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Araştırma Makalesi / Research Article

The Modified Trial Equation Method to the van der Waals Model

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Modified trial equation method; Van der Waals model; Wave solution; Exact solution

Abstract

In this research, the modified trial equation method (MTEM) is considered in order to find some exact solutions of the van der Waals model. In addition, to finding the solution of the van der Waals model this method can be used in the solution of nonlinear problems. Thus, some wave solutions for various situations are obtained. Also, three and two dimensional graphs were found with the help of Mathematica9 to analyze the physical behavior of the obtained solutions.

Van der Waals Modeline Modifiye Edilmiş Deneme Denklem Metodu

Anahtar kelimeler
Modifiye edilmiş deneme denklem metodu; Van der Waals model; Dalga çözümü; Tam çözüm

Öz

Bu araştırmada, van der Waals modelinin bazı tam çözümlerini bulmak için modifiye edilmiş deneme denklem metodu (MEDDM) ele alınmıştır. Van der Waals modelinin çözümünün bulunmasına ek olarak, bu metod lineer olmayan problemlerin çözümünde de kullanılabilir. Böylece çeşitli durumlar için bazı dalga çözümleri elde edilir. Ayrıca, elde edilen çözümlerin fiziksel davranışlarını analiz etmek için Mathematica9 yardımıyla üç ve iki boyutlu grafikler bulunmuştur.

1.Introduction

In relation to wave solutions, soliton theory has a great importance in applied sciences, physics and engineering. Since the problems discussed in this theory are generally modeled by nonlinear partial differential equations (NLPDE), many methods have been developed in the literature to find numerical, analytical and exact solutions of these equations. Especially in recent years, many scientists have published several articles on exact solution methods. Some of the exact methods developed are Hirota's bilinear method (Hirato 2004, Ma and Xia 2013), Bäcklund transformation method (Miura 1978), Exp-function method (He and Wu 2006, Zang 2007), the tanh function method (Malfliet and Hereman 1996, Duffy and Parkes 1996), Homogeneous balance method (Fan and Zhang 1998, Tang and Zhao 2002, Kaushal *et al*. 2010), sine-cosine method (Wazwaz 2004, Yan 1996), Extended trial equation method (Gurefe *et al*. 2013), Kudryashov method (Kudryashov 2012, Pandır *et al*. 2012) and so on.

In this study, MTEM is applied to the following van der Waals model (Bibi *et al*. 2018).

$$\frac{\partial^2 u}{\partial t^2}+\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 u}{\partial x^2}-\eta\frac{\partial u}{\partial t}-u^3-\varepsilon u\right)=0. \qquad (1)$$

Here, u defines the field which reflect correction to critical average vertical density, x is the horizontal direction of the granular system, ε and η are respectively the bifurcation parameter and effective viscosity. Many authors have obtained the solutions of the van der Waals Equation using different methods (Argentina *et al*. 2002, Abourabia *et al*. 2015, Lu *et al*. 2017, Zafar *et al*. 2020).

2.The Modified Trial Equation Method

We will explain in detail the MTEM which has various applications in the literature (Bulut *et al*. 2013, Bulut and Pandir 2013, Odabasi and Misirli 2018).

Step 1. In the form of let's consider NLPDE,

$$P\left(u,u_t,u_x,u_{xx},...\right)=0. \tag{2}$$

using traveling wave transformation

$$u\left(x,t\right)=u\left(\xi\right)\ ,\ \ \xi=kx-wt \tag{3}$$

Where w is a constant. Applying the transformation (3) to Eq.(2), we can observe the following nonlinear ordinary differential equation

$$O\left(t,x,u,u',u'',\cdots\right)=0, \tag{4}$$

where $u'=\dfrac{du}{d\xi}$.

Step 2. The first order trial Equation:

$$u'=\frac{J\left(u\right)}{K\left(u\right)}=\frac{\sum_{i=0}^{n}a_iu^i}{\sum_{j=0}^{l}b_ju^j}=\frac{a_0+a_1u+a_2u^2+\ldots+a_nu^n}{b_0+b_1u+b_2u^2+\ldots+b_lu^l}, \tag{5}$$

and

$$u''=\frac{J\left(u\right)\left[J'\left(u\right)G\left(u\right)-J\left(u\right)K'\left(u\right)\right]}{K^3\left(u\right)}, \tag{6}$$

where $J(u)$ and $K(u)$ are polynomials of u. Inserting Eqs.(5)-(6) into Eqs.(4) symplast an equation of $r\left(u\right)$ polynomial u .

$$r\left(u\right)=\chi_0+\chi_1u+\ldots+\chi_ru^r=0. \tag{7}$$

Step 3. Equating the coefficients of $r\left(u\right)$ to zero,

$$\chi_p=0\ ,\ \ p=0,\ldots,r. \tag{8}$$

Solving the system (8), we can find the values of $a_0,\ldots,a_n$ and $b_0,\ldots,b_l$.

Step 4. Consider Eq.(5), the following integral form can be written

$$\xi-\xi_0=\int\frac{K\left(u\right)}{J\left(u\right)}du. \tag{9}$$

Using the complete discrimination system with the roots of $J(u)$, we obtain exact solutions of Eq. (2).

3.Application to Van der Waals Model

By using Eq.(3), we get following

$$\frac{w^2-\varepsilon k^2}{k^2}+k^2u''+w\eta u'-u^3=0. \tag{10}$$

By balancing u'' and u^3, $n=l+2$ is obtained.

Case 1:

For $l=0$ is chosen for $n=2$ then

$$u'=\frac{a_0+a_1u+a_2u^2}{b_0}, \tag{11}$$

$$u''=\frac{\left(a_0+a_1u+a_2u^2\right)\left(a_1+2a_2u\right)}{{b_0}^2}, \tag{12}$$

where $a_2\neq 0$ and $b_0\neq 0$. Then we get following case us.

Case 1.1:

$$a_0=0\,,\,a_1=\frac{\sqrt{\varepsilon}\sqrt{\dfrac{\xi^2}{9k-2k\xi^2}}b_0}{\sqrt{k}}\,,\,a_2=-\frac{b_0}{\sqrt{2}k} \tag{13}$$

$$w=-\frac{3k^{3/2}\sqrt{\varepsilon}\sqrt{\dfrac{\xi^2}{9k-2k\xi^2}}}{\xi}$$

When we substitute the results in Eq. (13) into Eq. (9), we have

$$a_3 = \frac{\left(9-2\eta^2\right)^2 b_0 b_1}{\sqrt{18-4\eta^2}\sqrt{-k^2\left(-9+2\eta^2\right)^3 b_0^{\ 2}}}, w = -\frac{3k\sqrt{\varepsilon}}{\sqrt{9-2\eta^2}}$$

$$\int \frac{1}{\frac{a_2}{b_0}\left(\left(u+\frac{a_1}{2a_2}\right)^2+\frac{4a_0a_2-a_1^{\ 2}}{4a_2^{\ 2}}\right)}du = \xi - \xi_0. \quad (14)$$

Integrating Eq.(14), we construct the wave solution of Eq. (1) as foll

$$u(x,t) = \frac{\pm\frac{2b_0\sqrt{\varepsilon}\eta}{k\sqrt{9-2\eta^2}}\exp\left(\pm\frac{2\sqrt{\varepsilon}\eta}{\sqrt{9-2\eta^2}}\left(x\pm\frac{3\sqrt{\varepsilon}}{\sqrt{9-2\eta^2}}t\right)\right)}{1-\exp\left(\pm\frac{2\sqrt{\varepsilon}\eta}{\sqrt{9-2\eta^2}}\left(x\pm\frac{3\sqrt{\varepsilon}\eta}{\sqrt{9-2\eta^2}}t\right)\right)}. \quad (15)$$

Case 2:

For $l=1$ is chosen $n=3$.

$$u' = \frac{a_0+a_1u+a_2u^2+a_3u^3}{b_0+b_1u}, \quad (16)$$

and

$$u'' = \frac{\left(a_0+a_1u+a_2u^2+a_3u^3\right)\left(\left(b_0+b_1u\right)\left(a_1+2a_2u+3a_3u^2\right)-b_1\left(a_0+a_1u+a_2u^2+a_3u^3\right)\right)}{\left(b_0+b_1u\right)^3}$$

(17)

Thus, solve the system of algebraic equations by using matematica codes,

Case 2.1:

$$a_0 = 0, a_1 = \frac{\sqrt{\varepsilon}\eta b_0}{k\sqrt{9-2\eta^2}},$$

$$a_2 = -\frac{-\sqrt{2}\sqrt{-k^2\left(-9+2\eta^2\right)^3 b_0^{\ 2}}+2k\sqrt{\varepsilon}\eta\left(-9+2\eta^2\right)b_1}{2k^2\left(9-2\eta^2\right)^{3/2}} \quad (18)$$

Substituting the results in Eq. (18) into Eq. (9), we have

$$u_1(x,t) = -\frac{\vartheta\exp(\varphi)}{-\exp(-\vartheta)+\sqrt{2}k\left(9-2\eta^2\right)^2 b_0\exp(\varphi)}. \quad (19)$$

Where

$$\varphi = \left(\frac{3t\varepsilon\eta}{9-2\eta^2}+\frac{x\sqrt{\varepsilon}\eta}{\sqrt{9-2\eta^2}}\right), \vartheta = 2\sqrt{\varepsilon}\eta\sqrt{-k^2\left(-9+2\eta^2\right)^3 b_0^{\ 2}}$$

Case 2.2:

$$a_0 = 0, a_1 = 0, a_2 = \frac{\sqrt{\varepsilon}\eta b_1}{k\sqrt{9-2\eta^2}},$$
$$a_3 = -\frac{b_1}{\sqrt{2}k}, b_0 = 0, w = -\frac{3k\sqrt{\varepsilon}}{\sqrt{9-2\eta^2}}. \quad (20)$$

And from here

$$u_2(x,t) = \frac{2\sqrt{\varepsilon}\eta}{\sqrt{18-4\eta^2}-\exp\left(\sqrt{\varepsilon}\eta\left(-\frac{x}{\sqrt{9-2\eta^2}}+\frac{3t\sqrt{\varepsilon}}{-9+2\eta^2}+2\right)\right)} \quad (21)$$

Case2.3:

$$a_0 = 0,\ a_1 = \frac{\sqrt{\varepsilon}\eta b_0}{k\sqrt{9-2\eta^2}},$$
$$a_2 = \frac{2\left(k\eta\left(9-2\eta^2\right)b_0+\sqrt{-k^2(-9+2\eta^2)^3 b_0^{\ 2}}\right)}{k^2\left(18-4\eta^2\right)^{3/2}}$$
$$a_3 = \frac{\sqrt{-k^2\left(-9+2\eta\right)}}{2k^2\sqrt{\varepsilon}\left(9-2\eta^2\right)^{3/2}}, \quad (22)$$
$$b_1 = -\frac{b_0}{\sqrt{2\varepsilon}} = 0, w = -\frac{3k\sqrt{\varepsilon}}{\sqrt{9-2\eta^2}}.$$

$$u_3(x,t)=\frac{\sqrt{2\varepsilon}k\eta\left(-9+2\eta^2\right)b_0\exp\left(\dfrac{9\sqrt{\varepsilon}\eta\mu}{\left(9-2\eta^2\right)^{3/2}}+2k\sqrt{\varepsilon}\eta^3 b_0\right)}{\left(\psi b_0-\varpi b_0\right)\exp\left(\dfrac{2\sqrt{\varepsilon}\eta^3\mu}{\left(9-2\eta^2\right)^{3/2}}+\psi b_0\right)+\sqrt{-k^2\left(-9+2\eta^2\right)^3 b_0^{\,2}}\exp\left(\dfrac{9\sqrt{\varepsilon}\eta\mu}{\left(9-2\eta^2\right)^{3/2}}+\varpi b_0\right)} \tag{23}$$

wave solution is obtained. Where $\psi=9k\sqrt{\varepsilon}\eta$, $\varpi=2k\sqrt{\varepsilon}\eta^3$, $\mu=x+\dfrac{3k\sqrt{\varepsilon}}{\sqrt{9-2\eta^2}}t.$

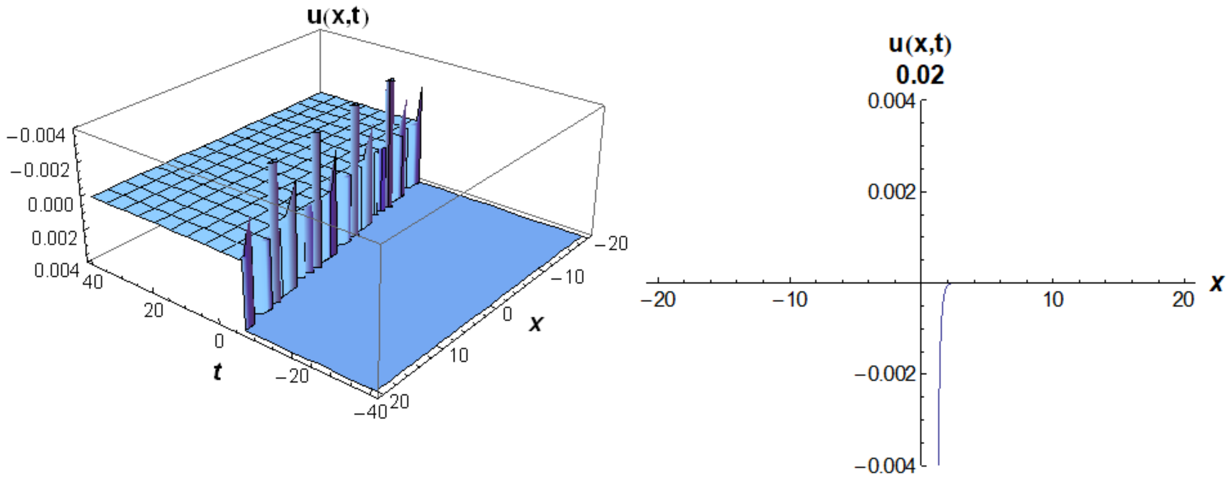

Figure 1. The 3D and 2D surfaces of real values of Eq.(15) for $\varepsilon=4,\eta=7$, $k=2,-15\le x\le 15$, $-10\le t\le 10$ and $t=0.01$ for 2D.

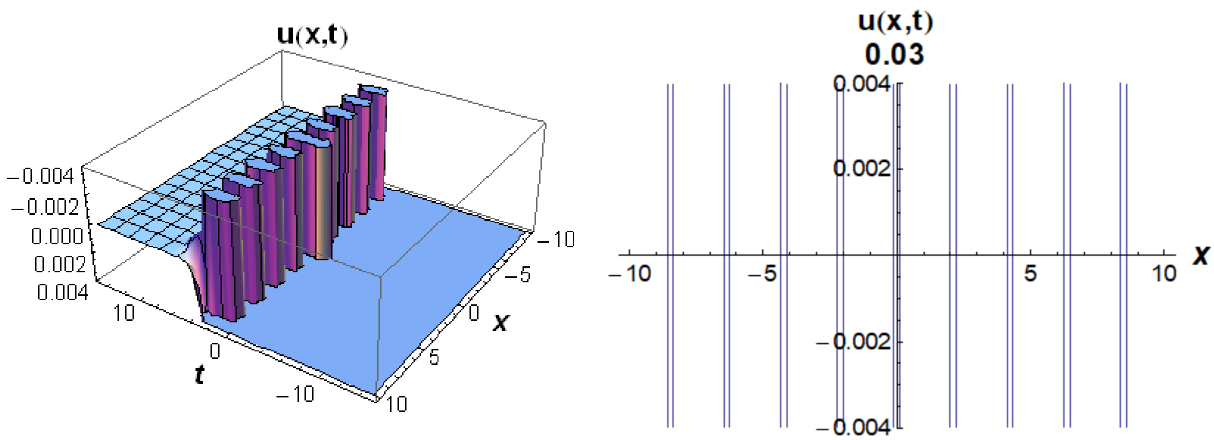

Figure 2. The 3D and 2D surfaces of imaginary values of Eq.(15) for $\varepsilon=-9$, $\eta=-3,k=2$, $b_0=4$, $-20\le x\le 20$, $-40\le t\le 40$ and $t=0.01$ for 2D.

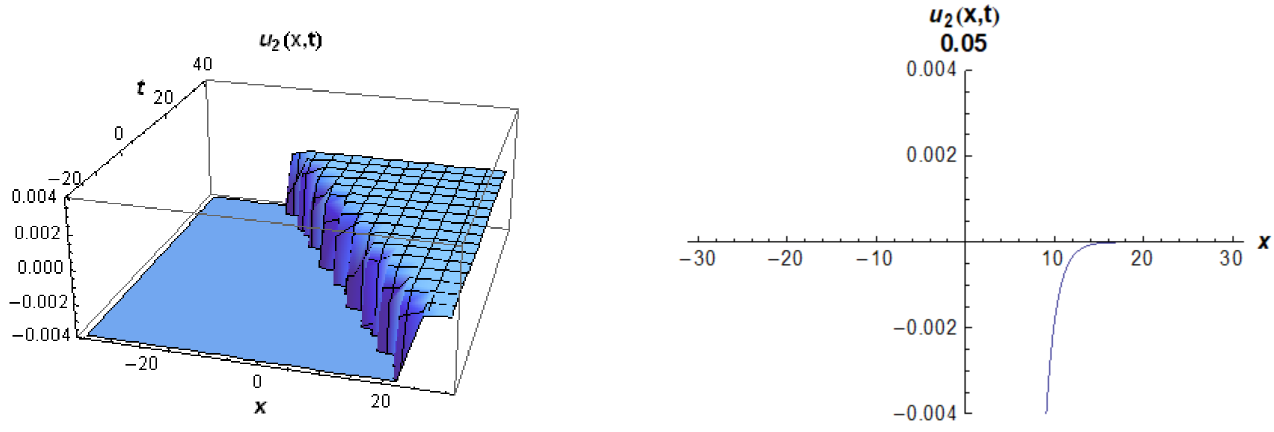

Figure 3. The 3D and 2D surfaces of real values of Eq.(21) for $\varepsilon=-1,\ \eta=-4,\ -30\le x\le 30,\ -20\le t\le 40$ and $t=0.05$ for 2D.

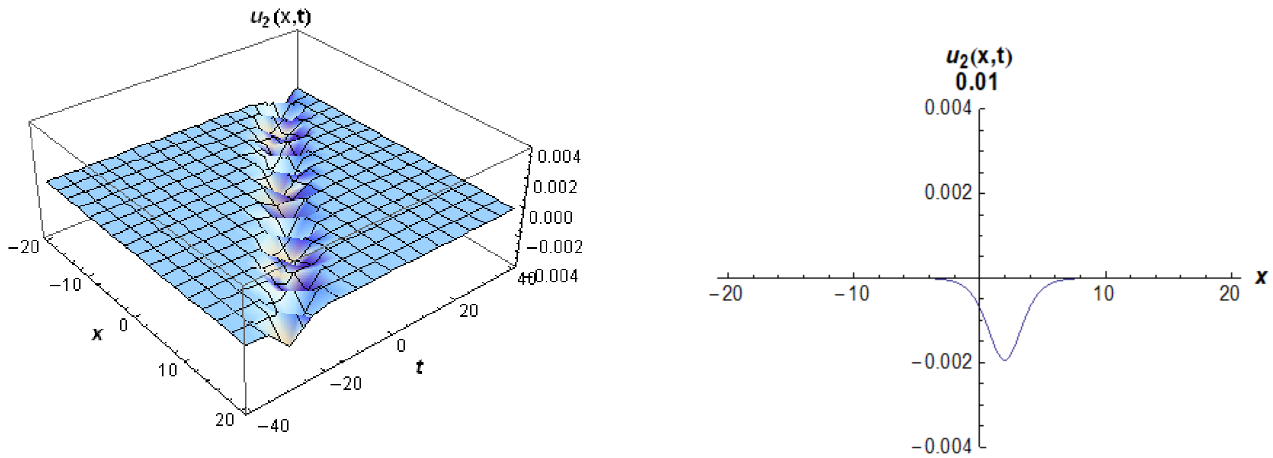

Figure 4. The 3D and 2D surfaces of imaginary values of Eq.(21) for $\varepsilon=-2,\ \eta=-5,\ -20\le x\le 20, -40\le t\le 40$ and $t=0.01$ for 2D.

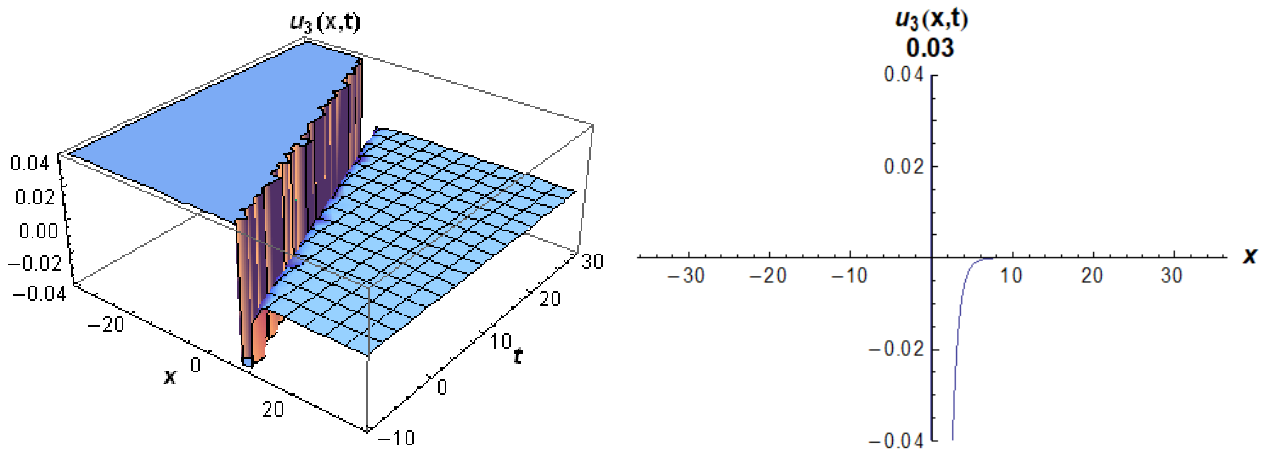

Figure 5. The 3D and 2D surfaces of real values of Eq. (23) for $\varepsilon=3,\ \eta=-6,\ b_0=1,\ k=2,$ $-35\le x\le 35, -10\le t\le 30$ and $t=0.03$ for 2D.

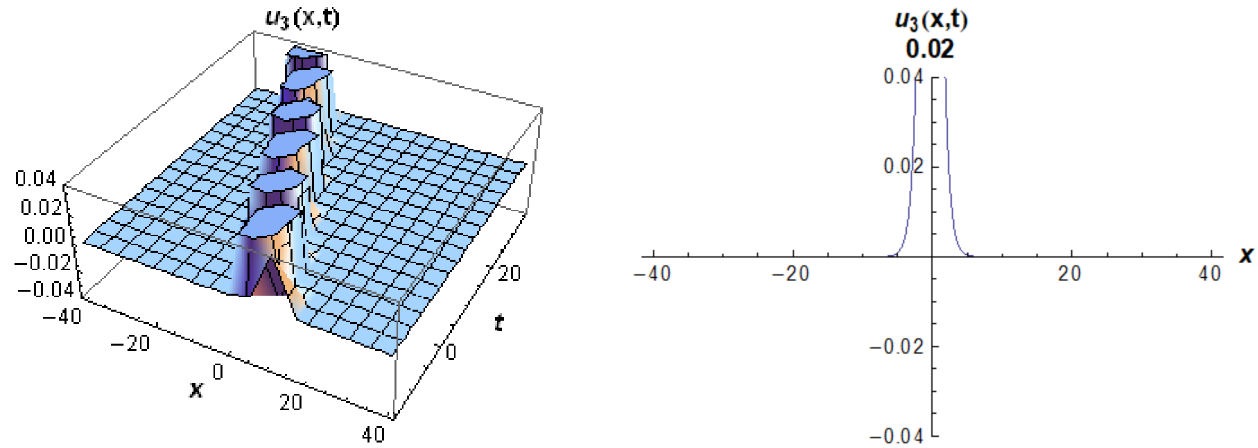

Figure 6. The 3D and 2D surfaces of imaginary values of Eq.(23) for $\varepsilon=-4,\eta=-7,b_0=3,k=1,-40\le x\le 40,$ $-15\le t\le 35$ and $t=0.02$ for 2D.

4. Conclusions

In this article, we use the MTEM in order to construct the exact traveling wave solutions of the van der Waals model. This method is suitable for van der Waals model. By this technique, we find some useful wave solutions to this problem.

Remark:

The solutions of Eq.(1) were procured by using MTEM. These solutions were controlled in Wolfram Mathematica 9. To our knowledge, these solutions that we obtained in this work, are new.

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