

# Aerodynamic Design Optimization of Residential Scale Wind Turbine Blades for Lower Wind Speeds

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**Abstract**—Sustainable power generation using wind energy has been actively pursued around the world in the last few decades. Typical off the shelf commercially available wind turbines are optimized to produce rated output power at design wind speeds of 9 m/s to 15 m/s and performance significantly degrades at off- design lower wind speeds. Such high wind speeds are not available in South East Asia in general and Pakistan in particular. Hence, commercially available high wind speed turbine solutions have not been able to produce the manufacturer's suggested power values at lower wind speeds of 6 m/s to 7 m/s. A need has been felt to aerodynamically optimize the wind turbine blades for lower wind speeds with an aim to increase the aerodynamic efficiency of power extraction as compared to commercially available high wind speed turbines. The blades for a domestic scale horizontal axis wind turbine have been aerodynamically optimized at wind speeds of 6 m/s to 7 m/s in order to produce maximum power output with a 1.52 m (5 ft) blade length. The power output of aerodynamically optimized blades has been compared with the power produced by commercially available wind turbine blades having same span designed for 10 m/sec wind speed. The optimized blades not only produced at least 10% increased power but have also shown lesser structural bending under the same operating conditions. This paper aims at highlighting the methodology adapted for aerodynamic design optimization of domestic scale wind turbine blades at lower wind speeds of 6 m/s to 7 m/s along with the design and analysis tools/techniques used during this research.

**Keywords**—Wind Turbine Design, Blade Optimization, Blade Element Momentum Theory, Generalized Dynamic Wake Modeling, Wind Energy

## 1. Introduction

With rapid increase in population and urbanization in developing countries, electric power requirements have grown tremendously across the globe. The dependence of power generation on depleting fossil fuels and natural gas resources along with ever growing environmental concerns demands exploration of renewable and environment friendly power generation methods. Renewable energy methods including hydroelectric, solar power conversion and wind power extraction are being vigorously pursued worldwide to help meet the ever growing electric power requirements. Hydroelectric and wind power extraction methods are not dependent on solar cycle and have the ability to provide reliable and consistent power throughout the day. However, solar energy can only be conserved during the sunlight hours

and has to rely on stored energy during the night. On the other hand, wind energy extraction is only possible in high speed wind corridors generally in coastal areas and hydroelectricity needs some kind of water stream for power generation. Based on the wind speed corridor available and specific power requirements of a geographical area, different wind turbine solutions varying from small turbines able to suffice one domestic unit requirement to huge wind turbine farms capable of fulfilling the requirements of bigger towns are already in use. Despite broad spectrum of application from less than a Kilowatt to tens of Megawatts, all these turbines are generally installed in corridors with wind speeds varying from 9 m/s to 15 m/s. Horizontal Axis wind turbines are widely in use but numerous vertical axis turbines have also been pursued in space constrained applications.

Typical design of a new wind turbine is based on rated power output, operating wind speed, size constraint if any and production cost. The geometric shape and aerodynamic profile of the wind turbine blades are selected based on the operating wind speed which in turn is dependent on geographical location and the installation wind corridor. The aerodynamic shape and size of a specific wind turbine also vary with the design value of power to be extracted due to the variation in Reynolds Number (Re) and rotational speed of the turbine blades. The corresponding aerodynamic loads for operating conditions determine the required structural strength. Many a times structural and manufacturing limitations result in minor modifications to the proposed aerodynamic shape and very rarely the design configuration is shelved due to structural concerns and manufacturing inadequacies. Material, structural elements and manufacturing processes dictate the production cost of a wind turbine. An aerodynamically optimized blade geometry produces rated output power within a specific design wind speed range and power output generally decreases at wind speeds below and above the design wind speed range. Aerodynamic efficiency is a measure of how much energy is extracted out of the total energy available in the air channel faced by the turbine and is expressed by the value of Coefficient of Power ( $C_p$ ). Similarly, optimized geometry for one design wind speed range can not be scaled up or down in size to extract different values of power output. Each specific power extraction value along with the operating wind speed will have a different aerodynamic geometry of blades to produce optimum power output.

The energy requirement in Pakistan has increased manifold in the last few decades as shown in Figure 1 and is expected to increase further in the coming years. In 2012, the deficit between consumption and generation increased to 6620 MW, while in 2013 deficit of 5250 MW was recorded [1]. Crude oil, natural gas, hydro and nuclear power are the primary resources utilized for electric power generation. At present crude oil accounts for approximately 37% of total commercial energy supply while hydroelectric power, natural gas and nuclear power contribute 31%, 26% and 4% respectively [2]. The share of alternate energy methods including wind power and solar energy remains less than 2%. Wind power not only provides the opportunity to reduce dependence on imported fossil fuel but also expands the power supply capacity to remote locations where grid expansion is not feasible/practical. Survey conducted by Pakistan Meteorological Department (PMD) indicate that coastal areas of Pakistan has the potential to generate 346,000 MW of power [3]. Gharo-Keti Bandar corridor of 170 km along the coastal line in provinces of Sindh and Baluchistan alone can produce more than 50,000 MW which is more than twice the current power requirement of Pakistan [4]. The

annual mean wind speed is approximately 6.86 m/s at 50 meters above ground level and the annual power density is 408.6 W/m<sup>2</sup>. [4]. Despite huge potential of wind power in Pakistan, no concrete research effort has been made to indigenously design wind turbines optimized for wind speeds available in Pakistan and only projects involving installation of commercially available turbines have been undertaken. Most of these off the shelf turbines are designed to produce rated power at higher wind speeds in excess of 9 m/s and the output power is significantly reduced from manufacturer's suggested rated values at lower wind speeds.

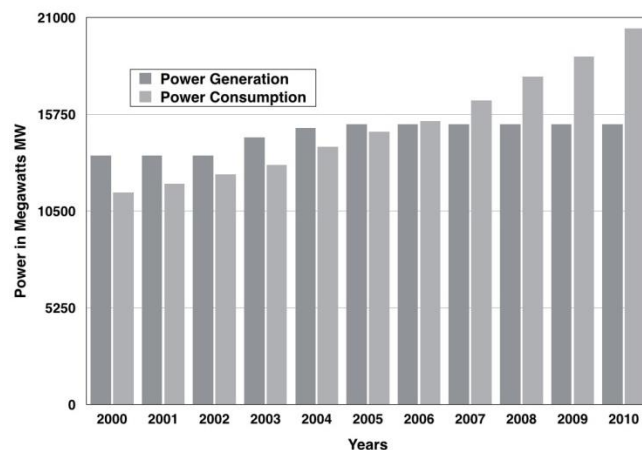


Fig. 1. Pakistan's Power Generation and Consumption in the last decade[1]

## 2. Aim & Scope

This research effort was launched with the primary aim to indigenously design wind turbine blades with optimum aerodynamic efficiency at the lower wind speeds in the range of 6 m/s to 7 m/s. Although, bigger wind turbines installed in formation as wind turbine farms are preferred due to cost effectiveness in terms of installation and operational cost per unit of energy but prototyping of such large turbine blades require tremendous development cost. Additionally, industrial setups capable of undertaking fabrication of large turbine blades are almost non-existent in Pakistan. Due to these limitations, the research scope was limited to design of a small residential scale wind turbine having same size constraints as that of a 1 kW power output high wind speed commercially available turbine. The research was funded by Higher Education Commission of Pakistan [5].

### 2.1. Research Methodology

The research methodology adapted for the conceptual design of wind turbine blades to maximize the aerodynamic efficiency is as follows:

- Identify and evaluate the aerodynamic performance prediction tools which could be utilized for the conceptual design of wind turbine
- Formulation of an automated Modeling & Simulation environment capable of analyzing any given wind turbine geometry/configuration operating at the desired wind speed.
- Optimization of wind turbine blades geometry to maximize the aerodynamic efficiency and power output.

## 2.2. Design Specifications

This finalized design specifications for this design and development effort include:-

- Domestic scale wind turbine with 1.52 m (5 ft) blade span operating at maximum aerodynamic efficiency ( $C_p$ ).
- Design wind speed range from 6 m/s to 7 m/s.
- Utilization of off the shelf commercially available hardware except the wind turbine blades.
- Comparison of optimized blades performance with those of a commercially available wind turbine having same size and same rated output power.

## 3. Modeling & Simulation

### 3.1 Aerodynamic Design and Analysis Models

Wind turbines are devices that harness and convert the kinetic energy available in the wind to useful mechanical/electrical energy. Laws of physics govern how much energy can be harnessed in an ideal case. Betz's law limits this to 16/27 or 59.3% for an ideal case. Most of the wind turbines are designed to achieve 70 to 80% of this ideal theoretical limit [6]. Researchers have utilized analytical tools varying from simple back of the envelope calculations to computationally intensive CFD (Computational Fluid Dynamics) analysis for aerodynamic modeling and performance prediction of horizontal axis wind turbines (HAWT). Few basic models utilized for the design and analysis of HAWTs are:-

#### 3.1.1 Linear Momentum Theory:

This theory assumes the fluid to be inviscid, incompressible and irrotational (with no swirl). The turbine is modeled as a homogeneous actuator disk with infinite number of blades which creates a discontinuity of pressure in the stream tube of air flowing through it. Since, the turbine extracts energy from the flowing wind, velocity decreases across the turbine or results in an induced velocity in opposite

direction. This change in momentum across the disk results in an axial thrust force on the turbine as shown in Figure 2. The energy dissipated per second is equal to the power output of the turbine.

$$\text{Thrust} = T = \text{Rate of Change of Momentum} = P = \text{Axial Thrust Force} \times \text{Velocity}$$

Continuity Equation yields

$$\rho A_\infty U_\infty = \rho A_d U_d = \rho A_e U_e$$

where  $\infty$  corresponds to inlet, d corresponds to disc/turbine station and e corresponds to wake far field

$$\text{Also mass flow rate} = \dot{m} = \rho A_d U_d$$

$$\text{where turbine disc area} = A_d = \pi R^2$$

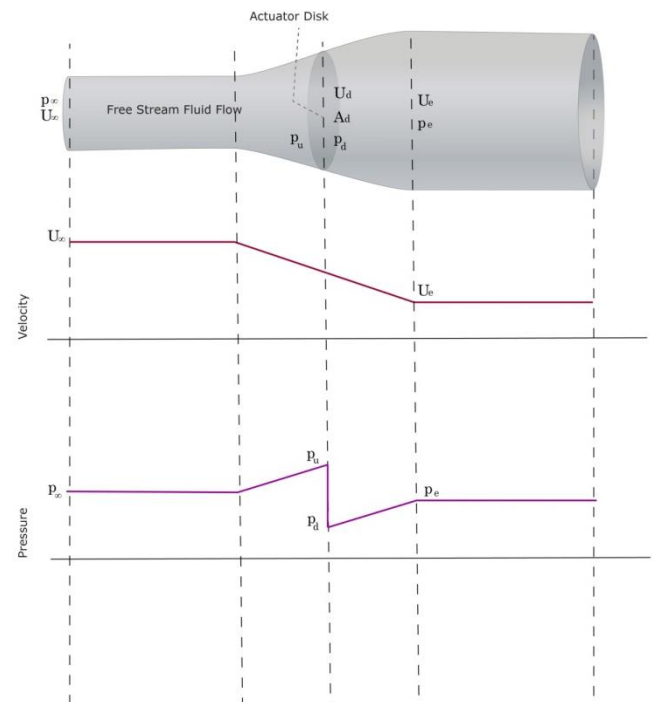


Fig. 2. Linear Momentum Theory

Applying Bernoulli's Equation to compute the pressure drop across the turbine gives

$$\dot{p} = \frac{1}{2} \rho A_d (U_\infty^2 - U_e^2)$$

$$\text{Thrust} = T = \dot{m} (U_\infty - U_e) = A_d \dot{p}$$

$$\text{Velocity at the disc} = U_d = \left( \frac{U_\infty + U_e}{2} \right)$$

$$\text{Axial velocity induction factor} = a = \left( \frac{U_\infty - U_d}{U_\infty} \right)$$

Power output of the turbine=  $P = TU_d$

$$P = \frac{1}{2}\rho(U_\infty^2 - U_e^2)U_d = 2\rho A_d a U_\infty^3 (1 - a)^2$$

Tip speed ratio=  $\lambda = \frac{R\Omega}{U_\infty}$

Total power available in the channel faced by the turbine is

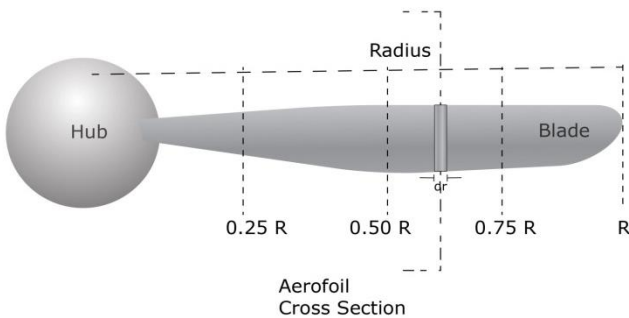
$$P_T = \frac{1}{2}\rho U_\infty^2 A_d U_\infty = \frac{1}{2}\rho \pi R^2 U_\infty^3$$

Coefficient of Power=  $C_p = \frac{P}{P_T} = \frac{2P}{\rho U_\infty^3 \pi R^2}$

$$C_p = 4a(1 - a)^2 P_T$$

### 3.1.2. Blade Element Theory:

Blade Element Theory (BET) assumes each turbine blade a rotating wing comprising of small blade elements along the blade span. Each blade element behaves like a two dimensional aerofoil to produce aerodynamic forces (lift and drag) and moment (pitching moment) due to the combined effect of incoming airspeed, rotational velocity and induced velocity. Since each blade element experiences different rotational velocity due to its radial placement, it experiences different airflow as compared to other elements as shown in Figure 3. Overall performance characteristics of the blade are calculated by numerical integration over the entire blade span. The 3D effects including finite span, wake behind the rotating wing and influence of rest of the rotor blades are all accounted for in the induced velocity term.



**Fig. 3.** Distribution of blade into small elements: Blade Element Theory

The forces and moments are constant at all the azimuth angles over an entire rotation through  $2\pi$  radians as shown in Figure 4.

For an individual blade element

$\theta_p$  = Elemental pitch angle

$\alpha$  = Effective angle of attack

$\phi$  = Angle b/w net velocity and plane of rotation

$U$  = Free stream velocity-axial induced velocity

$$U = U_\infty(1 - a)$$

$U_r$  = Rotational velocity + tang induced velocity

$$U_r = r\Omega(1 + \hat{a})$$

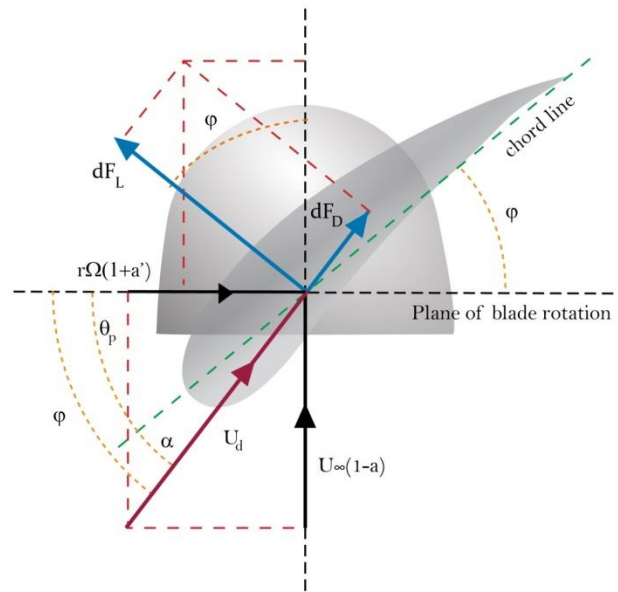
$U_d$  = Net velocity at blade element

$$U_d = \frac{U_\infty(1 - a)}{\sin \phi}$$

$$\tan \phi = \frac{U}{U_r} = \frac{U_\infty(1 - a)}{r\Omega(1 + \hat{a})} = \frac{(1 - a)}{\lambda(1 + \hat{a})}$$

Elemental Lift=  $dF_L = \frac{1}{2}\rho C_L U_d c dr$

Elemental Drag=  $dF_D = \frac{1}{2}\rho C_D U_d c dr$



**Fig. 4.** Angles and Forces on a Blade Element : Blade Element Theory

### 3.1.3 Blade Element Momentum Theory:

Blade Element Momentum (BEM) Theory is a combination of linear momentum theory and blade element theory (BET). This theory computes aerodynamic forces and moments by change of momentum of the air passing through the annulus swept by each element along with determination of axial and tangential induced velocities as shown in Figure 5. BEM assumes no radial interaction between the rotating

elements. The total forces and moments are calculated by integration over the entire blade span. BEM theory produces considerably accurate results when the local angles of attack at the blades are small (axial flow conditions). However, for conditions of high angles of attack or yawed flow conditions, BEM theory fails to predict accurately the blade aerodynamic load distributions. Furthermore, the 2D aerofoil static data fails to model the dynamic stall phenomenon and unsteady flow including separation at the blades, especially towards the tip. The tip and hub losses are generally modeled by empirical formulations in BEM.



Fig. 5. Blade Element Momentum Theory

For annulus swept by turbine blades element:

$$\text{Elemental Lift } dL = (dF_L \sin\phi - dF_D \cos\phi)B$$

$$\text{Elemental Thrust } dT = (dF_L \cos\phi + dF_D \sin\phi)B$$

$$\text{Elemental Torque} = dQ$$

$$dQ = \frac{1}{2} \rho B U_a^2 (C_L \sin\phi - C_D \cos\phi) r c dr$$

where  $c$ =chord length,  $B$ =no of blades and  $r$ =elemental radius

$$\text{Total turbine power} = P = \int_{r_h}^R dP = \int_{r_h}^R \Omega dQ$$

where  $r_h$  =Hub radius,  $R$ =turbine blade radius

### 3.1.4 Generalized Dynamic Wake Modeling:

Generalized Dynamic Wake (GDW) method is based on Potential Flow Solution to Laplace's Equations. It is more reliable than BEM but less computationally demanding than CFD techniques. GDW assumes flow to be incompressible

and inviscid, but works on the principle that vorticity formed at the blades is convected into the wake as trailing and shed vorticity as shown in Figure 6 with a local velocity that is the vectorial sum of the free stream velocity and that induced by all vorticity sources in the wake and from the blades. Circulation around the blade is modeled as a lifting line.

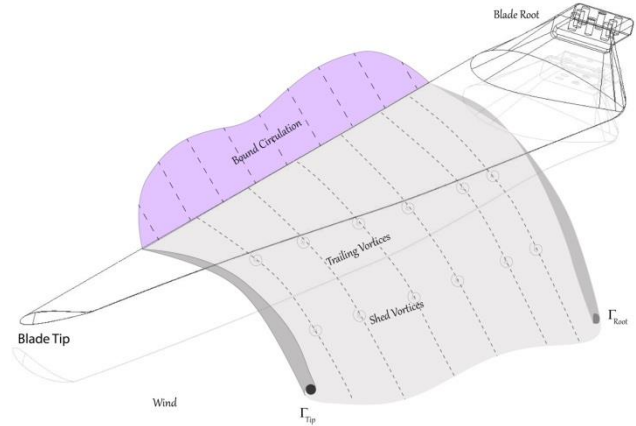


Fig. 6. Wake developed by a rotating blade of wind turbine[8]

The induced velocity at different points in the wake is computed using the BiotSavart law and vorticity in the wake is allowed to diffuse freely [7]. The evolution of the wake is calculated in time after defined time steps. The near wake is modeled as vortex sheets[8], one for each blade. Each vortex sheet is discretized by means of straight-line vortex filaments, the latter being interconnected by nodes to form vortex nodes as shown in Figure 7.

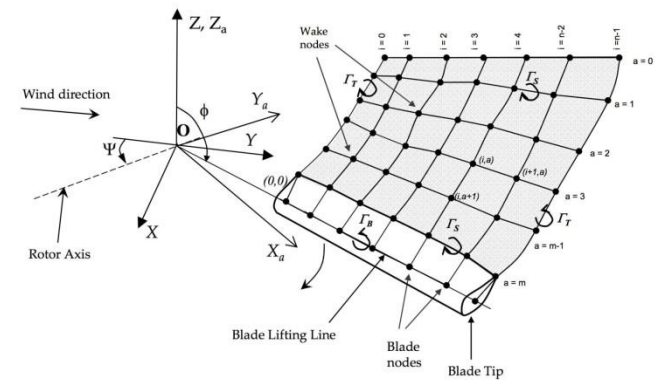


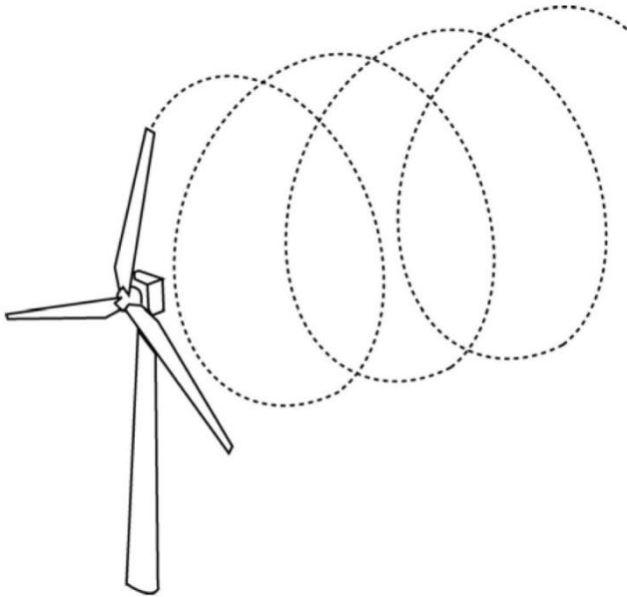
Fig. 7. Modeling of free wake behind a rotor blade [8]

The viscous effects are accounted by introduction of a viscous parameter in BiotSavart Equation. The induced velocity at each node is computed as:

$$\vec{du} = K_v \frac{r}{4\pi} \frac{d\vec{s} \times \vec{r}}{|\vec{r}|^3} \text{ where } K_v = \text{Viscous parameter}$$

The influence of far field on near wake is modeled by a prescribed single tip vortex helical model of constant

diameter per blade as shown in Figure 8. The near wake vortex sheet is modeled to rollup immediately into a concentrated tip vortex.



**Fig. 8.** Modeling of prescribed tip helical vortex behind a rotor blade [9]

### 3.2 Aerodynamic Modeling

AeroDyn [9] and YawDyn [10] codes developed by National Renewable Energy Laboratory (NREL) were used for the aerodynamic design and analysis of wind turbine blades in this research work. AeroDyn is a series of FORTRAN sub-routines written to evaluate the aerodynamic forces and moments for aeroelastic simulations of horizontal axis wind turbine configurations. AeroDyn utilizes Blade Element Momentum and Generalized Dynamic Wake theories for evaluation of aerodynamic forces and moments depending on the user selected parameters and operating wind conditions. BEM is used at wind speeds of less than 8 m/sec, while GDW is utilized at higher wind speeds. Various correction factors are incorporated in AeroDyn when using BEM to account for hub and tip losses in addition to skewed wake effects, while these effects are automatically accounted within the actual GDW model.

Prandtl's Tip loss factor in BEM is given as

$$f = \frac{2}{\pi} \cos^{-1} \left\{ \exp \left[ \frac{-\left(\frac{B}{2}\right) \left[1 - \left(\frac{r}{R}\right)\right]}{\left(\frac{r}{R}\right) \sin \phi} \right] \right\}$$

$$\text{Elem Torque} = Q = 4f\pi\rho U_{\infty} \Omega a(1-a)r^3 dr$$

$$\text{Elemental Thrust} = T = 4f\pi\rho U_{\infty}^2 a(1-a)r dr$$

AeroDyn by semi-empirical model of Beddoes-Leishman [11] to accurately predict aerodynamic loads at high angles of attack. The aerodynamic loading evaluated by AeroDyn is interfaced with YawDyn which is a dynamic aero-elastic simulation code. Basically, AeroDyn calculates the aerodynamic lift, drag, and pitching moment of airfoil sections along the wind turbine blades which are used by the YawDyn to calculate the distributed forces on the turbine blades. The aerodynamic forces and the turbine deflections are inter-related and are dependent on each other thus making this iterative interaction fully aero-elastic.

### 3.3. Blade Geometry and Aerofoil Selection

The aerodynamic performance, aero-elastic behavior and power output of wind turbine are dependent on blade geometry including aerofoil shape in addition to the operating wind conditions. Automated modeling and simulation environment requires definition of unique blade geometry and aerofoil shape based on the values of control variables selected. The selected variables constitute the blade geometry and aerofoil shape. Each unique selection of these design variables results in a different wind turbine shape. AeroDyn and YawDyn input files require these geometric variables for analysis to predict the aerodynamic loads and accompanied structural stresses.

#### 3.3.1 Aerofoil Shape Parameterization:

Different available methods were considered to define a unique aerofoil shape based on the values of certain selected geometric variables. NACA (National Advisory Committee for Aeronautics) 4-Series aerofoils geometry is based on analytical relationships, which requires three variables to generate the aerofoil geometry. This method is simple, but can generate only simple NACA 4-Series aerofoil shapes. Similarly, B-Spline method has fixed x-locations along the chord for which based on the given thicknesses as y-coordinates, spline curves are fitted to the data for unique definition of aerofoil geometry [12]. The method is complex and is limited to aerofoil shapes which could be generated by spline curve polynomials. In order to have flexibility to define any aerofoil shape including the ones typically utilized in modern wind turbines, Parsec method was chosen for aerofoil shape definition. Aerofoil shape is expressed as a linear combination of a suitable base function and 12 important geometric control variables. These geometric variables are shown in Figure 9 [13]. A simple Parsec method code was written in MATLAB to generate aerofoil coordinates from these 12 variables.

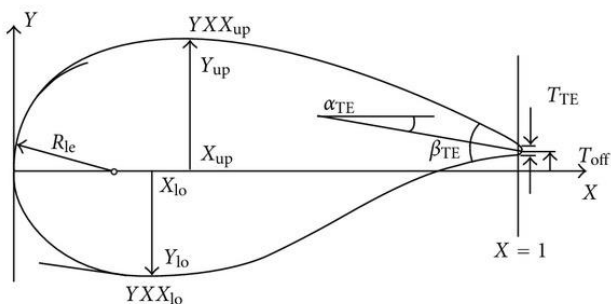


Fig. 9. Control Variables

The accuracy of Parsec method and written code was validated by reproducing few of the known aerofoil geometries as shown in Figure 10.

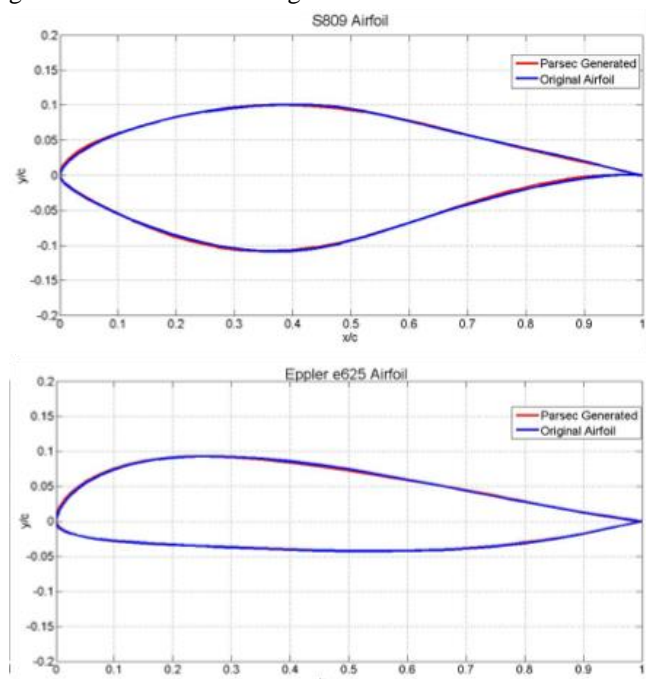


Fig. 10. Sample Aerofoils Generated by Parsec Method

### 3.3.2 Aerodynamic Characteristics of Aerofoils:

Aerodynamic analysis of wind turbine requires aerodynamic characteristics of aerofoil shape including the drag polar, lift curve slope and stall characteristics. Potential flow solver XFOIL [14] was used to determine aerodynamic characteristics of aerofoils. XFOIL is based on a 2<sup>nd</sup> order panel method with linearly varying velocity along with taking into consideration the boundary layer effects. The results are reasonable in the attached flow region but XFOIL fails to predict the aerodynamic characteristics accurately in the stall region. Furthermore, wind turbine analysis requires the values of aerofoil's aerodynamic coefficients over the complete angle of attack range from  $-180^\circ$  to  $+180^\circ$ . FoilCheck [15] another NREL program was used to extrapolate the static characteristics computed by XFOIL

over the complete angle of attack range. In addition to extrapolation, FoilCheck also estimates the dynamic stall performance as per the Beddoes dynamic stall model [11] based on initially computed static characteristics. Lift and Drag Coefficients of a typical airfoil generated over the complete angle of attack range are shown in Figure 11 [10].

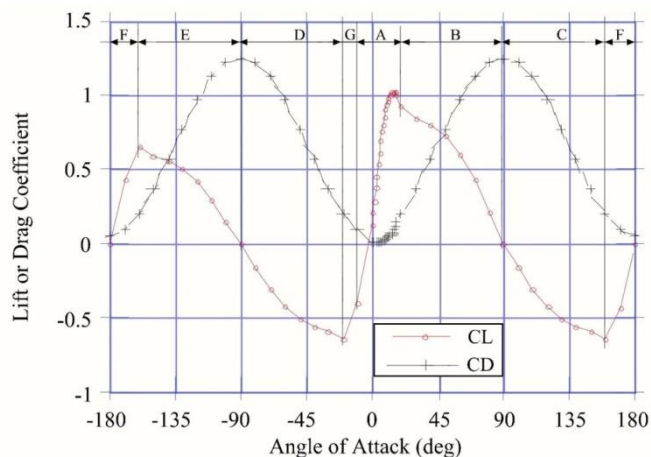


Fig. 11.  $C_L$  &  $C_D$  curves for a typical airfoil [10]

### 3.3.3 Blade Geometry:

In order to define the blade geometry for further aerodynamic analysis, the twist and chord distribution along the blade span are required. Each blade was divided into 15 elements and six stations were chosen as control points for twist and chord distribution of blade geometry. X locations of these six control points were fixed at 0.07 R, 0.13 R, 0.27 R, 0.47 R, 0.73 R and blade tip. MATLAB piecewise cubic spline interpolation was used to calculate the values of twist and chord at rest of the stations. In addition to these 12 variables for twist and chord distribution, blade pitch angle was selected as the 13<sup>th</sup> blade geometry variable.

### 3.4. Modeling & Simulation Environment

A fully automated batch style modeling & simulation (M&S) environment was formulated to facilitate design, analysis and optimization of wind turbine blades. The flow chart for data flow in the order of execution is shown in Figure 12. A MATLAB code is used to prepare input file for Parsec code aerofoil generator based on the values of design variables. XFOIL evaluates the static aerodynamic characteristics for the aerofoil geometry produced by Parsec code. The dynamic stall correction and extrapolation over the complete angle of attack range is then carried out by FoilCheck. The input files for AeroDyn and YawDyn are also prepared by the MATLAB code. The results of AeroDyn and YawDyn for all configurations are consolidated by another small MATLAB code.

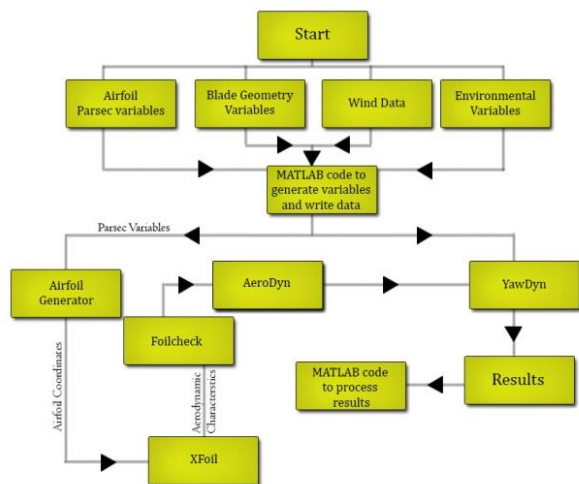


Fig. 12. Data Flow in Modeling & Simulation Environment

### 3.5 Validation

After successfully formulating the M&S Environment, a validation run was performed to determine accuracy of codes and methods being utilized in M&S Environment. A commercially available three bladed wind turbine with rated output of 1 kW was selected for the validation. This wind turbine is optimized for wind speeds in the range of 9 m/s and is not optimized for lower wind speeds of 7 m/s and below. The profile and blade planform of existing turbine blades were translated into values of design variables using coordinate measuring machine as shown in Figure 13. The same were given as input to M&S environment in order to determine the power curve of the wind turbine. The values thus obtained were compared with the available power data provided by the manufacturer in product data sheets. The comparison of results is given in Figure 14.

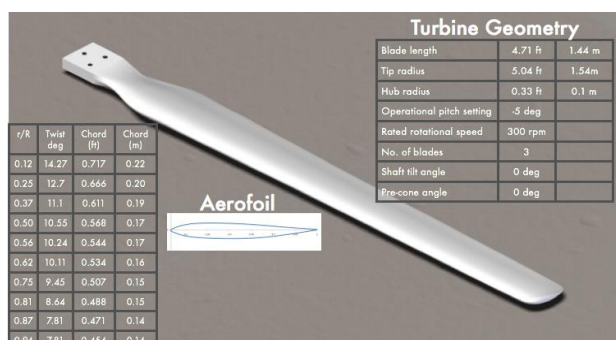


Fig. 13. Geometric Characteristics of Turbine Blades for M & S Validation

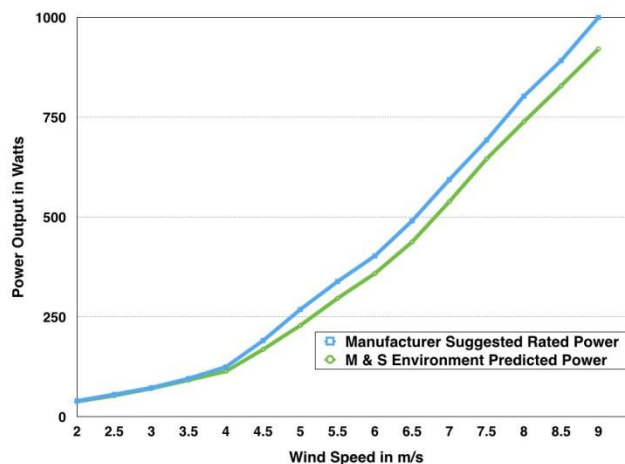


Fig. 14. Comparison of Power Output : M & S Validation

## 4. Aerodynamic Design Optimization

### 4.1 Design Variables

Blade geometry including aerofoil shape was defined by 25 variables. 12 Parsec variables for aerofoil shape and 13 variables were used for defining blade geometry. Ranges were selected for these 25 variables which set the upper and lower limits for the design space to be explored. The details of design variables are given in Table I. Coefficient of Power ( $C_p$ ) was taken as the optimization function for determining the blade geometry with optimal settings of design variables.

TABLE I  
 DESIGN RANGES & OPTIMIZED VALUES OF DESIGN VARIABLES

Variable	Description	Lower Limit	Upper Limit	Optimized Value
$R_{leu}$	Upper Leading Edge Radius	0.02	0.035	0.033
$R_{let}$	Lower Leading Edge Radius	0.01	0.02	0.019
$X_{up}$	Position of Upper Crest	0.25	0.4	0.319
$Y_{up}$	Upper Crest Point	0.1	0.14	0.117
$Y X X_{up}$	Upper Crest Curvature	-2	-1	-1.167
$X_{lo}$	Position of Lower Crest	0.2	0.3	0.259
$Y_{lo}$	Lower Crest Point	-0.1	-0.05	-0.099
$Y X X_{lo}$	Lower Crest Curvature	0.5	1.5	0.63
$\alpha_{TE}$	Trailing Edge Direction Angle	-25	0	-3.693
$\beta_{TE}$	Trailing Edge Wedge Angle	0	25	20.741
$Y_{TEup}$	Upper Surface TE Point	0	0.005	0
$Y_{TElo}$	Lower Surface TE Point	-0.005	0	0
$a_1$	Twist in degrees at 0.07R	3	8	5.9
$a_2$	Twist in degrees at 0.13R	13	18	17.9
$a_3$	Twist in degrees at 0.27R	6	13	8.9
$a_4$	Twist in degrees at 0.47R	3	6	4.9
$a_5$	Twist in degrees at 0.73R	-1	3	2.8
$a_6$	Twist in degrees at Tip	-5	-1	-1.03
$b_1$	Chord length in % R at 0.07R	6.5	7.5	7.1
$b_2$	Chord length in % R at 0.13R	7	8.5	7.63
$b_3$	Chord length in % R at 0.27R	6	7	6.76
$b_4$	Chord length in % R at 0.47R	5	6	5.6
$b_5$	Chord length in % R at 0.73R	4.5	5	4.6
$b_6$	Chord length in % R at Tip	3.5	4.5	3.7
$\theta_p$	Pitch Angle in degrees	-5	10	-1.0

### 4.2 Pareto Analysis

The design space for the wind turbine design consisted of 25 dimensions each corresponding to a design variable and bounded by the working ranges set for these variables. Initially it was assumed that all the design variables are



equally important and the variability of the power output is sensitive to the selected values of these variables. An effort was made to identify the design variables whose variability is more significant to change in the power output. A screening Design of Experiments (DoE) array was generated using fractional factorial technique. JMP (Statistical Data Software) was used to generate the fractional factorial orthogonal array for the 25 design variables. Using factors of level 2 an array of 500 runs was generated. The results of DoE runs were plotted on a Pareto Chart to study these effects as shown in Figure 15.

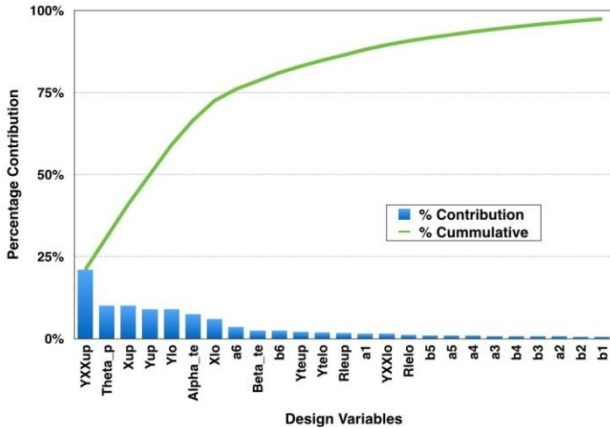


Fig. 15. Pareto Chart of Power Output variability with change in Design Variables

The results indicated that the variables which have greater influence on output power are airfoils' upper crest curvature, pitch angle of blade, position and location of upper and lower crests and trailing edge direction angle. The top ten variables encompassed 81% variability in the output power. The design space was reduced to these ten variables and working ranges for these 10 variables were kept same as before.

### 4.3 Optimization

The built in optimizer function of MATLAB FMINCON (Minimization of function with constraints) was used for optimization due to its ease of implementation and simple interface with the modeling & simulation environment. It is a first order gradient based optimizer which reaches the optimal function value by search in conjugate directions. This gradient based optimizer is very effective in cases with single optimal value but effectiveness reduces with multiple local optimum values. In this wind turbine optimization problem, the FMINCON showed limited success due to non linearity of the problem and presence of multiple local optimal points.

The optimizer was started from different initial values of variables but converged at different local optimum points when the convergence tolerances were satisfied. Hence, after reaching the optimum using FMINCON, the design ranges

were reduced to smaller windows and Genetic Algorithm (GA) was used to reach the global optima. Since each function call required less than a minute, the excessive function calls during GA were not a cause of concern. GA was not only simple to implement but also reached global optimum in this problem with multiple local optima. Population comprising of 1024 designs with 8 bit chromosome length given to each variable was generated for genetic algorithm optimization. After 13 to 14 generations with different starting points and seed values, optimum was achieved. The optimization problem was setup as

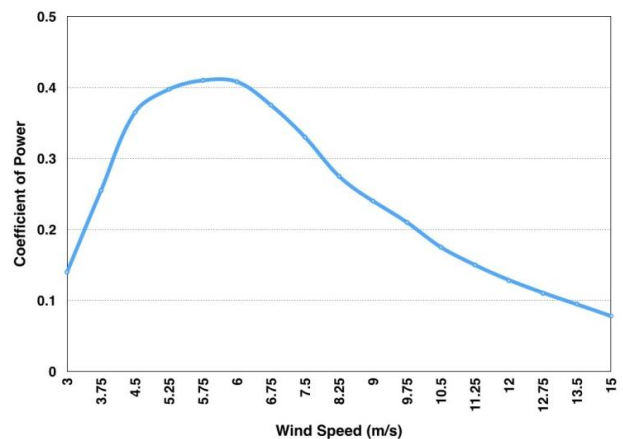
Maximize  $C_p$  =Coefficient of Power

subject to

$$\begin{aligned}
 -2 &\leq YXX_{up} \leq -1 \\
 -5 &\leq \theta_p \leq 10 \\
 0.25 &\leq X_{up} \leq 0.4 \\
 0.1 &\leq Y_{up} \leq 0.14 \\
 -0.1 &\leq Y_{lo} \leq -0.05 \\
 -25 &\leq \alpha_{TE} \leq 0 \\
 0.2 &\leq X_{lo} \leq 0.3 \\
 -5 &\leq a_6 \leq -1 \\
 0 &\leq \beta_{TE} \leq 25 \\
 -5 &\leq b_6 \leq -1
 \end{aligned}$$

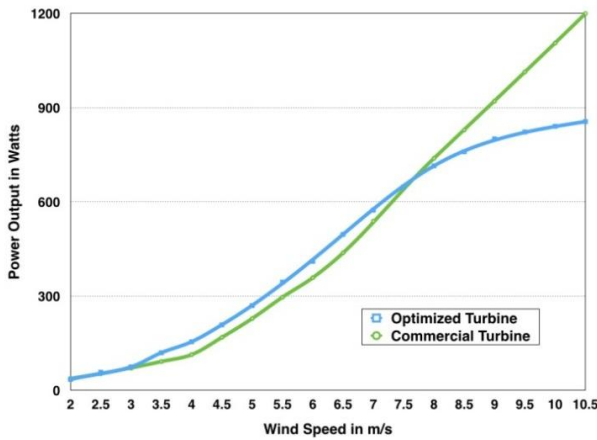
Following two cases were optimized by the optimization scheme mentioned above:-

4.3.1 Case I - Fixed Blade Length: The first optimization problem was based on the commercially available wind turbine used for validation of M&S environment with a blade length of 1.54 m (5.04 ft). It was decided that optimized blade geometry may be computed using the same blade length for maximum power output at 6 m/s. The performance can then be compared with that of the existing turbine to determine the enhanced performance after optimization. Maximum achieved value of  $C_p$  was 0.41 at wind speeds in the range of 5.5 m/s to 6 m/s as shown in Figure 16.



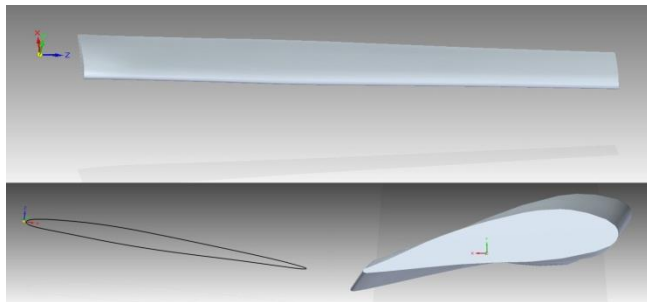
**Fig. 16.**  $C_p$  vs Wind Speed for Case I Optimized Turbine with blade length of 1.537m (5.04 ft)

The power output was compared with the installed commercial turbine and comparison is shown in Figure 17.



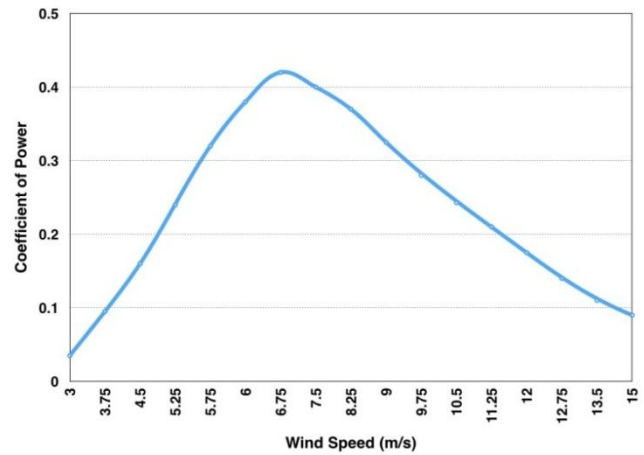
**Fig. 17.** Comparison of Power Output: Optimized Turbine and Commercial Turbine

The performance for both the turbines was predicted using the same M & S Environment. The optimized blades performed better at lower wind speeds till 7.75 m/s producing higher power output with the same geometric constraint, however, at higher wind speeds above 8 m/s the commercial turbine was able to achieve higher power output. The optimized values of variables are also annotated against each variable in Table I. The geometry of the optimized blade is shown in Figure 18.



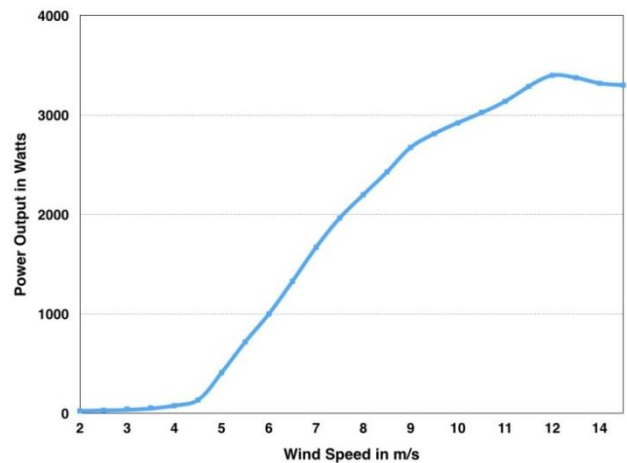
**Fig. 18.** Optimized Turbine Blade Geometry: Case I

**4.3.2 Case II - Power Output of 1 kW:** The turbine blades of 1.54 m (5.04 ft) could only achieve a maximum  $C_p$  value of 0.41 with corresponding power output of approximately 410 watts. In order to achieve the power output of 1 kW a second optimization problem was setup. The optimization function was to minimize blade length subject to achieving power output value of 1 kW. The optimization resulted in a blade length of 2.2 m (7.1 ft) with a  $C_p$  value of 0.42 within the wind speed range of 6 m/s to 7 m/s as shown in Figure 19.



**Fig. 19.**  $C_p$  vs Wind Speed for Case II Optimized Turbine with blade length of 2.134m (7.1ft)

The variation of power output with change in wind speed is shown in Figure 20.



**Fig. 20.** Power Output for Case II Optimized Turbine with blade length of 2.2 m (7.1 ft)

The ranges of variables for this optimization problem along with the optimum values are shown in Table II.

TABLE II  
 DESIGN RANGES & OPTIMIZED VALUES OF DESIGN VARIABLES : CASE II

Variable	Description	Lower Limit	Upper Limit	Optimized Value
$R_{leu}$	Upper Leading Edge Radius	0.02	0.035	0.025
$R_{lel}$	Lower Leading Edge Radius	0.01	0.02	0.017
$X_{up}$	Position of Upper Crest	0.25	0.4	0.32
$Y_{up}$	Upper Crest Point	0.1	0.14	0.1
$YX_{X_{up}}$	Upper Crest Curvature	-2	-1	-1.0
$X_{lo}$	Position of Lower Crest	0.2	0.3	0.28
$Y_{lo}$	Lower Crest Point	-0.1	-0.05	-0.074
$YXX_{lo}$	Lower Crest Curvature	0.5	1.5	1.29
$\alpha_{TE}$	Trailing Edge Direction Angle	-25	0	-21.6
$\beta_{TE}$	Trailing Edge Wedge Angle	0	25	1.12
$Y_{TE_{up}}$	Upper Surface TE Point	0	0.005	0.0
$Y_{TE_{lo}}$	Lower Surface TE Point	-0.005	0	0.0
$a_1$	Twist in degrees at 0.07R	3	8	7.2
$a_2$	Twist in degrees at 0.13R	13	18	17.0
$a_3$	Twist in degrees at 0.27R	6	13	9.2
$a_4$	Twist in degrees at 0.47R	3	6	5.4
$a_5$	Twist in degrees at 0.73R	-1	3	1.7
$a_6$	Twist in degrees at Tip	-5	-1	-1.9
$b_1$	Chord length in % R at 0.07R	9	10.5	10.2
$b_2$	Chord length in % R at 0.13R	10.5	12	10.9
$b_3$	Chord length in % R at 0.27R	9	10.5	9.4
$b_4$	Chord length in % R at 0.47R	8	9	8.2
$b_5$	Chord length in % R at 0.73R	7	8	7.6
$b_6$	Chord length in % R at Tip	6	7	6.8
$\theta_p$	Pitch Angle in degrees	-5	10	-0.78

The geometric shape of this turbine blade is shown in Figure 21.

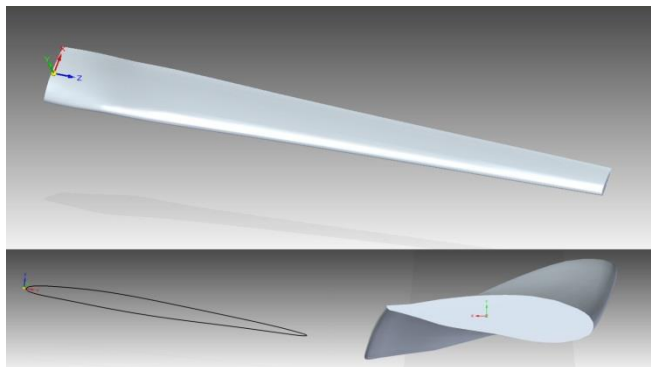


Fig. 21. Optimized Turbine Blade Geometry: Case II

#### 4.4 Trade Studies

With the optimal blade geometries determined for both the optimization problems, different trade studies were performed including effect of pitch angle change and effect of change in number of blades along with tip speed ratio (TSR) variation on power output. The results are as follows:-

**4.4.1 Effect of Pitch Angle:** The effect of different pitch settings was studied using three different pitch angle settings ( $-2^\circ$ ,  $-1^\circ$  and  $2^\circ$ ) on the optimized blade of 2.2 m (7.1 ft) length. The results are shown in Figure 22. At lower wind speeds, power output is independent of pitch setting, but effect of pitch setting is prominent at higher wind speeds in excess of 9 m/s. Pitch setting of  $-1^\circ$  was selected for the blade installation at which the turbine is expected to reach ( $C_p$ ) value of 0.42 or 42% aerodynamic efficiency.

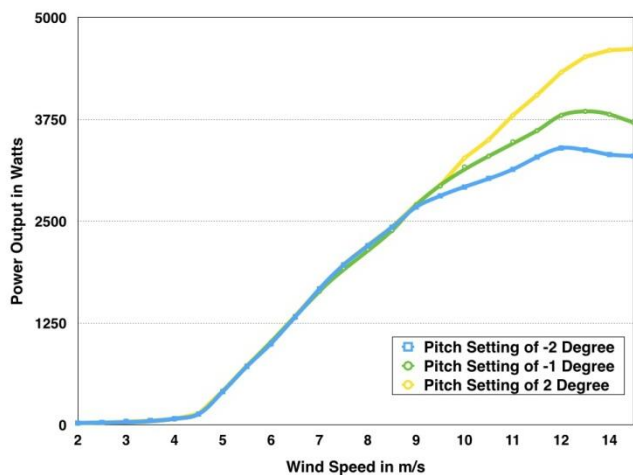


Fig. 22. Power Output Variation with different Pitch Settings for Case II Optimized Turbine with blade length of 2.2 m (7.1 ft)

**4.4.2 Effect of Number of Blades:** Tip speed ratio ( $\lambda$ ) varies with operational RPM for a fixed blade length operating at a specific wind speed. Increase in number of

blades results in higher values of  $C_p$  at lower values of TSR ( $\lambda$ ). Different turbine configurations based on optimized blade of 2.2 m (7.1 ft) length were analyzed with the number of blades varying from 2 to 5. The results are shown in Figure 23. Higher number of blades are preferred for lower values of TSR ( $\lambda$ ) or at higher wind speeds. Three bladed configuration is selected for the current domestic scale wind turbine of 1 kW output power.

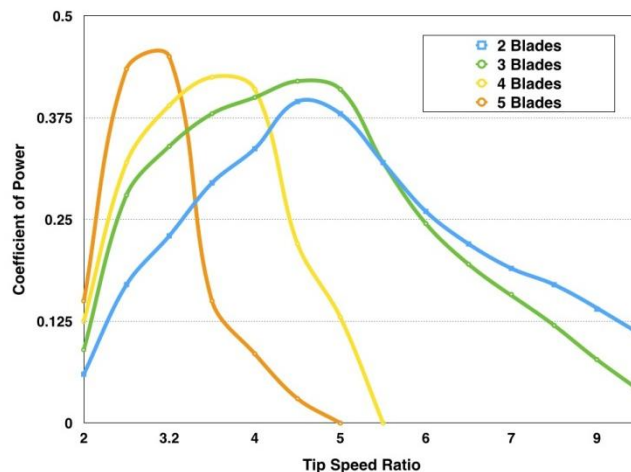


Fig. 23. Effect of number of blades for Case II Optimized Turbine with blade length of 2.2 m (7.1 ft)

#### 5. Structural Analysis

Aerodynamic loading on turbine blades varies with incoming wind velocity along with the rotational speed and generates structural bending moments on the blades. These bending moments based on the exposed area of the blades and axis of rotation cause structural stresses which are computed by YawDyn. These aero-elastic stresses will form the baseline for structural design of the blades. Depending on the factor of safety, materials, manufacturing process and structural components, the deformations are computed for different wind speeds. YawDyn performs structural analysis of the rotor in steady state rotation phase and turbine cut off state. The stresses produced during rotation phase are of three types as shown in Figure 24:-

- Out of plane bending stresses.
- In plane bending stresses.
- Stresses due to centrifugal force.

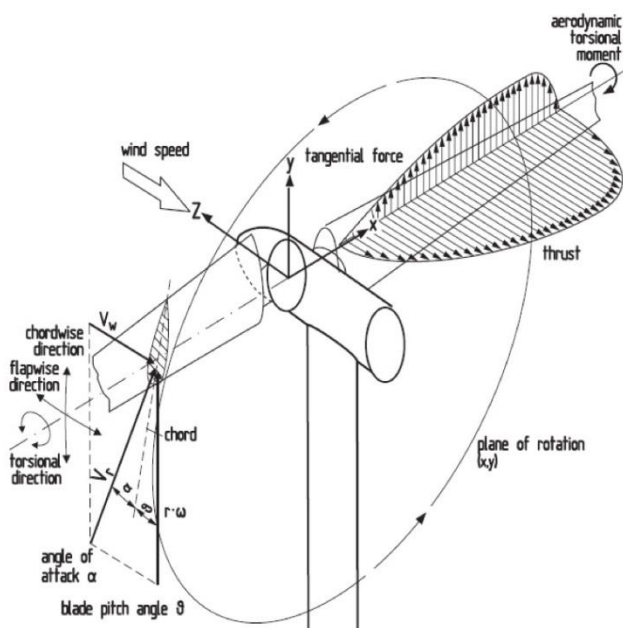


Fig. 24. Forces and Moments on rotating turbine blades

The stresses were computed for four span wise locations as shown in Figure 25. Vector sum of out of plane bending stresses, in plane bending stresses and centrifugal stresses in rotation phase gives the average stress in that specific section of blade.



Fig. 25. Span wise blade locations for Stress measurements

The stresses computed for Case I blade of 1.54 m (5.1 ft) length during rotation phase are given below. Integration along the span produces the average stress value in complete blade. The maximum value of stress was observed at the blade hub.

$$\begin{aligned} \sigma_{section1} &= 2.962MPa \\ \sigma_{section2} &= 3.241MPa \\ \sigma_{section3} &= 3.578MPa \\ \sigma_{blade} &= 4.002MPa \end{aligned}$$

Similarly, the stresses computed for Case II blade of 2.2 m (7.1 ft) length during rotation phase follow the same trend and are:-

$$\begin{aligned} \sigma_{section1} &= 2.3MPa \\ \sigma_{section2} &= 3.725MPa \\ \sigma_{section3} &= 4.85MPa \\ \sigma_{blade} &= 9.329MPa \end{aligned}$$

During the cut off phase, there are no centrifugal forces as the blade is not rotating, so only the out of plane and in plane bending stresses account for the stress in the sections and

blade. Based on the historical wind data of past 20 years in Pakistan, at wind speed of 47 m/s, the cut off stresses on the blades were evaluated and are as follows:

- For Case I blade of 54 m (5.1 ft)

$$\sigma_{blade} = 19.74MPa$$

- For Case II blade of 2.2 m (7.1 ft)

$$\sigma_{blade} = 35.486MPa$$

The stresses during cut off phase are higher than the rotation phase. Based on a factor of safety of 2.25, the structural strength required for the blades is as follows:-

- For Case I blade of 1.54 m (5.1 ft) Structural Strength required = 44.4 MPa
- For Case II blade of 2.2 m (7.1 ft) Structural Strength required = 79.8 MPa

## 6. Conclusion

In this paper, residential scale wind turbine blades operating at lower wind speeds in the range of 6 m/s to 7 m/s have been aerodynamically optimized for optimum power output. The comparison of power output of optimized blades turbine with commercially available turbine revealed higher outputs by optimized turbine, mainly due to the fact that off the shelf turbine is designed for higher wind speed operation. Furthermore, the turbines designed for higher wind speeds perform poorly on lower wind speeds and rated power significantly reduces with decrease in operating wind speed. A minimum of 2.2 m (7.1 ft) blade length is required to successfully achieve the power output of 1 kW at lower wind speeds in the range of 6 m/s to 7 m/s.

A fully automated modeling and simulation environment was generated for this research project to facilitate quick design iterations. AeroDyn and YawDyn routines by NREL, USA based on blade element momentum theory and generalized dynamic wake methods were utilized as basic tools for aero elastic computations. Tip losses, hub losses and skewed wake effects were accounted by empirical corrections in the AeroDyn. The results of M&S environment were validated by analysis of an existing installed wind turbine and were found in strong agreement with the manufacturer's suggested power values. Gradient based first order optimization search techniques were not very effective in this problem due to the presence of numerous local optima. Due to the lesser computational intensity involved, Genetic algorithm despite higher number of function calls yielded better optimization results in achieving the global optima. The formulated M&S environment along with the GA optimizer utilized for current

research work can be used for design and optimization of bigger scale wind turbines and similar design problems.

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