Araştırma Makalesi / Research Article

Analysis of Linear Consecutive-2-out-of-n: F Repairable System with Different Failure Rate

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Abstract
In the reliability analysis, when the reliability of a repairable consecutive-k-out-of-n system is considered, the components are generally supposed to have an equal failure rate. In practice, this assumption may fail. Therefore, in this paper it is adopted that the lifetime of each component are random variables, exponentially distributed, with different failure rates. The time required to repair is an exponential random variable and each component after repair is as durable as new. In this paper, we proposed a model for the state transition probability of this repairable system when components have unequal failure rates. The system mean time to first failure was also studied.

Keywords: Repairable consecutive system, Markov process, Mean time to first failure.

Farklı Hata Oranlı Doğrusal Ardışık n-den 2 Çıkışlı Tamir Edilebilir F Sistemin Analizi

Öz
Güvenilirlik analizinde, tamir edilebilir ardışık n-den k-çıkışlı sistemin güvenilirliği elde edilirken genellikle bileşenlerin eşit hata oranlarına sahip olduğu varsayılır. Uygulamada bu varsayım hatalı olabilir. Bu nedenden dolayı bu çalışmada her bir bileşenin ömrü farklı hata oranlarına sahip üstel tesadüfi değişken olarak ele alınmıştır. Tamir için gerekli süre üstel tesadüfi değişken olarak tanımlanmıştır ve tamirden sonra her bir bileşen yeni bir bileşen kadar iyi durumdadır. Bu çalışmada, bileşenlerin eşit olamayan hata olasılıklarına sahip olduklarında bu tamir edilebilir sistemin durum geçiş olasılıkları için bir model önerilmiştir. Ayrıca sistemin ilk ortalama arızalanma süresi de incelenmiştir.

Anahtar kelimeler: Tamir edilebilir ardışık sistem, Markov süreci, İlk ortalama arızalanma süresi.

1. Introduction

As the technology develops it becomes more complicated to deal with the reliability analysis of systems consisting of the mixture of series and parallel connected components. To overcome this problem, more convenient and more reliable consecutive k-out-of-n (Con-k-n) and related systems are proposed. These systems are called linear and circular taking into account the connection structures of the components in the system. In addition, if the system is designed on the functioning, it is called G system, and if the system is designed on the principle of failure, it is called F system. A Con-k-n: F system consists of n components, and this system fails if at least k consecutive components fail [1]. This system was presented by Kontoleon [1] in 1980. On the other hand, a Con-k-n: G system works if at least k consecutive components work. Both of them can be either linear or circular. In the literature, there are many studies on such system structures mentioned above. For further reference, see [2-8], among others. However, in most of these studies, the system is designed as a non-repairable system. There is limited

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number of studies on repairable consecutive-*k*-out-of-*n* systems (Rep-Con-*k*-*n*) in the literature. A linear Rep-Con-2/*n*: F system considering generalized transition probability (GTP) is studied by [9]. A linear Rep-Con-(*n*-1)/*n*: G system is analyzed by [10]. Later on, Rep-Con-*k*-*n*: G system with $k > n/2$ is considered by [11]. Assigning different priorities to the failed components, a circular Rep-Con-2-*n*: F system is studied by [12]. The work [9] is generalized by [13]. In literature, the components are generally supposed to have an equal failure rate. But this assumption is not justified in many situations. For example, components of the same brand that were made in different factories or times show varying life spans. Also because components may have been used in different systems and they may have worn out differently. Some recent works on k-out-of-n systems are in Navas et al. [14], Wang et al. [15], Gökdere and Güral [16], Gökdere et al. [17].

A Rep-Con-2-*n*: F system with Markov dependence was investigated by [18]. Then, a system with 1-step Markov dependence was analyzed by [19]. Later on, a system with (*k*-1)-step Markov dependence was considered by [20].

In this paper, a linear Rep-Con-2-*n*: F system is analyzed under the assumption that components do not have equal failure rates. A new model is developed for the state transition probabilities (STP) in the system. The mean time to first failure (MTTFF) of this system is obtained.

The organization of this paper is as follows. In Section 2, at first the preliminary assumptions are given and then the concept of STP is defined. In Section 3, a general methodology is presented for MTTFF of the repairable system and dynamic performance probabilities of the irreparable system, respectively. In Section 4, the numerical results are given for a special linear Rep-Con-2-n: F system with five components. In the last section, we summarize what we have done in the article.

2. Material and Method

For the linear Rep-Con-2-*n*: F system, first, we suggest the following model assumptions. Then, the state of the system at time t, $N(t)$, is given. Finally, we obtain some mathematical formulations for the STP. To start with, we give the following definition.

Let $\{N(t), t \geq 0\}$ be a continuous-time homogeneous Markov process with state space Ω. Also let $\rho_{\xi(m)}$ be the probability that the process starts in the mth case of state ξ under the condition that the state of the system at time t is equal to ξ and defined as

$$\rho_{\xi(m)} = P\{\text{It is in case } m \text{ of state } \xi | N(t) = \xi\} \tag{1}$$

where $\xi = 1,2,\dots,M_\xi$ and M_ξ is the number of all cases for given state $\xi \in \Omega$. Let $\rho_{\xi(m)j}(\Delta t)$ be the probability that the process starting in the mth case of state ξ will be in state j in time Δt and defined as

$$\rho_{\xi(m)j}(\Delta t) = P\{N(t+\Delta t) = j | \text{It is in case } m \text{ of state } \xi \text{ at time } t\}. \tag{2}$$

Then, the GTP from ξ to j in time Δt is defined as

$$\rho_{\xi j}(\Delta t) = P\{N(t+\Delta t) = j | N(t) = \xi\} = \sum_{m=1}^{M_\xi} \rho_{\xi(m)}\, \rho_{\xi(m)j}(\Delta t), \tag{3}$$

where

$$\sum_{m=1}^{M_\xi} \rho_{\xi(m)} = 1. \tag{4}$$

2.1. Model Assumptions

Assumption 1. In the system all components are new at the beginning.

Assumption 2. The lifetimes of component ξ $(\xi = 1,2,\dots,n)$ have an exponential distribution $\mathrm{Exp}(\lambda_\xi)$ whose probability density function is as follow

$$f_\xi(t) = \begin{cases} \lambda_\xi e^{-\lambda_\xi t} & t > 0 \\ 0 & t \leq 0 \end{cases}. \tag{5}$$

Assumption 3. There is only one repairman. Because of this, at the same time, only one component can be repaired.
Assumption 4. The repair discipline is first in first out. Moreover, repair completely restores all properties of failed components.
Assumption 5. The probability of failure of two or more components at the same time is zero.
Assumption 6. The repair time of the failed components has a constant failure rate μ;

$$g_\xi(t) = \begin{cases} \mu e^{-\mu t} & t > 0 \\ 0 & t \leq 0 \end{cases} \tag{6}$$

where $\xi = 1,2,\dots,n$.
Assumption 7. The components will not fail after the system fails.

2.2. Model Analysis

According to the assumptions, we have

$$N(t) = \begin{cases} 0, & \text{The system works when all components work at time } t \\ -1, & \text{The system works when one component fails at time } t \\ -2, & \text{The system works when two components fail at time } t \\ \vdots & \vdots \\ -d, & \text{The system still works when } d \text{ components fail at time } t \\ 2, & \text{The system fails when two components fail at time } t \\ 3, & \text{The system fails when three components fail at time } t \\ \vdots & \vdots \\ d+1, & \text{The system fails when } d+1 \text{ components fail at time } t \end{cases} \tag{7}$$

where $d = [(n+1)/2]$ is the greatest integer function. When the lifetimes of every component and the time required to repair are exponentially distributed, then $\{N(t), t \geq 0\}$ is a continuous-time homogeneous Markov process with finite state space $\Omega = \{0, -1, -2, -3, \dots, -d, 2, 3, \dots, d+1\}$. Obviously, $W = \{0, -1, -2, -3, \dots, -d\}$ is the set of working state, while $F = \{2, 3, 4, \dots, d+1\}$ is the set of failed state.

Example 1. Consider a linear consecutive-2-out-of-3: F repairable system with different failure rate, then the state of the system at time t can be wrote down in the form

$$N(t) = \begin{cases} 0, & \text{The system works when all components work at time } t \\ -1, & \text{The system works when one component fails at time } t \\ -2, & \text{The system works when two components fail at time } t \\ 2, & \text{The system fails when two components fail at time } t \\ 3, & \text{The system fails when three components fail at time } t. \end{cases}$$

Based on the results presented above, the state space $\Omega = \{0, -1, -2, 2, 3\}$, the set of working state is $W = \{0, -1, -2\}$ and the set of failed state is $F = \{2, 3\}$.

By using the definition and the model assumptions, we have:

$$\rho_{0j}(\Delta t) = \begin{cases} \sum_{k=1}^{n} \lambda_k \Delta t + O(\Delta t), & j = -1 \\ 1 - \sum_{k=1}^{n} \lambda_k \Delta t + O(\Delta t), & j = 0 \\ O(\Delta t), & \text{for all other } j \text{ values} \end{cases} \tag{8}$$

where $j \in \Omega$.

$$\rho_{-\xi-(\xi-1)}(\Delta t) = \mu\Delta t + O(\Delta t), \tag{9}$$

where $\xi = 1,2,\dots,d$.

$$\rho_{-\xi-(\xi+1)}(\Delta t) = \left[\left((\xi+1)\sum_{P^{a(\xi+1)}} \prod_{l=1}^{\xi+1} \lambda_{u_l}\right) / \sum_{P^{a(\xi)}} \prod_{l=1}^{\xi} \lambda_{u_l}\right] \Delta t + O(\Delta t), \tag{10}$$

where $\sum_{P^{a(\xi)}}$ denotes the summation over $(u_1, u_2, \dots, u_\xi)$ of $(1,2,\dots,n)$ under the conditions $u_2 - u_1 \geq 2$, $u_3 - u_2 \geq 2$,…,$u_\xi - u_{\xi-1} \geq 2$ and $\xi = 1,2,\dots,d-1$. When $\xi = 1$, then

$$\sum_{P^{a(i)}} \prod_{l=1}^{\xi} \lambda_{u_l} = \sum_{u=1}^{n} \lambda_u \,. \tag{11}$$

$$\rho_{-\xi(\xi+1)}(\Delta t) = \left[\left(2\sum_{m=1}^{\xi} \sum_{P^{b(m)}} \prod_{l=1}^{\xi+1} \lambda_{u_l} + \sum_{m=1}^{\xi-1} \sum_{P^{c(m)}} \prod_{l=1}^{\xi+1} \lambda_{u_l}\right) \Big/ \sum_{P^{a(\xi)}} \prod_{l=1}^{\xi} \lambda_{u_l}\right] \Delta t + O(\Delta t), \tag{12}$$

where $\sum_{P^{b(m)}}$ and $\sum_{P^{c(m)}}$ denote the summation over $(u_1, u_2, \dots, u_{\xi+1})$ of $(1,2,\dots,n)$ under the conditions $u_2 - u_1 \geq 2$, $u_3 - u_2 \geq 2$,…,$u_m - u_{m-1} \geq 2$, $u_{m+1} - u_m = 1$, $u_{m+2} - u_{m+1} \geq 2$,…,$u_{\xi+1} - u_\xi \geq 2$ and $u_2 - u_1 \geq 2$, $u_3 - u_2 \geq 2$,…,$u_m - u_{m-1} \geq 2$, $u_{m+1} - u_m = 1$, $u_{m+2} - u_{m+1} = 1$, $u_{m+3} - u_{m+2} \geq 2$,…, $u_{\xi+1} - u_\xi \geq 2$ respectively. Also where, $\xi = 1,2,\dots,d$.

$$\rho_{-\xi-\xi}(\Delta t) = 1 - \left(\rho_{-\xi-(\xi+1)}(\Delta t) + \rho_{-\xi(\xi+1)}(\Delta t) + \mu\right)\Delta t + O(\Delta t), \tag{13}$$

where $\xi = 1,2,\dots,d$.

$$\rho_{-\xi j}(\Delta t) = O(\Delta t), j \neq -(\xi-1), -(\xi+1), \xi+1, -\xi \tag{14}$$

where $\xi, j \in \Omega$.

$$\rho_{\xi j}(\Delta t) = \begin{cases} a\mu\Delta t + O(\Delta t), & j = \xi - 1 \\ b\mu\Delta t + O(\Delta t), & j = -(\xi-1) \\ 1 - \mu\Delta t + O(\Delta t), & j = \xi \\ O(\Delta t), & \text{for all other } j \text{ values} \end{cases} \tag{15}$$

where

$$a = \frac{2\sum_{m=1}^{\xi-1} \sum_{P^{b(m)}} \prod_{l=1}^{\xi} \lambda_{u_l}}{(\xi-1)\left(\sum_{m=1}^{\xi-2} \sum_{P^{c(m)}} \prod_{l=1}^{\xi} \lambda_{u_l} + 2\sum_{m=1}^{\xi-1} \sum_{P^{b(m)}} \prod_{l=1}^{\xi} \lambda_{u_l}\right)} \tag{16}$$

and

$$b = \frac{2(\xi-2)\sum_{m=1}^{\xi-1} \sum_{P^{b(m)}} \prod_{l=1}^{\xi} \lambda_{u_l} + (\xi-1)\sum_{m=1}^{\xi-2} \sum_{P^{c(m)}} \prod_{l=1}^{\xi} \lambda_{u_l}}{2\sum_{m=1}^{\xi-1} \sum_{P^{b(m)}} \prod_{l=1}^{\xi} \lambda_{u_l} + (\xi-1)\left(\sum_{m=1}^{\xi-2} \sum_{P^{c(m)}} \prod_{l=1}^{\xi} \lambda_{u_l}\right)}. \tag{17}$$

And also where $\xi = 3,4, \dots, d$.

Using the equations listed above for the STP in the system, we can obtain Ψ as follows:

$$\Psi = (\psi_{\xi j}), \tag{18}$$

where Ψ is the transition rate matrix,

$$\psi_{\xi j} = \lim_{\Delta t \to 0} \frac{\rho_{\xi j}(\Delta t)}{\Delta t}, \tag{19}$$

and

$$\psi_{j} = \lim_{\Delta t \to 0} \frac{1 - \rho_{jj}(\Delta t)}{\Delta t}, \tag{20}$$

for $i, j \in \Omega$ and $i \neq j$.

3. MTTFF of the Repairable System

In the following, by using matrix Ψ, we shall obtain the MTTFF of the linear Rep-Con-2-*n*: F system with independent and identically distributed components under the condition that components have different life spans.

For $j \in \Omega$,

$$\rho_j(t) = P\{N(t) = j\}. \tag{21}$$

Then, we can obtain as follows [17]:

$$\rho_j'(t) = \sum_{k \in \Omega} \rho_k(t) \psi_{kj}, \tag{22}$$

for $\rho_0(0) = 1$, $\rho_j(0) = 0$, $j \neq 0$ and $j \in \Omega$. Let $P(t) = \left(\rho_0(t), \rho_{-1}(t), \dots, \rho_{-d}(t), \rho_2(t), \dots, \rho_{d+1}(t)\right)$.

Then, using matrix Ψ , we can rewrite the above expression in the matrix form as follows:

$$P'(t) = P(t)\Psi, \tag{23}$$

with the initial condition $P(0) = (1,0, \dots, 0)$.

To obtain the MTTFF of the system by using the Laplace transform of the system's reliability $R(t)$, we consider $\{N(t), t \geq 0\}$ as a $\{\widetilde{N}(t), t \geq 0\}$ with finite state space $\widetilde{\Omega} = \{0, -1, -2, \dots, -d\}$.

Now let

$$\tilde{\rho}_j(t) = P\{\widetilde{N}(t) = j\}, \tag{24}$$

for $j \in \widetilde{\Omega}$. Then, the system reliability is

$$R(t) = \sum_{j \in \widetilde{\Omega}} \tilde{\rho}_j(t). \tag{25}$$

For $R(t)$, the matrix Ψ is redefined as follows:

$$\widetilde{\Psi} = (\psi_{\xi j}), \tag{26}$$

for $\xi, j \in \widetilde{\Omega}$ and $\xi \neq j$.

Let $\tilde{P}(t) = \left(\tilde{\rho}_0(t), \tilde{\rho}_{-1}(t), \dots, \tilde{\rho}_{-d}(t)\right)$. Then, it is easy to derive the following systems of differential equations

$$\tilde{P}'(t) = \tilde{P}(t)\widetilde{\Psi}, \tag{27}$$

with the initial condition $\tilde{P}(0) = (1,0, \dots ,0)$.

Now, using the Laplace transform in (27), with the help of the initial condition $\tilde{P}(0)$ we have the Laplace transform of $R(t)$ as follows

$$R^*(s) = \sum_{j\in\widetilde{\Omega}} \tilde{\rho}_j^*(s). \tag{28}$$

Thus the MTTFF of the system is as follow

$$\text{MTTFF} = \log_{s\to 0} R^*(s). \tag{29}$$

4. Special Model: The Linear Rep-Con-2-5: F System

Based on previous definitions, we know $\Omega = \{-3, -2, -1, 0, 2, 3, 4\}$, $\mathrm{W} = \{-3, -2, -1, 0\}$ and $\mathrm{F} = \{2, 3, 4\}$ of the linear Rep-Con-2-n: F system.

The matrix Ψ is:

$$\Psi = \left(\psi_{\xi j}\right) = \begin{bmatrix} \psi_{(0,0)} & \psi_{(0,-1)} & 0 & 0 & 0 & 0 & 0 \\ \mu & \psi_{(-1,-1)} & \psi_{(-1,-2)} & 0 & \psi_{(-1,2)} & 0 & 0 \\ 0 & \mu & \psi_{(-2,-2)} & \psi_{(-2,-3)} & 0 & \psi_{(-2,3)} & 0 \\ 0 & 0 & \mu & \psi_{(-3,-3)} & 0 & 0 & \psi_{(-3,4)} \\ 0 & \mu & 0 & 0 & -\mu & 0 & 0 \\ 0 & 0 & \psi_{(3,-2)} & 0 & \psi_{(3,2)} & -\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu & -\mu \end{bmatrix} \tag{30}$$

where

$$\psi_{(0,0)} = -(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5),$$

$$\psi_{(0,-1)} = \lambda_5 + \lambda_4 + \lambda_3 + \lambda_2 + \lambda_1 ,$$

$$\psi_{(-1,-1)} = -\frac{2(\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_1\lambda_4+\lambda_1\lambda_5+\lambda_2\lambda_3+\lambda_2\lambda_4+\lambda_2\lambda_5+\lambda_3\lambda_4+\lambda_3\lambda_5+\lambda_4\lambda_5)}{\lambda_5+\lambda_4+\lambda_3+\lambda_2+\lambda_1} - \mu ,$$

$$\psi_{(-1,-2)} = \frac{2(\lambda_1\lambda_3+\lambda_1\lambda_4+\lambda_1\lambda_5+\lambda_2\lambda_4+\lambda_2\lambda_5+\lambda_3\lambda_5)}{\lambda_5+\lambda_4+\lambda_3+\lambda_2+\lambda_1},$$

$$\psi_{(-2,-2)} = -\mu - \frac{\lambda_1\lambda_2\lambda_3+\lambda_2\lambda_3\lambda_4+\lambda_3\lambda_4\lambda_5+2(\lambda_1\lambda_2\lambda_4+\lambda_1\lambda_2\lambda_5+\lambda_1\lambda_3\lambda_4+\lambda_1\lambda_4\lambda_5+\lambda_2\lambda_3\lambda_5+\lambda_2\lambda_4\lambda_5)+3\lambda_1\lambda_3\lambda_5}{\lambda_1\lambda_3+\lambda_1\lambda_4+\lambda_1\lambda_5+\lambda_2\lambda_4+\lambda_2\lambda_5+\lambda_3\lambda_5},$$

$$\psi_{(-2,-3)} = \frac{3\lambda_1\lambda_3\lambda_5}{\lambda_1\lambda_3+\lambda_1\lambda_4+\lambda_1\lambda_5+\lambda_2\lambda_4+\lambda_2\lambda_5+\lambda_3\lambda_5},$$

$$\psi_{(-3,-3)} = -(\lambda_2 + \lambda_4) - \mu ,$$

$$\psi_{(-1,2)} = \frac{2(\lambda_1\lambda_2+\lambda_2\lambda_3+\lambda_3\lambda_4+\lambda_4\lambda_5)}{\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5},$$

$$\psi_{(-2,3)} = \frac{\lambda_1\lambda_2\lambda_3+\lambda_2\lambda_3\lambda_4+\lambda_3\lambda_4\lambda_5+2(\lambda_1\lambda_2\lambda_4+\lambda_1\lambda_2\lambda_5+\lambda_1\lambda_3\lambda_4+\lambda_1\lambda_4\lambda_5+\lambda_2\lambda_3\lambda_5+\lambda_2\lambda_4\lambda_5)}{\lambda_1\lambda_3+\lambda_1\lambda_4+\lambda_1\lambda_5+\lambda_2\lambda_4+\lambda_2\lambda_5+\lambda_3\lambda_5},$$

$\psi_{(-3,4)} = \lambda_2 + \lambda_4$,

$\psi_{(3,-2)} = \frac{\lambda_1\lambda_2\lambda_4+\lambda_1\lambda_2\lambda_5+\lambda_2\lambda_3\lambda_5+\lambda_1\lambda_3\lambda_4+\lambda_1\lambda_4\lambda_5+\lambda_2\lambda_4\lambda_5}{\lambda_1\lambda_2\lambda_3+\lambda_2\lambda_3\lambda_4+\lambda_3\lambda_4\lambda_5+2(\lambda_1\lambda_2\lambda_4+\lambda_1\lambda_2\lambda_5+\lambda_2\lambda_3\lambda_5+\lambda_1\lambda_3\lambda_4+\lambda_1\lambda_4\lambda_5+\lambda_2\lambda_4\lambda_5)}\mu$

and

$\psi_{(3,2)} = \frac{\lambda_1\lambda_2\lambda_3+\lambda_2\lambda_3\lambda_4+\lambda_3\lambda_4\lambda_5+\lambda_1\lambda_2\lambda_4+\lambda_1\lambda_2\lambda_5+\lambda_2\lambda_3\lambda_5+\lambda_1\lambda_3\lambda_4+\lambda_1\lambda_4\lambda_5+\lambda_2\lambda_4\lambda_5}{\lambda_1\lambda_2\lambda_3+\lambda_2\lambda_3\lambda_4+\lambda_3\lambda_4\lambda_5+2(\lambda_1\lambda_2\lambda_4+\lambda_1\lambda_2\lambda_5+\lambda_2\lambda_3\lambda_5+\lambda_1\lambda_3\lambda_4+\lambda_1\lambda_4\lambda_5+\lambda_2\lambda_4\lambda_5)}\ \mu.$

In the following, we study MTTFF of the linear Rep-Con-2-5: F system consisting of components with unequal failure rates.

4.1. The Repairable System's MTTFF

Let $\tilde{P}(t) = \left(\tilde{\rho}_0(t), \tilde{\rho}_{-1}(t), \tilde{\rho}_{-2}(t), \tilde{\rho}_{-3}(t)\right)$. Then the Laplace transform on both sides of (27) is given by

$-1 + s\rho_0^*(s) = \psi_{(0,0)}\rho_0^*(s) + \mu\rho_{-1}^*(s),$

$s\rho_{-1}^*(s) = \psi_{(0,-1)}\rho_0^*(s) + \psi_{(-1,-1)}\rho_{-1}^*(s) + \mu\rho_{-2}^*(s),$

$s\rho_{-2}^*(s) = \psi_{(-1,-2)}\rho_{-1}^*(s) + \psi_{(-2,-2)}\rho_{-2}^*(s) + \mu\rho_{-3}^*(s),$

$s\rho_{-3}^*(s) = \psi_{(-2,-3)}\rho_{-2}^*(s) + \psi_{(-3,-3)}\rho_{-3}^*(s).$

With the initial condition $\tilde{P}(0) = (1,0,0,0)$. The solutions are

$\rho_{-3}^*(s) = \frac{\psi_{(-2,-3)}}{s-\psi_{(-3,-3)}}\rho_{-2}^*(s),$

$\rho_{-1}^*(s) = \left(\frac{s-\psi_{(-2,-2)}}{\psi_{(-1,-2)}} - \frac{\psi_{(-2,-3)}\mu}{\psi_{(-1,-2)}(s-\psi_{(-3,-3)})}\right)\rho_{-2}^*(s),$

$\rho_0^*(s) = \left(\frac{(s-\psi_{(-1,-1)})(s-\psi_{(-2,-2)})}{\psi_{(0,-1)}\psi_{(-1,-2)}} - \frac{(s-\psi_{(-1,-1)})\psi_{(-2,-3)}\mu}{\psi_{(0,-1)}\psi_{(-1,-2)}(s-\psi_{(-3,-3)})} - \frac{\mu}{\psi_{(0,-1)}}\right)\rho_{-2}^*(s),$

$\rho_{-2}^*(s) = \left(\frac{(s-\psi_{(0,0)})(s-\psi_{(-1,-1)})(s-\psi_{(-2,-2)})}{\psi_{(0,-1)}\psi_{(-1,-2)}} - \frac{(s-\psi_{(0,0)})(s-\psi_{(-1,-1)})\psi_{(-2,-3)}\mu}{\psi_{(0,-1)}\psi_{(-1,-2)}(s-\psi_{(-3,-3)})}\right.$

$\left.-\frac{(s-\psi_{(0,0)})\mu}{\psi_{(0,-1)}} - \frac{(s-\psi_{(-2,-2)})\mu}{\psi_{(-1,-2)}} + \frac{\psi_{(-2,-3)}\mu^2}{\psi_{(-1,-2)}(s-\psi_{(-3,-3)})}\right)^{-1}.$

Thus, for the linear Rep-Con-2-5: F system, from (28) and (29), the MTTFF is given by

$$\begin{aligned}\mathrm{MTTFF} = {}&\left[\psi_{(0,-1)}\psi_{(-1,-2)}\left(\psi_{(-2,-3)} - \psi_{(-3,-3)}\right) + \psi_{(-2,-2)}\psi_{(-3,-3)}\left(\psi_{(0,-1)} - \psi_{(-1,-1)}\right)\right.\\ &+ \left(\psi_{(-1,-1)}\psi_{(-2,-3)} + \psi_{(-1,-2)}\psi_{(-3,-3)}\right.\\ &\left.\left.- \psi_{(0,-1)}\psi_{(-2,-3)}\right)\mu\right]\left[\psi_{(0,0)}\psi_{(-1,-1)}\psi_{(-2,-2)}\psi_{(-3,-3)}\right.\\ &- \left(\psi_{(0,-1)}\psi_{(-2,-2)}\psi_{(-3,-3)} + \psi_{(0,0)}\psi_{(-1,-1)}\psi_{(-2,-3)}\right.\\ &\left.\left.+ \psi_{(0,0)}\psi_{(-1,-2)}\psi_{(-3,-3)}\right)\mu + \psi_{(0,-1)}\psi_{(-2,-3)}\mu^2\right]^{-1}\end{aligned} \tag{31}$$

Note that, if $\mu = 0$ and in the system, all components have equal failure probability, λ, we find the MTTFF to be $\frac{7}{10\lambda}$. This result is equal to equation (13) in [9]. The MTTFF under different values of μ and the parameters λ are shown in Table 1.

Table 1. MTTFF under different values of μ and the parameters λ

λ_1	λ_2	λ_3	λ_4	λ_5	μ	MTTFF
1	1	1	1	1	0	0.7000
					1	0.7722
					2	0.8585
					3	0.9538
					4	1.0555
					5	1.1615
2	2	2	2	2		0.4526
3	3	3	3	3		0.2762
4	4	4	4	4		0.1982
5	5	5	5	5		0.1544
5	1	1	1	1		0.7506
1	5	1	1	1		0.5808
1	1	5	1	1		0.6170
1	1	1	5	1		0.5808
1	1	1	1	5		0.7506
5	4	3	2	1		0.2712
					6	0.2808
					7	0.2907
					8	0.3009
					9	0.3112
					10	0.3217

5. Conclusions

In this paper, we have studied linear Rep-Con-2-n: F system. In the system, the components are assumed to have unequal failure rates. In particular, we have introduced mathematical formulations for the STP in the system. In [9, p.606], a clear formula of the coefficients a and b were not given. It was stated that only these coefficients should meet the following two conditions:

$$0 \leq a, b \leq 1 \quad \text{and} \quad a + b = 1 \tag{32}$$

In this paper, the explicit formulas of the coefficients a and b were obtained for the case that the components in the system have independent but non-identical exponential distributions. Equation for one performance characteristic for the system which is the MTTFF is developed when $n = 5$.

Author' Contributions

Authors contributed equally.

Statement of Conflicts of Interest

No potential conflict of interest was reported by the authors.

Statement of Research and Publication Ethics

The authors declare that this study complies with Research and Publication Ethics.

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