The Application of Production-Inventory Balance Model on WMI

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ABSTRACT
In this study, at the solution of production-inventory problem the dynamic programming model was constructed, taking into consideration the time parameter. It was realized for West Machine Industry (WMI). After that the application of dynamic programming model was realized on the production-inventory problem at WMI. Following this, the results taken by application of the model were explained.
Keywords: *Dynamic Programming, Production, Inventory*

Batı Makine Sanayiinde Üretim-Envanter Denge Modelinin Uygulanması

ÖZET
Bu çalışmada; ilk olarak bir üretim-stok probleminin çözümünde zaman parametresi de dikkate alınarak, Batı Makine Sanayi (BMS) için bir dinamik programlama modeli kurulmuştur. Daha sonra; BMS' de üretim-stok problemi üzerinde deterministik dinamik programlama modeli uygulaması gerçekleştirilmiştir. Bunu takiben; modelin uygulamasından elde edilen sonuçlar yorumlanmıştır.
Anahtar Kelimeler: *Dinamik Programlama, Üretim, Envanter*

1. INTRODUCTION

In present days, different models especially used in spare parts production systems in factories in order to solve problems of manufacture planning were developed. The aims of these models to determine the values realize minimizing or maximizing of the target function. Because of this, these models are called on optimization models [Akdeniz, 2000].

In this research, the deterministic dynamic programming model will be constructed by also using time parameter in a solution of the balance problem of production-inventory.

Because of the deterministic dynamic programming model takes into problems directly, not by estimating, this model which is optimization technique was selected. In other side, besides dynamic programming is an optimization technique, it is a technique that parts the problems into series of such problems [Hiller&Lieberman, 1990].

In this study, first of all deterministic dynamic programming technique will be examined briefly. Then in a factory, this model will be constructed on a production-inventory problem. And then, application of this model will be realized using data belonged to four period of time in the year 1997. Following these, the results determined by application of model will be exalted.

2. DETERMINISTIC DYNAMIC MODELLING

In optimization problems, selected variables have values that minimize or maximize the objective functions. But in dynamic models, objective politics of some variables are taken into light in the basis of a variation frame. Optimization technique that will be applied to sub problems of dynamic programming may be as simple comparisons or mathematics programming, linear programming [Chiang, 1991].

Deterministic dynamic programming can be described diagrammatically as shown in figure l. Thus at stage n the process will be in some state S_n. Making policy decision X_n then moves the process to some state S_{n+1} at stage (n+1). The contribution there after to the objective function under an optimal policy has been previously calculated to be $f_{n+1}^*(S_{n+1})$. The policy decision X_n also makes some contribution to the objective function. Combining these two quantities in an appropriate way provides $f_n(S_n,X_n)$, the contribution of stages n onward to the objective function optimizing with respect to X_n then gives $f_n^*(S_n)=f_n(S_n,X_n^*)$. After finding X_n^* and $f_n^*(S_n)$ for each possible value of S_n, the solution procedure is ready to move back one stage [Tolunay, 1991].

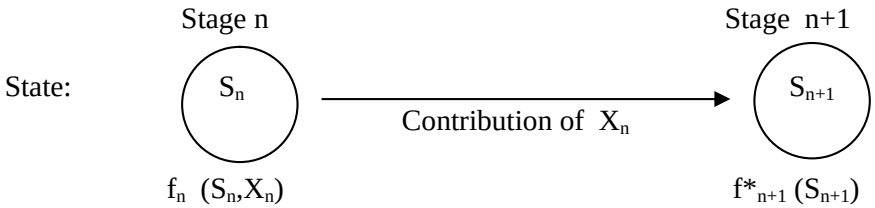

Figure 1. The Basic Structure For Deterministic Dynamic Programming.

Deterministic dynamic programming has different applications fields :

I) Investment problem
II) Loading problem
III) Production -inventory problem
IV) Distribution problem
V) Inventorying problem

3.PRODUCTION -INVENTORY MODEL OF WMI

Production planning is a study that is used to take series of decisions belonged to future parts of time in factories. One of the problems which is necessary for production planning is to balance the inventories.

Developed decision models related to production planning, can be solved special techniques related to their variables and functions. In solution of all dynamic problems, “Deterministic programming” approach is an effective method. Basic parameters of WMI production-inventory models are these:

n : 1,2,3,.... Planning period of time,

D_n : Demand values of every n period of time,
S_n : Inventory level of beginning of n^{th} period of time,
X_n : Production values in n^{th} period of time,
X : Maximum production capacity of a period,
d : Holding cost of a unit property,
$S(X_n)$: $d.S_n$ holding cost,
$f(X_n)$: Production cost
C(n) : Total cost.

Restrictions of the model:

I) Production capacity of WMI is 15.000 unit for a period of time.

II) Function of production cost is $f(X_n)= a + b X_n + c X_n^2$

III) Maximum inventory unit shouldn't increase than 4.000 unit in every period of time.

It is asked that production planning under the conditions mentioned before, should be realized with minimum costing in order to product for demand.

Steps of Model:

I) **Decision step**: Every period of time takes into consideration as decision step (n=1,2,3,...)

II) **State variable**: The variable that will be effect in decision is inventory value in the beginning of the period of time (S_n).

The inventory-value in a period of time is related to inventory, production and demand value in other periods.

$$S_{n+1}= S_n + X_n - D_n$$

III) **Decision variable**: Production value related to every period of time (X_n).

IV) **Optimum Decision Rule:** Optimal production which has the expression of $X_n^*(S_n)$ will be determined as the function beginning inventory value for every period of time.

V) **Total cost function related to every period of time is equal to sum of production and holding costs:**

$$C(n)= f(X_n) + S(X_n) = a + b X_n + c X_n^2 + d.S_n$$

VI) **Objective function related to every period of time f(n):**

$$f_n^*(S_n)= \min \{ Cn (X_n,S_n) + f_{n+1}^*(S_n +X_n - D_n) \} ,$$
$$0 \le X_n \le K$$
$$X_n + S_n \ge D_n$$

Taking an decision, production amount $X_n^* (S_n)$ which minimize the total cost for every value of S_n named condition variable is determined in every decision steps. Beginning with last period of time (n), An optimal decision rule that minimizes the total cost was developed for every beginning inventory starting from last to first periods of time.

4. APPLICATIONS

In this study; using deterministic dynamic programming method and WMI' s production-inventory parameters, preparing a production plan that meets

demand to their products in minimum cost was aimed. In application, data belonged to four periods of time that has three months in 1997 been used.

Amounts of demand, in four periods of time for products by WMI were estimated as given below: (see. Table 1).

Table 1. Amounts of demand according to periods

Periods (n)	**Amounts of Demand (D_n)**
1	12.000
2	13.500
3	15.000
4	10.000

Production capacity of WMI is 15.000 units for one period of time.

Function of production cost:

$f(X_n)= 6500 + 18\ X_n + (1/1.000)\ X_n^2$ (Million TL.)

It is wanted that amount of inventory at the end of every period of time should not exceed 4.000 units.

Cost of keeping a unit in inventory is $d = 1{,}2 \cdot 10^6$ TL

Decision model of problem under these circumstances is:

$S_{n+1}= S_n + X_n - D_n$ (n= 1, 2, 3, 4)

$S_n + X_n \geq D_n$

$S_n \leq 4.000$

$X_n \leq 15.000$

S_n, X_n and $D_n \geq 0$ and Integer

Goal Function:

$\min C_n = \{C_1 + C_2 + C_3 + C_4\}$

or

$\min C_n = \{ f(X_1) + d.S_1 + f(x_2) + d.S_2 + f(x_3) + d.S_3 + f(x_4) \}$

It was started to find a solution the problem by fourth period of time.

Taking into consideration that amount of inventory should not exceed 4.000 units at the end of period of time, production values and costs related to demands as 10.000 units were calculated and given in table 2.

Calculated values related to third, second and first periods of time expressed in Table 3, Table 4 and Table 5 respectively. As seen in Table 3, if it was started with inventories in 4.000 units at first period of time, amount of production realized in 11.000 units are enough for minimum cost.

Beginning with this, the values in every periods of time of optimum decision strategy which has total cost in 1.420.050 (Million TL) for four periods of time were shown in Table 6.

Table 2. Fourth Period

		COST			**END OF PER. INV.**
S_4	**X_4**	**PRODUCTION**	**HOLDING**	**TOTAL**	**S_5**
0	**10.000**	**286.500**	**0**	**286.500**	**0**

	11.000	325.500	0	325.500	1.000
	12.000	366.500	0	366.500	2.000
	13.000	409.500	0	409.500	3.000
	14.000	454.500	0	454.500	4.000
500	**9.500**	**267.750**	**600**	**268.350**	**0**
	10.500	305.750	600	306.350	1.000
	11.500	345.750	600	346.350	2.000
	12.500	387.750	600	388.350	3.000
	13.500	431.750	600	432.350	4.000
1.000	**9.000**	**249.500**	**1.200**	**250.700**	**0**
	10.000	286.500	1.200	287.700	1.000
	11.000	325.500	1.200	326.700	2.000
	12.000	366.500	1.200	367.700	3.000
	13.000	409.500	1.200	410.700	4.000
1.500	**8.500**	**231.750**	**1800**	**233.550**	**0**
	9.500	267.750	1.800	269.550	1.000
	10.500	305.750	1.800	307.550	2.000
	11.500	345.750	1.800	347.550	3.000
	12.500	387.750	1.800	389.550	4.000
2.000	**8.000**	**214.500**	**2.400**	**216.900**	**0**
	9.000	249.500	2.400	251.900	1.000
	10.000	286.500	2.400	288.900	2.000
	11.000	325.500	2.400	327.900	3.000
	12.000	366.500	2.400	368.900	4.000
2.500	**7.500**	**197.750**	**3.000**	**200.750**	**0**
	8.500	231.750	3.000	234.750	1.000
	9.500	267.750	3.000	270.750	2.000
	10.500	305.750	3.000	308.750	3.000
	11.500	345.750	3.000	348.750	4.000
3.000	**7.000**	**181.500**	**3.600**	**185.100**	**0**
	8.000	214.500	3.600	218.100	1.000
	9.000	249.500	3.600	253.100	2.000
	10.000	286.500	3.600	290.100	3.000
	11.000	325.500	3.600	329.100	4.000
3.500	**6.500**	**165.750**	**4.200**	**169.950**	**0**
	7.500	197.750	4.200	201.950	1.000
	8.500	231.750	4.200	235.950	2.000
	9.500	267.750	4.200	271.950	3.000
	10.500	305.750	4.200	309.950	4.000
4.000	**6.000**	**150.500**	**4.800**	**155.300**	**0**
	7.000	181.500	4.800	186.300	1.000
	8.000	214.500	4.800	219.300	2.000
	9.000	249.500	4.800	254.300	3.000
	10.000	286.500	4800	291.300	4.000

Table 3. Third Period

		COST			FUTURE COST		TOTAL
S_3	X_3	PRODUCTION	HOLDING	TOTAL	S_4	$F_4(S_4)$	COST
0	**15.000**	**501.500**	**0**	**501.500**	**0**	**286.500**	**788.000**

500	**14.500**	**477.750**	**600**	**478.350**	**0**	**286.500**	**764.850**
	15.000	501.500	600	502.100	500	268.350	770.450
1.000	**14.000**	**454.500**	**1.200**	**455.700**	**0**	**286.500**	**742.200**
	15.000	501.500	1.200	502.700	1.000	250.700	753.400
1.500	**13.500**	**431.750**	**1.800**	**433.550**	**0**	**286.500**	**720.050**
	14.500	477.750	1.800	479.550	1.000	250.700	730.250
	15.000	501.500	1.800	503.300	1.500	233.550	736.850
2.000	**13.000**	**409.500**	**2.400**	**411.900**	**0**	**286.500**	**698.400**
	14.000	454.500	2.400	456.900	1.000	250.700	707.600
	15.000	501.500	2.400	503.900	2.000	216.900	720.800
2.500	**12.500**	**387.750**	**3.000**	**390.750**	**0**	**286.500**	**677.250**
	13.500	431.750	3.000	434.750	1.000	250.700	685.450
	14.500	477.750	3.000	480.750	2.000	216.900	697.650
	15.000	501.500	3.000	504.500	2.500	200.750	705.250
3.000	**12.000**	**366.500**	**3.600**	**370.100**	**0**	**286.500**	**656.600**
	13.000	409.500	3.600	413.100	1.000	250.700	663.800
	14.000	454.500	3.600	458.100	2.000	216.900	675.000
	15.000	501.500	3.600	505.100	3.000	185.100	690.200
3.500	**11.500**	**345.750**	**4.200**	**349.950**	**0**	**286.500**	**636.450**
	12.500	387.750	4.200	391.950	1.000	250.700	642.650
	13.500	431.750	4.200	435.950	2.000	216.900	652.850
	14.500	477.750	4.200	481.950	3.000	185.100	667.050
	15.000	501.500	4.200	505.700	3.500	169.950	675.650
4.000	**11.000**	**325.500**	**4.800**	**330.300**	**0**	**286.500**	**616.800**
	12.000	366.500	4.800	371.300	1.000	250.700	622.000
	13.000	409.500	4.800	414.300	2.000	216.900	631.200
	14.000	454.500	4.800	459.300	3.000	185.100	644.400
	15.000	501.500	4.800	506.300	4.000	155.300	661.600

Table 4. Second Period

		COST			FUTURE COST		TOTAL
S_2	X_2	PRODUCTION	HOLDING	TOTAL	S_3	$F_3(S_3)$	COST
0	**13.500**	**431.750**	**0**	**431.750**	**0**	**788.000**	**1.219.750**
	14.500	477.750	0	477.750	1.000	742.200	1.219.950
	15.000	501.500	0	501.500	1.500	720.050	1.221.550
500	13.000	409.500	600	410.100	0	788.000	1.198.100
	14.000	**454.500**	**600**	**455.100**	**1.000**	**742.200**	**1.197.300**
	15.000	501.500	600	502.100	2.000	698.400	1.200.500
1.000	12.500	387.750	1.200	388.950	0	788.000	1.176.950
	13.500	**431.750**	**1.200**	**432.950**	**1.000**	**742.200**	**1.175.150**
	14.500	477.750	1.200	478.950	2.000	698.400	1.177.350
	15.000	501.500	1.200	502.700	2.500	677.250	1.179.950
1.500	12.000	366.500	1.800	368.300	0	788.000	1.156.300
	13.000	**409.500**	**1.800**	**411.300**	**1.000**	**742.200**	**1.153.500**
	14.000	454.500	1.800	456.300	2.000	698.400	1.154.700
	15.000	501.500	1.800	503.300	3.000	656.600	1.159.900

2.000	11.500	345.750	2.400	348.150	0	788.000	1.136.150
	12.500	**387.750**	**2.400**	**390.150**	**1.000**	**742.200**	**1.132.350**
	13.500	431.750	2.400	434.150	2.000	698.400	1.132.550
	14.500	477.750	2.400	480.150	3.000	656.600	1.136.750
	15.000	501.500	2.400	503.900	3.500	636.450	1.140.350
2.500	11.000	325.500	3.000	328.500	0	788.000	1.116.500
	12.000	366.500	3.000	369.500	1.000	742.200	1.111.700
	13.000	**409.500**	**3.000**	**412.500**	**2.000**	**698.400**	**1.110.900**
	14.000	454.500	3.000	457.500	3.000	656.600	1.114.100
	15.000	501.500	3.000	504.500	4.000	616.800	1.121.300
3.000	10.500	305.750	3.600	309.350	0	788.000	1.097.350
	11.500	345.750	3.600	349.350	1.000	742.200	1.091.550
	12.500	**387.750**	**3.600**	**391.350**	**2.000**	**698.400**	**1.089.750**
	13.500	431.750	3.600	435.350	3.000	656.600	1.091.950
	14.500	477.750	3.600	481.350	4.000	616.800	1.098.150
3.500	10.000	286.500	4.200	290.700	0	788.000	1.078.700
	11.000	325.500	4.200	329.700	1.000	742.200	1.071.900
	12.000	**366.500**	**4.200**	**370.700**	**2.000**	**698.400**	**1.069.100**
	13.000	409.500	4.200	413.700	3.000	656.600	1.070.300
	14.000	454.500	4.200	458.700	4.000	616.800	1.075.500
4.000	9.500	267.750	4.800	272.550	0	788.000	1.060.550
	10.500	305.750	4.800	310.550	1.000	742.200	1.052.750
	11.500	**345.750**	**4.800**	**350.550**	**2.000**	**698.400**	**1.048.950**
	12.500	387.750	4.800	392.550	3.000	656.600	1.049.150
	13.500	431.750	4.800	436.550	4.000	616.800	1.053.350

Table 5. First Period

		COST			FUTURE COST		TOTAL
S1	X1	PRODUCTION	HOLDING	TOTAL	S_2	$F_2(S_2)$	COST
0	12.000	366.500	0	366.500	0	1.219.750	1.586.250
	13.000	**409.500**	**0**	**409.500**	**1.000**	**1.175.150**	**1.584.650**
	14.000	454.500	0	454.500	2.000	1.132.350	1.586.850
	15.000	501.500	0	501.500	3.000	1.089.750	1.591.250
500	11.500	345.750	600	346.350	0	1.219.750	1.566.100
	12.500	**387.750**	**600**	**388.350**	**1.000**	**1.175.150**	**1.563.500**
	13.500	431.750	600	432.350	2.000	1.132.350	1.564.700
	14.500	477.750	600	478.350	3.000	1.089.750	1.568.100
	15.000	501.500	600	502.100	3.500	1.069.100	1.571.200
1.00	11.000	325.500	1.200	326.700	0	1.219.750	1.546.450
	12.000	**366.500**	**1.200**	**367.700**	**1.000**	**1.175.150**	**1.542.850**
	13.000	409.500	1.200	410.700	2.000	1.132.350	1.543.050

	14.000	454.500	1.200	455.700	3.000	1.089.750	1.545.450
	15.000	501.500	1.200	502.700	4.000	1.048.950	1.551.650
1.50	10.500	305.750	1.800	307.550	0	1.219.750	1.527.300
	11.500	345.750	1.800	347.550	1.000	1.175.150	1.522.700
	12.500	**387.750**	**1.800**	**389.550**	**2.000**	**1.132.350**	**1.521.900**
	13.500	431.750	1.800	433.550	3.000	1.089.750	1.523.300
	14.500	477.750	1.800	479.550	4.000	1.048.950	1.528.500
2.00	10.000	286.500	2.400	288.900	0	1.219.750	1.508.650
	11.000	325.500	2.400	327.900	1.000	1.175.150	1.503.050
	12.000	**366.500**	**2.400**	**368.900**	**2.000**	**1.132.350**	**1.501.250**
	13.000	409.500	2.400	411.900	3.000	1.089.750	1.501.650
	14.000	454.500	2.400	456.900	4.000	1.048.950	1.505.850
2.50	9.500	267.750	3.000	270.750	0	1.219.750	1.490.500
	10.500	305.750	3.000	308.750	1.000	1.175.150	1.483.900
	11.500	345.750	3.000	348.750	2.000	1.132.350	1.481.100
	12.500	**387.750**	**3.000**	**390.750**	**3.000**	**1.089.750**	**1.480.500**
	13.500	431.750	3.000	434.750	4.000	1.048.950	1.483.700
3.00	9.000	249.500	3.600	253.100	0	1.219.750	1.472.850
	10.000	286.500	3.600	290.100	1.000	1.175.150	1.465.250
	11.000	325.500	3.600	329.100	2.000	1.132.350	1.461.450
	12.000	**366.500**	**3.600**	**370.100**	**3.000**	**1.089.750**	**1.459.850**
	13.000	409.500	3.600	413.100	4.000	1.048.950	1.462.050
3.50	8.500	231.750	4.200	235.950	0	1.219.750	1.455.700
	9.500	267.750	4.200	271.950	1.000	1.175.150	1.447.100
	10.500	305.750	4.200	309.950	2.000	1.132.350	1.442.300
	11.500	**345.750**	**4.200**	**349.950**	**3.000**	**1.089.750**	**1.439.700**
	12.500	387.750	4.200	391.950	4.000	1.048.950	1.440.900
4.00	8.000	214.500	4.800	219.300	0	1.219.750	1.439.050
	9.000	249.500	4.800	254.300	1.000	1.175.150	1.429.450
	10.000	286.500	4.800	291.300	2.000	1.132.350	1.423.650
	11.000	**325.500**	**4.800**	**330.300**	**3.000**	**1.089.750**	**1.420.050**
	12.000	366.500	4.800	371.300	4.000	1.048.950	1.420.250

Table 6. Results

PERIOD	**Si**	**X4**	**END OF PERIOD INVENTORY**	**COST**		
				PRODUCTION	**HOLDING**	**TOTAL**

			(Si+1)			
1	4000	11,000	3,000	325,500	4800	330,300
2	3000	12,500	2,000	387,750	3600	391,350
3	2000	13,000	0	409,500	2400	411,900
4	0	10,000	0	286,500	0	286,500
T O T A L C O S T				**1.409.250**	**10.800**	**1.420.050**

5. CONCLUSION

The deterministic dynamic programming that applied in West Machine Industry is appropriate model than dynamic linear programming. The results determined by applying of multi-stage inventory model were given in table 6.

In the deterministic programming model dimension of time is also taken into consideration. Thus, this model gives a chance of using different parameters related to financial politics of company. And this model gives realistic solutions to problems. Following this; in order to approach to zero inventory in every period of time, the firm should decrease the costs of production and holding. But the quality of product should be the same, When West Machine Industry reach its purposes, the beginning of Just-in-time production could be done.

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