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Chaos synchronization in chaotic current modulated VCSELs by bidirectional coupling

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ABSTRACT This paper reports on the synchronization proprieties in bidirectional coupled current modulated vertical cavity surface-emitting lasers (CMVCSELs) based on the combined model of Danckaert et al.. Regular pulse packages and chaotic behaviors are found in CMVCSEL during the numerical results. The suitable coupling strength leading to high quality of synchronization is determined by numerical analysis. The consequence of the parameter mismatch and the duration of the synchronization process are also highlighted.

KEYWORDS
VCSEL
Modulation current
Bidirectional coupling
Synchronization

INTRODUCTION

Many researchers have proved that in certain conditions, current modulated vertical cavity surface-emitting lasers (VCSELs) are able to exhibit not only periodic and chaotic behaviors (Masoller *et al.* 2007; Valle *et al.* 2007; Mbé *et al.* 2010; Kingni *et al.* 2012). but also pulse packages (Mbé *et al.* 2010; Kingni *et al.* 2012; Tabaka *et al.* 2006). The compact light sources of chaotic VCSELs are desirable and can be used in chaos-based secure communications (Colet and Roy 1994). The fact of hiding a message carrying information in a noise and exploiting the synchronization of the both (receiver with the output) to recover the information signal constituting the idea of chaotic secure communications. Work on chaos synchronization has been demonstrated in several lasers, notably Nd: YAG (Roy and Thornburg Jr 1994), CO2 (Sugawara *et al.* 1994), fiber laser (Vanwiggeren and Roy 1998) and semiconductor edge-emitting lasers (Goedgebuer *et al.* 1998; Bindu and Nandakumaran 2000; Kouomou and Woafo 2003; Argyris *et al.* 2005; Kingni *et al.* 2020).

By contrast, it should be noted that studies remain scarce concerning VCSELs coupled with the synchronization of chaos (Takougang Kingni *et al.* 2012; Li *et al.* 2007; Sciamanna *et al.* 2007; Zhong *et al.* 2008; Xie *et al.* 2016; Wang *et al.* 2020; Roy *et al.* 2019). Many researches on the synchronization of chaos in coupled VCSELs found in the literature have been done using a complex mathematical model of VCSELs, mostly the Spin-Flip Model (SFM) (Li *et al.* 2007; Sciamanna *et al.* 2007; Zhong *et al.* 2008). According to the knowledge of the authors, the synchronization of chaos in coupled CMVCSELs based on the combined model of Danckaert et al. is scare (Takougang Kingni *et al.* 2012). In (Takougang Kingni *et al.* 2012), synchronization properties and communications of unidirectional coupled VCSELs based on the combined model of Danckaert et al. (Danckaert *et al.* 2002) and driven by chaotic oscillators with wide spectral frequency bandwidth has been studied numerically. The results showed that best quality synchronization was achieved and message transmission by using the chaos shift keying technique has been demonstrated.

The purpose of this article is to analyze the chaos synchronization in bidirectional coupled CMVCSELs described by the combined model of Danckaert et al. (Danckaert *et al.* 2002). The bidirectional coupling is used to achieve synchro-

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nization between chaotic coupled CMVCSELs due to the fact that it leads to high quality of synchronization and it is robust to the parameter mismatch. The paper is subdivided in three sections. Section 2 discusses the examination of chaos synchronization in two CMVCSELs by bidirectional coupling. Conclusion is given in section 3.

CHAOS SYNCHRONIZATION OF BIDIRECTIONAL COUPLED CMVCSELS

The system of two bidirectional coupled CMVCSELs based on the combined model of Danckaert et al. (Danckaert *et al.* 2002) is described by the following equations:

$$\frac{dP_{x,j}}{dt} = \left(\eta_j - \varepsilon_{xx}P_{x,j} - \varepsilon_{xy}P_{y,j}\right)P_{x,j} + \frac{R_{sp}}{2} \tag{1a}$$

$$\begin{aligned} \tfrac{dP_{y,j}}{dt} &= \left\{\eta_j + G\left[j\left(t\right)\right] - \varepsilon_{yy}P_{y,j} - \varepsilon_{yx}P_{x,j}\right\}P_{y,j} \\ &+ \tfrac{R_{sp}}{2} + k\left(P_{y,i\neq j} - P_{y,j}\right)\, H\left(t - T_0\right), \end{aligned} \tag{1b}$$

$$\begin{aligned} \tfrac{d\eta_j}{dt} &= \rho^{-1}\left[gj\left(t\right) - 1 - P_{x,j} - P_{y,j}\right] - \eta_j \\ &- \left(\eta_j - \varepsilon_{xx}P_{x,j} - \varepsilon_{xy}P_{y,j}\right)P_{x,j} \\ &- \left(\eta_j - \varepsilon_{yy}P_{y,j} - \varepsilon_{yx}P_{x,j}\right)P_{y,j} \end{aligned} \tag{1c}$$

where t, P_x, P_y and η are the time, the photon density in x and y polarization modes (PMs) and the carrier density, respectively. The parameters $\varepsilon_{xx} = 4$ and $\varepsilon_{yy} = 4$ are the self-gain saturation coefficients while the parameters $\varepsilon_{xyx} = 8$ and $\varepsilon_{yx} = 8$ are the cross-gain saturation coefficients. The parameter $R_{sp} = 0.001$ is the mean of the spontaneous emission above threshold and the parameter $\rho = 0.001$ is the ratio of photon lifetime $\tau_p = 1\,ps$ to carrier lifetime $\tau_c = 1\,ns$. The modulation current is $j(t) = j_{dc} + j_m \sin(2\pi f_m \tau_c t)$, j_{dc} is the dc bias current,j_m is the modulation amplitude and f_m is the modulation frequency. The parameter $G\left[j(t)\right] = g\left[1 - j\left(t\right)/j_{sw}\right]$ is the relative gain difference between the two modes, the parameter $j_{sw} = 0.15$ is the switching current and the parameter $g = 10$ is a positive coefficient. The index i and j represent the VCSEL number (i, $j \in \{1, 2\}$). The parameter K is the coupling strength; the parameter T_0 is the onset of synchronization time process and the the Heaviside function $H\left(t - T_0\right)$ is defined as:

$$H\left(t - T_0\right) = \begin{cases} 0 & for\, t \prec T_0 \\ 1 & for\ t\ \geq T_0 \end{cases}. \tag{2}$$

The uncoupled CMVCSEL can exhibit regular pulse packages and chaotic attractors as shown in Fig. 1.

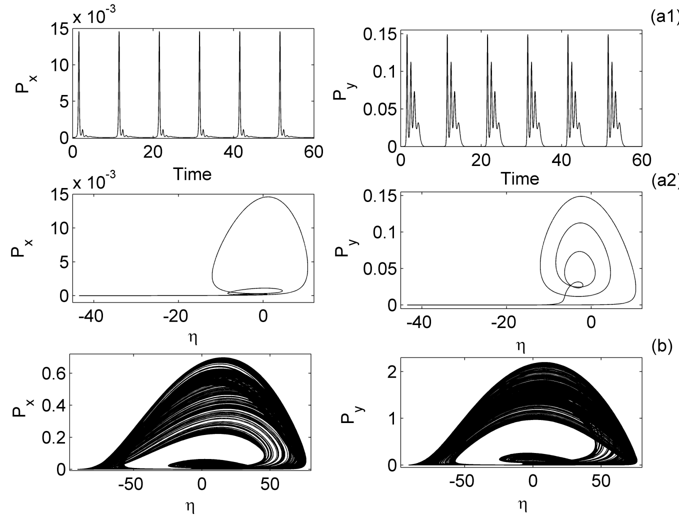

Figure 1 Pulse packages and chaotic attractors for given values of parameters j_{dc}, j_m, f_m: (a) $j_{dc} = 0.1$, $j_m = 0.005$, $f_m = 100\,MHz$ and (b) $j_{dc} = 0.12$, $j_m = 0.065$, $f_m = 3.2\,GHz$. The initial conditions are $\left(P_x\left(0\right), P_y\left(0\right), \eta\left(0\right)\right) = \left(0.01, 0.001, 0.1\right)$.

The photon densities display regular pulse packages in Fig. 1 (a) while in Fig. 1 (b) they exhibit chaotic attractors.

It is firstly assumed the case where the two coupled VCSELs are identical but with different initial conditions: $\left(P_{x1}\left(0\right), P_{y1}\left(0\right), \eta_1\left(0\right)\right) = \left(0.01, 0.001, 0.1\right)$ and $\left(P_{x2}\left(0\right), P_{y2}\left(0\right), \eta_2\left(0\right)\right) = \left(0.011, 0.001, 0.1\right)$. This means that the two VCSELs have the same threshold current, output power and relaxation oscillation frequency. In Figure 2, the higher synchronization error of both PMs as a function of coupling strength K in the chaotic regime are displayed.

In Fig. 2, when the maximal synchronization error of the absolute value of $\left(P_{x1,y1} - P_{x2,y2}\right)$ becomes equal to zero, this means that the two VCSELs are in a chaotic synchronization. This appears for $K \geq 1.33$ as shown in Fig. 2. The synchronization diagrams of photon densities of two coupled VCSELs are depicted in Fig. 3 in order to further emphasize the different synchronization properties found in Fig. 2.

For the coupling strength $K = 1$, there is no chaos synchronization between chaotic coupled CMVCSELs as seen in Fig. 3 (a) whereas in Fig. 3 (b) for $K = 1.4$, it is clear that CMVCSELs are well synchronized.

However, the high quality of synchronization mentioned here can only be achieved for the ideal condition. Regarding applications, parameters such as mismatch of device parameters, noise, coupling asymmetry, different bias current, etc. Moreover in a physical system, the parameters cannot remain constant in the course of its utilization. It may fluctuate due to the internal instabilities of the system or due to the perturbations from the environment. Fluctuations introduce parametric mismatches in coupled systems. Hence,

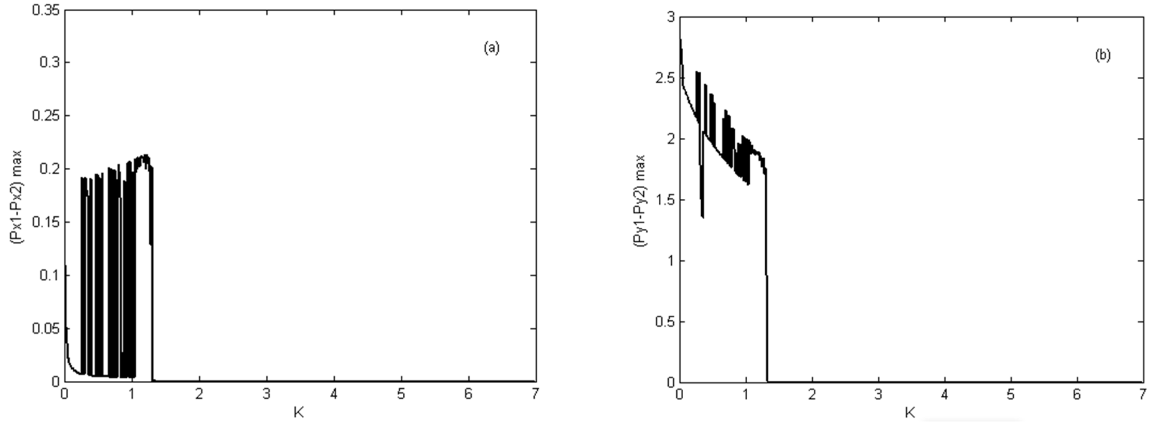

Figure 2 Variation of the maximal synchronization error of x-PM (a) and y-PM (b) versus the coupling strength K for $j_{dc} = 0.12$, $j_m = 0.065$, and $f_m = 3.2\,GHz$.

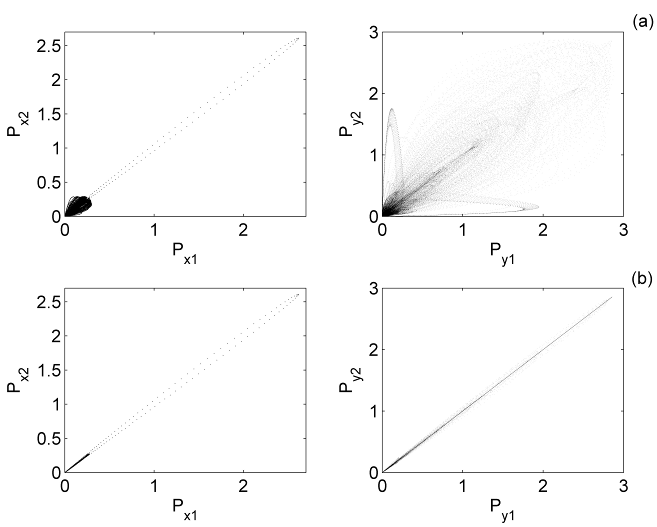

Figure 3 Synchronization diagrams for some values of the coupling strength : (a) $K = 1$ (b) $K = 1.4$ The initial conditions are $(P_{x1}(0), P_{y1}(0), \eta_1(0)) = (0.01, 0.001, 0.1)$ and $(P_{x2}(0), P_{y2}(0), \eta_2(0)) = (0.011, 0.001, 0.1)$.

it is relevant to check the robustness of synchronization in an environment where the parameters fluctuate. To analyze the influence of parameter mismatch, it is assumed that the parameters of VCSEL 2 are varied following the general rule:

$$a_2 = a_1 [\alpha\,\% (2\xi - 1) + 1], \tag{3}$$

where ξ is a random number, a_1 is the parameter of the VCSEL 1 which in this case coincides with those used in Fig. 2, a_2 corresponds to the parameters of the VCSEL 2 and $\alpha\,\%$ is the percentage of parameter mismatch. By using this variation, the maximal synchronization error versus the coupling strength K for different percentages of parameter mismatch is plotted in Fig. 4.

Figure 4 shows that the maximal synchronization error of both PMs effectively increases with the parameter mismatch. Chaos synchronization is lost for a parameter mismatch of 1%. A severe degradation of synchronization is noticed in x-PM above 1% (see Fig. 4 (a) than in y-PM (see Fig. 4 (b)).

The synchronization time is the duration from the launching of the synchronization process to the time where the synchronization is attained. In secure communication technologies, the synchronization time plays a central role since the range of time during which the chaotic VCSELs are not synchronized corresponds to the range of time during which the coded message can unfortunately not be recovered or sent. This loss of information can prove to be damaging in some circumstances. Hence, it clearly appears that T_{syn} has to be minimized, so that the chaotic VCSELs synchronize as fast as possible. The synchronization time is given as (Woafo and Kraenkel 2002):

$$T_{syn} = t_{syn} - T_0, \tag{4}$$

where t_{syn} is the time instant at which the trajectories of VCSEL 1 and VCSEL 2 are close enough to be considered as synchronized. Here, synchronization is achieved when the deviation $[\varepsilon_1$ obeys the following synchronization criterion:

$$\varepsilon_1 = |P_{x1,y1} - P_{x2,y2}| \prec h, \ \forall t \succ t_{syn}, \tag{5}$$

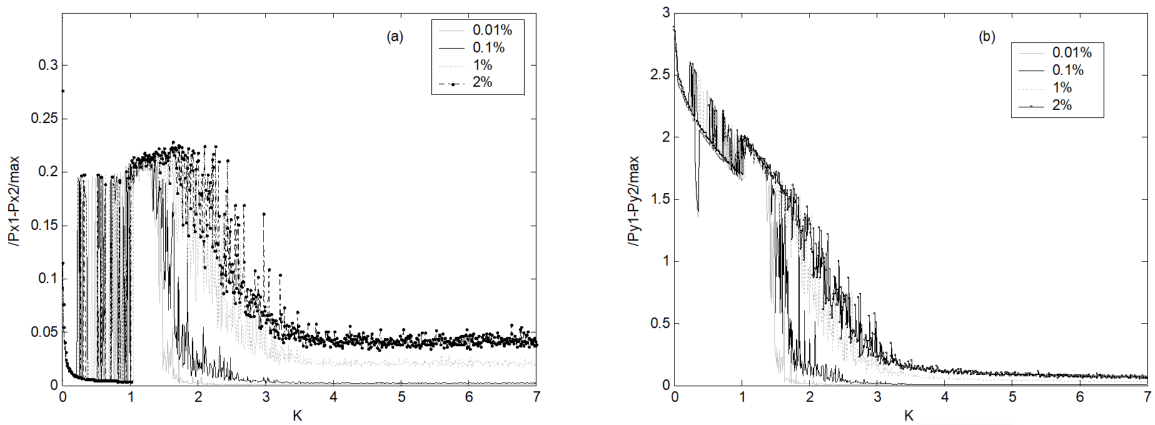

Figure 4 Variation of the maximal synchronization error of x-PM (a) and y-PM (b) versus the coupling strength K for different percentages of parameter mismatch.

where h is the synchronization precision or tolerance. The parameter T_{syn} is plotted versus the parameter K in Fig. 5.

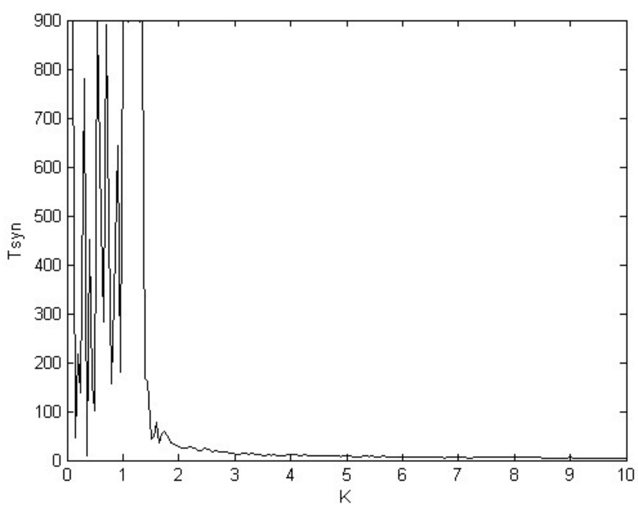

Figure 5 Synchronization time (T_{syn}) versus the coupling strength K in coupled CMVCSELs in the chaotic regime for synchronization precision $h = 10^{-6}$ and $T_0 = 100$.

It is noticed that for T_{syn} very close to the synchronization boundaries, its value is very large, however the coupling strength K approaches the limits, then T_{syn} decreases and for large K, it reaches a limit value of approximately about 4.0. Fig. 5 also shows that very large K values are not necessary to ensure the synchronization with approximately the minimum T_{syn}.

CONCLUSION

In this paper, the synchronization of two chaotic current modulated vertical cavity surface-emitting lasers based on the combined model of Danckaert et al. was carried out through a bidirectional coupling. A robust and quasi-perfect synchronization were found for a specific range of coupling strength. The quality of synchronization was influenced by parameter mismatch and it was found a severe degradation of synchronization for a parameter mismatch equal and above 1%. An asymptotic minimal value of the synchronization time was reached.

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CONFLICTS OF INTEREST

The authors declare that there is no conflict of interest regarding the publication of this paper.

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