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Brief Review of Soft Sets and Its Application in Coding Theory

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Abstract In this paper, we will focus to one of the recent applications of PU-algebras in the coding theory, namely the construction of codes by soft sets PU-valued functions. First, we shall introduce the notion of soft sets PU-valued functions on PU-algebra and investigate some of its related properties. Moreover, the codes generated by a soft sets PU-valued function are constructed and several examples are given. Furthermore, example with graphs of binary block code constructed from a soft sets PU-valued function is constructed.

Keywords *- PU-algebra, Soft PU-algebra, code soft PU-algebra*

1. Introduction

Imai and Is´eki [1] in 1966 introduced the notion of a BCK-algebra. Is´eki [2] introduced BCI-algebras as a super class of the class of BCK-algebras. In [3], Hu and Li introduced a wide class of abstract algebras, BCH-algebras. They are shown that the class of BCI-algebras is a proper subclass of the class of BCH-algebras. Elkabany et al, in [4] introduced a new algebraic structure called PU-algebra, and they investigated severed basic properties. Moreover, they derived new view of several ideals on PU-algebra. Molodtsov [5] introduced the concept of soft set as a new mathematical tool for dealing with uncertainties that is free form the difficulties that have troubled the usual theoretical approaches. Maji et al [6,7] described the application of soft theory and studied several operations on the soft sets. Many Mathematicians have studied the concept of soft set of some algebraic structures. For example, see [8-13]. Coding theory is a mathematical domain with many applications in information theory, for more details see [14]. Various type of codes and their connections with other mathematical objects have been intensively studied. One of the recent applications was given in the Coding theory are BCK/ Hilbert/ R_0-algebras see [15-18]. In [12,18] provided an algorithm which allows to find a BCK/-algebra starting from a given binary block code. In [17] the authors presented some new connections between BCK- algebras and binary block codes. In [19,20] the authors established block-codes by using the notion of KU-valued functions.
In this paper, we will focus to one of the recent applications of PU-algebras in the coding theory, namely the Construction of codes by soft sets PU-valued functions. First, we shall introduce the notion of soft sets PU-valued functions on PU-algebra and investigate some of its related properties. Moreover, the codes generated by a soft sets PU-valued function are constructed and several Examples are given. Furthermore, Example with graphs of binary block code constructed from a soft sets PU-valued function is constructed.

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2. **Preliminaries**

Now, we will recall some known concepts related to PU-algebra from the literature, which will be helpful in further study of this article.

Definition 2.1. [4] A PU-algebra is a non-empty set X with a constant $0 \in X$ and a binary operation $*$ satisfying the following conditions:

(i) $0 * x = x$,

(ii) $(x * z) * (y * z) = y * x$, for any $x, y, z \in X$

On X we can define a binary relation "$\leq$" by $x \leq y$ if and only if $y * x = 0$.

Example 2.2. [4] Let $X = \{0, 1, 2, 3, 4\}$ be a set and $*$ is defined by

$*$	0	1	2	3	4
0	0	1	2	3	4
1	4	0	1	2	3
2	3	4	0	1	2
3	2	3	4	0	1
4	1	2	3	4	0

Then, $(X,*,0)$ is a PU-algebra.

Proposition 2.3. [4] In a PU-algebra $(X,*,0)$ the following hold, for all $x, y, z \in X$,

(a) $x * x = 0$

(b) $(x * z) * z = x$

(c) $x * (y * z) = y * (x * z)$

(d) $x * (y * x) = y * 0$

(e) $(x * y) * 0 = y * x$

(f) If $x \leq y$, then $x * 0 = y * 0$

(g) $(x * y) * 0 = (x * z) * (y * z)$

(h) $x * y \leq z$ if and only if $z * y \leq x$

(i) $x \leq y$ if and only if $y * z \leq x * z$

(j) In a PU-algebra $(X,*,0)$, the following are equivalent:

(1) $x = y$, (2) $x * z = y * z$, (3) $z * x = z * y$

(k) The right and the left cancellation laws hold in X.

(l) $(z * x) * (z * y) = x * y$

(m) $(x * y) * z = (z * y) * x$

(n) $(x * y) * (z * u) = (x * z) * (y * u)$ for all x, y, z and $u \in X$

Lemma 2.4. [4] If $(X,*,0)$ is a PU-algebra, then $(X, \leq)$ is a partially ordered set.

Definition 2.5. [4] A non-empty subset S of a PU-algebra $(X,*,0)$ is called a sub-algebra of X if $x * y \in S$ whenever $x, y \in S$.

Definition 2.6. [4] A non-empty subset I of a PU-algebra $(X, *, 0)$ is called a ***new ideal*** of X if,

(i) $0 \in I$,

(ii) $(a * (b * x)) * x \in I$, for all $a, b \in I$ and $x \in X$.

Theorem 2.7 [4] Any sub-algebra S of a PU-algebra X is a ***new ideal*** of X.

Example 2.8 [4] Let $X = \{0, a, b, c\}$ be a set with $*$ is defined by the following table:

*	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Then, $(X, *, 0)$ is a PU-algebra. It is easy to show that $I_1 = \{0, a\}$, $I_2 = \{0, b\}$, $I_3 = \{0, c\}$ are ***new ideals*** of X.

3. Brief review of soft set with examples in coding and fuzzy

Definition 3,1. [5] Molodtsov defined the notion of a soft sets as follows. Let U be an initial universe and E be the set of parameters. The parameters are usually "attributes, characteristics or properties of an object". Let $P(U)$ denote the power set of U and A is a subset of E.A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$.In other words, a soft set over a universe is a U parameterized family of subsets of the universe U. For $e \in A$, $F(e)$ may be considered as the set of e-elements or e-approximate elements of the soft set (F, A). Thus $(F, A) = \{F(e) \in P(U) : e \in A \subseteq E\}$. As an illustration, let us consider the following:

Example 3.2. (***soft***). Suppose a universe U is the set of eight Cars $U = \{C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8\}$ be the set of Cars under consideration, E be a set of parameters.

$$E = \left\{ \begin{array}{l} e_1 = \text{exp}ensive, e_2 = beautiful, e_3 = manual\ gear, e_4 = cheap, \\ e_5 = automatic\ gear, e_6 = in\ good\ repair, e_7 = in\ bad\ \ repair \end{array} \right\}.$$

Then, the soft set (F, E) describes the attractiveness of the cars. which say Mr. X wants to buy. In this case, to define the soft set (F, E) means to point out the cars for each parameter, i.e. expensive, beautiful, manual gear, cheap, automatic gear, etc. Let $A = \{e_1, e_2, e_6\} \subseteq E$ and Now consider the mapping $F : A \to P(U)$ is given by $F(e_1) = \{C_2, C_3\}, F(e_2) = \{C_1, C_3, C_5\}, F(e_6) = \{C_1, C_3, C_6\}$.

Then, the soft set (F, A) is a parameterized family $\{F(e_i), i = 1,2,3\}$ of subsets of the universe given by

$$(F, A) = \{\{C_2, C_3\}, \{C_1, C_3, C_5\}, \{C_1, C_3, C_6\}\}$$

for example, $F(e_1)$ means car (expensive) whose functional value, called the e_1 - approximate value set, is the set $\{C_2, C_3\}$.Thus we can view the soft set (F, A) as consisting of a collection of approximations given by

$$(F,A)=\{F(e_1)=\{C_2,C_3\},F(e_2)=\{C_1,C_3,C_5\},F(e_6)=\{C_1,C_3,C_6\}\}$$

Each approximation has two parts:

(i) A predicate $F(e_1)\, or\, F(e_2)\ or\ F(e_6)$ and

(ii) The approximate set $\{C_2,C_3\}\ or\ \{C_1,C_3,C_5\}\ or\{C_1,C_3,C_6\}\}$, respectively.

The soft set (F,A) can also be represented by the set of ordered pairs given by

$$(F,A)=\{(e_1,F(e_1)),(e_2,F(e_2)),(e_6,F(e_6))\}$$

$$(F,A)=\{(e_1,\{C_2,C_3\}),(e_2,\{C_1,C_3,C_5\}),\ (e_6,\{C_1,C_3,C_6\})\}$$

It is worth noting that the sets $F(e), e\in A$ may be arbitrary, may be empty or may have non-empty intersection. Also, the soft set (F,A) can be divided by F_A.

Example 3.3 (coding soft) To store a soft set in a computer, a two-dimensional table is used to represent it. Table 1 (below) shows the tabular representation of the soft set (F,A), where if $h_i \in F(e_i)$, $then\, h_{i,j} = 1$, $otherwise\ h_{i,j} = 0$, where $h_{i,j}$ are the entries in the table.

U	e_1	e_2	e_6	Choice value
C_1	0	1	1	2
C_2	1	0	0	1
C_3	1	1	1	3
C_4	0	0	0	0
C_5	0	1	0	1
C_6	0	0	1	1
C_7	0	0	0	0
C_8	0	0	0	0

Let U be a universe and let A be a fuzzy set on the universe U, characterized by its membership function μ_A, such that $\mu_A : U \to [0,1], A \subseteq U$. Thus, the fuzzy set A can be completely defined as a set of ordered pairs given by $A=\{\langle x,\mu(x) : x\in U\rangle\}, \mu(x)\in[0,1]$.

Now let us consider the family of α -level sets for μ_A, given by $F(\alpha)=\{x\in U : \mu_A(x)\geq\alpha\},\alpha\in[0,1]$, such that given F, we can find $\mu_A(x)$ by the formula: $\mu_A(x)=\sup\{\alpha\in[0,1] : x\in F(\alpha)\}$. Then, every Zadeh's fuzzy set A may be considered as the soft set $(F,[0,1])$. As an illustration, let us consider the following example.

Example 3.4. (fuzzy soft). Suppose that $U=\{C_1,C_2,C_3,C_4,C_5,C_6,C_7,C_8\}$ and that we consider the single parameter quality of cars which are characterized by the value set whose terms are {expensive, beautiful, manual gear and cheap}. Let the terms beautiful and cheap for example be associated with its own fuzzy set as follows:

$$F_{beautiful} = \{(C_1,0.2),(C_2,0.7),(C_5,0.9),(C_6,0.1)\}$$

$$F_{cheap} = \{(C_1,0.9),(C_2,0.3),(C_3,0.1),(C_4,0.1),(C_5,0.2)\}$$

Then, the α – level set of $F_{beautiful}$ and F_{cheap} are given by;

$$F_{beautiful}(0.2) = \{C_1, C_2, C_5\},\ F_{cheap}(0.2) = \{C_1, C_2, C_4, C_5\}$$

$$F_{beautiful}(0.7) = \{C_2, C_5\}, F_{cheap}(0.7) = \{C_1\}$$

$$F_{beautiful}(0.9) = \{C_5\}, F_{cheap}(0.9) = \{C_1\}$$

$$F_{beautiful}(0.1) = \{C_1, C_2, C_5, C_6\}, F_{cheap}(0.1) = \{C_1, C_2, C_3, C_4, C_5\}$$

$F_{beautiful}(0.3) = \{C_2, C_5\}, F_{cheap}(0.3) = \{C_1, C_2\}$, where here $A = \{0.1, 0.2, 0.3, 0.7, 0.9\} \subset [0,1]$

which can be regarded as the parameter set such that $F_{beautiful} : A \to P(U)$ gives the approximate value set $F_{beautiful}(\alpha)$, $for\ \alpha \in A$ .Thus, the soft set for the fuzzy set $F_{beautiful}$ can be written as:

$$(F_{beautiful}, A) = \{\ (0.1, \{C_1, C_2, C_5, C_6\}), (0.2, \{C_1, C_2, C_5\}), (0.3, \{C_2, C_5\}), (0.7, \{C_2, C_5\}), (0.9, \{C_5\})\ \}.$$

Similarly, the soft set for the fuzzy set F_{cheap} is given by;

$$(F_{cheap}, B) = \{\ (0.1, \{C_1, C_2, C_3, C_4, C_5\}), (0.2, \{C_1, C_2, C_4, C_5\}), (0.3, \{C_1, C_2\}), (0.7, \{C_1\}), (0.9, \{C_1\})\ \},$$

where $A, B = \{0.1, 0.2, 0.3, 0.7, 0.9\} \subset [0,1]$

Definition 3.5. The complement of a soft set (F, A) is denoted by (F^C, A) and is defined by $(F, A)^C$, where $F^C : A \to P(U)$ is a mapping given by $F^C(x) = U - F(x)\ \forall\ x \in X$.

Let the terms beautiful and cheap for example be associated with its own fuzzy set as follows:

$$F_{beautiful} = \{(C_1,0.2),(C_2,0.7),(C_5,0.9),(C_6,0.1)\}$$

$$F_{cheap} = \{(C_1,0.9),(C_2,0.3),(C_3,0.1),(C_4,0.1),(C_5,0.2)\}, \text{ then}$$

$$F^C{}_{beautiful} = \{(C_1,0.8),(C_2,0.3),(C_5,0.1),(C_6,0.9)\}, \text{ and}$$

$$F^C{}_{cheap} = \{(C_1,0.1),(C_2,0.7),(C_3,0.9),(C_4,0.9),(C_5,0.8)\}.$$

4. Soft PU-algebras

Mostafa et al [21] introduced the concept soft PU-algebras. Let X and A be a PU-algebra and a nonempty set, respectively. A pair (F, A) is called a soft set over X if and only if F is a mapping from a set of A into the power set of X . That is, $F : A \to P(X)$ such that $F(x) = \phi$ if $x \notin A$. A soft set over X can be represented by the set of ordered pairs $\{(x, F(x)) : x \in A, F(x) \in P(X)\}$.

It is clear to see that a soft set is a parameterized family of subsets of the set X .

Definition 4.1. Let (F, A) be a soft set over X . Then, (F, A) is called a soft PU-algebra over X , if $F(x)$ is a ***new ideal*** of X , for all $x \in A$.

Example 4.2. Let X = {0, a, b, c} be a set provided in Example 2.8. Define a mapping $F : X \to P(X)$ by: F (0) = {0}, F (a) = {0, a}, F (b) = {0, b} and F (c) = {0, c}. It is clear that (F, X) is a soft PU-algebra over X .

Definition 4.3. Let (F, A) and (G, B) be two soft PU-algebras over X . Then, (F, A) is called a soft PU-subalgebra of (G, B), denoted by $(F, A) \prec (G, B)$, if it satisfies:

(i) $A \subset B$,

(ii) $F(x)$ is sub-algebra of $G(x)$, for all $x \in A$.

Proposition 4.4. A soft set (F, A) over X is a soft PU-algebra, if and only if each $\Phi \neq F\ (\varepsilon)$ is a ***new ideal*** of X , for all $\varepsilon \in A$.

Theorem 4.5. Let (F, A) and (G, B) be two soft PU-algebras over X . If $A \cap B \neq \phi$,then the intersection $(F, A) \tilde{\cap} (G, B)$ is a soft PU-algebra over X .

5. Codes generate by a soft PU-algebra

Definition 5.1. Let $F : A \to P(X)$ be a mapping from a set $A \subseteq X$ into the power set of X , then F is called a soft PU-function on A.

Definition 5.2. A cut function of F , for $p \in P(X)$ is defined to be a mapping $F_p : A \to \{0,1\}$ such that $F_p(x) = 1 \Leftrightarrow p \subseteq F(x)$, for all x in A.

Obviously, F_p is the characteristic function of p -level subset (or, a p -cut) $f_p = \{x \in A : F_p(x) = 1\}$.

Example 5.3 Let $A = \{e_1, e_2\} \subseteq E$ and let X= { C_0, C_1, C_2, C_3 } be the set of Cars with the following Cayley table:

$*$	C_0	C_1	C_2	C_3
C_0	C_0	C_1	C_2	C_3
C_1	C_1	C_0	C_3	C_2
C_2	C_2	C_3	C_0	C_1
C_3	C_3	C_2	C_1	C_0

Then, (X, ∗, 0) is a PU-algebra. Let $A = \{e_1, e_2\} \subseteq E, where$

$$E = \{e_1 = expensive, e_2 = beautiful, e_3 = manual\ gear, e_4 = cheap\}$$

be a set of parameters and $F\ : A \to P$ (X) and given by

$$F = \left\langle \begin{matrix} e_1 & e_2 \\ F(e_1) & F(e_2) \end{matrix} \right\rangle = \left\langle \begin{matrix} e_1 & e_2 \\ \{C_2, C_3\} & \{C_1, C_3\} \end{matrix} \right\rangle$$

and $F_p : A \to \{0,1\}$. Table 1 (below) shows the tabular representation of the soft set (F, A), where if $C_i \in F(e_i)$, *then* $F_p(x)=1$, *otherwise* $F_p(x)=0$, where $F_p(x)$ are the entries in the table.

X	e_1	e_2
C_0	0	0
C_1	0	0
C_2	1	1
C_3	1	1

Definition 5.4. Let $F : A \to P(X)$ be a soft PU-function on A, and ~ a binary relation on $P(X)$, such that $\forall p,q \in P(X), [p \sim q \Leftrightarrow f_p = f_q]$. Then, ~ is an equivalence relation on $P(X)$.

Let $F(A) = \{p \in P(X) : p = F(x), for some x \in A\}$, and for $p \in P(X)$, let $[p) = \{q \in P(X) : p \subseteq q\}$.

Lemma 5.5. If $F : A \to P(X)$ is a soft PU-function on A, then for $p,q \in P(X)$

$$p \sim q \Leftrightarrow [p) \cap F(A) = [q) \cap F(A)$$

Proof. $\forall p,q \in P(X), p \sim q \Leftrightarrow f_p = f_q$

$$(\text{for } x \in A), [p \subseteq F(x) \Leftrightarrow q \subseteq F(x)] \Leftrightarrow \{x \in A : F(x) \in [p)\} = \{x \in A : F(x) \in [q)\} \Leftrightarrow$$
$$[p) \cap F(A) = [q) \cap F(A).$$

Example 5.6. Let $X = \{0, a, b\}$ be a set with the operation $*$ defined by the following table.

*	0	a	b
0	0	a	b
a	b	0	a
b	a	b	0

and $P(X) = \{\phi, \{0\}, \{a\}, \{b\}, \{0,a\}, \{0,b\}, \{a,b\}, \{0,a,b\}\}$. Then, $(X, *, 0)$ is a PU-algebra.

Let $F : X \to P(X)$ be a soft PU-function on X given by $F = \begin{pmatrix} 0 & a & b \\ \{0\} & \{0,a\} & \{0,b\} \end{pmatrix}$, then a cut function of F is given by the following table:

	0 {0}	a {0, a}	b {0, b}
F_ϕ	1	1	1
$F_{\{0\}}$	1	1	1
$F_{\{a\}}$	0	1	0
$F_{\{b\}}$	0	0	1
$F_{\{0,a\}}$	0	1	0
$F_{\{0,b\}}$	0	0	1
$F_{\{a,b\}}$	0	0	0
$F_{\{0,a,b\}}$	0	0	0

Hence, $f_{\phi} = f_{\{0\}} = \{0,a,b\}$, $f_{\{a\}} = f_{\{0,a\}} = \{a\}$, $f_{\{b\}} = f_{\{0,b\}} = \{b\}$ and $f_{\{a,b\}} = f_X = \phi$.

Lemma 5.7. Let $F : A \to P(X)$ be a soft PU-function on A, for every $x \in A$, if $F(x) = p$, then $p \in P(X)$ is an infimum of the class to which it belongs, i.e. $p = \inf [p]_{\sim}$.

Proof. If $q \in [p]_{\sim}$, then $p = F(x) \subseteq q$. Hence, $p = \inf [p]_{\sim}$.

Theorem 5.8. If $F : A \to P(X)$ is a soft PU-function on A, then for all $x \in A$,

$$F(x) = \sup\{p \in P(X) : F_p(x) = 1\}$$

Proof. Let $F(x) = q \in P(X)$. Then, $F_q(x) = 1$. Now, if any $p \in P(X), F_p(x) = 1$, then $p \subseteq F(x)$, i.e., $p \subseteq q$. Also, $q \in \{p \in P(X) : F_p(x) = 1\}$, thus q is the greatest element of that family. Thus,

$$F(x) = q = \sup\{p \in P(X) : F_p(x) = 1\}$$

Proposition 5.9. Let $F : A \to P(X)$ be a soft PU-function on A. If $q \subseteq p$, for all $p, q \in P(X)$, then $f_p \subseteq f_q$.

Proof. Let $q \subseteq p$, for all $p, q \in P(X)$ and $x \in f_p$, then $p \subseteq F(x)$. It follows that $q \subseteq F(x)$ and so $x \in f_q$. Hence, $f_p \subseteq f_q$.

Proposition 5.10. Let $F : A \to P(X)$ be a soft PU-function on A. Then, $[F(x) \neq F(y) \Leftrightarrow A_{F(x)} \neq A_{F(y)}]$, $\forall x, y \in A$.

Proof. The sufficiency is obvious. Assume that $A_{F(x)} \neq A_{F(y)}$, $\forall x, y \in A$. Then,

$$A_{F(x)} = \{z \in A : F(x) \subseteq F(z)\} \neq \{z \in A : F(y) \subseteq F(z)\} = A_{F(y)}$$

Corollary 5.11. Let $F : A \to P(X)$ be a soft PU-function on A. Then,

$$\forall x, y \in A,\ [F(y) \subseteq F(x) \Leftrightarrow A_{F(x)} \subseteq A_{F(y)}]$$

Proof. Clear.

Let $F : A \to P(X)$ be a soft PU-function on A, and $\sim$ a binary relation on $P(X)$, such that $\forall p, q \in P(X), [p \sim q \Leftrightarrow f_p = f_q]$. Then, $\sim$ is an equivalence relation on $P(X)$ and $[p]_{\sim} = \{q \in P(X) : p \sim q\}$ is an equivalence class containing p. Every soft PU-function on A determines a binary block code c of length n, in the following way:

To every class $[p]_{\sim}$, where $p \in P(X)$, there are corresponds a codeword $c_{[p]} = w_1 ... w_n$ such that $w_i = w_j \Leftrightarrow F_r(i) = j$, for $i \in A, j \in \{0,1\}$.

We use the following defined order on the set of codeword's belonging to a binary block code C, for any $x, y \in C, x = x_1 ... x_n$, $y = y_1 ... y_n$, $x \leq_c y \Leftrightarrow x_1,..., x_n \leq_c y_1,..., y_n$.

Example 5.12. Let X = {0, a, b} be a set with the operation $*$ defined by the following table.

$*$	0	a	b
0	0	a	b
a	b	0	a
b	a	b	0

Then, $(X,*,0)$ is a PU-algebra. Let $F : X \to P(X)$ be a soft PU-function on X given by $F = \begin{pmatrix} 0 & a & b \\ \{0\} & \{0,a\} & \{0,b\} \end{pmatrix}$ and $F_p : X \to \{0,1\}$, then a cut function is given by the following table:

	0 {0}	a {0, a}	b {0, b}
$F_{\{0\}}$	1	1	1
$F_{\{0,a\}}$	0	1	0
$F_{\{0,b\}}$	0	0	1

Then, $C = \{111, 010, 001\}$, see Fig.1

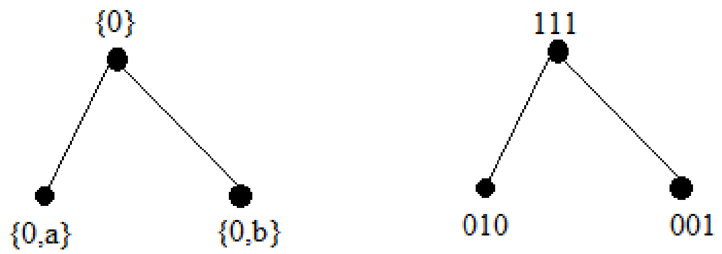

Fig 1. Graphs of the binary block code C

Theorem 5.13. Let X be a finite PU-algebra. Every $(P(X), \leq)$ determines a block-code C, such that $(P(X), \leq)$ is isomorphic with $(C, \leq_c)$.

Proof. Let $X = \{r_i : i = 1,..., n\}$ be a finite PU-algebra in which r_1 is the least element and let a mapping $H : P(X) \to P(X)$ be identify PU-valued function on $P(X)$. The decomposition of H gives a family $\{H_r : r \in X\}$ which is the required code, under the above defined order $x \leq_c y \Leftrightarrow x_1,..., x_n \leq_c y_1,..., y_n$. Let $f : X \to \{H_r : r \in X\}$ be a function defined by $f(r) = H_r$, for all $r \in X$. By Lemma 4.6 every class contains exactly one element, and thus f is one to one. If $r, m \in X$ and $r \leq m$, then $H_m \subseteq H_r$, which means that $H_m \leq H_r$, and f is an isomorphism.

Example 5.14. Let X = {0, a, b} be a set with the operation $*$ defined by the following table.

$*$	0	a	b
0	0	a	b
a	b	0	a
b	a	b	0

and $P(X) = \{\phi, \{0\}, \{a\}, \{b\}, \{0,a\}, \{0,b\}, \{a,b\}, \{0,a,b\}\}$. Then, $(X,*,0)$ is a PU-algebra.

Let $F: X \to P(X)$ be a soft PU-function on X given by $F = \begin{pmatrix} 0 & a & b \\ \{0\} & \{0,a\} & \{0,b\} \end{pmatrix}$.

Now, let $H: P(X) \to P(X)$ be identify PU-valued function on $P(X)$, then a cut function is given by the following table:

	ϕ	{0}	{a}	{b}	{0, a}	{0, b}	{a, b}	{0, a, b}
F_{ϕ}	1	1	1	1	1	1	1	1
$F_{\{0\}}$	0	1	0	0	1	1	0	1
$F_{\{a\}}$	0	0	1	0	1	0	1	1
$F_{\{b\}}$	0	0	0	1	0	1	1	1
$F_{\{0,a\}}$	0	0	0	0	1	0	0	1
$F_{\{0,b\}}$	0	0	0	0	0	1	0	1
$F_{\{a,b\}}$	0	0	0	0	0	0	1	1
$F_{\{0,a,b\}}$	0	0	0	0	0	0	0	1

$$C = \{11111111, 01001101, 00101011, 00010111, 00001001, 00000101, 00000011, 00000001\}$$

6. Conclusion

Coding Theory is a mathematical domain with many applications in Information theory. Various type of codes and their connections with other mathematical objects have been intensively studied. One of these applications, namely connections between binary block codes and BCK-algebras, was recently studied in [16,17]. In this paper, we focused to one of the recent applications of PU-algebras in the coding theory, namely the construction of codes by soft sets PU-valued functions. First, we introduced the notion of soft sets PU-valued functions, on a set and investigated some of its related properties. Moreover, the codes generated by a soft sets PU-valued function were constructed and several examples are given. Furthermore, example with graphs of binary block code were constructed from a soft sets PU-valued function.

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