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Research Article

Conceptual technique for comparison figures by geometric thinking in analysis level

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Abstract

In underdeveloped countries, research in mathematics education has been mostly focused on students' geometry abilities based on levels, learning approaches, and textbooks. But, thinking process and level are a problem relevant to the low quality of student achievement. The process and level of students' thinking are due to the conceptual system in operating. In this study, a geometry question at the analysis level was designed to investigate conceptual systems. Students represent and compare two figures by their techniques. Data obtained from the survey and narrative study. Data were analyzed based on three components of activity: input, internal processing, and output. Students represent by copying, revising symbols, rummaging objects, and reconstructing properties. They analyze property geometry on the building block or spatial representation. Students compare through one of the two process models of think, namely: object extraction techniques to structure-property connection and inter-object connection to property extraction. The systematic paths of the two models are different. One produces a creative conceptual formulation before extracting geometry properties. Its creativity is involved in comparisons so there is a leap to a more objective point of view. Therefore, conceptual systems and construction for the conceptual formulation are two ideas for learning situations or solving problems.

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Introduction

According to the international assessment report, the geometry achievements of Indonesian students in primary and secondary schools are still in the low international benchmark category (Alghadari, Herman, & Prabawanto, 2020; Sandy, Inganah, & Jamil, 2019), because most students are only able to work up to level 2 of 4 (Hidayah & Forgasz, 2020). The level of student achievement has to do with the status of geometry education in the curriculum and the learning environment related to what is provided to them (Silfverberg, 2019). In the Indonesian educational curriculum, geometry is introduced from elementary school students (Purnomo et al., 2019), to advanced levels (Alghadari & Noor, 2020; Alghadari et al., 2020; Jelatu, Sariyasa, & Ardana, 2018). There are different geometry concepts studied at each school level where these differences are based on considerations of cognitive development theories, such as Piaget and van Hiele's theory (Lesh & Harel, 2003), which are related to students' thinking level and ability. At the basic level, students learn geometry shapes (class of shapes) such as triangles or rectangles (Morales et al., 2018). At the intermediate level, students explore the geometry properties in a shape so that learning leads to conceptual matters (Mahendra et al., 2017). Whereas at an advanced level, students learn geometry concepts based on axioms and definitions. The difference between the three is the thinking stage of students who operate through the conceptual system based on the level of cognitive development required. Based on consideration of the international assessment reports, we can assume how students' geometry thinking ability at an advanced level is due to problems in the conceptual system and their development at the previous level. On the other hand, the modern style learning

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environment has emphasized the use of dynamic geometry with exploratory and experimental activities, but its effectiveness is still not widely known (Silfverberg, 2019).

The focus of our review has been on the question of why students' learning geometry was problematic so that the quality of their achievement was low. We summarize several causes and among them are because: students are less skilled for non-routine problems involving high order thinking (Hidayah & Forgasz, 2020), problems of synchronization between concepts in problem solving (Alghadari et al., 2020; Jupri, 2017; Sumule, Amin, & Fuad, 2018), the conception of the problem is due to limited conceptual knowledge (Alghadari & Herman, 2018; Alghadari & Noor, 2020), the problem of the association between contextual features on the problem and reality context of students understand (Purnomo et al., 2019), problems from students' understanding of the abstract things (Jelatu et al., 2018; Mahendra et al., 2017), as well as problems of the relevance between the level of geometry abstraction and the visualization ability to an abstract object (Riastuti et al., 2017; Sumule et al., 2018). The summary of these problems is relevant to the thinking process and level (Noor & Alghadari, 2021; Silfverberg, 2019). We quote an illustration related to students' thinking processes and levels by Byers (2020), about the conceptual system between integers and rational numbers, where there are children who say that no numbers between 2 and 3, but the other say that there are many numbers between that. Here, differences in processes and levels of students' abilities are due to the operating conceptual system while they think (Noor & Alghadari, 2021; Palmiero, 2020). This difference underlies our study because research has currently more focused on the students' achievement of geometry abilities based on level (e.g. Fitriyani, Widodo, & Hendroanto, 2018; Lutfi & Jupri, 2020; Prayito, Suryadi, & Mulyana, 2019), learning approach (e.g. Alghadari et al., 2020; Jelatu et al., 2018; Mahendra et al., 2017), and textbooks (Hidayah & Forgasz, 2020; Purnomo et al., 2019), so there have not been many studies on operating conceptual systems student do. The conceptual system is one of the causes of differences in students' geometry thinking levels and abilities (Battista et al., 2018; Heyd-Metzuyanim & Schwarz, 2017; Noor & Alghadari, 2021; Olkun et al., 2005).

A coherent conceptual system is the basis of planning (Sierpinska, 2005), and the resulting model in the problem-solving process (Lesh & Harel, 2003). By definition, a conceptual system consists of concepts and relationships at different levels, or a mixed conceptual space if it has physical or mental elements (Burgin & Díaz-Nafría, 2019). The conceptual system is a pathway based on the thought process. When the pathway contains a wide variety of concepts and relationships, as many concepts as considered involved show a structure with a higher degree of abstraction (Burgin & Díaz-Nafría, 2019; Lesh & Harel, 2003), so the conceptual system describes the level of thinking ability. We analyze it in the context when students have been thinking about answering the geometry question posed. The adopted thinking level on the geometry question is based on the relevance between the problems summarized and the geometry achievement category of Indonesian students according to the International assessment report. Therefore, we set this study at the analysis level of geometry thinking. In practice, the transitioning process of conceptual knowledge when students answer geometry questions at the analysis level begins with the class of shape as an object of thought to property of shape as a product they produce (Van de Walle et al., 2017).

Purpose of Study

Class and property of shape are concepts that will be analyzed to transparency the conceptual systems in students' thinking processes from the visual objects to abstract properties. The importance of the thinking process is as a first step towards abstraction thinking to the development of mathematical thinking at the axiomatic level. For this reason, this study aims to investigate the conceptual system when students think geometrically at the analysis level.

Method

Research Model

In this study, we investigate the coherence between the three components of human activity according to Tall (2013), namely input, internal processing, and output. We analyze how the response appears, what plans are made, and what is followed. We break down the systematic stages of the thinking process by students based on how they compare. A coherent and systematic thought process creates a schema, with the relationship between input and output, and is based on the appropriate concept. When the design thinking is coherent, the model is a structured schema that forms a conceptual system (Burgin & Díaz-Nafría, 2019; Lesh & Harel, 2003). Furthermore, Lesh & Harel (2003) stated that van Hiele's theory of conceptual development can be used to help, understand, and explain the many behaviors that students exhibit during the development process in selecting the necessary conceptual tools in modeling activities. Some of these references underlie this research model which we designed following Figure 1.

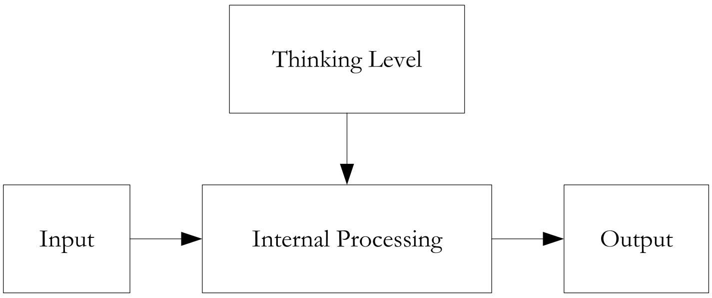


Figure 1.

Model for Synthesizing a Conceptual System in Thinking

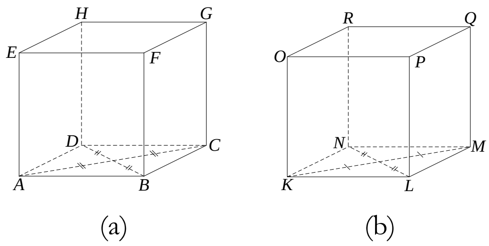
Participants

This study was conducted on 61 people from 12th-grade students at Indonesian Public High Schools in the 2019-2020 academic year. Students who participate are those who have studied the geometry of three-dimensional figures. Some students come from two study groups at a school located in a rural area in Belitung Regency. They were selected based on the major program and the cross-interest subject (minor program) by stratified random sampling.

Data Collection Tools

Various concepts have been studied by students. For example, differences in the properties and shapes of a plane that involve geometry elements such as points, lines, and planes. In the three dimensional-figure, such as cubes and prisms, many concepts are contained abstractly by their representation, including differences in their properties, shape, and relevancy based on the shaped planes of the two figures. Based on these different properties, we designed it to be a geometry question to use for investigating the conceptual system of students' thinking. The question refers to the analysis level according to van Hiele's theory with the classification of objects and product of thought following Van de Walle et al. (2017) namely class and property of shape respectively.

Question: Figure (a) is a cube *ABCD.EFGH*. Figure (b) is a rectangular prism *KLMN.OPQR* with two parallel sides that are rhombus. The lengths of all edges in both figures are the same. Write differences of properties in the model of the pictures, also state your reason why?



Process

Based on the question above, our first step in obtaining data is through a survey. The survey data were the students' responses for the differences of properties in the model of the pictures. Then, the survey results were followed up with an explanatory narrative study. The data from the results of the narrative study were the students' reasons for answering the survey questions. Both data were analyzed and synchronized. The domain that we identified from the response was conceptual knowledge so that we obtained class and property of shape. Its domain becomes the basic element for students to compare the geometry concepts in two figures. When students compare geometry concepts, there is an internal process in operating and the students' reasons clarify the existence of the comparison process. The underlying technique of conceptual comparison for the two figures, we call that is the conceptual technique for comparison.

Result and Discussion

Our findings are based on an analysis of the internalization of students' thinking contains several processes that occurred before they compare the two figures, namely the copying stage of Figure (a) to produce Figure (b), revising symbols, rummaging objects, which are then accompanied by reconstructing properties. Figure (a) is a familiar solid figure with students' learning experiences. Then, in their learning experiences at school, they often and mostly symbolize the vertices of the cube with *ABCD. EFGH*. However, Figure (b) is not a model they usually encounter when studying geometry or shapes. On the other hand, the two drawing models have been presented in front of them on the question sheet so that the model can be a visual mediator or reference for the construction process. These references support students thinking to represent (Battista et al., 2018; Silfverberg, 2019). Representations have invited students to recall or create experiences and continue with revising, rummaging, and reconstructing. Here, the process in the revision stage is simpler than rummage and can occur in parallel. The rummage stages are from a representation cube *ABCD. EFGH* to prism *KLMN. OPQR*, and at the same time it includes that there is a rummage process from square *ABCD* in Figure (a) to rhombus *KLMN* in Figure (b). Following these findings, Rowlands (2019) states that the process of constructing a new conceptual system is the process of dismantling or complementing the old. In this case, the reconstruction occurs because there is a change in shape in figure (b) and dismantling symbols at the vertices of Figure, from *ABCD* to *KLMN*.

The changes in the object are due to changes in shape and length on the two parallel diagonals in the rhombus as the base and roof of Figure (b). The reconstruction from Figure (a) to Figure (b) is by "rummaging object form", that is from square to rhombus. Relevant to the results of the study by Chotimah & Jannah (2020) that the aspect of internalization when students represent objects abstractly is constructing the appearance of building blocks from spatial shapes. In this internal process, the property figures other than the diagonals of the sides of the base and the roof along with the angles in these shapes are still in the capsulation. We view the process as a process-to-concept encapsulation, which should represent the Figure from the beginning of the construction process, but the processes are encapsulated as a concept from the unchanged Figure representation. Because demolition of property in the representation is not carried out in its entirety but is adjusted to the needs of the change, and the construction of the representation is not carried out from an early stage, so these activities become relevant when we say it with reconstruction. The reconstruction of the Figure representation, in this case, is an internal process for modeling the figure. This case is a model or student's way of representing figures of the same size and shape. Lesh & Harel (2003) stated that a model is a conceptual system that is generally expressed using interacting representations, which may involve writing symbols or experience-based metaphors. Therefore, this finding is a clue about one of the identities of the conceptual system in the geometry thinking process of high school students at the analysis level for the case of representing internally.

The geometry properties that construct two figures in each representation are compared by students. Students involve competence from cognitive factors, namely figural property construction (Chotimah & Jannah, 2020; Rivera, 2018), and figural conceptual constructs (Patsiomitou, 2018), which influence the appearance of mathematical structures. The base plane includes given information in the question which is represented by the rectangle in Figure (a) and the rhombus in Figure (b). A shape as a base of two Figures is a comparison object by students has been a basic design to elicit their response. Based on the students' responses, some of the products of thought they produced were about the base of solid as the class of shape, namely the edges or angles of their intersection, the diagonal of the sides or the length, the angle of intersection between the diagonals of the sides or the magnitude, and the area of the base. All of them are property of the building block in the figure. Then, another response from students is their point of view on other areas of the figure, such as the diagonal of space which is also one of the properties of the two figures. The diagonal of space is a spatial property. The results of our investigation show that the response to space diagonal is the focus of the next analysis after switching from the properties in the base plane. Based on these findings, the categories for the two classifications of property of figure as the product of thought are based on students' abilities in spatial dimensions, namely property on the building block or spatial representation (Jirout & Newcombe, 2015).

Conceptual Technique for Comparison

The results of the identification of objects and product of thought provided the basis for us to continue to investigate how students compared the two figures. This investigation is on the relationship between input, internal processing, and output as a component of human activity, namely the systematic operation of geometry concepts in figures. In the results of this investigation, we found two categories of students' techniques comparing Figures. First, the properties between the figures are then encapsulated in a geometry relationship. An example of student response from the first category is that the side diagonal of one figure is longer than the other. We categorize this based on the process of transparency of the properties of the shape of the two structures that precede the comparison process, so we address this fact with the product of thought in the analysis level of geometry thinking as the basis for comparison. In other words, thinking about geometry at the level of analysis precedes the process of mathematical connections. Therefore, an image can be considered alone, as a graphic object, or as one of the possible representations of a geometry object. In the case of this first category, the product appears through empirical abstraction because the student focus is on objects and their properties (Gray & Tall, 2007). Furthermore, Mithalal & Balacheff (2019) explained that figures are associations of theoretical references from the two determinants, namely the relationship between geometry properties and geometry objects. The comparison technique of the first category is based on theoretical references to geometry properties, and we categorize them as object extraction to structure-property connection. Meanwhile, the comparison technique determined by geometry objects, or inter-object connection to property extraction, is categorized as the second conceptual technique as follows.

The second, the properties of each object in capsulation and then compared. Examples of student responses from this second category are the diagonal lengths of the sides and spaces of the two different figures. The results of our confirmation of the students' responses that the response model from the second category is not based on the results of one-by-one analysis properties of shape in each figure but it is the consequences of analysis on the shape of the basic plane of the two figures. The property of the figure encapsulated in the object is reframed by the students because the question is the differences of properties that should direct the focus of their analysis. Byers (2020) calls such framing efforts a conceptual world of mathematics in which logical processes live exclusively and control that effort. Piaget calls cases from this second category an empirical pseudo-abstraction that focuses on actions on objects and the properties of the actions (Fitriani, Suryadi, & Darhim, 2018a,b; Scheiner & Pinto, 2014). The basis for comparison of this second category is the object so that students' geometry thinking, in this case, occurs after they compare the class of shape. Students compare the object of thought representations then think geometrically in the analysis level occurs after the mathematical connection process, and they formulate concepts to construct a concept that can be thought of. In this second case, the geometry properties are encapsulated in a class of shape and then compared between the two figures. Students see the entire properties of shape as individual objects which then recursively extract the properties. Students compare the shape of the base planes in two figures and not their properties directly. Because there are different classes of shape, of course, there are abstract properties that are also different. Silfverberg (2019) explains that conceptualization does require synthesis through association, abstraction, and differentiation between properties before analysis. Internal processes as in this second category are not easy enough for most people and virtually impossible for the many who simply learn the rules by rote, but it is a process that seems to be performed implicitly by those who make sense of the hierarchical structure (Tall, 2013; Tall & Witzke, 2020).

Conceptual System in Representing to Comparing

The next investigation is focused on the base of the figure, namely the process of representing the figure and two conceptual techniques for comparison. These two concepts form a model linked by systematic operation. Lesh & Harel (2003) stated that the model includes accompanying procedures to produce constructs, manipulations, or predictions that are useful for achieving recognized goals. Referring to these references and based on the findings of this study, the model of the representation process is copying, revising symbols, rummaging objects, and reconstructing properties. Models are also a way of treating concepts through thinking about objects, properties, and their relationships (Burgin & Díaz-Nafría, 2019) so that the model for comparison is object extraction to structure-property connection and inter-object connection to property extraction. From this case, there are two process models of think until students respond to questions and both are different because of the consideration of the concept formulation that students think that if there are differences in the class of shape, then there are abstract properties that are also different. The formulation of concepts from the results of students' thoughts becomes an additional concept for consideration in comparing figures. The conceptual formulation has become the first point of different parts in students' conceptual systems. By involving this formula, the comparison stage becomes more objective because there is a leap to a higher point of view, and the process is said by Byers (2020) as creativity. Schoevers et al. (2019) stated that mathematical creativity can be in the form of creating new and meaningful mathematical ideas or concepts through cognitive action combining known concepts in an adequate way. Furthermore, at the conceptualization stage where the situation is more complex, problems that were difficult at first but when brought to the original space with the right logical leaps can become more transparent (Morales et al., 2018). Therefore, conceptual considerations show structures with a higher degree of abstraction (Burgin & Díaz-Nafría, 2019; Lesh & Harel, 2003; Palmiero, 2020) so that the conceptual technique from the second category with the empirical pseudo-abstraction type is superior because it is considered more sophisticated than sensory experiences (Gray & Tall, 2007; Scheiner & Pinto, 2014; Tall, 2013; Tall & Witzke, 2020).

The concept formulation is in the operational phase of the geometry thinking system at the analysis level, occurs before extracting geometry properties, and is an additional concept to the inter-object connection to the property extraction model. Van Hiele's theory has detailed objects and product of thought at the level of analysis of geometry thinking, respectively, shape and property of shape (Van de Walle et al., 2017) where these details are the pathways of the geometry thinking system that underlies the two model process students' thinking as a study finding is subdivision part of the conceptual system for comparison. The operational phase of geometry thinking in the comparison stage is when the object was extracted or the product of thought has been connected. The extraction or connection process sequence in the comparison stage confirms the division path of the geometry thinking system. This case is an example of the progressive sophistication of students' intellectual means to control the representation of the phenomenon of geometry knowledge development, that requires a specific focus on relevant aspects of a situation to name and compress into a thinkable concept (Gray & Tall, 2007; Tall & Witzke, 2020). If the model includes a conceptual system to describe or explain relevant mathematical objects, relationships, actions, patterns, and regularities associated with problem solving situations Lesh & Harel (2003), then the models of the process of think in the findings of this study are two types of conceptual systems, because Burgin & Díaz-Nafría (2019) definitively explain that a conceptual system consists of concepts and relationships at different levels, as well as concepts with their construction based on certain concept models as their basic elements.

Furthermore, Burgin & Díaz-Nafría (2019) state that models of conceptual systems exist in conceptual space such as conceptual representation theory in mind. Therefore, based on the findings of this study, our idea for learning geometry so that students can achieve a higher degree of abstraction is to formulate concepts to take logical leaps into the right conceptual space when in problem solving situations. Lesh & Harel (2003) detail that the resulting model in the problem-solving process includes the conceptual constructs and systems needed to understand complex types of systems. However, in students' thinking, the construction community and conceptual systems often compete to dominate the interpretation to be emphasized. Here, it is explained that solving problems requires both conceptual systems and construction (Schoevers et al., 2019). The first, the conceptual system is a line of systematic thinking that is operated both when students learn and when solving problems. However, when learning only emphasizes systematic thinking, the result will give birth to procedural tendencies. Based on the notes from several research results, it shows that procedural knowledge controls students' conceptual knowledge and not the other way around (Alghadari & Noor, 2020; Morales et al., 2018). The second, construction that produces a conceptual formulation is a phase to enter another different conceptual system in the world of mathematics and an attempt to leap into a more transparent space. Creating new and meaningful ideas in mathematics is by break away from established mindsets (Morales et al., 2018; Palmiero, 2020; Schoevers et al., 2019). All this time, especially in studying geometry, the effective relevance of the learning approach and creative conceptual construction has not been seriously considered (Hidayat et al., 2017; Nugraheni et al., 2018). Thus, one of the pieces of information for researching special thinking skills in students' geometry learning for better achievement can be started by integrating the conceptual system and the construction of creative concept formulations. Whether efforts to improve students' thinking skills have involved the model of the two elements suggested by Lesh & Harel (2003) in learning geometry is a question that needs to be investigated at another stage.

Conclusion

In the geometry thinking process, especially at the analysis level, when students represent and compare figures, there are two types of models of conceptual systems in operating. The model for representing is no different, namely copying, revising symbols, rummaging objects, and reconstructing properties. But the model is different when students start comparing figures. The different models are based on the following conceptual techniques, object extraction to structure-property connection or inter-object connection to property extraction. There is another consideration with a conceptual formulation that appears in the conceptual technique from the type of inter-object connection to property extraction so that it becomes the first point of different parts in students' conceptual system. Concept formulation is an additional concept for figural comparing technique and has resulted in a leap of thinking to the right space and a more objective point of view. The problems are difficult but with a leap of logic so that can become more transparent. For learning or solving geometry problems so that students can achieve a higher degree of abstraction is by formulating concepts to take logical leaps into the right conceptual space. Therefore, it takes both the conceptual formulations constructed and the conceptual system for understanding the types of complex systems when in problem solving situations.

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