International Journal of Contemporary Educational Research (IJCER)

www.ijcer.net

Radical versus Social Constructivism: An Epistemological-Pedagogical Dilemma

Shashidhar Belbase[1]
[1]University of Wyoming, Laramie, Wyoming, USA

To cite this article:

Belbase, S. (2014). Radical versus social constructivism: An epistemological-pedagogical dilemma. *International Journal of Contemporary Educational Research, 1*(2), 98-112.

International Journal of Contemporary Educational Research
Volume 1, Number 2, July 2014, Page 98-112 ISSN: 2148-3868

Radical versus Social Constructivism: An Epistemological-Pedagogical Dilemma

Shashidhar Belbase*
University of Wyoming, Laramie, Wyoming, USA

Abstract

In this paper, the author has discussed the epistemological and the pedagogical dilemma he faced in the past and that he is still facing within radical and social constructivist paradigms. He built up an understanding of radical constructivism from the works of Ernst von Glasersfeld and social constructivism from the works of Paul Ernest. He introduced the notion of constructivism including both radical constructivism and social constructivism in brief. Then he reconceptualized these forms of constructivism in terms of epistemological and pedagogical motivation leading to a dilemma. He emphasized how the dilemma within these paradigms might impact one's actions and how resolving this dilemma leads to eclecticism. He summarized that one paradigm world does not function well in the context of teaching and learning of mathematics (and science). Finally, he concluded the dilemma issue with epistemological and pedagogical eclecticism.

Keywords: Radical Constructivism, Social Constructivism, Epistemological-pedagogical Dilemma, Epistemological Eclecticism, Pedagogical Eclecticism

Introduction

Constructivism in mathematics (and science) education is a very popular term. However, it is understood and used by different scholars in different ways. What is constructivism? Why constructivism is radical or social? Does it matter if it is radical or social? These questions encompass some degree of epistemological and pedagogical dilemma facing by this author. From the time when this author inclined toward constructivism in terms of epistemology and pedagogy, the dilemma of radical or social constructivism has significantly impacted his thinking, believing, and acting as a student, teacher, teacher educator, and researcher of mathematics (and science) education. To him, each method or approach of research, teaching and learning mathematics (and science) has a backdrop to a philosophy and theory of learning and knowing from the time of Socrates and even before. Different approaches of teaching mandate different ways of learning and knowing by students and vice versa. Different forms of constructivism, including radical and social constructivism, have influenced the epistemology and methodology of research and also pedagogy of practice. Therefore, '*How students learn and know mathematics?*' is an area of interest to many mathematics education researchers (Ernest, 2010; Noddings, 1990; von Glasersfeld, 1995), especially who focus on constructivist worldview.

Many philosophers and scholars contributed to the theory, epistemology, and philosophy in general (e.g., Ernest, 1991 & 1998; Steffe, 1995; Steffe & Thompson, 2000; von Glasersfeld, 1995) and subsequently, they impacted teaching and learning mathematics (and science) in particular. However, these impacts have not been realized widely beyond research and scholarly publications. It seems that the lives in the classrooms today in different parts of the world seriously lack the conditions of meaningful learning and teaching of mathematics (and science). This author's epistemological and pedagogical dilemma is a consequence of such meager educational foundations and pedagogical practices of the schools and colleges where he spent a significant amount of time as a student and teacher. Many students do not have the opportunity to construct mathematics (and science) knowledge for themselves. They are forced to rote learn facts, formulas, and procedures.

The argument in this paper is based on social constructivism (Ernest, 1991, 1995, 1998) and radical constructivism (von Glasersfeld, 1985, 1995, 1996) in relation to how these epistemological and philosophical

* Corresponding Author: Shashidhar Belbase, *belbaseshashi@gmail.com*

perspectives might influence approaches and methods of research, nature of teaching, and learning of mathematics (and science). At first, the author introduced the notion of constructivism, radical constructivism, and social constructivism in brief. He reconceptualized these forms of constructivism in terms of epistemological and pedagogical motivation leading to epistemological and pedagogical dilemma. Finally, he concluded the dilemma issue with epistemological and pedagogical eclecticism. The author draws some of the ideas in this paper from his manuscript in an online version in a clearinghouse (Please see Belbase, 2011).

Constructivism

It seems imperative to introduce the notion of constructivism in epistemology, philosophy, and theory of learning. Constructivism is a system of beliefs (worldview) in which the construction of knowledge or process of knowing is compared metaphorically with the construction of a building or furniture or an artifact (Ernest, 2010). The act of construction depends on what tools an individual already has. The tools are prior conceptions of the world through experiences. An individual may construct knowledge of something based on what he or she already knows about it and how he or she reconceptualizes the new experiences based on earlier experiences. The process of knowing is related to one's cognitive, affective, psychomotor, mental, and metacognitive responses to the change within those conceptions (von Glasersfeld, 1995). Hence, one's construction of knowledge stands on what he or she already has in the form of prior knowledge and conception, and how the new experiences adapt to the new conceptions, schemes, or actions (Steffe & Thompson, 2000).

The role of teacher is a facilitator or guide for the students through authentic situations, settings, tasks, and assessments (Christie, 2005). He or she creates the classroom as a learning community (Bielaczyc & Collins, 1999). The role of students is to be cognitive beings who learn through active and constructive engagement in tasks, situations, interactions, and problems. The students become active and creative members of the learning community and contribute in each other's learning (Cooperstein & Kocevar-Weidinger, 2004). The curriculum is flexible, adaptive, and supportive to create learning environment. It is a guide for modeling better teaching-learning environment based on students' cognitive ability and developmental stages (Confrey, 1990). The pedagogy is beyond just teaching and learning. It is a process of caring each other (Hackenberg, 2010). It is a process of connecting self with others and the environment. It is a process of raising awareness and wisdom in the students and also in the teachers (Richardson, 2003). Pedagogy is not a way of preaching by teacher for the students. Constructivist pedagogy dissolves in the process, action, interaction, and is mediated between different states of being with time, space, characters, and actions (Hatfield, 2013).

Constructivism has many faces- trivial, constructionist, cultural and social, radical, critical, feminist, and postmodern constructivism. One can be a trivial constructivist or a constructionist. Others can be critical or feminist or postmodern constructivist in terms of how they practice teaching and learning mathematics and how they conceive and implement the respective epistemologies. The intent of this paper is to highlight radical and social constructivism as competing paradigms for research and pedagogies for education. Hence, this paper focuses on radical and social constructivism as the two competing paradigms as most debated, contested, and used in research, teaching, and learning mathematics (and science).

Radical constructivism

Radical constructivism has been a major philosophical and psychological theory in mathematics and science education. Ernst von Glasersfeld and others have applied this philosophy and theory in research in mathematics and science education, especially in teaching experiments. This philosophical paradigm stands on two basic principles:

Knowledge is not passively received either through the senses or by way of communication, but it is actively built up by the cognizing subject. The function of cognition is adaptive and serves the subject's organization of the experiential world, not the discovery of an objective ontological reality. (von Glasersfeld, 1996, p. 2).

These two principles founded the basis of radical constructivism as an epistemology in research and pedagogy in teaching and learning of mathematics (and science). The paradigm of radical constructivism assumes that the mind is like an organism undergoing through an evolutionary process (Wuketits, 1984). The metaphor of the evolved mind, to this author, is the cognitive re-construction of the experiential world that continues adaptation to better and clearer conceptual or mental percepts or schemes through reorganization of this world (Ernest, 1995; von Glasersfeld, 1995). The mind is like an organism that undergoes continuous evolution, analogous to Darwin's theory of natural selection. The mental process takes the path toward more favorable thinking,

believing, and acting for a better survival, existence, and power. Here, the selection process is governed by the adaptation of the mind to the experiential world. The metaphor of the world is the subject's experiential world consisting of schemes, perceptions, values, and knowledge. The Newtonian absolute space is rejected and is replaced by the subjective world of the individual (Ernest, 1995). There might be an objective reality as absolute 'REAL', but there is no way to know it meticulously (von Glasersfeld, 1995). Whatever we claim as reality that we know about the world are mere subjective experiential world(s). The construction of meaning out of the world is an individual mental process. There might be a mediation of social and cultural activities in the process of knowing about the world. More viable cognitive processes are adapted into the mind, whereas counterproductive processes are suppressed and eliminated with time.

A radical constructivist teacher may adapt differentiated instruction based on students' cognitive, affective, and developmental stage. He or she uses creative and constructive situations to present, discuss, test, decide, and apply a model in problem solving. He or she tries to evoke students' interest to the subject matter and the context (von Glasersfeld, 2001). Students build their concepts of what they learn through active cognitive and adaptive process. They embrace the reflective and reflexive thinking and reasoning about content, process, and product (Leo, 1990). However, radical constructivism is not beyond limitations. These limitations are related to social and cultural adaptation of knowledge and knowing. The role of language and interactions among peers or community of practice has not been well conceived in this paradigm. The excessive focus on individual process of knowing and constructing knowledge has created a ground for dilemma. This process led this author toward adaptation of social constructivism.

Social constructivism

Ernest (2010, p. 43) stated, "Social constructivism regards individual learners and the reality of the social as indissolubly interconnected." That means social constructivism identifies individual efforts in learning or knowing in relation to the social context. He further asserted, "Human beings are formed through their interactions with each other as well as by their individual processes. Thus, there is no underlying model for the socially isolated individual mind" (p. 43). This view reflects back to Vygotsky's (1930, 1978) 'mind and society' and 'mind in society'. Social constructivism is the paradigm in which the metaphor of mind is like a connected network of self and others (Vygotsky, 1978). An interpersonal communication and interaction plays a significant cognitive role. The mind is seen not only in an individual context, but it is expanded to a broader social and cultural context, and construction of meaning is considered as a social phenomenon. The role of the individual mind in the construction of meaning is valued in a broader context in relation to others (persons and the environment). The mind constitutes a social entity which creates meaning through conversation, dialogue, interaction, and social and cultural exchanges of ideas (Ernest, 1995). The metaphor of the world is associated with socially constructed world. The individual world is a part of the collective social world. It assumes that there is no isolated individual reality far from socially and culturally constructed world. The metaphor of the world is like a socially and culturally connected experiential world. It does not deny the existence of absolute reality out of a shared world. However, social constructivism does not discuss the nature of reality out of a shared social and cultural world. Personal experiences of individuals become social and collective experiences when they are shared, interacted, transmitted, reconstructed, and retained as knowledge. Therefore, knowledge of the world is constructed out of shared experiences either from the society and culture and/or from the physical world. Even the physical world is interpreted in the context of society and culture. Teaching and learning of mathematics are then tied to the social responsibilities and values (Ernest, 1991 & 1998; Wilding-Martin, 2011).

The dynamic interplay of these metaphors (of the mind and the world) helps us to understand the multiplicities of epistemological and philosophical bases and interpretations of mind and body, being and existing, performing and acting, and relating and connecting things in the natural, social, and cultural arena. This author thinks that the metaphor of the mind and the world is helpful to understand the ontological, epistemological, methodological, and axiological standpoints of an individual or group. The role of teacher is to create constructive interactive environment for students to learn from discussion and peer or group work. Students construct knowledge through interaction in the class and out of class. They play an active role in learning and constructing knowledge through participation, negotiation, and shared values.

Constructivists believe that knowledge is constructed personally and socially based upon experiences. Noddings further asks "What has the assumption to do with judging the status of the general knowledge claim? How do we judge when one knows and when s/he does not?" (Noddigs, 1990, p. 11). To the author, these questions are significant indicators of epistemological and pedagogical dilemma incurred by radical and social constructivism.

The role of teachers, students, curriculum, and pedagogy needs further clarification. It seems that social constructivism lays excessive focus on the language game and interaction as means of constructing knowledge and knowing. The language and interactions can be means of knowledge and knowing, but they do not clearly state what happens in the mind and brain in terms of schemes, perceptions, values, and knowledge. Excessive dominance of social and cultural phenomena has a danger of limiting mental and brain interactions in one's creative efforts. Every knowledge in the world were initiated somewhere by somebody at a time and later it became shared knowledge. These issues influenced the author's ability in judging and distinguishing social and radical constructivist epistemologies and pedagogies through reflective and reflexive interplay within his experiences as a student and teacher. At this point, the author would like to go back and elaborate how he reconceptualized radical constructivism.

Reconceptualizing Radical Constructivism

The author's initial stage of teaching and learning mathematics (and science) began from behaviorism. This kind of practice focused heavily on realist-traditionalist approach of teaching, learning, and even doing research (Belbase, 2006; Belbase, Luitel, & Taylor, 2008). Following reflection on how he learned mathematics as a student at the school level shows a glimpse of this trend:

My learning of arithmetic was limited to numbers and purely bookish numbers, my algebra was limited to variables in the exercises and it never exemplified social and cultural issues. …. Mathematics should serve as an eye of the society in seeing the world, it should be a tool of society for analyzing the social justice and equity, and it should be a way of living, the practice of democracy starting in our classrooms. (Belbase, 2006, p. 66)

The sense of disgraceful classroom practices of mathematics led to his understanding of mathematics as a difficult, decontextualized, and disengaging subject irrelevant in day-to-day life (Luitel, 2009). The bookish knowledge did not relate to any of the practical aspects in social and cultural life. Learning meant simply reproducing the content knowledge even without knowing the meaning of what we did, how we did, and why we did the way we did in mathematics. The context of learning mathematics also influenced this author's teaching of mathematics at the early days of teaching career. Following reflection shows that his teaching was a kind of preaching of mathematical procedures to the students:

I asked Rupak about the day's lesson. He said that it was to start values of trigonometric ratios of standard angles. I made a chart on the board for the values of 0, 30, 45, 60 and 90 degrees of Sine, Cosine, Tangent, Cosecant, Secant and Cotangent ratios in a tabular form. …. When all the students finished their writing, I told them to read silently the values of trigonometric ratios for ten minutes. I moved front and back in the class while they were reading the values from the table. After ten minutes, I asked them to stop reading and be ready to reproduce. (Belbase, 2006, pp. 139-140)

The author reproduced the same notion of teaching as he was engaged in learning while he was a student. He taught mathematics the way he was taught when he was a student. He engaged students in the same process of rote memorization and reproduction of content and in many cases, students might not even know what they were doing and why they were doing. This scenario was how his personal journey of teaching began at the earlier stage of his teaching career. The epistemological and pedagogical significance of these reflections lies within the historical development of this author's personal philosophy of teaching and learning of mathematics from traditionalism to constructivism (Belbase, 2006).

This author participated in different professional development short courses offered by the Ministry of Education of Nepal and different degree courses at universities. These professional development trainings and education led him to develop his personal epistemology and pedagogy of mathematics (and science) oriented toward constructivism in general and radical constructivism in particular. He came to know that radical constructivism as a viable epistemology and pedagogy in mathematics (and science) education. He had a sense of realization that students can construct their mathematics (science) knowledge through individual practice, conceptualization, schematization, and creation of a model of their own learning path. The very notion of active construction, but not the transfer of knowledge, motivated this author toward radical constructivism as a viable philosophical and pedagogical practice.

A motivation toward radical constructivism

The epistemology and theory of radical constructivism have a backdrop on Piaget's constructivist theory of knowing. Von Glasersfeld (1995) clarified how he interpreted Piaget's constructivism and came up with the landmark epistemology of radical constructivism. He worked on the theory of schemes, perturbations, accommodations, and equilibrium. He summarized Piaget's learning theory, *"That cognitive change and learning in a specific direction takes place when a scheme, instead of producing the expected result, leads to perturbation, and perturbation, in turn, to accommodation that maintains or re-establishes equilibrium"* (von Glasersfeld, 1995, p. 68). The condition of perturbation is similar to von Glasersfeld's idea of subjective experiential constraint. These perturbations or constraints are associated with Vygotsky's (1978) zone of proximal development (ZPD). This stage could be the zone within which the learner feels a gap between what he or she knows and what is his or her potential to know. Because of these zones, an individual feels that his or her existing experience contradicts with new experiences and develops state of readiness for learning (by resolving the tensions between existing scheme and a new scheme). It seems that this readiness is associated with Steffe's idea of epistemic students. "Epistemic students are dynamic organizations of schemes of action and operation in the researcher's or the teacher's or students' mental life" (Steffe, n. d., p. 17; Steffe, 2011, p. 21). Steffe further clarifies that the schemes of actions and operations include accommodations in the schemes. This reorganization of the experience in terms of schemes and operations relates to knowledge and knowing in mathematics (and science) (Steffe, Moore, Hatfield, & Belbase, 2014).

A constructivist epistemology deals with what knowledge is, and from where it comes (Von Glasersfeld, 1991). Von Glasersfeld further claims that "the constructivist theory does not fit the conceptual patterns of traditional epistemology, precisely because it posits a different relation between knowledge and that 'real' outside world" (p. 170). Knowing is not about what is real in the world, but it is what one can conceptualize about the world. Von Glasersfeld (1989) stated, "Knowledge is not passively received, but actively built up by the cognizing subject" (p. 162). An individual actively builds up knowledge through reorganization of his or her mental state and brain state through a functional relation (Belbase, 2013). This view relates to what Ernest (1995) accepts that constructivism well acknowledges that knowing is active; it is an individual mental process that goes on while knowing, and it is a personal phenomenon. Largely knowing is based on previously constructed experiential knowledge. Ernest (1995) further points to politics and states that it has to do with giving respect to those positions with which we disagree. Ernest (1995) claims that radical constructivism values multifaceted pedagogy with its heart being sensitive to *individual construction*. This view has a great significance in pedagogy although he seems sceptic to the individual nature of construction, and he purports his arguments in favour of *social construction*.

A radical constructivist teacher does not assume his authoritative role in class. Rather, he or she brings democratic ideals in the classroom, providing enough opportunities to the students to learn from their participation in various activities. These activities relate to creating new experiences or re-adjusting prior experiences while constructing knowledge by students. The teacher considers that his or her role in the class is like a facilitator or a guide to the students (Belbase, 2011). He or she considers the students' active role in learning and creates such environment in which students feel free to learn at their pace, ability, and interest. The teacher acknowledges the role of students as co-authors or co-researchers or co-teachers in the class. Learning does not mean to just assimilating with new information, but it also involves constructing meaning and making sense of what they do, how they do and the why they do the way they do. Then the focus is on thinking and the meanings attributed to experience by the learners (Hein, 1991). Learning is an active process that involves the construction of meaning, and it is a mental process at an abstraction level and also a brain process at the physical bodily level. Other characteristics of constructivist learning are- "learning involves language; learning is a social activity; learning is contextual, and learning takes time" (Hein, 1991, subheading principles of learning, n. p.). The notion of social activity and context for learning are similar to what Ernest (1995) claims. He states that "all knowledge being constructed by the individual (learner) on the basis of his or her cognitive processes in dialogue with his or her experiential world" (Ernest, 1995, p. 14). This notion clearly means that learning is a self-cognition and re-organization of one's experiences while adapting to the social, cultural, and the natural environment. It seems that radical constructivism considers role of teacher and students as collaborators and the social, cultural and natural environment serve as a background for their (students') construction of knowledge.

The process of cognizing schemes, operations, and actions seems to be a progressive one since it always goes from simple to complex in an ordinary situation, whereas, in some cases, it may flow in any direction depending upon the maturity of the learner, self-directed goal of learning, and complexity in the learning environment. In this context, the author would like to emphasize what Noddings (1990) states about critical imagination in teaching and learning:

The great strength of constructivism is that it leads us to think critically and imaginatively about the teaching-learning process. Believing the premise of constructivism, we no longer look for simple solutions, and we have a powerful set of criteria by which to judge our possible choices of teaching method. (P. 18)

Noddings' statement about the constructivist approach takes us to think beyond the actual teaching and learning process, and to consider creative, critical, and imaginative thinking as overarching themes of constructivist pedagogy that Ernest (1995) emphasized. To this author, these qualities of constructivist teaching and learning are the things that he can imagine about any ideal method of teaching-learning that focuses on students' productive and constructive engagement in mathematics (and science) rather than teacher's imposition of contents to their brain or mind.

Radical constructivism, a person who is in favor of this epistemology, neither discards nor accepts the existence of an ideal equation that can truly represent the context (e.g., population growth/change of a species at a place). There is no way to know such ideal equation from the experiential world even if it exists. There is a limitation of human experience to know the ultimate reality. An equation or model can be viable representation of such reality in the world. Nonetheless, such equation may not portray an absolute representation. Hence, viability is a way to judge the usefulness and rationality of knowledge by "fitting within or sliding between its constraints" (Hardy & Taylor, 1997, p. 137).

Ernst von Glasersfeld's theory of radical constructivism considers two ways to look at viability as a legitimate process to establish knowledge. He draws the idea of viability from Piaget. He states that:

Piaget's theory of cognition involves two kinds of 'viability' and, therefore, two kinds of instrumentalism. One is at the sensory-motor level in which viable action schemes are instrumental in helping organisms to achieve goals through sensory equilibrium and survival in their interaction with the world they experience. The next is at the level of reflective abstraction. (von Glasersfeld, 1995, p. 68)

His second principle of radical constructivism assumes these viable functions at the sensory and abstract level to modify and accommodate with the new or existing mental schema or constructs in the mind. While doing this, we try to overcome the conceptual (or schematic) obstacle or constraint that may come to our experience. Our effort to overcome these cognitive, experiential constraints leads us to learning and accommodating new schemes. Then, reflective and reflexive thinking as a cognitive process (of thought experiments) becomes a major part in the construction of new knowledge. That means thought experiment can be a helpful tool to judge the value and applicability of the model. "Insofar as their results can be applied and lead to viable outcomes in practice, thought experiments constitute what is perhaps the most powerful learning procedure in the cognitive domain" (von Glasersfeld, 1995, p. 69). That means radical constructivists can begin making such model at conceptual level and experiment in the real world to judge whether the model produces desired outcome or not. The modification and recreation of such models and schemes continue forever in a teacher's life. This author thinks that viability is associated with establishing the usefulness of methods and findings of such procedures as alternatives to the traditional criteria of objective truth (or Truth). Ernest (1995) further indicates that "one's representations of the world and other human beings are personal and idiosyncratic" (p. 14). For him, "such a view makes it hard to establish a social basis for interpersonal communication, for shared feelings and concerns, let alone for shared values" (Ernest, 1995, p. 14).

The notion of subjective knowing and learning puts this author into a state of epistemological dilemma. How does an individual (if isolated from the social and cultural arena) construct knowledge? What tools are available to him or her? How does the individual knowledge contribute in the broader collective or shared knowledge? These questions led this author further toward epistemological dilemma in radical constructivism.

Epistemological dilemma in radical constructivism

This author thinks that radical constructivism does not explicitly take account of social interaction. There is no explicit discussion about the role of social interaction in the construction of meaning and self-adaptation of mental schemas while shaping one's knowing or learning from the experiential world. This view does not mean that it does not take account of social interaction at all. This author further thinks that there can be various ways to look at the issue. One of them is the role of language in the construction of knowledge and learning. Language acts through semiotics in construction of meaning and learning. Radical constructivism shares some aspects of the language game through semiotics (Uden, Liu, & Shank, 2001) through which a child makes meaning of any object or event is a social process. The semiotics perspective claims that knowledge construction is mediated through signs, but these signs are again schematized within mind and brain through active cognition of an individual. Without such cognition, there is no meaning of a sign or symbol. The signs are

visual, auditory or any other sensual forms that can be perceived by the brain and mind in terms of specific constructs (not just communicative structures). The semiotics within a language game is also a part of such cognitive and adaptive function toward learning.

There is interaction of a child with an object, or text, or any artifact through semiotics. Semiotics clearly takes meaning from social interaction in a direct way (face to face communication) or in an indirect way (communication through artifacts). That means social interaction is a necessary condition for learning and making sense of what one learns, but to this author, it is not a sufficient condition for learning. The active participation in community of practice brings individuals into a common platform to share each other's knowledge, ideas, and theories. After sharing they might go through some modifications if necessary. The shared knowledge, ideas, and theories become social knowledge. However, the social aspect is only an accessory part of learning as all social interactions do not result or confirm into a state of knowing or learning. Social interaction is an aspect of symbolic interaction (Thompson, 2000). One who participates in the interaction, to this author, does not mean that he or she is constructing knowledge. At least not, in the same way, as the speaker intends. The meaning of symbols through utterances (speech words, sentences) may change or may be different from the speaker to the listener. The structure may be the same, but the meaning can be different because the way meaning is constructed depends on how one conceives of those symbols (words, sentences, pitch of the sound, etc.). It is always a personal or individual matter that takes place differently, even within the same context. Then, it is clearly a secondary aspect of constructing knowledge or learning. True learning takes place in one's brain and mind. Interaction of mental state and brain state through an operator or function (whatever it may be) leads to conceptual change in mental state and some physical change in brain state (Belbase, 2013). Therefore, to some extent, learning is an individual responsibility, and teacher is simply a facilitator to help students in carrying out that responsibility. This view raises some questions in relation to radical constructivist epistemology. How does radical constructivism relate the individual and social interaction? Is knowing of mathematics a purely individual, idiosyncratic experience? How do we know one has learned something or not? As a teacher, how can one help others (students) to learn? Do students take responsibility for their learning? Is it possible at the early childhood or elementary level? These issues further extended to this author's pedagogical dilemma.

Pedagogical dilemma in radical constructivism

It seems that there is no direct control of a teacher in one's construction of knowledge or learning, though there can be an influence, in a way, how the students learn or do not learn as intended. Although radical constructivism acknowledges, adapts, and incorporates the role of social interactions in the process of constructing knowledge, this author thinks that it is not yet clear how such processes influence mind and brain. How individuals make sense of shared knowledge? The social and cultural norms, values, and practices are sometimes against individual growth and development. In such a case, individuals have to use their epistemic and pedagogical courage to move beyond social and cultural chains. However, this does not mean that radical constructivists ignore the role of social interaction, but it is not explicit. Piaget's idea of perturbation, disequilibrium, and coming into equilibrium; von Glasersfeld's idea of constraints, and Vygotsky's idea of ZPD are somehow associated with social interaction.

Feeling or experiencing a constraint may be due to personal experience toward an object or phenomena or it may arise due to social interaction when there is a state of disequilibrium in terms of experiences. Ernst von Glasersfeld (1991) admits that a social interaction plays a key role in the construction of individual knowledge, but as a radical constructivist, he argues that the understanding and making meaning of an object or phenomenon is purely an individual process. He states, "Experiential worlds belong to individuals, but in the course of social interaction these individual worlds become adapted to one another and come to form a *consensual domain*, i.e. an area where the interactor's mutual expectations are more or less regularly realized" (von Glasersfeld, 1991, p. 5). There might be some compromises in the meaning or sense of the object or the phenomenon even when individual differences still may exist. Individual interest, creativity, passion, perseverance, and efforts are key in construction of knowledge and knowing. Ontologically, an individual is the basis of social. Epistemologically, social is the basis of understanding an individual. Social cognition is the basis of individual cognition at first degree and then individual cognition may reflect back to the social cognition (Bandura, 1989). Therefore, teaching and learning and any form of knowledge construction at individual level is questionable.

The extreme form of individuation in the pedagogy of radical constructivism was questioned by many scholars at the Montreal meeting of the Psychology of Mathematics Education in 1987. The major questions were related

to- ontology, metaphysics, and solipsism (Steffe & Kieren, 1994). How teachers can help students construct mathematical ideas? Why teacher is necessary for the classroom? Why students need support from the teacher or others who know the subject matter? These issues raised a pedagogical dilemma within the author's mind. If knowing is subjectively abstracting the experiences, then what is the form of knowledge about the objective world beyond our experiences? How can we foster teaching and learning of mathematics and science at the level of generality? How can we develop a sense of shared values through individualistic knowing and learning? These dilemmas lead this author toward a reconceptualization of social constructivism.

Reconceptualizing Social Constructivism

Earlier, this author reflected on his early stage of learning and teaching mathematics that were heavily influenced by behaviorist and traditional-realist perspectives. His continued learning and teaching approach to be modified and somewhat improved with new experiences through trainings and further education. In this context, he reflects (in the following paragraph) on how he slowly moved from being a traditional behaviorist to a constructivist teacher.

Students came into the class at ten in the morning. I had already kept some cardboard boxes, some pencils, markers, cardboard papers, print papers and a roll of masking tape on a table. I welcomed the students in the class. I wrote the topic of the day "Algebraic thinking" on the white board..…… I let them discuss for fifteen minutes. The discussion on the topic continued for fifteen minutes. The students shared their views before reading the paper and after reading it. Then they summarized their views in print paper in three groups. Each group presented their views and opinions turn by turn by fixing the written print papers on the wall. (Belbase, 2006, pp. 185-186)

This reflection portrays how he was trying to change his epistemology and pedagogy from the traditionalism to the constructivism. The way he was teaching mathematics and mathematics education showed a shift in the paradigm, to some extent. The way he tried to facilitate the students' learning, the way he tried to engage the students in discussion, reflection, and sharing in the class involved some level of constructivism in general and social constructivism in particular.

This issue relates to '*how knowledge is constructed'* in the classroom through shared responsibilities between teacher and students. It may be a good idea to begin this discussion from mathematics and objects of mathematics. What is mathematics and what constitutes objects of mathematics? Mathematics as a process of systematic study of numbers, shapes, and various relations in the nature and natural phenomena may be a positivistic view. Whereas mathematics as the systematic study of human constructs of interpretation of various phenomena may be inclined toward the constructivist view. Domain of mathematics within positivist and constructivist epistemology may be different. The positivist epistemology considers mathematics as empirical studies of natural and social phenomena as its domain. The constructivist epistemology considers mathematics as interpretation and construction of models for various phenomena. Then from this point of view mathematical knowledge arises from negotiation of personal and social constructs and the shared meaning of any phenomena. Classroom teaching and learning is a context to create such knowledge and knowing of social, cultural and scientific phenomena as object of mathematics. This view goes further toward how we make sense of social constructivism and how it motivates us toward epistemological and pedagogical actions, as a metaphor of metamorphosis. The classroom process is a social reality. The teaching and learning involves multiple actors and hence it is also social reality. The research as a collective creative endeavour involves many aspects including social and cultural norms of the community of practice and hence the process and outcome is social reality. These ideas motivated this author toward social constructivism as an epistemology and pedagogy.

A motivation toward social constructivism

The metaphor of metamorphosis from traditionalism to constructivism seemed to be guided by Ernest's (1991) philosophy of mathematics and mathematics education (Belbase, 2006). Reading about constructivism, especially social constructivism, was a great eye opener to this author. Three major epistemological domains in the philosophy of mathematics- logicism, formalism, and constructivism – and how they view mathematics and mathematical processes provided this author an insight about his contemporary approach of teaching and learning of mathematics. Ernest (1991 & 1999) together with Driver, Asoko, Leach, Mortimer, and Scott (1994) helped this author in moving toward a constructivist approach in general and social constructivism in particular.

There are certain characteristics of social constructivism. The three grounds for social constructivism for mathematics (science) can be stated as: "*The basis of mathematical knowledge is linguistic knowledge, conventions and rules, and language is a social construction; interpersonal social processes are required to turn individual's subjective mathematical knowledge, after publication, into accepted objective mathematical knowledge; and objectivity itself will be understood to be social"* (Ernest, 1999, p. 42). These three criteria for mathematical knowledge as a social construction helped this author to orient toward social constructivist epistemology and pedagogy, to some extent. The linguistic basis, transformation of subjective to objective knowledge, and nature of objectivity as social formed the ground to develop social constructivism to a dominant philosophy and psychology of mathematics education. This view connects further to Driver et al. (1994). From a social constructivist perspective of Driver et al. (1994), mathematical and scientific knowledge originates from personal constructs of individual mathematicians, scientists or researchers in a raw form. The raw knowledge is brought to the scientific community for further processing. Processing of knowledge means bringing that raw knowledge into discussions among the members of the community (e.g., students, teachers, researchers, scientists, and parents) through publications, oral presentations, group discussions, and sharing among each other. This kind of practice may broaden the original knowledge with more inputs or comments or critiques from the stakeholders and scientific community (Ernest, 1999). The shared knowledge becomes a socially accepted and socially constructed knowledge that can be considered as 'taken for granted' knowledge within the community.

Hence, from a social constructivist perspective, the notion of learning is associated with the collective building of knowledge through negotiation, agreements, and common practices. While doing this, every person plays the role of a learner or a learned depending upon the context and complication of things that he or she deals with at the moment. A more experienced person guides a less experienced one in the process of learning. Therefore, learning is not just a personal or individual interpretation of things, nevertheless it is a collective interpretation of things through which the less experienced members get exposure to the community of practitioner from where he or she learns the culture of knowing, doing, thinking, and reasoning. This notion relates to social and cultural adaptation. The construction of knowledge depends on how individuals as members in a community play different roles actively (or passively) and make a contribution to the social process of generating knowledge. In this process, language plays a significant role. The construction of new knowledge is a language game by giving meaning to our words, actions, and experiences. This view seems somewhat counter-intuitive to radical constructivism. In radical constructivism, language, in the form of communication, does not carry meaning from person to person. Nonetheless, each individual actively constructs their meaning through individual cognitive, adaptive, and idiosyncratic process and experience. Language and communication (in the form of interaction) is not the end of construction of knowledge or learning, however it is only a part. The construction of knowledge is deeper than what is communicated, shared or interacted. The idea of social and cultural adaptation through language games pushed the author toward epistemological dilemma.

Epistemological dilemma in social constructivism

Ernest (1991 & 1999) and Driver et al. (1994) helped this author to change epistemological and pedagogical paradigm from traditionalism to constructivism in general and social constructivism in particular. Ernest (1995) compared different paradigms in terms of metaphor of mind and the world that helped the author to understand other constructivist paradigms. Later on, this author continued reading the different forms of constructivism that helped him to know more about radical constructivism of Ernst von Glasersfeld (von Glasersfeld, 1989, 1990, 1991 & 1995) and Steffe and Thompson (2000). These readings helped this author to compare the two paradigms- radical and social constructivism- and added in the growing dilemma.

The cycle of objective and subjective knowledge seems problematic to this author. The notion of objective and subjective interpretation of experiences, schemes, and personal constructs cannot be judged simply from the social phenomenon. Ernest (1991) states that-

Social constructivism links subjective and objective knowledge in a cycle in which each contributes to the renewal of the other. In this cycle, the path followed by new mathematical knowledge is from subjective knowledge (the personal creation of an individual), via publication of objective knowledge (by intersubjective scrutiny, reformulation and acceptance). Objective knowledge is internalized and reconstructed by individuals, during the learning of mathematics, to become the individuals' subjective knowledge. Using this knowledge, individuals create and publish new mathematical knowledge, thereby completing the cycle. Thus, subjective and objective knowledge of mathematics each contributes to the creation and re-creation of the other. (Ernest, 1991, p. 43).

The objective-subjective cycle as part of social constructivism in mathematics education seems contradicting within itself. It emphasizes the publication of individual knowledge of mathematics to convert it from subjective to objective. How do publication and dissemination make mathematical knowledge an objective? The publication is a way to share the knowledge from individual to the community of practitioners. Sharing in the community does not mean that it is accepted by the community. It does not mean that other mathematicians and researchers take such mathematical knowledge as taken for granted. There are always critiques and appraisals of any knowledge of mathematics (and science) when it is shared, interacted, and published. This is how the author's dilemma toward social constructivism grew further. This dilemma becomes strong with the notion of shared knowledge of Driver et al. (1994).

Knowledge is constructed through social conversations and activities about shared problems through which meaning is interpreted by involving persons-in-conversation with the help of skilled members (Driver et al., 1994). The process of appropriation through involvement in the social activities also may help individuals to gain control over the tools, to some extent. The metaphor of learning as discovery or a new invention assumes that learning is as an individual process through which individual makes meaning of things or phenomena, and constructs ideas out of them. This idea reminds me that "If learners are to be given access to the knowledge systems of science, the process of knowledge construction must go beyond personal empirical enquiry" (Driver et al., 1994, p. 5). In this statement, Driver et al. emphasized access to physical experiences, concepts, and models of conventional science. By this process intersubjective knowledge is converted to objective knowledge through public acceptance (Ernest, 1991). That means the public domain of mathematics as knowledge is an objective knowledge, even when it begins from subjective knowledge. Whereas, the author's understanding of objective knowledge is the one that is universally unique with time, place, society, and culture. At the beginning stage or in a crude form, to some extent, mathematical knowledge is localized and culture-dependent. It has a root somewhere in some cultures. It is originated at a place and in time with an individual effort. A group of practitioners of mathematics and research in mathematics within a community may have different mathematical developments than other groups. If other groups of practitioners do not agree with the initial group, then the conflicting knowledge cannot be taken for granted. That is why any knowledge in mathematics or in science is contestable, fallible, and subject to revision. Therefore, within this paradigm, epistemological and pedagogical notion of social knowledge of mathematics seems problematic.

Social constructivism is a populist term in mathematics (and science) education. Social and cultural adaptation, in many cases, kills individual creativity, and it simply helps individuals to follow the tradition as an unquestionable system. Probably, there should be a balance in construction and enculturation (in both social and cultural forms). Enculturation does not help society and individual to make a radical change or progress. Enculturation, to the author, is an analogous to Darwin's theory of evolution in which change is a gradual process. If we accept and continue following the same traditions or the same practices of knowledge as social and cultural, we will not be able to make progress further or even if it is made, then it will be a slow process. In this context, technological knowledge is growing so fast, and we never had a social and cultural bound to define it, explore it, and extend it. The radical progress in technology at present is through radical view of knowledge construction, dissemination, and sharing. The individuals who spend their lifetime in such development may not gain much from the social process except post development critiques and questions. If social and cultural process is adapted, then knowledge construction becomes evolution, nevertheless not a revolution. This view raises some questions. A social process of knowledge is possible only through individual efforts in terms of leadership in knowledge through research, publications, and interactions. The social nature of knowledge construction raises some questions: What is the nature of knowing as shared experience? How does the individual experience merge to the social experience to be generalized knowledge? How does the shared knowledge maintain its value across the communities of practices? Is knowing mathematics (and science) simply playing a language game? These questions put this author into a state of further dilemma. The dilemma extends further in the form of pedagogical dilemma.

Pedagogical dilemma in social constructivism

Ernest (1991& 1999) and Driver et al. (1994) claim that social constructivism emphasizes the construction of mathematics (and science) knowledge through interaction, communication, and sharing by means of a language game. This author may agree that one's interaction with the world provides a context to learn mathematics (and science). However, this context itself is not the knowledge. Ernest (1991) accepts that "The knowledge of the child develops through interaction with the world" (p. 181). Then, interaction with the peers, elders, juniors, and teachers provides such learning context to the students. Within this paradigm, the teacher is the one who at first intervenes in their learning in the classroom acting as a facilitator. The role of the teacher as an interventionist is

essential for enabling students to construct 'cultural tools' (Driver et al., 1994). These different cultural tools may be associated with certain ways of doing things (such as using a formula), writing symbols, defining things or phenomena, making assumptions, and using tools and techniques to solve problems. However, to this author, mere interactions do not generate knowledge, and they do not promote learning unless there are individual awareness and goal toward what to learn, how to learn, and why to learn. In many cases, teacher's intervention is necessary in order to make sure that students are spending their time in productive learning, not just wasting resources in the name of group projects. However, students' personal awareness toward what they are doing, how they are doing, and why they are doing the way they are doing (in mathematics and science class) seems more important.

Learning at the individual level can promote different experiences of the same objects or phenomena in different contexts, and consequently they may have differing 'conceptual profiles' developed in learners' mind (Driver et al., 1994). There can be existences of such competing conceptual profiles in the minds of students, and they can use these profiles wherever they can fit them depending upon contexts. Such differing conceptual profiles, even when they contradict each other, may exist as separate entities, and students can utilize appropriate profile depending upon the situations they come to deal with at time. These conceptual profiles may have different views of the same object or phenomenon, and they exhibit them wherever and whenever they feel comfortable. They may develop these conceptual profiles with respect to different ontology and epistemology. These conceptual profiles may represent layers of reasoning, understanding, and making sense of things or phenomena in different social and cultural contexts. Developing a new conceptual profile as a result of an amalgamation of existing profiles or an independent profile may not supersede or replace the existing profile. That means, the conceptual profile may exist in one's mind as a quantum of thinking or reasoning from a different perspective. Forming and retaining these conceptual profiles, to this author, is an individual process. How social constructivism influences such profiles is not clear.

The existence of such conceptual profile may be problematic about the notion of 'conceptual change' because students do not necessarily abandon their common-sense-ideas as a result of science (and mathematics) instruction, and they will have such ideas available to them for communication within appropriate social contexts (Solomon, 1983 as cited in Driver et al., 1994). "Human beings take part in multiple parallel communities of discourse, each with its specific practices and purposes" (Driver et al., 1994, p. 6). This process also helps to create such differing 'conceptual profiles' in students' mind. When learning science and mathematics is viewed from the perspective of 'conceptual change', then the existence of such 'conceptual profile' is a problem because, to the author, these conceptual profiles may be an obstacle in conceptual change. The existence of differing conceptual profiles may not lead to conceptual change. Nonetheless, it may generate a new conceptual profile. This author thinks that when students already have such differing conceptual profiles through experiences in different communities of practices, then 'conceptual change' may not happen in a real sense because students may not exhibit such change in a different context. Also, the notion of conceptual profile puts a shadow on the social constructivist notion of objectivity of mathematical knowledge as a shared and public entity. The existence of different conceptual profiles challenges the notion of shared knowledge as objective.

This author agrees with Gray (1997) when he states that "constructivist classrooms are structured in such a way that learners are immersed in experiences within which they may engage in meaning-making inquiry, action, imagination, interaction, hypothesizing, and personal reflection" (n. p.). Also, constructivist learning focuses on autonomy and ownership of learning by the learners in relation to what to learn, why to learn, when to learn, and how to learn. Then, obviously there may not be simple rules for learners to orient them in a more efficient learning. This issue raises a few questions- To what extent language game contributes in mathematics pedagogy? What are the parameters of teaching and learning mathematics beyond communicative function? How do social constructivist teaching and learning of mathematics contribute to reflective and reflexive thinking? These questions led this author further toward thinking about adapting to both radical and social constructivism. There is no one best way in making decision about what works and what does not work in classroom practice. Both radical and social constructivism has limitations in terms of epistemology, methodology, and pedagogy. Then making epistemological and pedagogical choice is based on one's wisdom.

Conclusion

This author came to realize through personal experiences and readings through relevant literature that there is no one best way to describe how we come to know what we claim to know and how we teach or facilitate the students' construction of knowledge. Therefore, eclecticism (a pluralistic approach) in epistemology and

pedagogy is necessary to use these theories and philosophies as tools not as ultimate paths to follow. This kind of thinking led the author toward epistemological and pedagogical eclecticism.

Epistemological eclecticism

The paradigmatic tensions between radical and social constructivism can be resolved through a balanced thinking and acting within the scope of radical and social constructivism and beyond. This author believes that the personal construction of knowledge is important for learning and making sense of the world (i.e. mathematics and science). This insight came from radical constructivism (by von Glasersfeld) which states that "the function of cognition is adaptive, and serves the subject's organization of her experiential world, not the discovery of an objective ontological reality" (Heylighen, 1997, n. p.). The subjective role in knowing can be linked with awareness and consciousness. One's awareness and consciousness are reflected through his or her relation to other selves and the world. Then, knowing mathematics (and science) is always connected to the world. Hence, the radical constructivist notion of knowing is not limited to one 'self', however it is more connected to the other 'selves' of the same individual or other individuals. Whereas, the social constructivist notion of knowing is related to shared consciousness and awareness among different selves. The notion of the language game of constructing shared meaning of mathematics and science is a way to bring individual selves into a greater "SELF" through social and cultural semiotics. Then, mathematics (and science) knowledge becomes 'collective commons' despite critiques and uncertainties. This greater 'SELF' is a cognizing subject, either at individual or community level, and its function of cognition is dynamic that serves reorganization with new experiences and it seeks justification of knowledge through legibility. This greater 'SELF' is realized through common uses of language, common social and cultural values, and common adaptation to new knowledge. The notion of common is not blind common but a critical common. Understanding of the two paradigms in terms of conflicting epistemological and pedagogical practices may take us to an unhelpful dilemma leading to confused state and frustration. However, bringing ideas from both paradigms as a unified theory of knowing and teaching-learning might help us in using them in diverse social and educational contexts.

The author concludes that one can never be a perfect or complete social or radical constructivist, but there is an overlap and shift from one paradigm to the other and vice versa in practical life. Construction of knowledge through research, teaching, and learning may not always be only a social or individual phenomenon. There could be a dynamic tradeoff between these paradigmatic movements. In some cases, knowledge construction may begin from an individual effort through laborious experimentation, observation, or intuition and then becomes social after sharing in public or community of practice. In other cases, this process could begin as a social phenomenon with interaction in groups (e.g., focus group discussion) and then becomes a source of individual knowledge for the researcher after his or her analysis and interpretation of the raw data. Again, publication and dissemination could make it social or public knowledge through critiques and comments by others. Students, teachers, teacher educators, and researchers may utilize the multiple ways for construction, validation, and dissemination of knowledge, no matter either the origin is an individual or a community of practice.

Pedagogical eclecticism

For this author, teaching is a social function, and learning could be a private one. Even the social interactions are not learning in themselves, but they are only context in which an individual is challenged or critiqued or suggested through which he or she can conceptualize knowledge. Either empirical or intuitive, experiences are always private and hence construction of knowledge can be a private function. The social function enhances and motivates this private domain of construction. Hence, an individual learns something means he or she constructs knowledge through the cognitive function of self-adaptation to the new ideas or schemes through perceptual experience or introspection. "The role of the teacher is then assumed to provide guidance to the students, but their guidance is tentative and cannot ever approach absolute determination" (von Glasersfeld, 1990, p. 37). The teacher, in the process, becomes a part of intervening force for the child's cognitive adaptation. His or her intervention in the learning process is simply as a guide to encompass the direction and pace of learning. The teacher may help students to orient toward the task, arouse their interest on tasks, and help them organize their tasks. This author thinks that the main pedagogical implication of radical constructivism is associated with clarity and description of the role of the teacher and students. The role of a teacher is simply as a facilitator or guide for learning and creating a learning environment, and the role of the students is like active learners who construct knowledge by active involvement in the learning process. The students' role as learners is more

important as constructors or co-constructors of knowledge with the teacher and peers than simply as receivers of knowledge. Both students and teachers take their responsibilities and understand their respective roles.

The construction or co-construction of knowledge and knowing is more a personal, subjective phenomenon than objective reality, though the students get involved in social interaction within or out of class. Ellerton and Clements (1992) discussed radical constructivism in terms of "ownership of mathematics learning by the learner, quality of social interaction as a basis for quality of mathematics learning, and principles for improving the quality of mathematics teaching and learning" (pp. 4-7). These ideas can be further extended to social constructivism. The issue of ownership, quality of learning, and social interactions are also concerns of social constructivism. Only level of interpretation to the contexts might be different. This view further can be linked to Cobb (1990) that students construct their mathematics, this construction is a process of externalization of what they already constructed with flexibility of problem solving and progressive abstraction of conceptual objects of mathematics. This objective can be achieved through teaching and learning mathematics with collaborative action, shared responsibility, shared ownership, effective, communicative function, formal and informal contexts for learning, and autonomy. Hence, pedagogical eclecticism may contribute back to the epistemological eclecticism with a more delicate and inclusive method of knowing and constructing new knowledge.

References

Bandura, A. (1989). Social cognitive theory. In R. Vasta (Ed.), *Annals of child development, vol. 6: Six theories of child development* (pp. 1-60). Greenwich, CT: JAI Press.

Belbase, S. (2006). *My journey of learning and teaching mathematics from traditionalism to constructivism: A portrayal of pedagogical metamorphosis*. (Unpublished M. Phil. Dissertation). Kathmandu University. Online available from: http://files.eric.ed.gov/fulltext/ED502252.pdf

Belbase, S. (2011). *Radical versus social constructivism: Dilemma, dialogue, and defense*. Online available from: http://files.eric.ed.gov/fulltext/ED525159.pdf

Belbase, S. (2013). A unified theory of mind-brain relationship: Is it possible? *Open Journal of Philosophy, 3*(4), 443-450. Online available from: http://file.scirp.org/Html/37629.html

Belbase, S., Luitel, B. C., & Taylor, P. C. (2008). Autoethnography: A method of research and teaching for transformative education. *Journal of Education and Research, 1*(1), 86-95. Online available from: http://www.nepjol.info/index.php/JER/article/view/7955

Bielaczyc, K., & Collins, A. (1999). Learning communities in classrooms: A reconceptualization of educational practice. In C. Reigeluth (Ed.), *Instructional design theories and models: A new paradigm of instructional theory* (vol. 2) (pp. 269-292). Mahwah, NJ: Lawrence Erlbaum Associates, Inc.

Christie, A. (2005). *Constructivism and its implications for educators.* Online available from the World Wide Web: http://alicechristie.com/edtech/learning/constructivism/index.htm

Cobb, P. (1990, February). *Reconstructing elementary school mathematics.* Paper presented at the annual meeting of the Research Council for Diagnostic and Prescriptive Mathematics, Washington D.C.

Confrey, J. (1990). What constructivism implies for teaching, in Davis, Maher, and Noddings (Eds.). *Constructivist views on the teaching and learning of mathematics.* Monograph of the Journal of Research in Mathematics Education. Reston, VA: NCTM.

Cooperstein, S. E., & Kocevar-Weidinger, E. (2004). Beyond active learning: A constructivist approach. *Reference Services Review, 32*(2), 141-148. Online available in the following World Wide Web: http://www.unc.edu/~bwilder/inls111/111beyondactivelearningWED.pdf

Driver, R., Asoko, H., Leach, J., Mortimer, E., & Scott, P. (1994). Constructing scientific knowledge in the classroom. *Educational Researcher, 23*(7), 5-12.

Ellerton, N.F., & Clements, M. A. (1992). Some pluses and minuses of radical constructivism in mathematics education. *Mathematics Education Research Journal, 4*(2), 1-22.

Ernest, P. (1991). *The philosophy of mathematics education*. London, U. K.: Falmer.

Ernest, P. (1995). The one and the many. In L. P. Steffe & J. Gale (Eds.), *Constructivism in education* (pp. 459–486). Hillsdale, NJ: Lawrence Erlbaum.

Ernest, P. (1998). *Social constructivism as a philosophy of mathematics*. New York, NY: State University of New York Press.

Ernest, P. (2010). Reflections on theories of learning. In B. Sriraman & L. English (Eds.), *Theories of mathematics education: Seeking new frontiers* (pp. 39-47). New York, NY: Springer. DOI: 10.1007/978-3-642-00742-2_4

Ernest, P. (March 23, 1999*). Social constructivism as a philosophy of mathematics: Radical constructivism rehabilitated*. Online available from: http://people.exeter.ac.uk/PErnest/soccon.htm

Gray, A. (1997). *Constructivist teaching and learning.* SSTA Research Center Report. Online retrieved from: http://saskschoolboards.ca/research/instruction/97-07.htm#What is Constructivism?

Hackenberg, A. J. (2010). Mathematical caring relations in action. *Journal for Research in Mathematics Education, 41*(3), 236-273.

Hardy, M., & Taylor, P. C. (1997). Von Glasersfeld's radical constructivism: A critical review. *Science & Education, 6*, 135-150.

Hatfield, L. L. (2013). Some pedagogical and psychological aspects of mathematical modeling in school mathematics. In R. L. Mayes, L. L. Hatfield, & M. V. Mackritis (Eds.), *Quantitative reasoning in mathematics and science education: Papers from an international STEM research symposium* (WISDOM^e Monograph, Vol. 3, pp. 149-167). Laramie, WY: University of Wyoming.

Hein, G. E. (1991). *Constructivist learning theory*. Paper presented at the International Committee of Museum Educators Conference, Jerusalem Israel, (15-22 October). Online available from the World Wide Web: http://www.exploratorium.edu/ifi/resources/constructivistlearning.html

Heylighen, F. (1997). *Epistemological constructivism*. Online retrieved on July 10, 2011 from the World Wide Web: http://pespmc1.vub.ac.be/CONSTRUC.html

Leo, C. (1990). Reflexive thinking in mathematics: Formal and informal aspects. In A. Diez, J. Echeverria, A. Ibarra (Eds.), *Structures in mathematical theories* (383-389). San Sebastian International Symposium. Online available from: http://www.tau.ac.il/~corry/publications/articles/pdf/reflexive.pdf

Luitel, B. C. (2009). *Narrative explorations of lived mathematical curriculum: An epic journey*. Saarbrucken, Germany: VDM Verlag.

Noddings, N. (1990). Constructivism in mathematics education. In R.B. Davis, C.A. Maher, & N. Noddings (Eds.), *Monographs of the Journal for Research in Mathematics Education, 4*, 19-29. Reston, VA: NCTM.

Richardson, V. (2003). Constructivist pedagogy. *Teachers College Record, 105*(9), 1623-1640. New York, NY: Teachers College, Columbia University.

Solomon, J. (1983). Learning about energy: How pupils think in two domains. *European Journal of Science Education, 5*(1), 49-59.

Steffe, L. P. (2011). Perspectives on collaborative research in mathematics education with interdisciplinary connections. In S. A. Chamberlin, L. L. Hatfield, & S. Belbase (Eds.), *New perspectives and directions for collaborative research in mathematics education: Papers from a planning conference for WISDOM^e* (WISDOM^e Monograph, Vol. 1, pp. 11-28). Laramie, WY: University of Wyoming.

Steffe, L. P. (n. d.). *Basic problems in mathematics education research.* Retrieved from the World Wide Web: http://www.wiziq.com/tutorial/1892-Basic-Problems-in-Mathematics-Education-Research1

Steffe, L. P., & Kieren, T. (1994). Radical constructivism and mathematics education. *Journal for Research in Mathematics Education, 25*(6), 711-733.

Steffe, L. P., Moore, K. C., Hatfield, L. L., & Belbase, S. (Eds.). (2014). *Epistemic algebraic students: Emerging models of students' algebraic knowing: Papers from an invitational international conference* (vol. 4). Laramie, WY: College of Education, University of Wyoming.

Steffe, L. P., & Thompson, P. W. (Eds.). (2000). *Radical constructivism in action: Building on pioneering work of Ernst von Glasersfeld*. New York, NY: RoutledgeFalmer.

Thompson, P. W. (2000). Radical constructivism: Reflections and directions. In L. P. Steffe & P. W. Thompson (Eds.), *Radical constructivism in action: Building on the pioneering work of Ernst von Glasersfeld* (pp. 291-315). New York, NY: RoutledgeFalmer.

Uden, L., Liu, K., & Shank, G. (2001). Linking radical constructivism and semiotics to design a constructivist learning environment. *Journal of Computing in Higher Education, 12*(2), 34-51.

Von Glasersfeld, E. (1985a) Reconstructing the concept of knowledge. *Archives de Psychologie, 53*, 91–101.

Von Glasersfeld, E. (1989). Constructivism in education. In T. Husen & T. N. Postlethwaite (Eds.), *The international encyclopedia of education: Research and studies* (pp. 162-163). Supplementary Volume 1. Oxford, U. K.: Pergamon Press.

Von Glasersfeld, E. (1990). Environment and communication. In L.P. Steffe & T. Wood (Eds.), *Transforming children's mathematics education: International perspectives* (pp. 30-38). Hillsdale, NJ: Lawrence Erlbaum.

Von Glasersfeld, E. (1991). Questions and answers about radical constructivism. In M.K. Pearsall (Ed.), *Scope, sequence, and coordination of secondary school science, Vol. II: Relevant research*, (pp. 169-182). Washington, D.C.: The National Science Teachers Association.

Von Glasersfeld, E. (1995). *Radical constructivism: A way of knowing and learning*. New York, NY: RoutledgeFalmer.

Von Glasersfeld, E. (1996). *Aspects of radical constructivism.* Earlier Published as: Aspectos del constructivismo radical in M. Pakman (Ed.), Construcciones de la experiencia humana (23-49).

Barcelona, Spain: Gedisa Editorial. Online available from the following World Wide Web: http://www.univie.ac.at/constructivism/EvG/papers/191.pdf

Von Glasersfeld, E. (2001). Radical constructivism and teaching. *Perspectives, 31*(2), 191-2004. Online available from: http://www.univie.ac.at/constructivism/EvG/papers/244.2.pdf

Vygotsky, L. S. (1930). *Mind and society*. Cambridge, MA: Harvard University Press.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.

Wilding-Martin, E. C. (2011). *Paul Ernest's social constructivist philosophy of mathematics education*. ProQuest, UMI Dissertation Publishing.

Wuketits, F. M. (1984). Evolutionary epistemology: A challenge to science and philosophy. In F. M. Wuketits (Ed.), *Concepts and approaches in evolutionary epistemology* (pp. 1-29). Dordrecht, The Netherlands: Reidel.