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A Comparative Analysis of the Ranking Functions for the IVIFVs and A New Score Function

Melda KOKOC[1,*], Suleyman ERSOZ[2]

[1]*Gazi University, Rectorate, 06560, Ankara, Turkey*
[2]*Kırıkkale University, Department of Industrial Engineering, 71451, Kırıkkale, Turkey*

Highlights

- A new score function of IVIFVs is introduced.
- The limitations of the existing score and accuracy functions are determined.
- A comparative analysis for the ranking functions of the IVIFVs is performed.

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Abstract

The ranking of interval-valued intuitionistic fuzzy values (IVIFVs) has an important role in real-life decision-making problems. Even though there are many approaches related to the ranking methods of the IVIFVs, some of them have some shortcomings. In this study, the disadvantages of the existing ranking functions of the IVIFVs are discussed. It is revealed that both the most popular ranking functions and their recently improved versions may lead to unacceptable results. Furthermore, in this article, a new score function is offered to cope with the shortcomings of the ranking functions. The importance and performance of this new score function are proven with the help of many examples. Also, the decision-making algorithm adapted by integrating the new function is presented to illustrate the applicability of the new score function in the decision-making problems.

1. INTRODUCTION

In case there is uncertainty and lack of information, it is difficult to evaluate alternatives correctly by considering several criteria. How to cope with vagueness and ambiguity is an interesting and significant issue [1]. To deal with vagueness and ambiguity Zadeh [2] offered fuzzy sets (FSs) theory. In the fuzzy set theory, the belonging of the element to the set is represented by the membership degree. Atanassov [3] put forward that the belonging of the element to the set should be defined by the membership, non-membership, and hesitant degrees instead of just its membership degree. So, he suggested the intuitionistic fuzzy sets (IFSs) theory as an extension of fuzzy sets. IFSs theory is more flexible and effective than traditional FSs theory in dealing with the vagueness and ambiguity of the objectives [4-7]. But, in some complex decision-making situations, decision-makers may not have adequate information to assign crisp values to membership and non-membership degrees. Hence, Atanassov and Gargov [8] proposed the interval-valued intuitionistic fuzzy sets (IVIFSs) theory in which the membership degree and non-membership degree of an element are described with closed intervals instead of crisp values. Owing to the fact that the membership degree and the non-membership degree of an element are defined by intervals, researchers have considered IVIFSs as a suitable and realistic tool to express ambiguous and vague information in real-life problems [4, 5, 9-11]. One of these real-life problems is the multi-criteria decision-making (MCDM) problem. The main goal in solving MCDM problems is to rank the alternatives by taking into consideration criteria. But, when the evaluate the alternatives in the IVIF environment, the problem of how to compare the IVIF values (IVIFVs) is revealed. So, the ranking of the IVIFVs is an attention-grabbing topic.

*Corresponding author, e-mail: meldakokoc@gazi.edu.tr

The problem of ranking IVIFVs, though, has been extensively studied, there is an agreed conclusion that there is no unique best approach to do this. Therefore, it is important to develop the single-stage ranking function that offers the best possible performance. Several approaches have been developed for ranking IVIFVs. Among these, score functions [6, 9, 10, 12-19] and accuracy functions [5, 7, 12, 15, 17, 18, 20-25] are frequently preferred approaches. However, in some cases, the existing score or accuracy functions for ranking IVIFVs do not yield sufficient performance and they cannot distinguish comparable IVIFVs. Therefore, there is a need for an improved score function that will handle the shortcomings of the existing score functions and give sufficient information about the IVIFVs. With this motivation, in this study, a new score function, which can rank or distinguish two IVIFVs more efficiently than the existing functions, is developed. This study stands out as the most comprehensive study comparing existing score functions and accuracy functions, in addition to proposing a high-performance score function.

This paper is organized as follows. The basic concepts of interval-valued intuitionistic fuzzy sets and monotonicity property of score function and of accuracy function are given in Section 2. In Section 3, existing ranking functions are presented and their shortcomings are proven with examples. The proposed score function is introduced in Section 4. A comparative performance analysis is performed with twenty IVIFV pairs to show that the proposed function is more reasonable in the ranking of IVIFVs in Section 5. In Section 6, Şahin [23]'s decision-making algorithm is adapted to show the application of the new score function in MCDM problems. Conclusions are presented in the last section.

2. BASIC CONCEPTS

Some basic definitions of IVIFSs are presented in this section.

Definition 1.[8] Let *IVIFS(X)* denotes the family of all the *IVIFSs* over the universe of discourse $X=\{x_1,x_2,...,x_n\}$ and $\tilde{A}\in IVIFS(X)$ be an IVIFV given by $\tilde{A}=\langle[\mu_{\tilde{A}}^L(x),\mu_{\tilde{A}}^U(x)],[v_{\tilde{A}}^L(x),v_{\tilde{A}}^U(x)]\rangle$.

Definition 2. [8] Let $\tilde{A}$ be an IVIFV. The hesitant degree of $\tilde{A}$ is defined as follows
$\pi_{\tilde{A}}=\left[\pi_{\tilde{A}}^L(x),\pi_{\tilde{A}}^U(x)\right]=\left\langle\left[1-\mu_{\tilde{A}}^U(x)-v_{\tilde{A}}^U(x)\right],\left[1-\mu_{\tilde{A}}^L(x)-v_{\tilde{A}}^L(x)\right]\right\rangle$.

Definition 3. [8] Let $\tilde{A},\tilde{B}\in IVIFS(X)$. A subset relation $\tilde{A}\subset\tilde{B}\Leftrightarrow\mu_{\tilde{A}}^L(x)\le\mu_{\tilde{B}}^L(x)$, $\mu_{\tilde{A}}^U(x)\le\mu_{\tilde{B}}^U(x)$, $v_{\tilde{A}}^L(x)\ge v_{\tilde{B}}^L(x)$ and $v_{\tilde{A}}^U(x)\ge v_{\tilde{B}}^U(x)$, $\forall x\in X$. In addition, $\tilde{A}=\tilde{B}\Leftrightarrow\tilde{A}\subset\tilde{B}$ and $\tilde{A}\supset\tilde{B}$.

Definition 4. [12] Let $\tilde{A}_j$ for $j=1,2,...,n$ is a collection of IVIFVs. The IVIF weighted arithmetic (IVIFWA) operator is defined as:

$$IVIFWA_\omega(A_1,A_2,...,A_n)=\sum_{j=1}^{n}\omega_jA_j=\left(\begin{array}{l}\left[1-\prod_{j=1}^{n}\left(1-\mu_{A_j}^L\right)^{\omega_j},1-\prod_{j=1}^{n}\left(1-\mu_{A_j}^U\right)^{\omega_j}\right],\\ \left[\prod_{j=1}^{n}\left(v_{A_j}^L\right)^{\omega_j},\prod_{j=1}^{n}\left(v_{A_j}^U\right)^{\omega_j}\right]\end{array}\right).$$

Definition 5. [18] Let $\tilde{A}$ be an IVIFV. The monotonicity property for the score function $S(\tilde{A})$ of the $\tilde{A}$ as below.

1) If *b, c, d* are fixed, then $\partial S(\tilde{A})/\partial a>0$,

2) If *a, c, d* are fixed, then $\partial S(\tilde{A})/\partial b>0$,

3) If a, *b, d* are fixed, then $\partial S(\tilde{A})/\partial c<0$,

4) If a, *b, c* are fixed, then $\partial S(\tilde{A})/\partial d<0$.

Definition 6. [18] Let $\tilde{A}$ be an IVIFV. The monotonicity property for the accuracy function $A(\tilde{A})$ of the $\tilde{A}$ as below.

1) If *b, c, d* are fixed, then $\partial A(\tilde{A}) / \partial a > 0$,
2) If *a, c, d* are fixed, then $\partial A(\tilde{A}) / \partial b > 0$,
3) If a, *b, d* are fixed, then $\partial A(\tilde{A}) / \partial c > 0$,
4) If a, *b, c* are fixed, then $\partial A(\tilde{A}) / \partial d > 0$.

3. EXISTING RANKING FUNCTIONS

The basic principle of ranking functions is that the greater score value or accuracy value of $\tilde{A}$, the greater IVIFV $\tilde{A}$. In the light of this principle, it is determined that the existing score and accuracy functions have counter-intuitive cases on the ranking of IVIFVs. So, these functions' capacity cannot be enough to distinguish some IVIFV pairs. These counter-intuitive cases are illustrated with examples under the related functions' titles.

3.1. Xu[12]'s Ranking Method of the IVIFV

To rank the IVIFVs, Xu [12] developed the score function $S(.)$ and the accuracy function $H(.)$ as follows:

$$S(\tilde{A}) = \frac{\mu_{\tilde{A}}^{L} - \nu_{\tilde{A}}^{L} + \mu_{\tilde{A}}^{U} - \nu_{\tilde{A}}^{U}}{2}, \tag{1}$$

$$H(\tilde{A}) = \frac{\mu_{\tilde{A}}^{L} + \mu_{\tilde{A}}^{U} + \nu_{\tilde{A}}^{L} + \nu_{\tilde{A}}^{U}}{2}. \tag{2}$$

Example 1. Let $\tilde{A}_1 = [0.65, 0.70], [0.03, 0.10]$ and $\tilde{A}_2 = [0.57, 0.78], [0.06, 0.07]$ be two IVIFVs. It is obtained that $S(\tilde{A}_1) = S(\tilde{A}_2) = 0.61$ and $H(\tilde{A}_1) = H(\tilde{A}_2) = 0.74$ by using Equations (1) and (2). Thus, there is a ranking problem with Xu [12]'s score function and accuracy function because of the inadequate use of the upper and lower bounds of the membership and non-membership.

3.2. Lee's [13] Ranking Method of the IVIFV

Due to the fact that Xu's [12] ranking techniques cannot give sufficient information in some cases, Lee [13] offered a new score function $S(.)$

$$S(\tilde{A}) = \frac{2 + \mu_{\tilde{A}}^{L} + \mu_{\tilde{A}}^{U} - \nu_{\tilde{A}}^{L} - \nu_{\tilde{A}}^{U}}{3 - \mu_{\tilde{A}}^{L} - \mu_{\tilde{A}}^{U} - \nu_{\tilde{A}}^{L} - \nu_{\tilde{A}}^{U}}. \tag{3}$$

Example 2. Let $\tilde{A}_1 = [0.07, 0.17], [0.27, 0.37]$ and $\tilde{A}_2 = [0.10, 0.17], [0.12, 0.52]$ be two IVIFVs. Using Equation (3), score values are obtained as $S(\tilde{A}_1) = S(\tilde{A}_2) = 0.75$, and so, Lee's [13] score function is inadequate to rank.

3.3. Ye's [5] Ranking Method of the IVIFV

Ye [5] proposed a new accuracy function by considering the hesitant degree of IVIFVs, and he put forward that this function is more reasonable than the accuracy function developed by Xu

$$M(\tilde{A}) = \mu_{\tilde{A}}^L + \mu_{\tilde{A}}^U - 1 + \frac{v_{\tilde{A}}^L + v_{\tilde{A}}^U}{2}. \tag{4}$$

Example 3. Let that $\tilde{A}_1 = [0.50, 0.60], [0.20, 0.20]$ and $\tilde{A}_2 = [0.40, 0.70], [0.10, 0.30]$ be two IVIFVs. Using Equation (4), accuracy values are obtained as $M(\tilde{A}_1) = M(\tilde{A}_2) = 0.30$, and so, Ye's [5] accuracy function cannot distinguish these two IVIFVs.

3.4. Nayagam et al.'s [20] Ranking Method of the IVIFV

Nayagam et al. [20] offered an accuracy function $L(.)$ that can provide a useful way to help to make a decision

$$L(\tilde{A}) = \frac{\mu_{\tilde{A}}^L + \mu_{\tilde{A}}^U - v_{\tilde{A}}^U(1-\mu_{\tilde{A}}^U) - v_{\tilde{A}}^L(1-\mu_{\tilde{A}}^L)}{2}. \tag{5}$$

Example 4. Let $\tilde{A}_1 = [0.55, 0.67], [0.12, 0.24]$ and $\tilde{A}_2 = [0.58, 0.68], [0.16, 0.28]$ be two IVIFVs and $\tilde{A}_1 \supset \tilde{A}_2$. By applying $L(\tilde{A})$ presented in Equation (5), accuracy values are calculated as $L(\tilde{A}_1) = 0.54$ and $L(\tilde{A}_2) = 0.55$. That is, the rank obtained by Nayagam et al.'s [20] accuracy function is the reverse of the expected rank $\tilde{A}_1 > \tilde{A}_2$.

3.5. Nayagam and Sivaraman [21]'s Ranking Method of the IVIFV

A new general accuracy function $LG(.)$ was presented by Nayagam and Sivaraman [21] to defeat that ranking functions arise illogical results where $0 \le \delta \le 1$

$$LG(\tilde{A}) = \frac{\mu_{\tilde{A}}^L + \mu_{\tilde{A}}^U + \delta(2 - \mu_{\tilde{A}}^L - \mu_{\tilde{A}}^U - v_{\tilde{A}}^L - v_{\tilde{A}}^U)}{2}. \tag{6}$$

Example 5. Let $\tilde{A}_1 = [0.58, 0.70], [0.28, 0.30]$ and $\tilde{A}_2 = [0.35, 0.50], [0.05, 0.10]$ be two IVIFVs. Using Equation (6), when $\delta = 0.5$, accuracy values are calculated as $LG(\tilde{A}_1) = LG(\tilde{A}_2) = 0.57$, and so, Nayagam and Sivaraman [21]'s score function has drawback.

3.6. Tu and Chen [6]'s Ranking Method of the IVIFV

Tu and Chen [6] proposed two score functions based on the opinion that the minimum of membership and the maximum of non-membership showed the least prioritized ranking

$$D_o(\tilde{A}) = \mu_{\tilde{A}}^L \left(2 - \mu_{\tilde{A}}^L - 2v_{\tilde{A}}^U\right), \tag{7}$$

$$D_p(\tilde{A}) = 1 - v_{\tilde{A}}^L \left(2 - v_{\tilde{A}}^L - 2\mu_{\tilde{A}}^U\right). \tag{8}$$

Example 6. Let $\tilde{A}_1 = [0.10, 0.25], [0.38, 0.45]$ and $\tilde{A}_1 = [0.10, 0.45], [0.42, 0.45]$ be two IVIFVs. By applying $D_o(\tilde{A})$ presented in Equation (7), score values are obtained as $D_o(\tilde{A}_1) = D_o(\tilde{A}_2) = 0.10$. That is, $D_o(\tilde{A})$ produces equal score value when $\mu_{\tilde{A}_1}^L = \mu_{\tilde{A}_2}^L$ and $v_{\tilde{A}_1}^U = v_{\tilde{A}_2}^U$ without affecting $\mu_{\tilde{A}}^U$ and $v_{\tilde{A}}^L$, and it cannot separate IVIFVs.

Example 7. Let $\tilde{A}_1 = \left[0.16, 0.50\right], \left[0.25, 0.29\right]$ and $\tilde{A}_2 = \left[0.37, 0.50\right], \left[0.25, 0.36\right]$ be two IVIFVs. By applying $D_p(\tilde{A})$ presented in Equation (8), score values are obtained as $D_p(\tilde{A}_1) = D_p(\tilde{A}_2) = 0.813$. That is, $D_p(\tilde{A})$ produces equal score value when $\mu^U_{\tilde{A}_1} = \mu^U_{\tilde{A}_2}$ and $v^L_{\tilde{A}_1} = v^L_{\tilde{A}_2}$ without affecting $\mu^L_{\tilde{A}}$ and $v^U_{\tilde{A}}$, and it cannot distinguish IVIFVs.

3.7. Bai [14]'s Ranking Method of the IVIFV

Bai [14] introduced an improved score function $I(.)$ for the ranking order of IVIFVs

$$I(\tilde{A}) = \frac{\mu^L_{\tilde{A}} + \mu^L_{\tilde{A}}(1 - \mu^L_{\tilde{A}} - v^L_{\tilde{A}}) + \mu^U_{\tilde{A}} + \mu^U_{\tilde{A}}(1 - \mu^U_{\tilde{A}} - v^U_{\tilde{A}})}{2}. \quad (9)$$

Example 8. If corresponding to two different IVIFVs $\tilde{A}_1$ =[0.094, 0.262], [0.439, 0.687], $\tilde{A}_2$ =[0.088, 0.27], [0.437, 0.683] and $\tilde{A}_1 \subset \tilde{A}_2$. Using Equation (9), score values are obtained as $I(\tilde{A}_1) = 0.2062$ and $I(\tilde{A}_2) = 0.2058$, and so, the Bai (2013)'s score function $I(\tilde{A})$ fails to rank these two IVIFVs.

3.8. Wang and Niu [15]'s Ranking Method of the IVIFV

Wang and Niu [15] developed the score function $W_S(.)$ and the accuracy function $W_H(.)$ to overcome the disadvantages of existing functions

$$W_S(\tilde{A}) = \left(\frac{\mu^L_{\tilde{A}} + \mu^U_{\tilde{A}}}{2} - \frac{v^L_{\tilde{A}} + v^U_{\tilde{A}}}{2}\right)\left(1 + \frac{\left(1 - \mu^U_{\tilde{A}} - v^U_{\tilde{A}}\right) + \left(1 - \mu^L_{\tilde{A}} - v^L_{\tilde{A}}\right)}{2}\right), \quad (10)$$

$$W_H(\tilde{A}) = \left(\frac{\mu^L_{\tilde{A}} + \mu^U_{\tilde{A}}}{2} + \frac{v^L_{\tilde{A}} + v^U_{\tilde{A}}}{2}\right)\left(1 + \frac{\left(1 - \mu^U_{\tilde{A}} - v^U_{\tilde{A}}\right) + \left(1 - \mu^L_{\tilde{A}} - v^L_{\tilde{A}}\right)}{2}\right). \quad (11)$$

Example 9. Let $\tilde{A}_1 = \left[0.38, 0.38\right], \left[0.20, 0.40\right]$ and $\tilde{A}_2 = \left[0.26, 0.50\right], \left[0.15, 0.45\right]$ be two IVIFVs. Using Equations (10) and (11), score values are obtained as $W_S(\tilde{A}_1) = W_S(\tilde{A}_2) = 0.106$ and accuracy values are obtained as $W_H(\tilde{A}_1) = W_H(\tilde{A}_2) = 0.898$. Score values and accuracy values shows that, Wang and Niu [15]'s score function and accuracy function cannot distinguish IVIFVs. Also, if the midpoints of the membership and the non-membership coincide, the score function $W_S(.)$ will equal to 0.

3.9. Joshi and Kharayat [7]'s Ranking Method of the IVIFV

Joshi and Kharayat [7] gave an accuracy function $P(.)$ as follows

$$P(\tilde{A}) = \frac{\mu^L_{\tilde{A}} + \mu^U_{\tilde{A}} + \mu^L_{\tilde{A}}\mu^U_{\tilde{A}} - v^L_{\tilde{A}}v^U_{\tilde{A}}}{3}. \quad (12)$$

Example 10. Let $\tilde{A}_1 = \left[0.20, 0.20\right], \left[0.36, 0.42\right]$ and $\tilde{A}_2 = \left[0.20, 0.20\right], \left[0.24, 0.63\right]$ be two IVIFVs. By applying $P(.)$ in Equation (12), accuracy values are obtained as $P(\tilde{A}_1) = P(\tilde{A}_2) = 0.096$. Thus, this accuracy function fails to separate IVIFVs.

3.10. Kang et al. [22]'s Ranking Method of the IVIFV

A new accuracy function $H_K(.)$ was defined by Kang et al. [22]

$$H_K(\tilde{A}) = \frac{\mu_{\tilde{A}}^L - v_{\tilde{A}}^L + \mu_{\tilde{A}}^U - v_{\tilde{A}}^U - v_{\tilde{A}}^L\left(1-\mu_{\tilde{A}}^U - v_{\tilde{A}}^U\right)}{2} - \frac{v_{\tilde{A}}^U\left(1-\mu_{\tilde{A}}^L - v_{\tilde{A}}^L\right)}{2}. \quad (13)$$

Example 11. Let $\tilde{A}_1 = \left[0.13, 0.41\right], \left[0.43, 0.43\right]$ and $\tilde{A}_2 = \left[0.22, 0.32\right], \left[0.43, 0.43\right]$ be two IVIFVs. By applying $H_K(.)$ in Equation (13), accuracy values are calculated as $H_K(\tilde{A}_1) = H_K(\tilde{A}_2) = -0.289$. In this case, ranking cannot be made because both values are equal to each other.

3.11. Garg [9]'s Ranking Method of the IVIFV

Garg [9] presented a new generalized improved score function $GIS(.)$ by considering the weighted average of the degree of hesitation

$$GIS(\tilde{A}) = \frac{\mu_{\tilde{A}}^L + \mu_{\tilde{A}}^U}{2} + k_1\mu_{\tilde{A}}^L(1-\mu_{\tilde{A}}^L - v_{\tilde{A}}^L) + k_2\mu_{\tilde{A}}^U(1-\mu_{\tilde{A}}^U - v_{\tilde{A}}^U). \quad (14)$$

Example 12. Let $\tilde{A}_1 = \left[0.08, 0.16\right], \left[0.17, 0.19\right]$ and $\tilde{A}_2 = \left[0.10, 0.14\right], \left[0.10, 0.26\right]$ be two IVIFVs. By applying $GIS(\tilde{A})$ presented in Equation (14), score values are calculated as $GIS(\tilde{A}_1) = GIS(\tilde{A}_2) = -0.04$. Therefore, it is not known which IVIFV is bigger. Moreover, if $\mu_{\tilde{A}}^L = \mu_{\tilde{A}}^U = 0$, then whatever $v_{\tilde{A}}^L, v_{\tilde{A}}^U$, k_1 and k_2 are $GIS(\tilde{A})$ is always equal to 0.

3.12. Şahin [23]'s Ranking Method of the IVIFV

Şahin [23] proposed an accuracy function $K(.)$ by taking into account the hesitant degree of IVIFV

$$K(\tilde{A}) = \frac{\mu_{\tilde{A}}^L + \mu_{\tilde{A}}^U(1-\mu_{\tilde{A}}^L - v_{\tilde{A}}^L) + \mu_{\tilde{A}}^U + \mu_{\tilde{A}}^L(1-\mu_{\tilde{A}}^U - v_{\tilde{A}}^U)}{2}. \quad (15)$$

*Example 13***.** If corresponding to two different IVIFVs $\tilde{A}_1 = \left[0,0\right], \left[0.27, 0.37\right]$ and $\tilde{A}_2 = \left[0,0\right], \left[0.19, 0.45\right]$. Using Equation (15), accuracy values are obtained as $I(\tilde{A}_1) = I(\tilde{A}_2) = 0$.When $\mu_{\tilde{A}}^L = \mu_{\tilde{A}}^U = 0$, then $K(\tilde{A}) = 0$, $v_{\tilde{A}}^L$ and $v_{\tilde{A}}^U$ have no effect on $K(\tilde{A})$.

3.13. Nayagam et al. [10]'s Ranking Method of the IVIFV

Nayagam et al. [10] introduced a non-hesitance score function $J(.)$ to cope with the shortcomings of familiar methods

$$J(\tilde{A}) = \frac{\mu_{\tilde{A}}^L + \mu_{\tilde{A}}^U + v_{\tilde{A}}^L - v_{\tilde{A}}^U + \mu_{\tilde{A}}^L\mu_{\tilde{A}}^U + v_{\tilde{A}}^L v_{\tilde{A}}^U}{3}. \quad (16)$$

Example 14. Let $\tilde{A}_1 = \left[0.15, 0.25\right], \left[0.30, 0.55\right]$ and $\tilde{A}_2 = \left[0.05, 0.15\right], \left[0.50, 0.55\right]$ be two IVIFVs, and $\tilde{A}_1 \supset \tilde{A}_2$**.** Using Equation (16), score values are obtained as $J(\tilde{A}_1) = 0.118$ and $I(\tilde{A}_2) = 0.144$, and so, the Nayagam et al. [10]'s score function is unsuccessful.

3.14. Wang and Chen [16]'s Ranking Method of the IVIFV

The score function $S_{WC}(.)$ was defined by Wang and Chen [16] as follows

$$S_{WC}(\tilde{A}) = \frac{\mu_{\tilde{A}}^{L} + \mu_{\tilde{A}}^{U} + \sqrt{\mu_{\tilde{A}}^{U} v_{\tilde{A}}^{U}}\,(1 - \mu_{\tilde{A}}^{L} - v_{\tilde{A}}^{L})}{2} + \sqrt{\mu_{\tilde{A}}^{L} v_{\tilde{A}}^{L}}\,(1 - \mu_{\tilde{A}}^{U} - v_{\tilde{A}}^{U}). \tag{17}$$

Considering the Example 17, it is observed that in this function also if $\mu_{\tilde{A}}^{L} = \mu_{\tilde{A}}^{U} = 0$ then $S_{WC}(.) = 0$. This function has drawbacks for ranking.

3.15. Zhang and Xu [24]'s Ranking Method of the IVIFV

Zhang ve Xu [24] suggested an accuracy function $F(.)$.

$$F(\tilde{A}) = \frac{1}{2}\left(\frac{(\mu_{\tilde{A}}^{L} - v_{\tilde{A}}^{L}) + (\mu_{\tilde{A}}^{U} - v_{\tilde{A}}^{U})(1 - \mu_{\tilde{A}}^{L} - v_{\tilde{A}}^{L})}{2} + \frac{(\mu_{\tilde{A}}^{U} - v_{\tilde{A}}^{U}) + (\mu_{\tilde{A}}^{L} - v_{\tilde{A}}^{L})(1 - \mu_{\tilde{A}}^{U} - v_{\tilde{A}}^{U})}{2} \right) \tag{18}$$

Example 15. Let $\tilde{A}_1 = [0.11, 0.21], [0.11, 0.21]$ and $\tilde{A}_2 = [0.07, 0.33], [0.07, 0.33]$ be two IVIFVs. Using Equation (18), $F(\tilde{A}_1) = F(\tilde{A}_2) = 0$ is obtained. That is, if $\mu_{\tilde{A}}^{L} = v_{\tilde{A}}^{L}$ and $\mu_{\tilde{A}}^{U} = v_{\tilde{A}}^{U}$, then $F(\tilde{A}) = 0$. Therefore, it cannot distinguish the IVIFVs.

3.16. Joshi and Kumar [25]'s Ranking Method of the IVIFV

Joshi and Kumar [25] suggest the accuracy function $T(.)$ to distinguish IVIFVs.

$$T(\tilde{A}) = \frac{\mu_{\tilde{A}}^{L}(1 - v_{\tilde{A}}^{L}) + \mu_{\tilde{A}}^{U}(1 - v_{\tilde{A}}^{U})}{2} \tag{19}$$

Example 16. Let $\tilde{A}_1 = [0.45, 0.67], [0.14, 0.26]$ and $\tilde{A}_2 = [0.50, 0.62], [0.09, 0.31]$ be two IVIFVs. By applying $T(\tilde{A})$ presented in Equation (18) we get $T(\tilde{A}_1) = T(\tilde{A}_2) = 0.441$. Therefore, it is not known which IVIFV is bigger. Moreover, if $\mu_{\tilde{A}}^{L} = \mu_{\tilde{A}}^{U} = 0$, then whatever $v_{\tilde{A}}^{L}, v_{\tilde{A}}^{U}$, k_1 and k_2 are $T(\tilde{A})$ is always equal to 0.

3.17. Wang and Chen [17]'s Ranking Method of the IVIFV

Wang and Chen [23] offered a score function $S_{NWC}(.)$ and an accuracy function $H_{NWC}(.)$ of IVIFVs.

$$S_{NWC}(\tilde{A}) = \frac{(\mu_{\tilde{A}}^{L} + \mu_{\tilde{A}}^{U})(\mu_{\tilde{A}}^{L} + v_{\tilde{A}}^{L})}{2} - \frac{(v_{\tilde{A}}^{L} + v_{\tilde{A}}^{U})(\mu_{\tilde{A}}^{U} + v_{\tilde{A}}^{U})}{2} \tag{20}$$

$$H_{NWC}(\tilde{A}) = \frac{(1 - \mu_{\tilde{A}}^{L} + \mu_{\tilde{A}}^{U})(1 - \mu_{\tilde{A}}^{L} - v_{\tilde{A}}^{L})}{2} + \frac{(1 - v_{\tilde{A}}^{L} + v_{\tilde{A}}^{U})(1 - \mu_{\tilde{A}}^{U} - v_{\tilde{A}}^{U})}{2} \tag{21}$$

Example 17. Let $\tilde{A}_1 = [0.15, 0.25], [0.18, 0.30]$ and $\tilde{A}_2 = [0.35, 0.45], [0.38, 0.50]$ be two IVIFVs and $\tilde{A}_1 \subset \tilde{A}_2$. Using Equations (20) and (21), score values and accuracy values are calculated respectively as $S_{NWC}(\tilde{A}_1) = -0.07$, $S_{NWC}(\tilde{A}_2) = -0.13$ and $H_{NWC}(\tilde{A}_1) = 0.62$, $H_{NWC}(\tilde{A}_2) = 0.18$. Both functions rank IVIFVs as $\tilde{A}_1 > \tilde{A}_2$ and fail to present the expected ranking. Also, it is important mentioning that when $\mu_{\tilde{A}}^{L} = \mu_{\tilde{A}}^{U} = v_{\tilde{A}}^{L} = v_{\tilde{A}}^{U}$, $S_{NWC}(\tilde{A})$ and $H_{NWC}(\tilde{A})$ cannot rank IVIFVs.

3.18. Gong and Ma [18]'s Ranking Method of the IVIFV

Gong and Ma [18] developed a score function $S_{GM}(.)$ and an accuracy function $H_{GM}(.)$ of the IVIFV as below

$$S_{GM}(\tilde{A}) = \frac{v_{\tilde{A}}^{U} + v_{\tilde{A}}^{L} - \mu_{\tilde{A}}^{U} - \mu_{\tilde{A}}^{L}}{2} + \frac{\mu_{\tilde{A}}^{L} + \mu_{\tilde{A}}^{U} + 2(\mu_{\tilde{A}}^{L}\mu_{\tilde{A}}^{U} - v_{\tilde{A}}^{L}v_{\tilde{A}}^{U})}{\mu_{\tilde{A}}^{L} + \mu_{\tilde{A}}^{U} + v_{\tilde{A}}^{L} + v_{\tilde{A}}^{U}}, \tag{22}$$

$$H_{GM}(\tilde{A}) = \mu_{\tilde{A}}^{U} + v_{\tilde{A}}^{U} - \frac{\left(\mu_{\tilde{A}}^{U} - \mu_{\tilde{A}}^{L}\right)^2}{2\mu_{\tilde{A}}^{U}} - \frac{\left(v_{\tilde{A}}^{U} - v_{\tilde{A}}^{L}\right)^2}{2v_{\tilde{A}}^{U}}. \tag{23}$$

Example 18. If corresponding to two different IVIFVs $\tilde{A}_1 = [0.18, 0.27], [0.19, 0.40]$ and $\tilde{A}_2 = [0.13, 0.32], [0.29, 0.30]$, and $\tilde{A}_1 \subset \tilde{A}_2$**.** Using Equations (22) and (23), score values and accuracy values are obtained respectively as $S_{GM}(\tilde{A}_1) = 0.45$, $S_{GM}(\tilde{A}_2) = 0.42$ and $H_{GM}(\tilde{A}_1) = 0.64$, $H_{GM}(\tilde{A}_2) = 0.60$. It is clear that these functions aren't valid sufficiently for the ranking of IVIFVs.

3.19. Jia et al. [19]'s Ranking Method of the IVIFV

Jia et al. [19] introduced a score function $J(.)$ in Equation (24) utilized to quantify the information included in IVIFV with the idea of p-norm.

$$J(\tilde{A}) = \left[\frac{e^{\frac{\mu_{\tilde{A}}^{U} - v_{\tilde{A}}^{U} + \mu_{\tilde{A}}^{L} - v_{\tilde{A}}^{L}}{2}} - 1}{e^{\frac{\mu_{\tilde{A}}^{U} - v_{\tilde{A}}^{U} + \mu_{\tilde{A}}^{L} - v_{\tilde{A}}^{L}}{2}} + 1}\right]\left[\frac{1}{2^{\frac{1}{p}} + 1}\left[\left(\left(\frac{\left(\mu_{\tilde{A}}^{L}\right)^p + \left(\mu_{\tilde{A}}^{U}\right)^p + \left(v_{\tilde{A}}^{L}\right)^p + \left(v_{\tilde{A}}^{U}\right)^p}{2}\right) + \left(\frac{\left(\mu_{\tilde{A}}^{L} + v_{\tilde{A}}^{L}\right)^p + \left(\mu_{\tilde{A}}^{U} + v_{\tilde{A}}^{U}\right)^p}{2}\right)\right)^{\frac{1}{p}} + \left|\left(\frac{\left(\mu_{\tilde{A}}^{U}\right)^p - \left(v_{\tilde{A}}^{U}\right)^p + \left(\mu_{\tilde{A}}^{L}\right)^p - \left(v_{\tilde{A}}^{L}\right)^p}{2}\right)\right|^{\frac{1}{p}}\right]\right] \tag{24}$$

Example 19. Let $\tilde{A}_1 = [0.02, 0.03], [0.02, 0.03]$ and $\tilde{A}_2 = [0.41, 0.44], [0.41, 0.44]$ be two IVIFVs. By applying $J(\tilde{A})$ presented in Equation (24), score values are obtained as $J(\tilde{A}_1) = J(\tilde{A}_2) = 0$. Hence, when $\mu_{\tilde{A}}^{L} = v_{\tilde{A}}^{L}$ and $\mu_{\tilde{A}}^{U} = v_{\tilde{A}}^{U}$, then $J(\tilde{A}) = 0$ and it and cannot distinguish such IVIFVs.

4. A NEW SCORE FUNCTION FOR IVIFSs

Let $\tilde{A} = [\mu_{\tilde{A}}^{L}, \mu_{\tilde{A}}^{U}], [(v_{\tilde{A}}^{L}, v_{\tilde{A}}^{U}]$ be an IVIFV. The score function $MK(\tilde{A})$ and the properties of the new score function of IVIFV are explained as below

$$MK(\tilde{A}) = \frac{(\mu_{\tilde{A}}^{L} + 2\mu_{\tilde{A}}^{U} - v_{\tilde{A}}^{U} - 1) + (v_{\tilde{A}}^{U})(1 - \mu_{\tilde{A}}^{U} - v_{\tilde{A}}^{L})}{2}. \tag{25}$$

Property 1. $MK(\tilde{A}) = 1$ if and only if $\tilde{A} = ([1,1],[0,0])$.

Proof 1. If $MK(\tilde{A}) = 1$, then it is obtained that $\mu_{\tilde{A}}^{L} + 2\mu_{\tilde{A}}^{U} - \mu_{\tilde{A}}^{U}v_{\tilde{A}}^{U} - v_{\tilde{A}}^{L}v_{\tilde{A}}^{U} = 3$. In order to provide this equality under condition $0 \le \mu_{\tilde{A}}^{L}, \mu_{\tilde{A}}^{U}, v_{\tilde{A}}^{L}, v_{\tilde{A}}^{U} \le 1$, then $\mu_{\tilde{A}}^{L} = 1$, $2\mu_{\tilde{A}}^{U} = 2$, $\mu_{\tilde{A}}^{U}v_{\tilde{A}}^{U} = 0$ and $v_{\tilde{A}}^{L}v_{\tilde{A}}^{U} = 0$. It can be easily observed that $\tilde{A} = ([1,1],[0,0])$ provides these equations.

Property 2. $MK(\tilde{A}) = -1$ if and only if $\tilde{A} = ([0,0],[1,1])$.

Proof 2. If $MK(\tilde{A})=-1$, then it is obtained that $\mu_{\tilde{A}}^{L}+2\mu_{\tilde{A}}^{U}-\mu_{\tilde{A}}^{U}v_{\tilde{A}}^{U}-v_{\tilde{A}}^{L}v_{\tilde{A}}^{U}=-1$. In order to provide this equality under condition $0\le \mu_{\tilde{A}}^{L},\mu_{\tilde{A}}^{U},v_{\tilde{A}}^{L},v_{\tilde{A}}^{U}\le 1$, then $\mu_{\tilde{A}}^{L}=0$, $2\mu_{\tilde{A}}^{U}=0$, $\mu_{\tilde{A}}^{U}v_{\tilde{A}}^{U}=0$ and $v_{\tilde{A}}^{L}v_{\tilde{A}}^{U}=1$. It can be easily observed that $\tilde{A}=([0,0],[1,1])$ provides these equations.

Property 3. $MK(\tilde{A})$ has monotonicity property, i.e. $MK(\tilde{A})$ on $\mu_{\tilde{A}}^{L}$ and $\mu_{\tilde{A}}^{U}$ monotonously increases; $MK(\tilde{A})$ on $v_{\tilde{A}}^{L}$ and $v_{\tilde{A}}^{U}$ monotonously decreases.

Proof 3. When the partial derivatives of the $MK(\tilde{A})$ with respect to $\mu_{\tilde{A}}^{L}$, $\mu_{\tilde{A}}^{U}$, $v_{\tilde{A}}^{L}$ and $v_{\tilde{A}}^{U}$ are examined, it is seen that $MK(\tilde{A})$ provides monotonicity property

$$\frac{\partial MK(\tilde{A})}{\partial \mu_{\tilde{A}}^{L}}=\frac{1}{2}>0,\ \frac{\partial MK(\tilde{A})}{\partial \mu_{\tilde{A}}^{U}}=\frac{2-v_{\tilde{A}}^{U}}{2}>0,\ \frac{\partial MK(\tilde{A})}{\partial v_{\tilde{A}}^{L}}=\frac{-v_{\tilde{A}}^{U}}{2}<0,\ \frac{\partial MK(\tilde{A})}{\partial v_{\tilde{A}}^{U}}=\frac{-v_{\tilde{A}}^{L}}{2}<0.$$

Therefore, if $\mu_{\tilde{A}}^{L}>\mu_{\tilde{B}}^{L}$, $\mu_{\tilde{A}}^{U}>\mu_{\tilde{B}}^{U}$, $v_{\tilde{A}}^{L}<v_{\tilde{B}}^{L}$ and $v_{\tilde{A}}^{U}<v_{\tilde{B}}^{U}$ then $MK(\tilde{A})>MK(\tilde{B})$.

Property 4. Let $\tilde{A}$ and $\tilde{B}$ be two IVIFVs. $MK(\tilde{A})=MK(\tilde{B})$ if and only if $\tilde{A}=\tilde{B}$.

Proof 4. If $MK(\tilde{A})=MK(\tilde{B})$, then we obtain

$(\mu_{\tilde{A}}^{L}+2\mu_{\tilde{A}}^{U}-\mu_{\tilde{A}}^{U}v_{\tilde{A}}^{U}-v_{\tilde{A}}^{L}v_{\tilde{A}}^{U}-1)/2=(\mu_{\tilde{B}}^{L}+2\mu_{\tilde{B}}^{U}-\mu_{\tilde{B}}^{U}v_{\tilde{B}}^{U}-v_{\tilde{B}}^{L}v_{\tilde{B}}^{U}-1)/2$.

It is obtained that $\mu_{\tilde{A}}^{L}=\mu_{\tilde{B}}^{L}$, $\mu_{\tilde{A}}^{U}=\mu_{\tilde{B}}^{U}$, $\mu_{\tilde{A}}^{U}v_{\tilde{A}}^{U}=\mu_{\tilde{B}}^{U}v_{\tilde{B}}^{U}$, $v_{\tilde{A}}^{L}v_{\tilde{A}}^{U}=v_{\tilde{B}}^{L}v_{\tilde{B}}^{U}$ from above equality. Since $\mu_{\tilde{A}}^{U}=\mu_{\tilde{B}}^{U}$, then $v_{\tilde{A}}^{U}=v_{\tilde{B}}^{U}$ from $\mu_{\tilde{A}}^{U}v_{\tilde{A}}^{U}=\mu_{\tilde{B}}^{U}v_{\tilde{B}}^{U}$. Moreover, since $v_{\tilde{A}}^{U}=v_{\tilde{B}}^{U}$ then $v_{\tilde{A}}^{L}=v_{\tilde{B}}^{L}$ from $v_{\tilde{A}}^{L}v_{\tilde{A}}^{U}=v_{\tilde{B}}^{L}v_{\tilde{B}}^{U}$. Evidently, if $\mu_{\tilde{A}}^{L}=\mu_{\tilde{B}}^{L}$, $\mu_{\tilde{A}}^{U}=\mu_{\tilde{B}}^{U}$, $v_{\tilde{A}}^{L}=v_{\tilde{B}}^{L}$ and $v_{\tilde{A}}^{U}=v_{\tilde{B}}^{U}$ then $\tilde{A}=\tilde{B}$, and so, if $\tilde{A}=\tilde{B}$ then $MK(\tilde{A})=MK(\tilde{B})$.

5. A COMPARISON OF RANKING FUNCTIONS FOR IVIFS

Properties of the ranking functions mentioned in Section 3 such as monotonicity, max value, and min value are summarized in Tables 1 and 2. While the value range of the score function and accuracy function developed by Xu is [-1,1] and [0.1], respectively, it is observed that the value ranges of the score functions and accuracy functions developed later differed. While the maximum values obtained by accuracy ranking functions in [17, 24] and score function [19] are smaller than 1, the maximum value obtained by score function [13] is bigger than 1. Besides, minimum and maximum values calculated by accuracy functions in [12, 15, 17, 18] are equal, and this equality is unacceptable.

According to the containment relation of IVIFVs, the score function should monotonically increase, while the accuracy function should monotonically decrease [18]. The monotonicity properties of the ranking functions are examined with partial derivatives by considering Definition 5 and Definition 6. The partial derivatives results illustrated that the ranking functions in [6, 7, 9, 10, 13, 17, 20-25] do not satisfy the monotonicity property. On the other hand, the proposed score function satisfies the monotonicity property.

In order to compare ranking functions and show the superiority of the new score function, the comparative analysis is conducted by using twenty IVIFV pairs compiled from several studies [4, 5, 7, 10, 18, 20, 23, 25-29]. Using the new score function and the existing ranking functions, ranking values are calculated. These values are shown in respectively Tables 3 and 4. It is seen from Tables 3 and 4, that the existing ranking functions produce counter-intuitive results for two different IVIFVs. When all examples are investigated, the proposed score function is the only one that does not yield counter-intuitive cases. Hence, it is determined that the new score function can distinguish the IVIF values that the existing ranking functions are not capable of distinguishing.

Table 1. *Properties of the score functions*

Score Function		Monotonicity	Minimum Value	Maximum Value
S(.)	[12]	Monotonic	-1	1
S(.)	[13]	**Nonmonotonic**	0	4
Do(.)	[6]	**Nonmonotonic**	0	1
Dp(.)	[6]	**Nonmonotonic**	0	1
I(.)	[14]	Monotonic	0	1
W_S(.)	[15]	Monotonic	-1	1
GIS(.)	[9]	**Nonmonotonic**	0	1
J(.)	[10]	**Nonmonotonic**	0.33	1
S_{WC}(.)	[16]	Monotonic	0	1
S_{NWC}(.)	[17]	**Nonmonotonic**	-1	1
S_{GM}(.)	[18]	Monotonic	0	1
J(.)	[19]	Monotonic	-0.46	0.46

Table 2. *Properties of the accuracy functions*

Accuracy Function		Monotonicity	Minimum Value	Maximum Value
H(.)	[12]	Monotonic	**1**	**1**
M(.)	[5]	Monotonic	0	1
L(.)	[20]	**Nonmonotonic**	-1	1
LG(.)	[21]	**Nonmonotonic**	0	1
W_H(.)	[15]	Monotonic	**1**	**1**
P(.)	[7]	**Nonmonotonic**	-0.33	1
H_K(.)	[22]	**Nonmonotonic**	-1	1
K(.)	[23]	**Nonmonotonic**	0	1
F(.)	[24]	**Nonmonotonic**	-0.50	0.50
T(.)	[25]	**Nonmonotonic**	0	1
H_{NWC}(.)	[17]	**Nonmonotonic**	0	0
H_{GM}(.)	[18]	Monotonic	**1**	**1**

In order to compare ranking functions and show the superiority of the new score function, the comparative analysis is conducted by using twenty IVIFV pairs compiled from several studies [4, 5, 7, 10, 18, 20, 23, 25-29]. Using the new score function and the existing ranking functions, ranking values are calculated. These values are shown in respectively Tables 3 and 4. It is seen from Tables 3 and 4, that the existing ranking functions produce counter-intuitive results for two different IVIFVs. When all examples are investigated, the proposed score function is the only one that does not yield counter-intuitive cases. Hence, it is determined that the new score function can distinguish the IVIF values that the existing ranking functions are not capable of distinguishing.

The theorems are presented below to show that the cases described with examples in Tables 3 and 4 are not limited to the examples selected in this study. In these theorems, it is proved that the proposed score function overcomes the limited situations that the existing ranking functions have.

Theorem 1. Ranking function $D_o(.)$ proposed by Tu and Chen [6] produces equal score value when $\mu_{\tilde{A}}^L = \mu_{\tilde{B}}^L$ and $v_{\tilde{A}}^U = v_{\tilde{B}}^U$ without affecting $\mu_{\tilde{A}}^U$, $\mu_{\tilde{B}}^U$, $v_{\tilde{A}}^L$ and $v_{\tilde{B}}^L$. But, $MK(\tilde{A}) \neq MK(\tilde{B})$ when $\mu_{\tilde{A}}^L = \mu_{\tilde{B}}^L$ and $v_{\tilde{A}}^U = v_{\tilde{B}}^U$ under the condition $\mu_{\tilde{A}}^U \neq \mu_{\tilde{B}}^U$ and $v_{\tilde{A}}^L \neq v_{\tilde{B}}^L$.

Proof. In case $\mu_{\tilde{A}}^L = \mu_{\tilde{B}}^L$ and $v_{\tilde{A}}^U = v_{\tilde{B}}^U$, then we obtain $MK(\tilde{A}) = (\mu_{\tilde{A}}^L + 2\mu_{\tilde{A}}^U - \mu_{\tilde{A}}^U v_{\tilde{A}}^U - v_{\tilde{A}}^L v_{\tilde{A}}^U - 1)/2$ and $MK(\tilde{B}) = (\mu_{\tilde{A}}^L + 2\mu_{\tilde{B}}^U - \mu_{\tilde{B}}^U v_{\tilde{A}}^U - v_{\tilde{B}}^L v_{\tilde{A}}^U - 1)/2$. If $MK(\tilde{A}) \neq MK(\tilde{B})$ then $(\mu_{\tilde{A}}^L + 2\mu_{\tilde{A}}^U - \mu_{\tilde{A}}^U v_{\tilde{A}}^U - v_{\tilde{A}}^L v_{\tilde{A}}^U - 1)/2 \neq (\mu_{\tilde{A}}^L + 2\mu_{\tilde{B}}^U - \mu_{\tilde{B}}^U v_{\tilde{A}}^U - v_{\tilde{B}}^L v_{\tilde{A}}^U - 1)/2$. When this inequality is simplified as $(2\mu_{\tilde{A}}^U - v_{\tilde{A}}^U(\mu_{\tilde{A}}^U + v_{\tilde{A}}^L)) \neq (2\mu_{\tilde{B}}^U - v_{\tilde{A}}^U(\mu_{\tilde{B}}^U + v_{\tilde{B}}^L))$ it is seen that $MK(\tilde{A}) \neq MK(\tilde{B})$ if $\mu_{\tilde{A}}^U \neq \mu_{\tilde{B}}^U$ and $v_{\tilde{A}}^L \neq v_{\tilde{B}}^L$.

Theorem 2. Ranking function $D_p(\tilde{A})$ proposed by Tu and Chen [6] produces equal score value when $\mu_{\tilde{A}}^U = \mu_{\tilde{B}}^U$ and $v_{\tilde{A}}^L = v_{\tilde{B}}^L$ without affecting $\mu_{\tilde{A}}^L$, $\mu_{\tilde{B}}^L, v_{\tilde{A}}^U$ and $v_{\tilde{B}}^U$. But $MK(\tilde{A}) \neq MK(\tilde{B})$ when $\mu_{\tilde{A}}^U = \mu_{\tilde{B}}^U$ and $v_{\tilde{A}}^L = v_{\tilde{B}}^L$ under the condition $\mu_{\tilde{A}}^L \neq \mu_{\tilde{B}}^L$ and $v_{\tilde{A}}^U \neq v_{\tilde{B}}^U$.

Proof. In case $\mu_{\tilde{A}}^U = \mu_{\tilde{B}}^U$ and $v_{\tilde{A}}^L = v_{\tilde{B}}^L$, then we obtain $MK(\tilde{A}) = (\mu_{\tilde{A}}^L + 2\mu_{\tilde{A}}^U - \mu_{\tilde{A}}^U v_{\tilde{A}}^U - v_{\tilde{A}}^L v_{\tilde{A}}^U - 1)/2$ and $MK(\tilde{B}) = (\mu_{\tilde{B}}^L + 2\mu_{\tilde{A}}^U - \mu_{\tilde{A}}^U v_{\tilde{B}}^U - v_{\tilde{A}}^L v_{\tilde{B}}^U - 1)/2$. If $MK(\tilde{A}) \neq MK(\tilde{B})$ then $(\mu_{\tilde{A}}^L + 2\mu_{\tilde{A}}^U - \mu_{\tilde{A}}^U v_{\tilde{A}}^U - v_{\tilde{A}}^L v_{\tilde{A}}^U - 1)/2 \neq (\mu_{\tilde{B}}^L + 2\mu_{\tilde{A}}^U - \mu_{\tilde{A}}^U v_{\tilde{B}}^U - v_{\tilde{A}}^L v_{\tilde{B}}^U - 1)/2$. When this inequality is simplified as $(\mu_{\tilde{A}}^L - v_{\tilde{A}}^U(\mu_{\tilde{A}}^U + v_{\tilde{A}}^L)) \neq (\mu_{\tilde{B}}^L - v_{\tilde{B}}^U(\mu_{\tilde{A}}^U + v_{\tilde{A}}^L))$ it is seen that $MK(\tilde{A}) \neq MK(\tilde{B})$ if $\mu_{\tilde{A}}^L \neq \mu_{\tilde{B}}^L$ and $v_{\tilde{A}}^U \neq v_{\tilde{B}}^U$.

Theorem 3. The score functions in [12, 15, 19] and accuracy function in [24] give ranking value 0 when the midpoints of the membership and the non-membership coincide, namely $\mu_{\tilde{A}}^L = v_{\tilde{A}}^L$ and $\mu_{\tilde{A}}^U = v_{\tilde{A}}^U$. But, $MK(\tilde{A}) \neq 0$ when $\mu_{\tilde{A}}^L = v_{\tilde{A}}^L$ and $\mu_{\tilde{A}}^U = v_{\tilde{A}}^U$ under the condition $\mu_{\tilde{A}}^L \neq \mu_{\tilde{A}}^U \neq 0$.

Proof. When $\mu_{\tilde{A}}^L = v_{\tilde{A}}^L$ and $\mu_{\tilde{A}}^U = v_{\tilde{A}}^U$, we obtain $MK(\tilde{A}) = (\mu_{\tilde{A}}^L + 2\mu_{\tilde{A}}^U - \left(\mu_{\tilde{A}}^U\right)^2 - \mu_{\tilde{A}}^L \mu_{\tilde{A}}^U - 1)/2$. So, $MK(\tilde{A}) \neq 0$ when $\mu_{\tilde{A}}^L \neq \mu_{\tilde{A}}^U \neq 0$.

Theorem 4. Score functions proposed in [6, 9, 14, 16] and accuracy functions proposed in [23, 25] give ranking value 0 when $\mu_{\tilde{A}}^L = \mu_{\tilde{A}}^U = 0$. So, $v_{\tilde{A}}^L$ or $v_{\tilde{A}}^U$ have no effect on ranking value. But, score value $MK(\tilde{A})$ is affected from $v_{\tilde{A}}^L$ and $v_{\tilde{A}}^U$ when $\mu_{\tilde{A}}^L = \mu_{\tilde{A}}^U = 0$.

Proof. If $\mu_{\tilde{A}}^L = \mu_{\tilde{A}}^U = 0$, then $MK(\tilde{A}) = (-v_{\tilde{A}}^L v_{\tilde{A}}^U - 1)/2$. So, $MK(\tilde{A})$ is affected from $v_{\tilde{A}}^L$ and $v_{\tilde{A}}^U$.

6. THE MCDM METHOD USING THE PROPOSED SCORE FUNCTION

In this section, a MCDM algorithm is adapted from Sahin [23]'s study in order to provide the effectiveness of the new score function in MCDM problems. Two illustrative examples introduced in [4, 25] are used to demonstrate the application of the algorithm.

Algorithm 1. Step 1: Compute the aggregated IVIFVs γ_i for each alternative A_i $(i = 1, 2, ..., m)$ using IVIFWA operator considering criteria' weights w_j $(j = 1, 2, ..., n)$ according to each row in decision matrix R_{mxn}.

Step 2: Calculate the ranking value for the aggregated IVIF value γ_i $(i = 1, 2, ..., m)$ using ranking function.

Step 3: Rank the alternatives $A_1, A_2, ..., A_m$ by using their ranking values and select the best alternative according to higher values.

Example 20. [4] There are four possible alternatives for company investment: (A_1) car company, (A_2) food company, (A_3) computer company, and (A_4) arms company. The company wants to make a decision considering five criteria: (C1) productivity, (C2) technological innovation capability, (C3) marketing capability, (C4) management, and (C5) risk avoidance. The criteria' weights are $w = [0.2, 0.3, 0.15, 0.1, 0.25]^T$. The decision matrix R_{4X5} (Table 5) includes information related to the evaluation of alternatives on criteria.

Table 3.*The* ***c****omparison of the score functions (counter-intuitive cases are in bold type)*

IVIF Values		Rank	Xu [12]	Lee [13]	Tu and Chen,1 [6]	Tu and Chen,2 [6]	Bai [14]	Wang and Niu [15]	Garg [9]	Nayagam et al. [10]	Wang and Chen [16]	Wang and Chen [17]	Gong and Ma [18]	Jia et al. [19]	This study
A1	[0.20,0.30],[0.30,0.60]	1	-0.200	1.000	0.120	0.670	0.315	-0.260	0.315	**0.147**	0.368	-0.280	0.386	-0.054	-0.280
B1	[0.10,0.20],[0.50,0.60]	2	-0.400	0.750	0.070	0.450	0.190	-0.520	0.190	**0.173**	0.242	-0.350	0.214	-0.118	-0.460
A2	[0.30,0.60],[0.20,0.30]	1	**0.200**	**1.500**	0.330	**0.880**	**0.555**	**0.260**	**0.555**	0.347	**0.568**	**0.000**	**0.614**	0.054	0.130
B2	[0.40,0.50],[0.10,0.40]	2	**0.200**	**1.500**	0.320	**0.910**	**0.575**	**0.260**	**0.575**	0.280	**0.572**	**0.000**	**0.671**	0.052	0.080
A3	[0.40,0.50],[0.20,0.30]	2	0.200	1.500	0.400	0.840	0.580	0.260	0.580	0.353	0.556	0.070	0.643	0.052	0.095
B3	[0.50,0.60],[0.10,0.20]	1	0.400	1.750	0.550	0.930	0.710	0.520	0.710	0.440	0.642	0.210	0.786	0.118	0.280
A4	[0.18,0.60],[0.10,0.30]	1	**0.190**	**1.308**	**0.220**	**0.930**	**0.485**	**0.268**	**0.485**	0.239	**0.549**	**-0.071**	**0.603**	0.048	0.085
B4	[0.28,0.50],[0.00,0.40]	2	**0.190**	**1.308**	**0.258**	**1.000**	**0.516**	**0.268**	**0.516**	0.173	**0.551**	**-0.071**	**0.708**	0.044	0.040
A5	[0.50,0.60],[0.10,0.30]	2	0.350	1.800	0.450	0.930	0.680	0.438	0.680	0.410	0.646	0.150	0.743	0.106	0.245
B5	[0.60,0.70],[0.05,0.15]	1	0.550	2.067	0.660	0.973	0.808	0.688	0.808	0.543	0.720	0.338	0.867	0.183	0.444
A6	[0.35,0.45],[0.20,0.30]	2	**0.150**	**1.353**	**0.368**	0.820	**0.535**	**0.203**	**0.535**	**0.306**	0.516	**0.033**	**0.615**	0.035	0.028
B6	[0.30,0.50],[0.15,0.35]	1	**0.150**	**1.353**	**0.300**	0.873	**0.520**	**0.203**	**0.520**	**0.268**	0.531	**-0.033**	**0.615**	0.036	0.036
A7	[0.00,0.00],[0.10,0.10]	1	-0.100	0.643	**0.000**	0.810	**0.000**	-0.190	**0.000**	**0.003**	**0.000**	-0.010	**0.000**	-0.005	-0.505
B7	[0.00,0.00],[0.90,0.90]	2	-0.900	0.167	**0.000**	0.010	**0.000**	-0.990	**0.000**	**0.270**	**0.000**	-0.810	**0.000**	-0.380	-0.905
A8	[0.30,0.50],[0.30,0.50]	2	**0.000**	1.429	0.210	**0.790**	0.460	**0.000**	0.460	0.300	**0.500**	-0.160	**0.500**	**0.000**	-0.050
B8	[0.40,0.50],[0.40,0.50]	1	**0.000**	1.667	0.240	**0.760**	0.490	**0.000**	0.490	0.400	**0.500**	-0.090	**0.500**	**0.000**	-0.025
A9	[0.40,0.60],[0.00,,0.30]	2	**0.350**	**1.588**	**0.400**	**1.000**	**0.650**	**0.473**	**0.650**	0.313	**0.627**	**0.065**	**0.788**	0.097	0.210
B9	[0.30,0.70],[0.10,0.20]	1	**0.350**	**1.588**	**0.390**	**0.950**	**0.625**	**0.473**	**0.625**	0.377	**0.621**	**0.065**	**0.712**	0.101	0.270
A10	[0.00,0.00],[0.10,0.30]	2	-0.200	0.615	**0.000**	0.810	**0.000**	-0.360	**0.000**	**-0.057**	**0.000**	-0.060	0.050	-0.022	-0.515
B10	[0.00,0.00],[0.00,0.20]	1	-0.100	0.643	**0.000**	1.000	**0.000**	-0.190	**0.000**	**-0.067**	**0.000**	-0.020	0.100	-0.007	-0.500
A11	[0.20,0.40],[0.20,0.40]	2	**0.000**	**1.111**	**0.200**	0.800	**0.400**	**0.000**	**0.400**	**0.187**	0.440	**-0.120**	**0.500**	**0.000**	-0.120
B11	[0.15,0.45],[0.15,0.45]	1	**0.000**	**1.111**	**0.143**	0.858	**0.375**	**0.000**	**0.375**	**0.145**	0.465	**-0.180**	**0.500**	**0.000**	-0.110
A12	[0.45,0.55],[0.02,0.18]	2	**0.400**	**1.556**	**0.536**	0.982	**0.694**	**0.560**	**0.694**	**0.364**	0.596	**0.162**	**0.840**	0.105	0.224
B12	[0.40,0.60],[0.01,0.19]	1	**0.400**	**1.556**	**0.488**	0.992	**0.681**	**0.560**	**0.681**	**0.354**	0.606	**0.126**	**0.830**	0.107	0.242
A13	[0.10,0.20],[0.60,0.80]	2	-0.550	0.692	0.030	0.400	0.165	-0.633	0.165	0.200	0.210	-0.595	0.185	-0.202	-0.570
B13	[0.40,0.50],[0.00,0.10]	1	0.400	1.400	0.560	1.000	0.670	0.600	0.670	0.333	0.517	0.150	0.900	0.093	0.175
A14	[0.10,0.20],[0.30,0.40]	2	**-0.200**	**0.800**	**0.110**	0.610	**0.220**	**-0.300**	**0.220**	**0.113**	0.269	**-0.150**	**0.300**	-0.039	-0.350
B14	[0.00,0.30],[0.20,0.50]	1	**-0.200**	**0.800**	**0.000**	0.760	**0.180**	**-0.300**	**0.180**	**0.033**	0.305	**-0.250**	**0.300**	-0.043	-0.325
A15	[0.13,0.26],[0.17,0.43]	1	**-0.105**	**0.891**	**0.131**	**0.777**	**0.281**	**-0.158**	**0.281**	0.079	**0.335**	-0.149	**0.420**	**-0.020**	-0.267
B15	[0.17,0.22],[0.09,0.51]	2	**-0.105**	**0.891**	**0.138**	**0.868**	**0.288**	**-0.158**	**0.288**	0.018	**0.336**	-0.168	**0.482**	**-0.022**	-0.274
A16	[0.20,0.20],[0.40,0.80]	2	-0.400	0.857	0.040	0.520	0.240	-0.480	0.240	**0.120**	0.280	-0.480	0.300	-0.136	-0.440
B16	[0.20,0.20],[0.20,0.35]	1	-0.075	0.902	0.220	0.720	0.305	-0.114	0.305	**0.120**	0.324	-0.071	0.433	-0.012	-0.270
A17	[0.20,0.30],[0.50,0.60]	2	-0.300	1.000	0.120	0.550	0.295	-0.360	0.295	**0.253**	0.329	-0.320	0.313	-0.092	-0.340
B17	[0.30,0.40],[0.20,0.40]	1	0.050	1.235	0.270	0.800	0.465	0.067	0.465	**0.233**	0.474	-0.065	0.550	0.010	-0.070
A18	[0.20,0.40],[0.20,0.30]	2	0.050	1.105	0.240	0.800	0.420	0.073	0.420	0.213	0.434	-0.055	0.532	0.009	-0.090
B18	[0.30,0.50],[0.10,0.20]	1	0.250	1.316	0.390	0.910	0.565	0.363	0.565	0.290	0.521	0.055	0.714	0.057	0.090
A19	[0.60,0.70],[0.20,0.30]	2	0.400	**2.333**	0.480	0.920	0.710	0.440	0.710	**0.560**	0.696	**0.270**	0.722	0.143	0.365
B19	[0.50,0.70],[0.10,0.20]	1	0.450	**1.933**	0.550	0.950	0.735	0.563	0.735	**0.490**	0.686	**0.225**	0.790	0.145	0.370
A20	[0.50,0.50],[0.063,0.25]	1	**0.344**	**1.593**	0.500	**0.941**	**0.672**	**0.461**	**0.672**	0.360	**0.599**	0.164	**0.775**	**0.093**	0.180
B20	[0.50,0.50],[0.031,0.28]	2	**0.344**	**1.593**	0.468	**0.970**	**0.672**	**0.461**	**0.672**	0.336	**0.602**	0.143	**0.786**	**0.093**	0.175

***Table 4**. The comparison of the accuracy functions (counter-intuitive cases are in bold type)*

IVIF Values		Rank	Xu [12]	Ye[5]	Nayagam and Sivaraman [21]	Nayagam et al. [20]	Wang and Niu [15]	Kang et al.[22]	Joshi and Kharayat [7]	Şahin [23]	Zhang and Xu [24]	Joshi and Kumar [25]	Wang and Chen [17]	Gong and Ma [18]
A1	[0.20,0.30],[0.30,0.60]	*1*	**0.700**	-0.050	0.400	-0.080	**0.910**	-0.365	0.127	0.335	-0.118	0.130	0.340	0.850
B1	[0.10,0.20],[0.50,0.60]	*2*	**0.700**	-0.150	0.300	-0.315	**0.910**	-0.570	0.007	0.200	-0.250	0.065	0.330	0.790
A2	[0.30,0.60],[0.20,0.30]	*1*	**0.700**	**0.150**	**0.600**	**0.320**	**0.910**	0.115	**0.340**	0.615	0.133	**0.330**	0.380	**0.850**
B2	[0.40,0.50],[0.10,0.40]	*2*	**0.700**	**0.150**	**0.600**	**0.320**	**0.910**	0.095	**0.353**	0.595	0.113	**0.330**	0.340	**0.850**
A3	[0.40,0.50],[0.20,0.30]	*2*	**0.700**	0.150	0.600	0.315	**0.910**	0.120	0.347	0.590	0.125	0.335	**0.330**	**0.790**
B3	[0.50,0.60],[0.10,0.20]	*1*	**0.700**	0.250	0.700	0.485	**0.910**	0.350	0.460	0.720	0.250	0.465	**0.330**	**0.790**
A4	[0.18,0.60],[0.10,0.30]	1	**0.590**	**-0.020**	**0.595**	**0.289**	**0.832**	0.077	**0.286**	0.615	0.136	0.291	0.571	**0.792**
B4	[0.28,0.50],[0.00,0.40]	2	**0.590**	**-0.020**	**0.595**	**0.290**	**0.832**	0.046	**0.307**	0.584	0.110	0.290	0.509	**0.796**
A5	[0.50,0.60],[0.10,0.30]	*2*	**0.750**	0.300	0.675	0.465	**0.938**	0.285	0.457	0.695	0.200	0.435	**0.280**	**0.875**
B5	[0.60,0.70],[0.05,0.15]	*1*	**0.750**	0.400	0.775	0.618	**0.938**	0.520	0.571	0.818	0.330	0.583	**0.275**	**0.840**
A6	[0.35,0.45],[0.20,0.30]	*2*	**0.650**	**0.050**	**0.575**	0.253	**0.878**	**0.058**	**0.299**	0.545	**0.098**	**0.298**	0.385	0.740
B6	[0.30,0.50],[0.15,0.35]	*1*	**0.650**	**0.050**	**0.575**	0.260	**0.878**	**0.043**	**0.299**	0.560	**0.094**	**0.290**	0.420	0.810
A7	[0.00,0.00],[0.10,0.10]	*1*	**0.100**	**-0.900**	0.450	-0.100	**0.190**	-0.190	-0.003	**0.000**	-0.095	**0.000**	0.900	**0.100**
B7	[0.00,0.00],[0.90,0.90]	*2*	**0.900**	**-0.100**	0.050	-0.900	**0.990**	-0.990	-0.270	**0.000**	-0.495	**0.000**	0.100	**0.900**
A8	[0.30,0.50],[0.30,0.50]	*2*	0.800	0.200	**0.500**	0.170	0.960	-0.100	0.267	**0.500**	**0.000**	0.230	**0.240**	0.960
B8	[0.40,0.50],[0.40,0.50]	*1*	0.900	0.350	**0.500**	0.205	0.990	-0.050	0.300	**0.500**	**0.000**	0.245	**0.110**	0.990
A9	[0.40,0.60],[0.00,,0.30]	*2*	**0.650**	**0.150**	**0.675**	**0.440**	**0.878**	0.260	**0.413**	0.700	0.208	0.410	0.425	**0.835**
B9	[0.30,0.70],[0.10,0.20]	*1*	**0.650**	**0.150**	**0.675**	**0.435**	**0.878**	0.285	**0.397**	0.725	0.243	0.415	0.475	**0.815**
A10	[0.00,0.00],[0.10,0.30]	*2*	**0.200**	**-0.800**	0.400	-0.200	**0.360**	-0.370	-0.010	**0.000**	-0.170	**0.000**	0.870	**0.280**
B10	[0.00,0.00],[0.00,0.20]	*1*	**0.100**	**-0.900**	0.450	-0.100	**0.190**	-0.200	0.000	**0.000**	-0.090	**0.000**	0.980	**0.180**
A11	[0.20,0.40],[0.20,0.40]	*2*	**0.600**	**-0.100**	**0.500**	0.100	**0.840**	**-0.140**	**0.200**	0.440	**0.000**	**0.200**	0.480	0.760
B11	[0.15,0.45],[0.15,0.45]	*1*	**0.600**	**-0.100**	**0.500**	0.113	**0.840**	**-0.165**	**0.200**	0.465	**0.000**	**0.188**	0.520	0.810
A12	[0.45,0.55],[0.02,0.18]	*2*	**0.600**	**0.100**	**0.700**	0.454	**0.840**	**0.350**	**0.415**	0.707	**0.263**	**0.446**	0.448	0.712
B12	[0.40,0.60],[0.01,0.19]	*1*	**0.600**	**0.100**	**0.700**	0.459	**0.840**	**0.343**	**0.413**	0.719	**0.263**	**0.441**	0.478	0.754
A13	[0.10,0.20],[0.60,0.80]	*2*	**0.850**	**0.000**	0.225	-0.440	**0.978**	-0.670	-0.053	0.180	-0.290	0.040	0.165	**0.975**
B13	[0.40,0.50],[0.00,0.10]	*1*	**0.500**	**-0.050**	0.700	0.425	**0.750**	0.370	0.367	0.680	0.290	0.425	0.550	**0.590**
A14	[0.10,0.20],[0.30,0.40]	*2*	**0.500**	**-0.350**	**0.400**	-0.145	**0.750**	**-0.380**	**0.067**	0.230	-0.145	**0.095**	0.550	0.590
B14	[0.00,0.30],[0.20,0.50]	*1*	**0.500**	**-0.350**	**0.400**	-0.125	**0.750**	**-0.420**	**0.067**	0.270	-0.135	**0.075**	0.650	0.710
A15	[0.13,0.26],[0.17,0.43]	*1*	**0.495**	**-0.310**	**0.448**	-0.038	**0.745**	-0.282	**0.117**	0.306	**-0.074**	**0.128**	0.591	0.648
B15	[0.17,0.22],[0.09,0.51]	*2*	**0.495**	**-0.310**	**0.448**	-0.041	**0.745**	-0.306	**0.127**	0.299	**-0.070**	**0.131**	0.580	0.641
A16	[0.20,0.20],[0.40,0.80]	*2*	**0.800**	**0.000**	0.300	-0.280	**0.960**	-0.560	0.040	0.240	-0.200	0.080	0.200	**0.920**
B16	[0.20,0.20],[0.20,0.35]	*1*	**0.475**	**-0.325**	0.463	-0.020	**0.724**	-0.225	0.123	0.305	-0.054	0.145	0.559	**0.539**
A17	[0.20,0.30],[0.50,0.60]	*2*	**0.800**	**0.050**	0.350	-0.160	**0.960**	-0.415	0.087	0.305	-0.173	0.110	0.220	**0.890**
B17	[0.30,0.40],[0.20,0.40]	*1*	**0.650**	**0.000**	0.525	0.160	**0.878**	-0.070	0.247	0.480	0.030	0.240	0.395	**0.775**
A18	[0.20,0.40],[0.20,0.30]	*2*	**0.550**	-0.150	0.525	0.130	**0.798**	-0.070	0.207	0.450	0.038	0.220	**0.525**	**0.675**
B18	[0.30,0.50],[0.10,0.20]	*1*	**0.550**	-0.050	0.625	0.315	**0.798**	0.175	0.310	0.595	0.178	0.335	**0.525**	**0.675**
A19	[0.60,0.70],[0.20,0.30]	*2*	**0.900**	**0.550**	0.700	**0.565**	**0.990**	0.370	**0.553**	0.720	0.210	0.485	0.110	**0.990**
B19	[0.50,0.70],[0.10,0.20]	*1*	**0.750**	**0.350**	0.725	**0.545**	**0.938**	0.405	**0.510**	0.765	0.273	0.505	0.295	**0.875**
A20	[0.50,0.50],[0.063,0.25]	1	**0.657**	**0.157**	**0.672**	**0.422**	**0.882**	0.281	**0.411**	**0.672**	0.215	**0.422**	**0.367**	**0.733**

B20	[0.50,0.50],[0.031,0.28]	2	**0.657**	**0.157**	**0.672**	**0.422**	**0.882**	0.274	**0.414**	**0.672**	0.209	**0.422**	**0.371**	**0.750**

Table 5. *Decision matrix* R_{4X5}

	C1	C2	C3	C4	C5
A1	[0.4,0.5], [0.1,0.3]	[0.5,0.6], [0.1,0.2]	[0.3,0.4], [0.2,0.3]	[0.7,0.8], [0.1,0.2]	[0.5,0.6], [0.1,0.2]
A2	[0.5,0.6], [0.1,0.2]	[0.3,0.4], [0.1,0.3]	[0.7,0.8], [0.1,0.2]	[0.3,0.4], [0.3,0.4]	[0.4,0.5], [0.1,0.2]
A3	[0.6,0.7], [0.1,0.2]	[0.7,0.8], [0.1,0.2]	[0.5,0.6], [0.3,0.4]	[0.4,0.5], [0.3,0.4]	[0.3,0.5], [0.3,0.4]
A4	[0.5,0.6], [0.2,0.3]	[0.4,0.5], [0.3,0.4]	[0.6,0.7], [0.2,0.3]	[0.6,0.7], [0.2,0.3]	[0.6,0.7], [0.1,0.3]

The aggregated IVIF value γ_i for each alternatives A_i is obtained using IVIFWA operator as γ_1 =[0.48, 0.48], [0.11, 0.23], γ_2 =[0.37, 0.47], [0.11, 0.24], γ_3 =[0.50, 0.61], [0.17, 0.28], γ_4 =[0,41, 0.50], [0.19, 0.33]. Then ranking values are calculated as MK(A_1)=0.15, MK(A_2)=0.08, MK(A_3)=0.25, MK(A_4)=0.09 by using score function MK(.). Thus, the rank of the alternatives is obtained as $A_3 > A_1 > A_4 > A_2$.

Example 21. [25] The performance of four teachers (A1, A2, A3, A4) are evaluated on the basis the five criteria: (C1) subject command of the teacher, (C2) ability to communicate, (C3) ability to create interest in the course, (C4) ability to ensure course coverage, (C5) ability to give suitable illustrations and to create linkage. The criteria' weights are $w = [0.1, 0.2, 0.25, 0.3, 0.15]^T$. The decision matrix D_{4X5} (Table 6) includes information related to the evaluation of teachers on criteria.

Table 6. *Decision matrix* D_{4X5}

	C1	C2	C3	C4	C5
A1	[0.16,0.16], [0.17,0.50]	[0.24,0.76], [0,00,0.00]	[0.00,0.00], [0.50,0.50]	[0.24,0.24], [0.26,0.26]	[0,00,0.00], [0.25,0.75]
A2	[0.16,0.16], [0.17,0.50]	[0.14,0.45], [0.15,0.25]	[0.24,0.24], [0.26,0.26]	[0.21,0.21], [0.22,0.36]	[0.16,0.16], [0.17,0.50]
A3	[0.24,0.24], [0.26,0.26]	[0.24,0.24], [0.26,0.26]	[0.24,0.24], [0.26,0.26]	[0.00,0.00], [0.50,0.50]	[0.00,0.00], [0.25,0.75]
A4	[0.10,0.30], [0.30,0.30]	[0.13,0.38], [0.13,0.38]	[0.12,0.38], [0.13,0.38]	[0.16,0.50], [0.17,0.17]	[0.00,0.00], [0.25,0.75]

The aggregated IVIF value γ_i for each alternative A_i is calculated using IVIFWA operator as γ_1 =[0.14,0.32], [0,0], γ_2 =[0.17, 0.24], [0.20, 0.33], γ_3 =[0.14, 0.14], [0.31, 0.37], γ_4 =[0.11,0.37], [0.17, 0.32]. Then ranking values are calculated as MK(A_1)=-0.11, MK(A_2)=-0.25, MK(A_3)=-0.37, MK(A_4)=-0.16 by using score function MK(.). Thus, the rank of the alternatives is obtained as $A_1 > A_4 > A_2 > A_3$.

To validate the applicability and feasibility of the proposed new approach, a comparative study with existing ranking functions is conducted on the basis of Example 20 and Example 21. The ranking results obtained from the existing ranking functions are listed in Table 7 for both examples. Table 7 shows that the ranking results derived from the score functions in [10, 13, 16, 19] and accuracy functions in [7] are identical to the ranking results ($A_3 > A_1 > A_4 > A_2$) of the new score function for Example 20. In addition, the ranking of the first two alternatives ($A_3 > A_1$) obtained from the score functions in [9, 12, 14, 17] and accuracy functions in [20-23, 25] are the same as the rank obtained from the proposed function. It is seen from Table 7 that ranking results obtained from the score functions in [6, 9, 10, 12, 14, 15, 17-19] and accuracy functions in [7, 20-25] are identical to the ranking results $\left(A_1 > A_4 > A_2 > A_3\right)$ of the proposed new score function for Example 21.

Table 7. *Ranking results obtained from different functions*

References	Example 20		Example 21	
	Ranking values	Rank	Ranking values	Rank
Xu [12]	$S(A_1)$=0.31, $S(A_2)$=0.24, $S(A_3)$=0.33, $S(A_4)$=0.19	$A_3>A_1>A_2>A_4$	$S(A_1)$=0.23, $S(A_2)$=−0.06, $S(A_3)$=−0.20, $S(A_4)$=0.00	$A_1>A_4>A_2>A_3$
	$H(A_1)$=0.65, $H(A_2)$=0.60, $H(A_3)$=0.78, $H(A_4)$=0.71	$A_3>A_4>A_1>A_2$	$H(A_1)$=0.23, $H(A_2)$=0.47, $H(A_3)$=0.483, $H(A_4)$=0.484	$A_4>A_3>A_2>A_1$
Lee [13]	$S(A_1)$=1.54, $S(A_2)$=1.37, $S(A_3)$=1.85, $S(A_4)$=1.51	$A_3>A_1>A_4>A_2$	$S(A_1)$=0.97, $S(A_2)$=0.91, $S(A_3)$=0.78, $S(A_4)$=0.98	$A_4>A_1>A_2>A_3$
Ye [5]	$M(A_1)$=0.13, $M(A_2)$=0.01, $M(A_3)$=0.34, $M(A_4)$=0.16	$A_3>A_4>A_1>A_2$	$M(A_1)$=−0.54, $M(A_2)$=−0.32, $M(A_3)$=−0.38, $M(A_4)$=−0.28	$A_4>A_2>A_3>A_1$
Nayagam et al. [20]	$L(A_1)$=0.39, $L(A_2)$=0.32, $L(A_3)$=0.46, $L(A_4)$=0.31	$A_3>A_1>A_2>A_4$	$L(A_1)$=0.23, $L(A_2)$=0.00, $L(A_3)$=−0.15, $L(A_4)$=0.06	$A_1>A_4>A_2>A_3$
Nayagam and Sivaraman [21]	$LG(A_1)$=0.65, $LG(A_2)$=0.62, $LG(A_3)$=0.66, $LG(A_4)$=0.60	$A_3>A_1>A_2>A_4$	$LG(A_1)$=0.62, $LG(A_2)$=0.47, $LG(A_3)$=0.40, $LG(A_4)$=0.50	$A_1>A_4>A_2>A_3$
Tu and Chen [6]	$D_o(A_1)$=0.51, $D_o(A_2)$=0.42, $D_o(A_3)$=0.47, $D_o(A_4)$=0.38	$A_1>A_3>A_2>A_4$	$D_o(A_1)$=0.27, $D_o(A_2)$=0.20, $D_o(A_3)$=0.16, $D_o(A_4)$=0.14	$A_1>A_2>A_3>A_4$
	$D_p(A_1)$=0.90, $D_p(A_2)$=0.8935, $D_p(A_3)$=0.8934, $D_p(A_4)$=0.85	$A_1>A_2>A_3>A_4$	$D_p(A_1)$=1.00, $D_p(A_2)$=0.74, $D_p(A_3)$=0.56, $D_p(A_4)$=0.82	$A_1>A_4>A_2>A_3$
Bai [14]	$I(A_1)$=0.65, $I(A_2)$=0.582, $I(A_3)$=0.67, $I(A_4)$=0.578	$A_3>A_1>A_2>A_4$	$I(A_1)$=0.40, $I(A_2)$=0.31, $I(A_3)$=0.21, $I(A_4)$=0.34	$A_1>A_4>A_2>A_3$
Wang and Niu [15]	$W_S(A_1)$=0.42, $W_S(A_2)$=0.34, $W_S(A_3)$=0.40, $W_S(A_4)$=0.25	$A_1>A_3>A_2>A_4$	$W_S(A_1)$=0.41, $W_S(A_2)$=−0.09, $W_S(A_3)$=−0.31, $W_S(A_4)$=−0.01	$A_1>A_4>A_2>A_3$
	$W_H(A_1)$=0.88, $W_H(A_2)$=0.84, $W_H(A_3)$=0.95, $W_H(A_4)$=0.92	$A_3>A_4>A_1>A_2$	$W_H(A_1)$=0.41, $W_H(A_2)$=0.72, $W_H(A_3)$=0.732, $W_H(A_4)$=0.734	$A_4>A_3>A_2>A_1$
Joshi and Kharayat [7]	$P(A_1)$=0.39, $P(A_2)$=0.33, $P(A_3)$=0.45, $P(A_4)$=0.35	$A_3>A_1>A_4>A_2$	$P(A_1)$=0.17, $P(A_2)$=0.13, $P(A_3)$=0.06, $P(A_4)$=0.16	$A_1>A_4>A_2>A_3$
Kang et al. [22]	$H_K(A_1)$=0.25, $H_K(A_2)$=0.16, $H_K(A_3)$=0.27, $H_K(A_4)$=0.11	$A_3>A_1>A_2>A_4$	$H_K(A_1)$=0.23, $H_K(A_2)$=−0.21, $H_K(A_3)$=0.38, $H_K(A_4)$=−0.15	$A_1>A_4>A_2>A_3$
Garg [9]	$GIS(A_1)$=0.65, $GIS(A_2)$=0.582, $GIS(A_3)$=0.67, $GIS(A_4)$=0.578	$A_3>A_1>A_2>A_4$	$GIS(A_1)$=0.40, $GIS(A_2)$=0.34, $GIS(A_3)$=0.31, $GIS(A_4)$=0.21	$A_1>A_4>A_2>A_3$
Şahin [23]	$K(A_1)$=0.65, $K(A_2)$=0.593, $K(A_3)$=0.68, $K(A_4)$=0.589	$A_3>A_1>A_2>A_4$	$K(A_1)$=0.42, $K(A_2)$=0.32, $K(A_3)$=0.21, $K(A_4)$=0.39	$A_1>A_4>A_2>A_3$
Nayagam et al. [10]	$J(A_1)$=0.37, $J(A_2)$=0.30, $J(A_3)$=0.45, $J(A_4)$=0.34	$A_3>A_1>A_4>A_2$	$J(A_1)$=0.17, $J(A_2)$=0.14, $J(A_3)$=0.13, $J(A_4)$=0.12	$A_1>A_4>A_2>A_3$
Wang and Chen [16]	$S_{WC}(A_1)$=0.58, $S_{WC}(A_2)$=0.54, $S_{WC}(A_3)$=0.64, $S_{WC}(A_4)$=0.56	$A_3>A_1>A_4>A_2$	$S_{WC}(A_1)$=0.23, $S_{WC}(A_2)$=0.33, $S_{WC}(A_3)$=0.25, $S_{WC}(A_4)$=0.38	$A_4>A_2>A_3>A_1$
Zhang and Xu [24]	$F(A_1)$=0.20, $F(A_2)$=0.16, $F(A_3)$=0.19, $F(A_4)$=0.12	$A_1>A_3>A_2>A_4$	$F(A_1)$=0.21, $F(A_2)$=−0.05, $F(A_3)$=−0.15, $F(A_4)$=0.00	$A_1>A_4>A_2>A_3$
Joshi and Kumar [25]	$T(A_1)$=0.40, $T(A_2)$=0.34, $T(A_3)$=0.43, $T(A_4)$=0.33	$A_3>A_1>A_2>A_4$	$T(A_1)$=0.23, $T(A_2)$=0.15, $T(A_3)$=0.09, $T(A_4)$=0.17	$A_1>A_4>A_2>A_3$
Wang and Chen [17]	$S_{NWC}(A_1)$=0.16, $S_{NWC}(A_2)$=0.08, $S_{NWC}(A_3)$=0.17, $S_{NWC}(A_4)$=0.06	$A_3>A_1>A_2>A_4$	$S_{NWC}(A_1)$=0.03, $S_{NWC}(A_2)$=−0.08, $S_{NWC}(A_3)$=−0.11, $S_{NWC}(A_4)$=−0.10	$A_1>A_2>A_4>A_3$
	$H_{NWC}(A_1)$=0.37, $H_{NWC}(A_2)$=0.45, $H_{NWC}(A_3)$=0.24, $H_{NWC}(A_4)$=0.32	$A_2>A_1>A_4>A_3$	$H_{NWC}(A_1)$=0.84, $H_{NWC}(A_2)$=0.58, $H_{NWC}(A_3)$=0.53, $H_{NWC}(A_4)$=0.63	$A_1>A_4>A_2>A_3$
Gong and Ma [18]	$S_{GM}(A_1)$=0.74, $S_{GM}(A_2)$=0.706, $S_{GM}(A_3)$=0.709, $S_{GM}(A_4)$=0.64	$A_1>A_3>A_2>A_4$	$S_{GM}(A_1)$=0.97, $S_{GM}(A_2)$=0.44, $S_{GM}(A_3)$=0.29, $S_{GM}(A_4)$=0.47	$A_1>A_4>A_2>A_3$
	$H_{GM}(A_1)$=0.702, $H_{GM}(A_2)$=0.696, $H_{GM}(A_3)$=0.88, $H_{GM}(A_4)$=0.81	$A_3>A_4>A_1>A_2$	$H_{GM}(A_1)$=0.30, $H_{GM}(A_2)$=0.56, $H_{GM}(A_3)$=0.51, $H_{GM}(A_4)$=0.64	$A_4>A_2>A_3>A_1$
Jia, et al. [19]	$J(A_1)$=0.08, $J(A_2)$=0.06, $J(A_3)$=0.10, $J(A_4)$=0.05	$A_3>A_1>A_4>A_2$	$J(A_1)$=0.03, $J(A_2)$=−0.01, $J(A_3)$=−0.04, $J(A_4)$=0.00	$A_1>A_4>A_2>A_3$
This study	$MK(A_1)$=0.15, $MK(A_2)$=0.08, $MK(A_3)$=0.25, $MK(A_4)$=0.09	$A_3>A_1>A_4>A_2$	$MK(A_1)$=−0.11, $MK(A_2)$=−0.25, $MK(A_3)$=−0.37, $MK(A_4)$=−0.16	$A_1>A_4>A_2>A_3$

It is remarkable that the ranking functions of the IVIFVs affect the ranking results, and so, ranking orders may be obtained differently with different functions. For this reason, in decision-making problems, it is necessary to use ranking functions that have the monotonicity property and do not have drawbacks in the ranking of the IVIFVs. Otherwise, misleading results may be produced for the decision-maker. The proposed new score function has the monotonicity property. It also copes with the shortcomings of the existing functions. Thus, the new score function may present an acceptable rank for IVIFVs, and it can be used effectively for decision-making problems.

7. CONCLUSION

The ranking of IVIFVs is one of the most popular fields in several real-world decision-making problems. It is well recognized that many researchers have been working on the ranking of IVIFVs. As far as we know, there is no general approach that ranks any two IVIFVs. Many researchers have suggested numerous score functions and accuracy functions. But it is pointed out that in some cases, these mentioned functions are not always effective. In this study, a new score function is developed and a ranking method based on this score function is introduced under the IVIF environment. The main contributions of this study are presented:
(i) It is shown that both the most popular ranking functions and their improved versions may produce counter-intuitive ranking results.
(ii) A new score function which can rank any two IVIFVs is developed. This function has monotonicity proper-ty, and so, score value increases with the increasing of μ^L, μ^U whereas it decreases with the increasing of v^L, v^U. Furthermore, it is noteworthy that the new function overcomes the shortcomings of the other ranking functions.
(iii) Decision-making process has ambiguity or vagueness. The proposed decision algorithm is suitable for ambiguity or vagueness environments.
(iv) Two illustrative examples are given to show the applicability and effectiveness of the proposed approach.

CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

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