Developing a Formula for Optimum Power of an Inverted Piston-in-Cylinder Wave Engine

Mohammad Sameti*‡, Alibakhsh Kasaeian*

*Department of Renewable Energies, Faculty of New Sciences & Technologies, University of Tehran, Iran

(muh.sameti@ut.ac.ir, akasa@ut.ac.ir)

‡ Corresponding Author; Mohammad Sameti, Tel: +98 913 212 4120,

Fax: +98 21 8861 7087, muh.sameti@ut.ac.ir

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Abstract - Harvesting energy from the ocean waves has two significant advantages over other renewables: more attractive mean power density and integration into coastal structures. A commonly deployed device is the oscillating water column (OWC), which has so far been mounted on shore and proposed for floating plants. It is an inverted piston-in-cylinder which consists of an air chamber in contact with the sea so that the water column in the chamber oscillates with the waves and makes the air flow in and out of the chamber which turns a turbine. The objective of this study is to propose simple formulas for estimating the output power, efficiency and their maximum values of an inverted piston-in-cylinder converter based on the engine geometry and the most occurring sea state wavelength. Two mathematical models were developed for a floating inverted piston-in-cylinder wave engine based on the theoretical fluid dynamics and expressions were proposed for its output power and overall efficiency. The optimum values were found for both the power and efficiency. The relationship between two models is investigated and it is showed that however the second model is simpler than the first one but, it can be used when the wavelength is more than a specific value.

Keywords Ocean Renewable Energy, Wave Energy, Inverted Piston-in-Cylinder Wave Engine, Power Optimization

1. Introduction

The ocean renewable energy resource includes wide-range energy conversion potentials. The energy can be harvested from its waves and tides, temperature difference between cold deep waters and warm surface waters, salinity differences at river mouths, ocean currents, salinity gradients and ocean-grown biomass [1]. Among these resources of ocean renewable energies, wave energy, with its vast energy resource, is considered as one of the most promising technologies due to its most reliability between other renewable energy resources. Ocean waves are the result of transferring of the solar energy to the wind and then water. The wind created by the solar energy blows over the ocean which converts the wind energy to wave energy. The arisen ocean waves can travel with little energy loss for thousands of kilometres (table 1). The most important aspect of wave energy is its regular power source with an intensity that can be exactly predicted days before their arrival. The time availability of the wave power at a potential site is up to 90%, while its value for solar and wind energies tend to be just about one third of that of the wave (20% or 30%) [2]. According to the World Energy Council, the ocean wave energy represents a net available quantity of 140 to 700TWh per year which equals to 1% to 5% of the annual world electricity demand. Considering the recoverable energy and if the efficient conversion systems are utilized, the annual quantity can even reach 2000TWh [3]. Variability in several time-scales is the main disadvantage of the wave energy which includes wave to wave, with sea state, and month to month variability [4].

Ocean wave energy engines take advantage of the contained kinetic and potential energy in the natural oscillations of ocean and sea waves to convert them into electricity. Fig. (1) shows a variety of mechanisms existing for the utilization of the ocean wave energy source. As fig. (2) depicts, an Oscillating Water Column (OWC) comprises of a fixed or floating buoy with an air chamber and an air-driven combined turbine-generator.

When the waves hit the body, the chamber water level increases and this increase creates a pressure difference between the air in the chamber and the ambient air. The pressurized air applies a force to the air turbine and rotates it which in turn, drives the electric generator and creates electrical power at its output terminals. The water level in the chamber decreases when the waves are pulled back to the ocean. This time, the syringe effect causes the air turbine to rotate in the same direction and a continuous electrical power is produced.

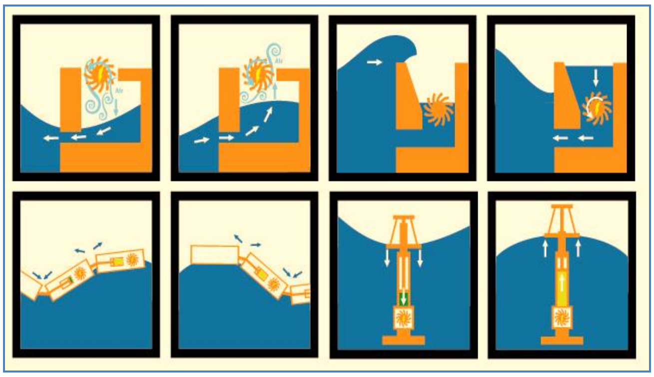

Fig. 1. Popular concepts for wave energy conversion: oscillating water columns, attenuators, overtoppings and point absorbers [5].

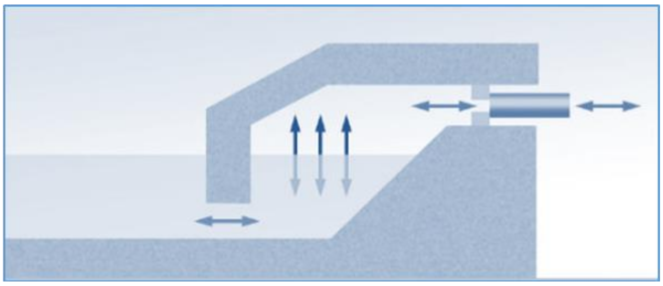

Fig. 2. Operation of a single off-shore OWC converter [13].

Falcao [4] and Drew et al. [6] published the most recent review on the wave energy conversion technologies. Heath [7] reviewed the history of OWC systems from whistling buoys to generation systems which are connected to the grid. He mentioned in his paper that on the practical level, the advantages of the OWCs are:

- very few moving parts and no moving parts in the water;
- adaptability of the concept and its utilization in the near shore region or floating offshore;
- eliminating the need for gearboxes by using an air turbine;
- reliability and ease of maintenance and
- Utilization of the sea space in an efficient way.

Josset and Climent [8] presented a time-domain numerical simulation of OWC wave power plants and estimated the annual performance of the European wave energy power plant seated on Pico Island, Azores. The linearized problems of radiation and scattering for a hollow cylinder with an open bottom are solved by Martins-Rivas and Mei [9] using usual method of Eigen function expansions and integral equations and a theoretical study on a single OWC installed at the tip of a long and thin breakwater is performed. Brito-Melo et al. [10] investigated the influence of the Wells turbine aerodynamic design on the overall OWC power plant performance by numerical simulation. Delaure and Lewis [11] used first order mixed distribution panel method and solved the steady state potential flow problem due to regular wave interaction with the OWC. The study of floating OWC wave energy converter performance including analysis of the dynamic coupling of the water column and the floating structure was performed by Stappenbelt and Cooper [12] and a mechanical oscillator model was proposed.

Despite the above studies which are usually based on the differential approach, a simple control volume approach is used in this study to derive a formula for approximating the output power and the efficiency of a single OWC converter. The air in the chamber is considered as the control volume and the mass balance equation is used to obtain the instantaneous velocity of the outgoing or incoming air. Then the output power can be achieved using energy balance for the turbine.

Table 1. Appearance properties of accepted manuscripts

Wave Type	Wavelength λ	Disturbing Force
Capillary wave	Less than 0.02m	Wind
Wind wave	60m-150m	Wind
Tsunami	$2\times10^5 m$	Faulting of sea floor, volcanic eruption, landslide

2. Models and Solution Methods

A simple model to harvest energy from the up-down motion of the waves is to use the inverted cylinder and piston assembly, which is depicted in fig. (3). The buoyant piston is moved vertically with the wave motion. Since the water surface in the chamber doesn't have the same shape as the ocean sinusoidal wave, two different types of piston models are considered and compared in the point of output power and efficiency: deformable sinusoidal piston and solid plane piston models. Any losses are neglected to achieve the maximum available power. Further assumption for the analysis is the incompressibility of the air in the chamber which means that the density is uniform and equals to the outdoor density. The general traveling wave equation of amplitude a, wavelength λ and velocity c is [15]:

$$y(x,t) = a\sin\frac{2\pi}{\lambda}(x-ct) \quad (1)$$

where y is the vertical displacement of the water particles. The wave height H from the top of a crest to the bottom of a trough is twice the amplitude H = 2a. By defining the wave number as k=2π/λ and the wave speed as c=ω/k proportional to radiant frequency ω, equation (1) can easily be written in the form of equation (2) as:

$$y(x,t) = \frac{H}{2}\sin(kx-\omega t) \quad (2)$$

The volume of the air at any point of the time V(t) is the total chamber volume V_c minus the volume occupied by the water $V_w(t)$:

$$V(t) = V_c - V_w(t) = V_c - \iint y(x,t)dA \tag{3}$$

The multiple integral is calculated on the cross section of the chamber.

2.1. Deformable Sinusoidal Piston

For the first case, the water surface profile y(x,t) in fig. (3) is considered to be sinusoidal. It is assumed that the cross section is a rectangular with length w perpendicular to wave direction and length l parallel to the wave direction. Then, the multiple integral in equation (3) is transformed to a simple integral and the air volume can be calculated as:

$$V(t) = V_c - w\int_{-\frac{l}{2}}^{\frac{l}{2}} y(x,t)dx$$

$$= V_c + \frac{wH}{2k}\left[\cos(\frac{kl}{2} - \omega t) - \cos(\frac{kl}{2} + \omega t)\right] \tag{4}$$

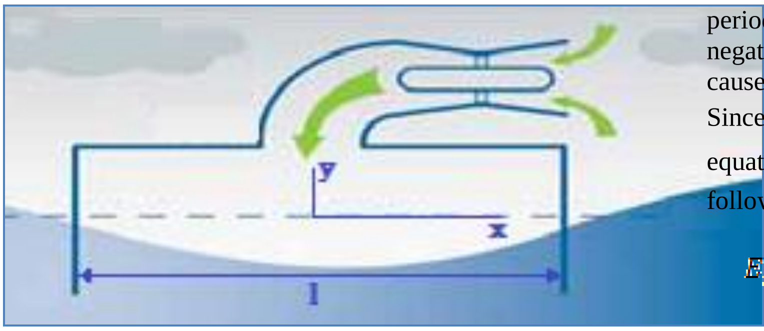

Fig. 3. Selection of the coordinate system in the analysis

The trigonometric expression in the bracket can be simplified further using the cosine subtraction rule, $\cos(p+q) - \cos(p-q) = -2\sin p \sin q$ and an expression for the instantaneous air volume is achieved:

$$V(t) = V_c + \frac{wH}{k}\sin\frac{kl}{2}\sin\omega t \tag{5}$$

Continuity equation for the control volume (CV) with mass m_{CV} can be written for air chamber as:

$$\dot{m}_{in} - \dot{m}_{out} = \frac{dm_{CV}}{dt} \tag{6}$$

where dot (.) denotes the time derivative and the indices in and out stand for incoming and outgoing flow, respectively. Therefore, assuming an incompressible fluid flow, the rate of change of the air volume $\dot{V}_a(t)$ which is the derivative with respect to the time t is the air volume rate passing the turbine:

$$\dot{V}_a(t) = \frac{dV(t)}{dt} = \frac{wH\omega}{k}\sin\frac{kl}{2}\cos\omega t = wHc\sin\frac{kl}{2}\cos\omega t \tag{7}$$

The turbine extracts the kinetic energy of the moving air with speed v and mass flow rate $\dot{m}_a$ at maximum rate of $\frac{16}{27}$ which is known as the Betz's limit [16] and converts it to the mechanical rotational power $\dot{E}_t$:

$$\dot{E}_t = \frac{1}{2}\frac{16}{27}\dot{m}_a v^2 = \frac{8\varrho}{27A_c^2}\dot{V}_a(t)^3 \tag{8}$$

$$\dot{E}_t = \frac{8\varrho}{27A_c^2}w^3H^3c^3\sin^3(\frac{kl}{2})\cos^3(\omega t) \tag{9}$$

where ϱ is the air density and A_c is the turbine cross section. The total delivered periodic energy E_t during one cycle or period T of the incident wave can be expressed as:

$$E_t = 4\int_0^{T/4}\dot{E}_t dt$$

$$= \frac{32\varrho}{27A_c^2}w^3H^3c^3\sin^3(\frac{kl}{2})\left[\frac{1}{\omega}\sin\frac{\omega T}{4} - \frac{1}{3\omega}\sin^3\frac{\omega T}{4}\right]. \tag{10}$$

where the integral is calculated in one quarter of the period instead of one period. The reason for that is the negative sign appears for the air volume in fig. (3) and caused by the surface water lower than the mean surface line. Since $\omega T = 2\pi$, the delivered periodic energy E_t (from equation 10) and the average power P_t can be calculated as follows:

$$E_t = \frac{32}{81\pi A_c^2}\varrho T w^3H^3c^3\sin^3(\frac{kl}{2}) \tag{11}$$

$$P_t = \frac{32\varrho}{81\pi A_c^2 T^3}w^3H^3\lambda^3\sin^3(\frac{\pi l}{\lambda}) \tag{12}$$

The power of a perfect sinusoidal and unidirectional monochromic traveling ocean wave can be calculated quite easily if the depth of the ocean in which it travels is supposed to be mathematically infinite. In practical purposes, it is supposed that the depth h is larger than half a wavelength λ. It can be shown under the above conditions [17] that the mean power transmitted for width w of the wave front in the direction of propagation is expressed as:

$$P_w = \frac{\rho g^2}{32\pi}H^2Tw \tag{13}$$

where ρ is the ocean water density (usually $1024\frac{kg}{m^3}$) and g is the gravity acceleration. The more generalized equation for wave power transfer is [18]:

$$P_w = Encw \tag{14}$$

$$n = \frac{1}{2}(1 + \frac{2kh}{\sinh 2kh}) \tag{15}$$

and E is the time-averaged, wave-induced energy (potential E_p plus kinetic E_k) per unit horizontal area:

$$E = E_k + E_p = \frac{1}{2}\rho g^2 a \tag{16}$$

Considering equations (12) and (13) or (11) and (16), the efficiency η of the wave engine can be easily found by:

$$\eta = \frac{P_t}{P_w} = 0.131\frac{\varrho w^2 H c^3}{\rho A_c^2 T}\sin^3\left(\frac{kl}{2}\right) \tag{17}$$

$$\eta = 0.131\frac{\varrho w^2 H \lambda^3}{\rho A_c^2 T^4}\sin^3\left(\frac{\pi l}{\lambda}\right) \tag{18}$$

2.2. Solid-Plane Pistons

In this case, the surface water profile is not a function of the wavelength or wave number and thus x so it can be expressed as:

$$y(t) = a\sin\omega t = \frac{H}{2}\sin\omega t \tag{19}$$

It means that the water surface in the chamber is always a flat surface which oscillates vertically with a frequency equal to that of the traveling ocean waves. Similar to the analysis in section (2.1), the air volume flow rate, the mechanical rotational power, periodic energy, average power and efficiency are as follows:

$$\dot{V}_a = lw\frac{dy(t)}{dt} = \frac{lw\omega H}{2}\cos\omega t \tag{20}$$

$$\dot{E}_t = \frac{\varrho}{27A_c^2}(lw\omega H)^3\cos^3\omega t \tag{21}$$

$$E_t = \frac{8\varrho}{81A_c^2}(lwH)^3\omega^2 \tag{22}$$

$$P_t = 3.9\frac{\varrho}{A_c^2}\frac{(lwH)^3}{T^3} \tag{23}$$

$$\eta = 4.06\frac{\varrho l^3 H w^2}{\rho A_c^2 T^4} \tag{24}$$

It should be kept in mind that for the deep regions of the ocean (as discussed before for $2h > \lambda$), period and wavelength are functionally dependent [16,18]:

$$T_{deep} = \sqrt{\frac{2\pi}{g}\lambda} \tag{25}$$

$$c_{shallow} = \sqrt{\frac{g}{k}} \tag{26}$$

3. Results and Discussions

For the case $A_c = 4m^2$ (2 turbines are used), $w = 12m$ and $l = 18m$ which equation (12) results a $450kW$ maximum power output for the sea state $T_{max} = 5.7s$, $H = 6m$ and $\lambda_{max} = 50m$. This maximum output power corresponds to 18.5% efficiency. The trends are illustrated in figs (4) and (5) for power and efficiency, respectively. It is obvious that there is maximum point for each graph but, these points do not occur in the same wavelength. The reason for this is that the term before the sinusoidal expression is $\lambda^{1.5}$ for the power but λ for the efficiency. To obtain it, one can introduce equation (25) into equations (12) and (18). However, both power and efficiency decreases smoothly by increasing the wavelength. As it can be seen from fig. (4), two model diverge as the wavelength increases and for $\lambda > 110$ (difference between two models is less than 10%), there is no significant differences between the power estimations. Mathematically we can write:

$$\sin^3\left(\frac{\pi l}{\lambda}\right) \cong \left(\frac{\pi l}{\lambda}\right)^3 \text{ as } \lambda \to \infty \tag{27}$$

and the average power in equation (12) becomes:

$$P_{t,\lambda\to\infty} = \frac{32\pi^2}{81A_c^2T^2}\varrho w^3 H^3 l^3 \tag{28}$$

which is exactly the derived expression in equation (23). The same relation exists between equations (24) and (18) for the efficiencies. Since the second model is independent from wave number, the formulas are rough estimators for power and efficiency and are only useful when the wave number or wavelength are not available. It is only useful in high wavelengths where the difference between two models in power estimation decreases significantly. In fig. (4), this amount is less than 25 kW.

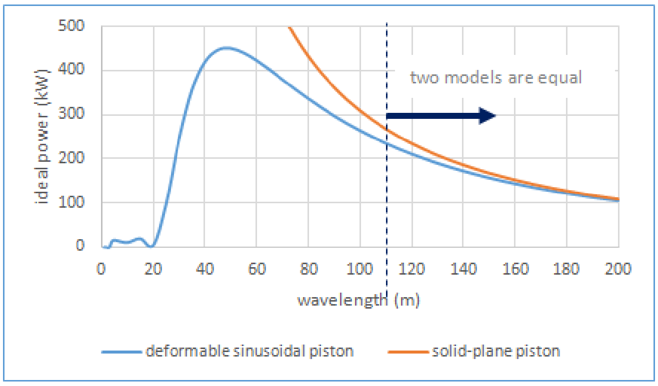

Fig. 4. Variations of output power versus ocean wavelength for two proposed models

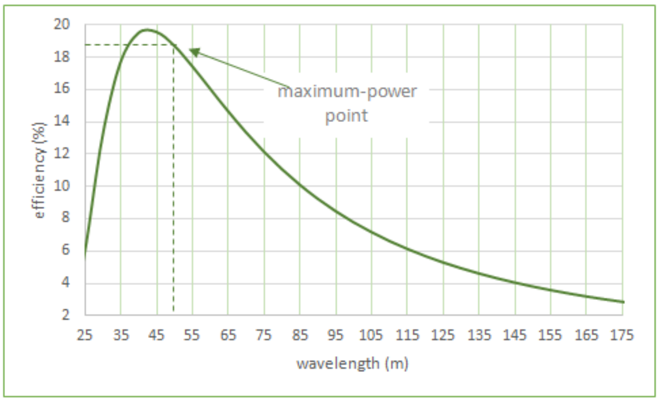

Fig. 5. Variations of efficiency versus ocean wavelength

As a physical viewpoint, when the wavelength increases, the sinusoidal shape of the traveling wave approaches a straight line and since the device is extracting energy only in a very small part of the curve ($l \ll \lambda$) the physical problem dependence to the wave number decreases considerably. This is the case that was assumed in the solid-plane piston model. The percentage of power estimation difference due to using the second model formula instead of utilizing the first one is shown in fig. (6). It should be noted that for a special design problem, the equation:

$$\sin^3\left(\frac{\pi l}{\lambda}\right) - 3\frac{\pi l}{\lambda}\sin^2\left(\frac{\pi l}{\lambda}\right)\cos\left(\frac{\pi l}{\lambda}\right) = 0 \qquad (29)$$

or

$$\tan\left(\frac{\pi l}{\lambda}\right) = 3\frac{\pi l}{\lambda} \qquad (30)$$

should be solved for finding design parameter l to obtain the maximum power output for the most occurring sea state. Equation (29) can be derived by differentiating equation (12) with respect to wavelength λ. Defining the non-dimensional parameter $l^* = \frac{\pi l}{\lambda}$ for length of the wave engine, equation (30) has a graphical solution as it is depicted in fig. (7). The first two answers illustrated in fig. (7) are:

$$l_1^* = 1.33 \text{ and } l_2^* = 4.64 \qquad (31)$$

which means that for a specific wavelength, the optimum power output occurs when the design parameter l is almost 42% of the ocean wavelength. Then, the maximum output power is:

$$P_t = 1.93\frac{\varrho}{A_c^2}w^3H^3\lambda \qquad (32)$$

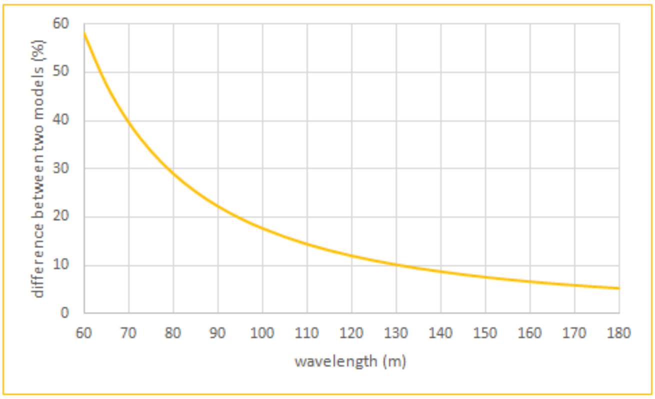

Fig. 6. Percentage of power estimation difference of two models

However, the values in equation (31) are not the optima of the efficiency, since the proper optimum problem which can be derived from equation (18) and with a similar method to that mentioned above is:

$$\tan\left(\frac{\pi l}{\lambda}\right) = 6\frac{\pi l}{\lambda} \qquad (33)$$

The first solution for equation (33) is $l_1^* = 1.457$ which means that for a specific wavelength, the optimum power output occurs when the design parameter l is almost 46% of the ocean wavelength. The maximum instantaneous power is illustrated in fig. (8).

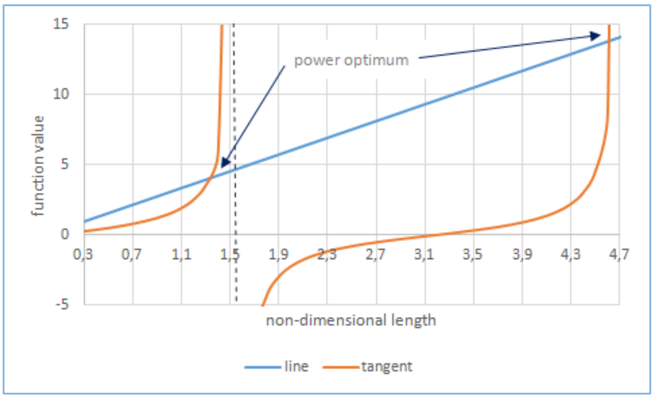

Fig. 7. Graphical solution for the maximum power as a function of non-dimensional length.

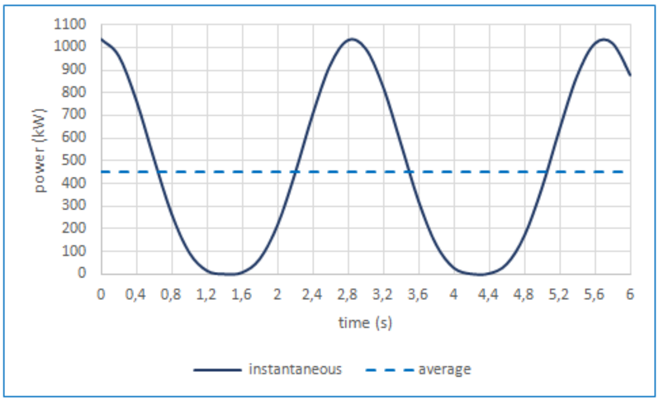

Fig. 8. Maximum instantaneous power and average power.

4. Conclusions

Wave energy studies are usually based on the differential approach however, control volume approach is used in this study to derive a formula for approximating the output power and the efficiency of a single OWC converter. Two models were presented based on the general inverted piston-in-cylinder model: deformable sinusoidal piston model and solid-plane piston model. There are two benefits for the proposed method: (1) Equation (12) or (23) can simply be used to estimate the power output of a near-shore inverted piston-in-cylinder power plant. (2) The design parameter l can be obtained to harvest the maximum power for the most occurring sea state wavelength, in other word there is a maximum power point (MPP). The output power is proportional to the third power of wave height and device width. It is inversely proportional to the square of the outlet/inlet air cross section area. However, the power independence to the wave length is not linear. Comparison between these approaches yields that two models diverge as the wavelength increases and for a specific wavelength, there is no significant differences between the power and efficiency estimations. There are maximum points for both the power and the efficiency graphs but, these points do not occur in the same wavelengths. The optimum power output occurs when the design parameter l is almost 42% of the ocean wavelength while this value is 46% for the efficiency. Thus, for a specific region the most occurring wavelength can be used to determine the optimum length of the wave engine. One case is also solved to illustrate the ideal maximum output of an existing plant and it is obtained that the plant produces 450 kW power. If this cross section area was used in a 100%-efficient solar thermal collector, assuming the solar constant to be $G_{0=}1360$ W/m^2, it would obtained that only 294 kW thermal energy can be extracted; furthermore, the output should pass the second cycle to be converted into turbine rotational energy. It can be easily understood that wave energy has the more attractive mean power density among other renewables. It is recommended that the formulas are improved by considering the water-air interaction using reference [19] and compared with a real world power plant.

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