}{m_k c^2}\left[\vec{P}^{(kn)}\times\left(\vec{\sigma}^{(k)}\times\vec{\xi}^{(kn)}\right) + \tau_{kn}\vec{P}^{(kn)}\times\left(\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)}\right) - \vec{\xi}^{(kn)}\times\left(\vec{\sigma}^{(k)}\times\vec{P}^{(kn)}\right) + \right.$$
$$\left. + L_{kn}\vec{\xi}^{(kn)}\times\left(\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)}\right)\right] + \frac{{e_k}^2}{2m_k c^2}\left\{\left[\vec{P}^{(k)ret}\times\left(\vec{\sigma}^{(k)}\times\vec{\xi}^{(k)ret}\right) + {\tau_k}^{ret}\vec{P}^{(k)ret}\times\left(\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)}\right) - \right.\right.$$
$$\left. - \vec{\xi}^{(k)ret}\times\left(\vec{\sigma}^{(k)}\times\vec{P}^{(k)ret}\right) + L_{k,ret}\vec{\xi}^{(k)ret}\times\left(\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)}\right)\right] -$$
$$-\left[\vec{P}^{(k)adv}\times\left(\vec{\sigma}^{(k)}\times\vec{\xi}^{(k)adv}\right) + {\tau_k}^{adv}\vec{P}^{(k)adv}\times\left(\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)}\right) - \vec{\xi}^{(k)adv}\times\left(\vec{\sigma}^{(k)}\times\vec{P}^{(k)adv}\right) + \right.$$
$$\left.\left. + L_{k,adv}\vec{\xi}^{(k)adv}\times\left(\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)}\right)\right]\right\}.$$

Transformation of the spin equation (3.4)

$$\frac{d\sigma_{14}^{(k)}}{ds_k} = \frac{e_k}{m_k c^2}\left[\sum_{n=1,n\neq k}^{3}\left(F_{1m}^{(kn)}\sigma_{m4}^{(k)} - F_{m4}^{(kn)}\sigma_{1m}^{(k)}\right) + F_{1m}^{(k)rad}\sigma_{m4}^{(k)} - F_{m4}^{(k)rad}\sigma_{1m}^{(k)}\right];$$

$$F_{1m}^{(kn)}\sigma_{m4}^{(k)} - F_{m4}^{(kn)}\sigma_{1m}^{(k)} = e_n\left(P_1^{(kn)}\xi_m^{(kn)}\sigma_{m4}^{(k)} - P_m^{(kn)}\sigma_{m4}^{(k)}\xi_1^{(kn)} - P_m^{(kn)}\sigma_{1m}^{(k)}\xi_4^{(kn)} + P_4^{(kn)}\sigma_{1m}^{(k)}\xi_m^{(kn)}\right);$$

$$\begin{aligned} P_1^{(kn)}\xi_m^{(kn)}\sigma_{m4}^{(k)} &= P_1^{(kn)}\left(\xi_1^{(kn)}\sigma_{14}^{(k)} + \xi_2^{(kn)}\sigma_{24}^{(k)} + \xi_3^{(kn)}\sigma_{34}^{(k)}\right) \\ &= P_1^{(kn)}\left(\xi_1^{(kn)}\frac{i}{c}\begin{vmatrix}\lambda_2^{(k)} & \lambda_3^{(k)}\\ \sigma_2^{(k)} & \sigma_3^{(k)}\end{vmatrix} - \xi_2^{(kn)}\frac{i}{c}\begin{vmatrix}\lambda_1^{(k)} & \lambda_3^{(k)}\\ \sigma_1^{(k)} & \sigma_3^{(k)}\end{vmatrix} + \xi_3^{(kn)}\frac{i}{c}\begin{vmatrix}\lambda_1^{(k)} & \lambda_2^{(k)}\\ \sigma_1^{(k)} & \sigma_2^{(k)}\end{vmatrix}\right) \\ &= \frac{iP_1^{(kn)}}{c}\begin{vmatrix}\xi_1^{(kn)} & \xi_2^{(kn)} & \xi_3^{(kn)}\\ \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)}\\ \sigma_1^{(k)} & \sigma_2^{(k)} & \sigma_3^{(k)}\end{vmatrix};\end{aligned}$$

$$
\begin{aligned}
-\xi_1^{(kn)}P_m^{(kn)}\sigma_{m4}^{(k)} &= -\xi_1^{(kn)}\left(P_1^{(kn)}\sigma_{14}^{(k)}+P_2^{(kn)}\sigma_{24}^{(k)}+P_3^{(kn)}\sigma_{34}^{(k)}\right)\\
&= -\xi_1^{(kn)}\left(\frac{iP_1^{(kn)}}{c}\begin{vmatrix}\lambda_2^{(k)} & \lambda_3^{(k)}\\ \sigma_2^{(k)} & \sigma_3^{(k)}\end{vmatrix}-\frac{iP_2^{(kn)}}{c}\begin{vmatrix}\lambda_1^{(k)} & \lambda_3^{(k)}\\ \sigma_1^{(k)} & \sigma_3^{(k)}\end{vmatrix}+\frac{iP_3^{(kn)}}{c}\begin{vmatrix}\lambda_1^{(k)} & \lambda_2^{(k)}\\ \sigma_1^{(k)} & \sigma_2^{(k)}\end{vmatrix}\right)\\
&= -\frac{i\xi_1^{(kn)}}{c}\begin{vmatrix}P_1^{(kn)} & P_2^{(kn)} & P_3^{(kn)}\\ \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)}\\ \sigma_1^{(k)} & \sigma_2^{(k)} & \sigma_3^{(k)}\end{vmatrix};
\end{aligned}
$$

$$
\begin{aligned}
-\xi_4^{(kn)}P_m^{(kn)}\sigma_{1m}^{(k)} &= -\xi_4^{(kn)}\left(P_2^{(kn)}\sigma_{12}^{(k)}+P_3^{(kn)}\sigma_{13}^{(k)}+P_4^{(kn)}\sigma_{14}^{(k)}\right)\\
&= -\xi_4^{(kn)}\left(P_2^{(kn)}\sigma_3^{(k)}-P_3^{(kn)}\sigma_2^{(k)}+P_4^{(kn)}\sigma_{14}^{(k)}\right)\\
&= -\xi_4^{(kn)}\left(-\begin{vmatrix}\sigma_2^{(k)} & \sigma_3^{(k)}\\ P_2^{(kn)} & P_3^{(kn)}\end{vmatrix}+\frac{iP_4^{(kn)}}{c}\begin{vmatrix}\lambda_2^{(k)} & \lambda_3^{(k)}\\ \sigma_2^{(k)} & \sigma_3^{(k)}\end{vmatrix}\right)\\
&= ic\tau_{kn}\begin{vmatrix}\sigma_2^{(k)} & \sigma_3^{(k)}\\ P_2^{(kn)} & P_3^{(kn)}\end{vmatrix}-ic\tau_{kn}L_{kn}\begin{vmatrix}\lambda_2^{(k)} & \lambda_3^{(k)}\\ \sigma_2^{(k)} & \sigma_3^{(k)}\end{vmatrix};
\end{aligned}
$$

$$
\begin{aligned}
P_4^{(kn)}\sigma_{1m}^{(k)}\xi_m^{(kn)} &= P_4^{(kn)}\left(\sigma_{12}^{(k)}\xi_2^{(kn)}+\sigma_{13}^{(k)}\xi_3^{(kn)}+\sigma_{14}^{(k)}\xi_4^{(kn)}\right)\\
&= P_4^{(kn)}\left(\sigma_3^{(k)}\xi_2^{(kn)}-\sigma_2^{(k)}\xi_3^{(kn)}+\sigma_{14}^{(k)}\xi_4^{(kn)}\right)\\
&= -P_4^{(kn)}\begin{vmatrix}\sigma_2^{(k)} & \sigma_3^{(k)}\\ \xi_2^{(kn)} & \xi_3^{(kn)}\end{vmatrix}+P_4^{(kn)}ic\tau_{kn}\frac{i}{c}\begin{vmatrix}\lambda_2^{(k)} & \lambda_3^{(k)}\\ \sigma_2^{(k)} & \sigma_3^{(k)}\end{vmatrix}\\
&= icL_{kn}\begin{vmatrix}\sigma_2^{(k)} & \sigma_3^{(k)}\\ \xi_2^{(kn)} & \xi_3^{(kn)}\end{vmatrix}+icL_{kn}\tau_{kn}\begin{vmatrix}\lambda_2^{(k)} & \lambda_3^{(k)}\\ \sigma_2^{(k)} & \sigma_3^{(k)}\end{vmatrix}.
\end{aligned}
$$

Then

$$
\begin{aligned}
F_{1\,m}^{(kn)}\sigma_{m4}^{(k)}-F_{m4}^{(kn)}\sigma_{1m}^{(k)} &= e_n\left(\frac{iP_1^{(kn)}}{c}\begin{vmatrix}\xi_1^{(kn)} & \xi_2^{(kn)} & \xi_3^{(kn)}\\ \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)}\\ \sigma_1^{(k)} & \sigma_2^{(k)} & \sigma_3^{(k)}\end{vmatrix}-\frac{i\xi_1^{(kn)}}{c}\begin{vmatrix}P_1^{(kn)} & P_2^{(kn)} & P_3^{(kn)}\\ \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)}\\ \sigma_1^{(k)} & \sigma_2^{(k)} & \sigma_3^{(k)}\end{vmatrix}+\right.\\
&\quad +ic\tau_{kn}\begin{vmatrix}\sigma_2^{(k)} & \sigma_3^{(k)}\\ P_2^{(kn)} & P_3^{(kn)}\end{vmatrix}-ic\tau_{kn}L_{kn}\begin{vmatrix}\lambda_2^{(k)} & \lambda_3^{(k)}\\ \sigma_2^{(k)} & \sigma_3^{(k)}\end{vmatrix}+icL_{kn}\begin{vmatrix}\sigma_2^{(k)} & \sigma_3^{(k)}\\ \xi_2^{(kn)} & \xi_3^{(kn)}\end{vmatrix}\\
&\quad \left.+icL_{kn}\tau_{kn}\begin{vmatrix}\lambda_2^{(k)} & \lambda_3^{(k)}\\ \sigma_2^{(k)} & \sigma_3^{(k)}\end{vmatrix}\right)\\
&= e_n\left(\frac{iP_1^{(kn)}}{c}\begin{vmatrix}\xi_1^{(kn)} & \xi_2^{(kn)} & \xi_3^{(kn)}\\ \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)}\\ \sigma_1^{(k)} & \sigma_2^{(k)} & \sigma_3^{(k)}\end{vmatrix}-\frac{i\xi_1^{(kn)}}{c}\begin{vmatrix}P_1^{(kn)} & P_2^{(kn)} & P_3^{(kn)}\\ \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)}\\ \sigma_1^{(k)} & \sigma_2^{(k)} & \sigma_3^{(k)}\end{vmatrix}\right.\\
&\quad \left.+ic\tau_{kn}\begin{vmatrix}\sigma_2^{(k)} & \sigma_3^{(k)}\\ P_2^{(kn)} & P_3^{(kn)}\end{vmatrix}+icL_{kn}\begin{vmatrix}\sigma_2^{(k)} & \sigma_3^{(k)}\\ \xi_2^{(kn)} & \xi_3^{(kn)}\end{vmatrix}\right)
\end{aligned}
$$

or

$$\frac{i}{c}\frac{d(\vec{\lambda}\times\vec{\sigma})_1}{ds_k}=$$

$$=\sum_{n=1,n\neq k}^{3}\frac{e_ke_n}{m_kc^2}\left(\frac{iP_1^{(kn)}}{c}\begin{vmatrix}\xi_1^{(kn)} & \xi_2^{(kn)} & \xi_3^{(kn)}\\ \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)}\\ \sigma_1^{(k)} & \sigma_2^{(k)} & \sigma_3^{(k)}\end{vmatrix}-\frac{i\xi_1^{(kn)}}{c}\begin{vmatrix}P_1^{(kn)} & P_2^{(kn)} & P_3^{(kn)}\\ \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)}\\ \sigma_1^{(k)} & \sigma_2^{(k)} & \sigma_3^{(k)}\end{vmatrix}+\right.$$
$$\left.+ic\tau_{kn}\begin{vmatrix}\sigma_2^{(k)} & \sigma_3^{(k)}\\ P_2^{(kn)} & P_3^{(kn)}\end{vmatrix}+icL_{kn}\begin{vmatrix}\sigma_2^{(k)} & \sigma_3^{(k)}\\ \xi_2^{(kn)} & \xi_3^{(kn)}\end{vmatrix}\right)$$

Transformation of the spin equation (3.5)

$$\frac{d\sigma_{24}^{(k)}}{ds_k}=\frac{e_k}{m_kc^2}\left[\sum_{n=1,n\neq k}^{3}\left(F_{2m}^{(kn)}\sigma_{m4}^{(k)}-F_{m4}^{(kn)}\sigma_{2m}^{(k)}\right)+F_{2\,m}^{(k)rad}\sigma_{m4}^{(k)}-F_{m4}^{(k)rad}\sigma_{2m}^{(k)}\right];$$

$$F_{2m}^{(kn)}\sigma_{m4}^{(k)}-F_{m4}^{(kn)}\sigma_{2m}^{(k)}=e_n\left(P_2^{(kn)}\xi_m^{(kn)}\sigma_{m4}^{(k)}-P_m^{(kn)}\sigma_{m4}^{(k)}\xi_2^{(kn)}-P_m^{(kn)}\sigma_{2m}^{(k)}\xi_4^{(kn)}+P_4^{(kn)}\xi_m^{(kn)}\sigma_{2m}^{(k)}\right);$$

$$\begin{aligned}P_2^{(kn)}\xi_m^{(kn)}\sigma_{m4}^{(k)}&=P_2^{(kn)}\left(\xi_1^{(kn)}\sigma_{14}^{(k)}+\xi_2^{(kn)}\sigma_{24}^{(k)}+\xi_3^{(kn)}\sigma_{34}^{(k)}\right)\\&=P_2^{(kn)}\left(\frac{i\xi_1^{(kn)}}{c}\begin{vmatrix}\lambda_2^{(k)} & \lambda_3^{(k)}\\ \sigma_2^{(k)} & \sigma_3^{(k)}\end{vmatrix}-\frac{i\xi_2^{(kn)}}{c}\begin{vmatrix}\lambda_1^{(k)} & \lambda_3^{(k)}\\ \sigma_1^{(k)} & \sigma_3^{(k)}\end{vmatrix}+\frac{i\xi_3^{(kn)}}{c}\begin{vmatrix}\lambda_1^{(k)} & \lambda_2^{(k)}\\ \sigma_1^{(k)} & \sigma_2^{(k)}\end{vmatrix}\right)\\&=\frac{iP_2^{(kn)}}{c}\begin{vmatrix}\xi_1^{(kn)} & \xi_2^{(kn)} & \xi_3^{(kn)}\\ \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)}\\ \sigma_1^{(k)} & \sigma_2^{(k)} & \sigma_3^{(k)}\end{vmatrix};\end{aligned}$$

$$\begin{aligned}\xi_2^{(kn)}P_m^{(kn)}\sigma_{m4}^{(k)}&=\xi_2^{(kn)}\left(P_1^{(kn)}\sigma_{14}^{(k)}+P_2^{(kn)}\sigma_{24}^{(k)}+P_3^{(kn)}\sigma_{34}^{(k)}\right)\\&=\xi_2^{(kn)}\left(\frac{iP_1^{(kn)}}{c}\begin{vmatrix}\sigma_2^{(k)} & \sigma_3^{(k)}\\ \lambda_2^{(k)} & \lambda_3^{(k)}\end{vmatrix}-\frac{iP_2^{(kn)}}{c}\begin{vmatrix}\sigma_1^{(k)} & \sigma_3^{(k)}\\ \lambda_1^{(k)} & \lambda_3^{(k)}\end{vmatrix}+\frac{iP_3^{(kn)}}{c}\begin{vmatrix}\sigma_1^{(k)} & \sigma_2^{(k)}\\ \lambda_1^{(k)} & \lambda_2^{(k)}\end{vmatrix}\right)\\&=\frac{i}{c}\xi_2^{(kn)}\begin{vmatrix}P_1^{(kn)} & P_2^{(kn)} & P_3^{(kn)}\\ \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)}\\ \sigma_1^{(k)} & \sigma_2^{(k)} & \sigma_3^{(k)}\end{vmatrix};\end{aligned}$$

$$\begin{aligned}\xi_4^{(kn)}P_m^{(kn)}\sigma_{2m}^{(k)}&=\xi_4^{(kn)}\left(P_1^{(kn)}\sigma_{21}^{(k)}+P_3^{(kn)}\sigma_{23}^{(k)}+P_4^{(kn)}\sigma_{24}^{(k)}\right)\\&=\xi_4^{(kn)}\left(\begin{vmatrix}\sigma_1^{(k)} & \sigma_3^{(k)}\\ P_1^{(kn)} & P_3^{(kn)}\end{vmatrix}-\frac{iP_4^{(kn)}}{c}\begin{vmatrix}\lambda_1^{(k)} & \lambda_3^{(k)}\\ \sigma_1^{(k)} & \sigma_3^{(k)}\end{vmatrix}\right)\\&=\xi_4^{(kn)}\left(\begin{vmatrix}\sigma_1^{(k)} & \sigma_3^{(k)}\\ P_1^{(kn)} & P_3^{(kn)}\end{vmatrix}-L_{kn}\begin{vmatrix}\lambda_1^{(k)} & \lambda_3^{(k)}\\ \sigma_1^{(k)} & \sigma_3^{(k)}\end{vmatrix}\right);\end{aligned}$$

$$\begin{aligned}P_4^{(kn)}\xi_m^{(kn)}\sigma_{2m}^{(k)}&=P_4^{(kn)}\left(\xi_1^{(kn)}\sigma_{21}^{(k)}+\xi_3^{(kn)}\sigma_{23}^{(k)}+\xi_4^{(kn)}\sigma_{24}^{(k)}\right)\\&=P_4^{(kn)}\left(\begin{vmatrix}\sigma_1^{(k)} & \sigma_3^{(k)}\\ \xi_1^{(kn)} & \xi_3^{(kn)}\end{vmatrix}-\frac{i}{c}\xi_4^{(kn)}\begin{vmatrix}\lambda_1^{(k)} & \lambda_3^{(k)}\\ \sigma_1^{(k)} & \sigma_3^{(k)}\end{vmatrix}\right)\\&=P_4^{(kn)}\left(\begin{vmatrix}\sigma_1^{(k)} & \sigma_3^{(k)}\\ \xi_1^{(kn)} & \xi_3^{(kn)}\end{vmatrix}+\tau_{kn}\begin{vmatrix}\lambda_1^{(k)} & \lambda_3^{(k)}\\ \sigma_1^{(k)} & \sigma_3^{(k)}\end{vmatrix}\right);\end{aligned}$$

$$\begin{aligned} F^{(kn)}_{2m}\sigma^{(k)}_{m4}-F^{(kn)}_{m4}\sigma^{(k)}_{2m} &= e_n\left(-\frac{iP^{(kn)}_2}{c}\begin{vmatrix}\xi^{(kn)}_1 & \xi^{(kn)}_2 & \xi^{(kn)}_3\\ \lambda^{(k)}_1 & \lambda^{(k)}_2 & \lambda^{(k)}_3\\ \sigma^{(k)}_1 & \sigma^{(k)}_2 & \sigma^{(k)}_3\end{vmatrix}+\frac{i\xi^{(kn)}_2}{c}\begin{vmatrix}P^{(kn)}_1 & P^{(kn)}_2 & P^{(kn)}_3\\ \lambda^{(k)}_1 & \lambda^{(k)}_2 & \lambda^{(k)}_3\\ \sigma^{(k)}_1 & \sigma^{(k)}_2 & \sigma^{(k)}_3\end{vmatrix}-\right.\\ &-ic\tau_{kn}\begin{vmatrix}\sigma^{(k)}_1 & \sigma^{(k)}_3\\ P^{(kn)}_1 & P^{(kn)}_3\end{vmatrix}-ic\tau_{kn}L_{kn}\begin{vmatrix}\lambda^{(k)}_1 & \lambda^{(k)}_3\\ \sigma^{(k)}_1 & \sigma^{(k)}_3\end{vmatrix}-icL_{kn}\begin{vmatrix}\sigma^{(k)}_1 & \sigma^{(k)}_3\\ \xi^{(kn)}_1 & \xi^{(kn)}_3\end{vmatrix}+\\ &\left.+ic\tau_{kn}L_{kn}\begin{vmatrix}\lambda^{(k)}_1 & \lambda^{(k)}_3\\ \sigma^{(k)}_1 & \sigma^{(k)}_3\end{vmatrix}\right)\\ &= e_n\left(-\frac{iP^{(kn)}_2}{c}\begin{vmatrix}\xi^{(kn)}_1 & \xi^{(kn)}_2 & \xi^{(kn)}_3\\ \lambda^{(k)}_1 & \lambda^{(k)}_2 & \lambda^{(k)}_3\\ \sigma^{(k)}_1 & \sigma^{(k)}_2 & \sigma^{(k)}_3\end{vmatrix}+\frac{i\xi^{(kn)}_2}{c}\begin{vmatrix}P^{(kn)}_1 & P^{(kn)}_2 & P^{(kn)}_3\\ \lambda^{(k)}_1 & \lambda^{(k)}_2 & \lambda^{(k)}_3\\ \sigma^{(k)}_1 & \sigma^{(k)}_2 & \sigma^{(k)}_3\end{vmatrix}-\right.\\ &\left.-ic\tau_{kn}\begin{vmatrix}\sigma^{(k)}_1 & \sigma^{(k)}_3\\ P^{(kn)}_1 & P^{(kn)}_3\end{vmatrix}-icL_{kn}\begin{vmatrix}\sigma^{(k)}_1 & \sigma^{(k)}_3\\ \xi^{(kn)}_1 & \xi^{(kn)}_3\end{vmatrix}\right)\end{aligned}$$

or

$$\begin{aligned} -\frac{i}{c}\frac{d}{\Delta_k dt}\begin{vmatrix}\sigma^{(k)}_1 & \sigma^{(k)}_3\\ u^{(k)}_1 & u^{(k)}_3\end{vmatrix} &= \sum_{n=1,n\neq k}^{3}\frac{e_ke_n}{m_kc^2}\left(-\frac{iP^{(kn)}_2}{c}\begin{vmatrix}\xi^{(kn)}_1 & \xi^{(kn)}_2 & \xi^{(kn)}_3\\ \lambda^{(k)}_1 & \lambda^{(k)}_2 & \lambda^{(k)}_3\\ \sigma^{(k)}_1 & \sigma^{(k)}_2 & \sigma^{(k)}_3\end{vmatrix}+\frac{i\xi^{(kn)}_2}{c}\begin{vmatrix}P^{(kn)}_1 & P^{(kn)}_2 & P^{(kn)}_3\\ \lambda^{(k)}_1 & \lambda^{(k)}_2 & \lambda^{(k)}_3\\ \sigma^{(k)}_1 & \sigma^{(k)}_2 & \sigma^{(k)}_3\end{vmatrix}-\right.\\ &\left.-ic\tau_{kn}\begin{vmatrix}\sigma^{(k)}_1 & \sigma^{(k)}_3\\ P^{(kn)}_1 & P^{(kn)}_3\end{vmatrix}-icL_{kn}\begin{vmatrix}\sigma^{(k)}_1 & \sigma^{(k)}_3\\ \xi^{(kn)}_1 & \xi^{(kn)}_3\end{vmatrix}\right);\end{aligned}$$

$$\begin{aligned} \frac{i}{c}\frac{d(\vec{\lambda}\times\vec{\sigma})_2}{ds_k} &= \sum_{n=1,n\neq k}^{3}\frac{e_ke_n}{m_kc^2}\left(\frac{iP^{(kn)}_2}{c}\begin{vmatrix}\xi^{(kn)}_1 & \xi^{(kn)}_2 & \xi^{(kn)}_3\\ \lambda^{(k)}_1 & \lambda^{(k)}_2 & \lambda^{(k)}_3\\ \sigma^{(k)}_1 & \sigma^{(k)}_2 & \sigma^{(k)}_3\end{vmatrix}-\frac{i\xi^{(kn)}_2}{c}\begin{vmatrix}P^{(kn)}_1 & P^{(kn)}_2 & P^{(kn)}_3\\ \lambda^{(k)}_1 & \lambda^{(k)}_2 & \lambda^{(k)}_3\\ \sigma^{(k)}_1 & \sigma^{(k)}_2 & \sigma^{(k)}_3\end{vmatrix}+\right.\\ &\left.+ic\tau_{kn}\begin{vmatrix}\sigma^{(k)}_1 & \sigma^{(k)}_3\\ P^{(kn)}_1 & P^{(kn)}_3\end{vmatrix}+icL_{kn}\begin{vmatrix}\sigma^{(k)}_1 & \sigma^{(k)}_3\\ \xi^{(kn)}_1 & \xi^{(kn)}_3\end{vmatrix}\right).\end{aligned}$$

Transformation of the spin equation (3.6)

$$\frac{d\sigma^{(k)}_{34}}{ds_k}=\frac{e_k}{m_kc^2}\left[\sum_{n=1,n\neq k}^{3}\left(F^{(kn)}_{3m}\sigma^{(k)}_{m4}-F^{(kn)}_{m4}\sigma^{(k)}_{3m}\right)+F^{(k)rad}_{3m}\sigma^{(k)}_{m4}-F^{(k)rad}_{m4}\sigma^{(k)}_{3m}\right];$$

$$F^{(kn)}_{3m}\sigma^{(k)}_{m4}-F^{(kn)}_{m4}\sigma^{(k)}_{3m}= e_n\left(P^{(kn)}_3\xi^{(kn)}_m\sigma^{(k)}_{m4}-P^{(kn)}_m\sigma^{(k)}_{m4}\xi^{(kn)}_3-P^{(kn)}_m\sigma^{(k)}_{3m}\xi^{(kn)}_4+P^{(kn)}_4\xi^{(kn)}_m\sigma^{(k)}_{3m}\right);$$

$$P^{(kn)}_3\xi^{(kn)}_m\sigma^{(k)}_{m4}=P^{(kn)}_3\left(\xi^{(kn)}_1\sigma^{(k)}_{14}+\xi^{(kn)}_2\sigma^{(k)}_{24}+\xi^{(kn)}_3\sigma^{(k)}_{34}\right)=\frac{i}{c}P^{(kn)}_3\begin{vmatrix}\xi^{(kn)}_1 & \xi^{(kn)}_2 & \xi^{(kn)}_3\\ \lambda^{(k)}_1 & \lambda^{(k)}_2 & \lambda^{(k)}_3\\ \sigma^{(k)}_1 & \sigma^{(k)}_2 & \sigma^{(k)}_3\end{vmatrix};$$

$$P^{(kn)}_m\sigma^{(k)}_{m4}\xi^{(kn)}_3=\left(P^{(kn)}_1\sigma^{(k)}_{14}+P^{(kn)}_2\sigma^{(k)}_{24}+P^{(kn)}_3\sigma^{(k)}_{34}\right)\xi^{(kn)}_3=\frac{i}{c}\xi^{(kn)}_3\begin{vmatrix}P^{(kn)}_1 & P^{(kn)}_2 & P^{(kn)}_3\\ \lambda^{(k)}_1 & \lambda^{(k)}_2 & \lambda^{(k)}_3\\ \sigma^{(k)}_1 & \sigma^{(k)}_2 & \sigma^{(k)}_3\end{vmatrix};$$

$$\begin{aligned}
\xi_4^{(kn)} P_m^{(kn)} \sigma_{3m}^{(k)} &= \xi_4^{(kn)} \left(P_1^{(kn)} \sigma_{31}^{(k)} + P_2^{(kn)} \sigma_{32}^{(k)} + P_4^{(kn)} \sigma_{34}^{(k)} \right) \\
&= \xi_4^{(kn)} \left(-ic\tau_{kn} \begin{vmatrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ P_1^{(kn)} & P_2^{(kn)} \end{vmatrix} + ic\tau_{kn} L_{kn} \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix} \right) \\
P_4^{(kn)} \xi_m^{(kn)} \sigma_{3m}^{(k)} &= P_4^{(kn)} \left(\xi_1^{(kn)} \sigma_{31}^{(k)} + \xi_2^{(kn)} \sigma_{32}^{(k)} + \xi_4^{(kn)} \sigma_{34}^{(k)} \right) \\
&= P_4^{(kn)} \left(icL_{kn} \begin{vmatrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ \xi_1^{(kn)} & \xi_2^{(kn)} \end{vmatrix} - ic\tau_{kn} L_{kn} \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix} \right)
\end{aligned}$$

or

$$\begin{aligned}
\tfrac{i}{c} \tfrac{d(\vec{\lambda}\times\vec{\sigma})_3}{ds_k} &= \sum_{n=1,n\neq k}^{3} \frac{e_k e_n}{m_k c^2} \left(\frac{i}{c} P_3^{(kn)} \begin{vmatrix} \xi_1^{(kn)} & \xi_2^{(kn)} & \xi_3^{(kn)} \\ \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} - \frac{i}{c} \xi_3^{(kn)} \begin{vmatrix} P_1^{(kn)} & P_2^{(kn)} & P_3^{(kn)} \\ \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} \right. \\
&\left. + ic\tau_{kn} \begin{vmatrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ P_1^{(kn)} & P_2^{(kn)} \end{vmatrix} + icL_{kn} \begin{vmatrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ \xi_1^{(kn)} & \xi_2^{(kn)} \end{vmatrix} \right) .
\end{aligned}$$

Now we are able to write in a vector form of the last three spin equations with radiation terms.

The last three equations can be written in a vector form

$$\begin{aligned}
\tfrac{d\left(\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)}\right)}{ds_k} &= \sum_{n=1,n\neq k}^{3} \frac{e_k e_n}{m_k c^2} \left(\vec{P}^{(kn)} \left\langle \vec{\xi}^{(kn)} \times \vec{\lambda}^{(k)}, \vec{\sigma}^{(k)} \right\rangle - \vec{\xi}^{(kn)} \left\langle \vec{P}^{(kn)} \times \vec{\lambda}^{(k)}, \vec{\sigma}^{(k)} \right\rangle + \right. \\
&\left. + c^2 \tau_{kn} \left(\vec{\sigma}^{(k)} \times \vec{P}^{(kn)} \right) + c^2 L_{kn} \left(\vec{\sigma}^{(k)} \times \vec{\xi}^{(kn)} \right) \right) + \\
&+ \tfrac{{e_k}^2}{2m_k c^2} \left\{ \left(\vec{P}^{(k)ret} \left\langle \vec{\xi}^{(k)ret} \times \vec{\lambda}^{(k)}, \vec{\sigma}^{(k)} \right\rangle - \vec{\xi}^{(k)ret} \left\langle \vec{P}^{(k)ret} \times \vec{\lambda}^{(k)}, \vec{\sigma}^{(k)} \right\rangle + \right. \right. \\
&\left. + c^2 {\tau_k}^{ret} \left(\vec{\sigma}^{(k)} \times \vec{P}^{(k)ret} \right) + c^2 L_{k,ret} \left(\vec{\sigma}^{(k)} \times \vec{\xi}^{(k)ret} \right) \right) - \\
&- \left(\vec{P}^{(k)adv} \left\langle \vec{\xi}^{(k)adv} \times \vec{\lambda}^{(k)}, \vec{\sigma}^{(k)} \right\rangle - \vec{\xi}^{(k)adv} \left\langle \vec{P}^{(k)adv} \times \vec{\lambda}^{(k)}, \vec{\sigma}^{(k)} \right\rangle + \right. \\
&\left. \left. + c^2 {\tau_k}^{adv} \left(\vec{\sigma}^{(k)} \times \vec{P}^{(k)adv} \right) + c^2 L_{k,adv} \left(\vec{\sigma}^{(k)} \times \vec{\xi}^{(k)adv} \right) \right) \right\} .
\end{aligned}$$

5. The Last Three Equations are Consequences of the First Three Ones

First, we rewrite the equations of motion in the form

$$\begin{aligned}
\tfrac{d\vec{\lambda}^{(k)}}{ds_k} &= -\tfrac{e_k^2}{m_k c^2} \left[\vec{P}^{(kn)} \left\langle \lambda^{(k)}, \xi^{(kn)} \right\rangle_4 - \vec{\xi}^{(kn)} \left\langle \lambda^{(k)}, P^{(kn)} \right\rangle_4 \right] ; \\
\tfrac{d\vec{\lambda}^{(k)}}{ds_k} &= -\tfrac{e_k^2}{m_k c^2} \left[\vec{P}^{(kn)} \tfrac{\left\langle \vec{u}^{(k)}, \vec{\xi}^{(kn)} \right\rangle - c^2 \tau_{kn}}{\Delta_k} - \vec{\xi}^{(kn)} \tfrac{\left\langle \vec{u}^{(k)}, \vec{P}^{(kn)} \right\rangle + c^2 L_{kn}}{\Delta_k} \right]
\end{aligned} \tag{5.1}$$

and

$$\begin{aligned}
\tfrac{d\vec{\sigma}^{(k)}}{ds_k} &= \textstyle\sum_{n=1,n\neq k}^{3} \tfrac{e_k e_n}{m_k c^2} \left[\vec{P}^{(kn)} \times \left(\vec{\sigma}^{(k)} \times \vec{\xi}^{(kn)} \right) + \tau_{kn} \vec{P}^{(kn)} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) - \vec{\xi}^{(kn)} \times \left(\vec{\sigma}^{(k)} \times \vec{P}^{(kn)} \right) + \right. \\
&\left. + L_{kn} \vec{\xi}^{(kn)} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \right] + \\
&+ \tfrac{{e_k}^2}{2m_k c^2} \left\{ \left[\vec{P}^{(k)ret} \times \left(\vec{\sigma}^{(k)} \times \vec{\xi}^{(k)ret} \right) + {\tau_k}^{ret} \vec{P}^{(k)ret} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) - \vec{\xi}^{(k)ret} \times \left(\vec{\sigma}^{(k)} \times \vec{P}^{(k)ret} \right) + \right. \right. \\
&\left. + L_{k,ret} \vec{\xi}^{(k)ret} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \right] - \left[\vec{P}^{(k)adv} \times \left(\vec{\sigma}^{(k)} \times \vec{\xi}^{(k)adv} \right) + {\tau_k}^{adv} \vec{P}^{(k)adv} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) - \right. \\
&\left. \left. - \vec{\xi}^{(k)adv} \times \left(\vec{\sigma}^{(k)} \times \vec{P}^{(k)adv} \right) + L_{k,adv} \vec{\xi}^{(k)adv} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \right] \right\} .
\end{aligned} \tag{5.2}$$

Our main goal is to check that if the last two systems have a solution then possess a solution $\left(\vec{\lambda}^{(k)},\vec{\sigma}^{(k)}\right)$, i.e.

$$\frac{d\left(\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)}\right)}{ds_k}=\sum_{n=1,n\neq k}^{3}\frac{e_ke_n}{m_kc^2}\left(\vec{P}^{(kn)}\begin{vmatrix}\xi_1^{(kn)}&\xi_2^{(kn)}&\xi_3^{(kn)}\\\lambda_1^{(k)}&\lambda_2^{(k)}&\lambda_3^{(k)}\\\sigma_1^{(k)}&\sigma_2^{(k)}&\sigma_3^{(k)}\end{vmatrix}-\vec{\xi}^{(kn)}\begin{vmatrix}P_1^{(kn)}&P_2^{(kn)}&P_3^{(kn)}\\\lambda_1^{(k)}&\lambda_2^{(k)}&\lambda_3^{(k)}\\\sigma_1^{(k)}&\sigma_2^{(k)}&\sigma_3^{(k)}\end{vmatrix}+\right.$$
$$\left.+c^2\tau_{kn}\left(\vec{\sigma}^{(k)}\times\vec{P}^{(kn)}\right)+c^2L_{kn}\left(\vec{\sigma}^{(k)}\times\vec{\xi}^{(kn)}\right)\right)\equiv\vec{R}_2$$

has a solution too. We verify this implication just for the Lorentz part.

Indeed, multiplying (5.1) from the right by $\vec{\sigma}^{(k)}$ and (5.2) from the left by $\vec{\lambda}^{(k)}$ we get:

$$\frac{d\vec{\lambda}^{(k)}}{ds_k}\times\vec{\sigma}^{(k)}=-\frac{e_k^2}{m_kc^2}\left[\left(\vec{P}^{(kn)}\times\vec{\sigma}^{(k)}\right)\frac{\left\langle\vec{u}^{(k)},\vec{\xi}^{(kn)}\right\rangle-c^2\tau_{kn}}{\Delta_k}-\left(\vec{\xi}^{(kn)}\times\vec{\sigma}^{(k)}\right)\frac{\left\langle\vec{u}^{(k)},\vec{P}^{(kn)}\right\rangle+c^2L_{kn}}{\Delta_k}\right];$$

$$\vec{\lambda}^{(k)}\times\frac{d\vec{\sigma}^{(k)}}{ds_k}=\sum_{n=1,n\neq k}^{3}\frac{e_ke_n}{m_kc^2}\vec{\lambda}^{(k)}\times\left[\vec{P}^{(kn)}\times\left(\vec{\sigma}^{(k)}\times\vec{\xi}^{(kn)}\right)+\tau_{kn}\vec{P}^{(kn)}\times\left(\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)}\right)-\right.$$
$$\left.-\vec{\xi}^{(kn)}\times\left(\vec{\sigma}^{(k)}\times\vec{P}^{(kn)}\right)+L_{kn}\vec{\xi}^{(kn)}\times\left(\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)}\right)\right].$$

We add the above last equations

$$\frac{d\vec{\lambda}^{(k)}}{ds_k}\times\vec{\sigma}^{(k)}+\vec{\lambda}^{(k)}\times\frac{d\vec{\sigma}^{(k)}}{ds_k}$$
$$=-\frac{e_k^2}{m_kc^2}\left[\left(\vec{P}^{(kn)}\times\vec{\sigma}^{(k)}\right)\frac{\left\langle\vec{u}^{(k)},\vec{\xi}^{(kn)}\right\rangle-c^2\tau_{kn}}{\Delta_k}-\left(\vec{\xi}^{(kn)}\times\vec{\sigma}^{(k)}\right)\frac{\left\langle\vec{u}^{(k)},\vec{P}^{(kn)}\right\rangle+c^2L_{kn}}{\Delta_k}\right]+$$
$$+\sum_{n=1,n\neq k}^{3}\frac{e_ke_n}{m_kc^2}\vec{\lambda}^{(k)}\times\left[\vec{P}^{(kn)}\times\left(\vec{\sigma}^{(k)}\times\vec{\xi}^{(kn)}\right)+\tau_{kn}\vec{P}^{(kn)}\times\left(\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)}\right)-\right.$$
$$\left.-\vec{\xi}^{(kn)}\times\left(\vec{\sigma}^{(k)}\times\vec{P}^{(kn)}\right)+L_{kn}\vec{\xi}^{(kn)}\times\left(\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)}\right)\right]\vec{R}_1$$

or

$$\frac{d\vec{\lambda}^{(k)}}{ds_k}\times\vec{\sigma}^{(k)}+\vec{\lambda}^{(k)}\times\frac{d\vec{\sigma}^{(k)}}{ds_k}=-\frac{e_k^2}{m_kc^2}\left[\left(\vec{P}^{(kn)}\times\vec{\sigma}^{(k)}\right)\frac{\left\langle\vec{u}^{(k)},\vec{\xi}^{(kn)}\right\rangle-c^2\tau_{kn}}{\Delta_k}-\left(\vec{\xi}^{(kn)}\times\vec{\sigma}^{(k)}\right)\frac{\left\langle\vec{u}^{(k)},\vec{P}^{(kn)}\right\rangle+c^2L_{kn}}{\Delta_k}\right]+$$
$$+\sum_{n=1,n\neq k}^{3}\frac{e_ke_n}{m_kc^2}\left[\left\langle\vec{\lambda}^{(k)},\vec{\sigma}^{(k)}\times\vec{\xi}^{(kn)}\right\rangle\vec{P}^{(kn)}-\left\langle\vec{\lambda}^{(k)},\vec{P}^{(kn)}\right\rangle\left(\vec{\sigma}^{(k)}\times\vec{\xi}^{(kn)}\right)-\tau_{kn}\left\langle\vec{\lambda}^{(k)},\vec{P}^{(kn)}\right\rangle\left(\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)}\right)-\right.$$
$$\left.-\left\langle\vec{\lambda}^{(k)},\vec{\sigma}^{(k)}\times\vec{P}^{(kn)}\right\rangle\vec{\xi}^{(kn)}+\left\langle\vec{\lambda}^{(k)},\vec{\xi}^{(kn)}\right\rangle\left(\vec{\sigma}^{(k)}\times\vec{P}^{(kn)}\right)-L_{kn}\left\langle\vec{\lambda}^{(k)},\vec{\xi}^{(kn)}\right\rangle\left(\vec{\lambda}^{(k)}\times\vec{\sigma}^{(k)}\right)\right].$$

Denote by $\vec{R}_1{}^{(k)}$ the right-hand side of the last equality and form the scalar product

$$\left\langle\vec{R}_1{}^{(k)},\vec{\sigma}^{(k)}\right\rangle=\sum_{n=1,n\neq k}^{3}\frac{e_ke_n}{m_kc^2}\left(\left\langle\vec{\lambda}^{(k)},\vec{\sigma}^{(k)}\times\vec{\xi}^{(kn)}\right\rangle\left\langle\vec{P}^{(kn)},\vec{\sigma}^{(k)}\right\rangle-\left\langle\vec{\lambda}^{(k)},\vec{\sigma}^{(k)}\times\vec{P}^{(kn)}\right\rangle\left\langle\vec{\xi}^{(kn)},\vec{\sigma}^{(k)}\right\rangle\right).$$

We have to show that the right-hand side $\vec{R}_1{}^{(k)}$ of the last equation coincides (in weak sense) with the right-hand side $\vec{R}_2{}^{(k)}$ of the following vector equation corresponding to the last three equations. For $\vec{R}_2{}^{(k)}$ we have

$$\left\langle\vec{R}_2{}^{(k)},\vec{\sigma}^{(k)}\right\rangle=\sum_{n=1,n\neq k}^{3}\frac{e_ke_n}{m_kc^2}\left(\left\langle\vec{P}^{(kn)},\vec{\sigma}^{(k)}\right\rangle\left\langle\vec{\xi}^{(kn)}\times\vec{\lambda}^{(k)},\vec{\sigma}^{(k)}\right\rangle-\left\langle\vec{\xi}^{(kn)},\vec{\sigma}^{(k)}\right\rangle\left\langle\vec{P}^{(kn)}\times\vec{\lambda}^{(k)},\vec{\sigma}^{(k)}\right\rangle\right).$$

Then we conclude that $\left\langle \vec{R}_1{}^{(k)}, \vec{\sigma}^{(k)} \right\rangle - \left\langle \vec{R}_2{}^{(k)}, \vec{\sigma}^{(k)} \right\rangle = \left\langle \vec{R}_1{}^{(k)} - \vec{R}_2{}^{(k)}, \vec{\sigma}^{(k)} \right\rangle = 0$. It is easy to see that the solutions of the last equation are $\vec{R}_1{}^{(k)} - \vec{R}_2{}^{(k)} = \vec{0}$ or $\vec{R}_1{}^{(k)} - \vec{R}_2{}^{(k)} = \vec{q} \times \frac{\vec{\sigma}^{(k)}}{\left\langle \vec{\sigma}^{(k)}, \vec{\sigma}^{(k)} \right\rangle}$, where $\vec{q}$is an arbitrary chosen vector.

The transformation of the radiation parts can be accomplished in a similar way.

6. Appendix

(A1) We verify that $\sigma_{ij}{}^{(k)} \lambda_j{}^{(k)} = 0$ (cf. [10]. [11]) $(i = 1, 2, 3, 4)$ in view of $\sigma_{ii}{}^{(k)} = 0$:

$$\sigma_{11}{}^{(p)}\lambda_1{}^{(p)} + \sigma_{12}{}^{(p)}\lambda_2{}^{(p)} + \sigma_{13}{}^{(p)}\lambda_3{}^{(p)} + \sigma_{14}{}^{(p)}\lambda_4{}^{(p)} = \sigma_{12}^{(p)}\lambda_2{}^{(p)} + \sigma_{13}^{(p)}\lambda_3{}^{(p)} + \sigma_{14}^{(p)}\lambda_4{}^{(p)} =$$

$$= \sigma_3{}^{(k)}\lambda_2{}^{(k)} - \sigma_2{}^{(k)}\lambda_3{}^{(k)} + i\,\frac{\lambda_2^{(k)}\sigma_3^{(k)} - \lambda_3^{(k)}\sigma_2^{(k)}}{c} ic = -\begin{vmatrix} \sigma_2{}^{(k)} & \sigma_3{}^{(k)} \\ \lambda_2{}^{(k)} & \lambda_3{}^{(k)} \end{vmatrix} + \begin{vmatrix} \sigma_2{}^{(k)} & \sigma_3{}^{(k)} \\ \lambda_2{}^{(k)} & \lambda_3{}^{(k)} \end{vmatrix} = 0;$$

$$\sigma_{21}{}^{(k)}\lambda_1{}^{(k)} + \sigma_{22}{}^{(k)}\lambda_2{}^{(k)} + \sigma_{23}{}^{(k)}\lambda_3{}^{(k)} + \sigma_{24}{}^{(k)}\lambda_4{}^{(k)} = \sigma_{21}^{(k)}\lambda_1{}^{(k)} + \sigma_{23}^{(k)}\lambda_3{}^{(k)} + \sigma_{24}^{(k)}\lambda_4{}^{(k)} =$$

$$= \sigma_2{}^{(k)}\lambda_1{}^{(k)} + \sigma_1{}^{(k)}\lambda_3{}^{(k)} + i\,\frac{\lambda_3^{(k)}\sigma_1^{(k)} - \lambda_1^{(k)}\sigma_3^{(k)}}{c} ic = \begin{vmatrix} \sigma_1{}^{(k)} & \sigma_3{}^{(k)} \\ \lambda_1{}^{(k)} & \lambda_3{}^{(k)} \end{vmatrix} - \begin{vmatrix} \sigma_1{}^{(k)} & \sigma_3{}^{(k)} \\ \lambda_1{}^{(k)} & \lambda_3{}^{(k)} \end{vmatrix} = 0;$$

$$\sigma_{31}{}^{(k)}\lambda_1{}^{(k)} + \sigma_{32}{}^{(k)}\lambda_2{}^{(k)} + \sigma_{33}{}^{(k)}\lambda_3{}^{(k)} + \sigma_{34}{}^{(k)}\lambda_4{}^{(k)} = \sigma_{31}^{(k)}\lambda_1{}^{(k)} + \sigma_{32}^{(k)}\lambda_2{}^{(k)} + \sigma_{34}^{(k)}\lambda_4{}^{(k)} =$$

$$= \sigma_2{}^{(k)}\lambda_1{}^{(k)} - \sigma_1{}^{(k)}\lambda_2{}^{(k)} + i\,\frac{\lambda_1^{(k)}\sigma_2^{(k)} - \lambda_2^{(k)}\sigma_1^{(k)}}{c} ic = -\begin{vmatrix} \sigma_1{}^{(k)} & \sigma_2{}^{(k)} \\ \lambda_1{}^{(k)} & \lambda_2{}^{(k)} \end{vmatrix} + \begin{vmatrix} \sigma_1{}^{(k)} & \sigma_2{}^{(k)} \\ \lambda_1{}^{(k)} & \lambda_2{}^{(k)} \end{vmatrix} = 0;$$

$$\sigma_{41}{}^{(k)}\lambda_1{}^{(k)} + \sigma_{42}{}^{(k)}\lambda_2{}^{(k)} + \sigma_{43}{}^{(k)}\lambda_3{}^{(k)} + \sigma_{44}{}^{(k)}\lambda_4{}^{(k)} = \sigma_{41}{}^{(k)}\lambda_1{}^{(k)} + \sigma_{42}{}^{(k)}\lambda_2{}^{(k)} + \sigma_{43}{}^{(k)}\lambda_3{}^{(k)} =$$

$$= -\frac{i\,(\lambda_2^{(k)}\sigma_3^{(k)} - \lambda_3^{(k)}\sigma_2^{(k)})\lambda_1{}^{(k)}}{c} - \frac{i(\lambda_3^{(k)}\sigma_1^{(k)} - \lambda_1^{(k)}\sigma_3^{(k)})\lambda_2{}^{(k)}}{c} - \frac{i\,(\lambda_1^{(k)}\sigma_2^{(k)} - \lambda_2^{(k)}\sigma_1^{(k)})\lambda_3{}^{(k)}}{c} =$$

$$= \frac{i}{c}\begin{vmatrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ \lambda_2^{(k)} & \lambda_3^{(k)} \end{vmatrix}\lambda_1{}^{(k)} - \frac{i}{c}\begin{vmatrix} \sigma_1^{(k)} & \sigma_3^{(k)} \\ \lambda_1^{(k)} & \lambda_3^{(k)} \end{vmatrix}\lambda_2{}^{(k)} + \frac{i}{c}\begin{vmatrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ \lambda_1^{(k)} & \lambda_2^{(k)} \end{vmatrix}\lambda_3{}^{(k)} = \frac{i}{c}\begin{vmatrix} \lambda_1{}^{(k)} & \lambda_2{}^{(k)} & \lambda_3{}^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} & \sigma_3^{(k)} \\ \lambda_1{}^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)} \end{vmatrix} = 0.$$

(A2) Let us consider the equations

$$\frac{d\sigma_{ij}^{(k)}}{ds_k} = \frac{e_k}{m_k c^2}\left(\left(\sum_{n=1,n\neq k}^{3} F_{im}^{(kn)} + F_{im}^{(k)rad}\right)\sigma_{mj}^{(k)} - \sigma_{im}^{(k)}\left(\sum_{n=1,n\neq k}^{3} F_{mj}^{(kn)} + F_{mj}^{(k)rad}\right)\right) \equiv \frac{e_k \tilde{F}_{ij}^{(k)}}{m_k c^2}.$$

We notice that $F_{im}^{(kn)} = -F_{mi}^{(kn)}$; $F_{im}^{(k)rad} = -F_{mi}^{(k)rad}$; $\sigma_{mj}^{(k)} = -\sigma_{jm}^{(k)}$.
Therefore

$$\tilde{F}_{ij}^{(k)} = \left(\sum_{n=1,n\neq k}^{3} F_{im}^{(kn)} + F_{im}^{(k)rad}\right)\sigma_{mj}^{(k)} - \sigma_{im}^{(k)}\left(\sum_{n=1,n\neq k}^{3} F_{mj}^{(kn)} + F_{mj}^{(k)rad}\right) -$$

$$-\left(\left(\sum_{n=1,n\neq k}^{3} F_{mi}^{(kn)} + F_{mi}^{(k)rad}\right)\sigma_{jm}^{(k)} - \sigma_{mi}^{(k)}\left(\sum_{n=1,n\neq k}^{3} F_{jm}^{(kn)} + F_{jm}^{(k)rad}\right)\right) = -\tilde{F}_{ji}^{(k)}.$$

It follows $\tilde{F}_{ii}^{(k)} = 0$ $(i = 1, 2.3, 4)$.

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