An Exact Analytical Method for Calculating the Parameters of a Real Solar Cell Using Special Trans Function Theory (STFT)

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Abstract-An exact analytical method using special trans function theory (STFT) is presented to calculate the parameters of a real solar cell. Calculated values are compared with other methods. Various characteristic curves are also drawn using the analytical expressions to validate this approach. The effect of the parasitic resistances in the fill factor is studied in detail. Relative percentage accuracy of STFT method is also calculated to show the significance of the method over other methods.

Keywords-STFT, Estimation of Parameters, Solar Cell

1. Introduction

With the advent of photovoltaic in recent years when its global production saw another year of extraordinary growth with a total capacity of 70 gigawatts worldwide [1], estimation of solar cell parameters becomes a crucial issue for describing their non-linear electrical behavior. These parameters are important not only for evaluating solar cell performances and quality control, but also play an important role in fabrication process optimization and scientific research [2].

To address the issue of the estimation of these parameters, various methods have been suggested. Many of the methods are either based on iterative or analytical approximations [2-5]; none of the reported methods present the explicit analytical solution of the transcendental I-V relation for single diode solar cell model till Jain and Kapoor [6] deduced the same using Lambert W-function. Currently, much research initiatives are on to improve the existing methods for the extraction of solar cell parameters with utmost accuracy [7, 8].

Recently Special Trans Function Theory (STFT) [9] had been used to determine the junction ideality factor of a solar cell [10]. STFT properties are well documented for solving transcendental equations. Compared with other methods, the advantage of W-function and STFT technique is the availability of analytical explicit solution. A numerical comparison analysis between STFT and classical approach (using W-function) is documented and it is found that STFT method has superior accuracy and computational efficiency over the famous Lambert W- function. The differentiability of the trans function in STFT makes it possible to analyze analytically any problem rigorously and is applicable for arbitrary nonlinear forms [9].

The present work describes a potential theoretical investigation of current-voltage characteristics of solar cells using STFT. In this the single diode equation of a solar cell is written in a suitable form to comply with STFT. The proposed method, based on explicit analytical expressions, enables the determination of its various characteristic parameters. Comparisons are also made with the results of other methods as well as with experimental data to prove the superiority of the method. It must be pointed out that the principal motivation of this work consists of developing an analytical approach that gives a convenient calculation methodology for the determination of solar cell parameters very accurately.

2. Theory

The most exploited transcendental I-V relation of a real solar cell under illumination is

$$I = I_0\left(\exp\left(\frac{V-IR_s}{nV_{th}}\right)-1\right)+\frac{V-IR_s}{R_{sh}}-I_{ph} \tag{1}$$

where I and V are the current and voltage output of the cell respectively, V_{th} (=k_BT/q) is the thermal voltage, I_{ph} is the light generated photocurrent, I_0 is the reverse saturation current of the diode, n is the diode ideality factor, R_s and R_{sh} are the parasitic series and shunt resistances respectively. The explicit analytical expression for current in terms of V and the newly defined trans function (*trans*$_+$(*D*)) and the model's parameters using STFT is [10]

$$I = \left(\frac{\frac{V}{R_{sh}}-\left(I_{ph}+I_0\right)}{1+\frac{R_s}{R_{sh}}}\right)\left(\frac{nV_{th}\left(1+\frac{R_s}{R_{sh}}\right)}{R_s\left(I_0+I_{ph}-\frac{V}{R_{sh}}\right)}trans_+\left(D\right)-1\right) \tag{2}$$

or

$$I = \left(\frac{\left(I_{ph}+I_0\right)-\frac{V}{R_{sh}}}{1+\frac{R_s}{R_{sh}}}\right)+\frac{nV_{th}}{R_s}\left(D\left(\frac{\sum_{m=0}^{[x]}\frac{D^m\left(x-m\right)^m}{m!}}{\sum_{m=0}^{[x+1]}\frac{D^m\left(x+1-m\right)^m}{m!}}\right)\right) \tag{3}$$

The trans function is defined by (*trans*$_+$(*D*))

$$Z=trans_+(D) \tag{4}$$

where *trans*$_+$(*D*) is a new special trans function [9, 10] defined as

$$trans_+\left(D\right)=\lim_{x\to\infty}\left[D\left(\frac{\varphi_+\left(D,x\right)}{\varphi_+\left(D,x+1\right)}\right)\right] \tag{5}$$

with

$$\varphi_+\left(D,x\right)=\sum_{m=0}^{[x]}\frac{D^m\left(x-m\right)^m}{m!} \tag{6}$$

so that

$$trans_+\left(D\right)=\lim_{x\to\infty}\left(D\left(\frac{\sum_{m=0}^{[x]}\frac{D^m\left(x-m\right)^m}{m!}}{\sum_{m=0}^{[x+1]}\frac{D^m\left(x+1-m\right)^m}{m!}}\right)\right) \tag{7}$$

and D as given in [10]

$$D = \frac{I_0R_s\exp\left(\frac{V}{nV_{th}}\right)\exp\left(\frac{R_s\left(I_0+I_{ph}-\frac{V}{R_{sh}}\right)}{nV_{th}\left(1+\frac{R_s}{R_{sh}}\right)}\right)}{nV_{th}\left(1+\frac{R_s}{R_{sh}}\right)} \tag{8}$$

Here [x] denotes the greatest integer less than or equal to x. The number of accurate digits in the numerical structure of the parameter will depend on [x] [9, 10]. We obtain the short-circuit current, I_{sc} , by substituting V=0 and the open-circuit voltage, V_{oc} , by putting I=0 in Eq. (3).

In the STFT we have the possibility to obtain different gradients of the newly defined trans$_+$ function [9] and hence make it useful in the analytical analysis of any physical problem. That is

$$\frac{\partial Z}{\partial V}=\frac{trans_+(D)}{D\left(1+trans_+(D)\right)}\frac{\partial D}{\partial V} \tag{9}$$

Using this property of differentiability of the trans$_+$ function, the respective dynamic resistances R_{so} and R_{sho} at the open-circuit voltage and short-circuit current can be evaluated. Both R_{so} and R_{sho} are also the slopes of I–V curve at open- and short-circuit conditions. That is

$$\frac{1}{R_{so}}=\left(\frac{\partial I}{\partial V}\right)_{V=V_{oc}} \tag{10}$$

and

$$\frac{1}{R_{sho}}=\left(\frac{\partial I}{\partial V}\right)_{I=I_{sc}}=\left(\frac{\partial I}{\partial V}\right)_{V=0} \tag{11}$$

with

$$\left(\frac{\partial I}{\partial V}\right)=\frac{1}{R_{sh}\left(1+\frac{R_s}{R_{sh}}\right)}+\frac{1}{R_s\left(1+\frac{R_s}{R_{sh}}\right)}\frac{trans_+\left(D\right)}{\left(1+trans_+\left(D\right)\right)} \quad 12)$$

Substituting the values of the model parameters in Eq. (12), we can determine the values of R_{so} and R_{sho} with respect to the conditions of Eq. (10) and Eq. (11) respectively.

The power at the output terminal can be expressed as

$$P= IV \tag{13}$$

Corresponding to maximum power point, voltage (V_{mp}) and current (I_{mp}) can be obtained by determining the maxima of the power relation

$$\left(\frac{\partial P}{\partial V}\right)_{V=V_{mp}}=0 \tag{14}$$

Differential of power with respect to voltage may be expressed as

$$\left(\frac{\partial P}{\partial V}\right)=\frac{\frac{2V}{R_{sh}}-\left(I_{ph}+I_0\right)}{1+\frac{R_s}{R_{sh}}}+\frac{nV_{th}}{R_s}trans_+\left(D\right)+\frac{V}{R_s\left(1+\frac{R_s}{R_{sh}}\right)}\frac{trans_+\left(D\right)}{\left(1+trans_+\left(D\right)\right)} \tag{15}$$

Solving Eq. (15) with the condition of Eq. (14), we can obtain the optimum voltage, V_{mp}, and correspondingly we can evaluate the optimum current, I_{mp}, at this point.

The fill-factor (FF), a measure of the squareness of the I-V curve, is calculated as

$$FF=\frac{V_{mp}I_{mp}}{V_{oc}I_{sc}} \tag{16}$$

3. Calculations

To investigate the validity of STFT method it is applied to the experimental solar cell I-V characteristics based on the single diode model including the parasitic series and shunt resistances, and the results are summarized in Table 1, 2 and 3. Various parameters for two experimental solar cells (namely blue solar cell and grey solar cell), using the data of Charles et al [3] and Phang et al [4], are calculated using the above analytical equations obtained through STFT and compared it with the results of other methods using the same reference parameters. The estimated results are compared with the experimental data and the relative accuracies are also determined. The calculations were performed in a Maple environment using 20-digit precision on Windows XP platform.

Table 1. Comparison between experimental and calculated data for Grey Solar Cell

Parameters	Exp. Data [3, 4]	W-function [6]	Present method(STFT)	Accuracy (%) (W-Function)	Accuracy (%)(STFT)
V_{oc} (V)	0.524	0.52093	0.52248	0.585	0.290
I_{sc} (A)	0.561	0.55931	0.55931	0.301	0.301
R_{so} (Ω)	0.162	0.16121	0.16148	0.487	0.320
R_{sho}(Ω)	25.9	25.896	25.767	0.015	0.513
V_{mp} (V)	0.390	0.38473	0.38596	1.35	1.035
I_{mp} (A)	0.481	0.48335	0.48334	0.488	0.486
FF	0.638	0.63824	0.63837	0.037	0.057
T (K)	307				

Table 2. Comparison between experimental and calculated data for Blue Solar Cell

Parameters	Exp. data[3,4]	W-function [6]	Present method(STFT)	Accuracy (%) (W-Function)	Accuracy (%)(STFT)
V_{oc} (V)	0.536	0.53465	0.53618	0.251	0.033
I_{sc} (A)	0.1023	0.10229	0.10229	0.009	0.009
R_{so}(Ω)	0.45	0.44298	0.44998	1.56	0.004
R_{sho}(Ω)	1000	997.4018	996.909	0.259	0.309
V_{mp} (V)	0.437	0.43191	0.43317	1.16	0.876
I_{mp} (A)	0.0925	0.093396	0.093394	0.968	0.966
FF	0.736	0.73759	0.7373	0.216	0.176
T (K)	300				

Table 3. Values of characteristic parameters obtained by different methods of a Plastic Solar Cell

Parameters	Co-content function [12]	W-function method	STFT method
V_{oc}(V)	0.755	0.754	0.754
I_{sc}(mA cm^{-2})	7.61	7.608	7.608
R_{so} (Ω cm^2)	22.12	22.11	22.11
R_{sho} (Ω cm^2)	205.76	205.8	205.8
V_{mp}(V)	0.548	0.547	0.548
I_{mp}(mA cm^{-2})	4.71	4.708	4.703
FF	0.4492	0.4489	0.4492
T (K)	300		

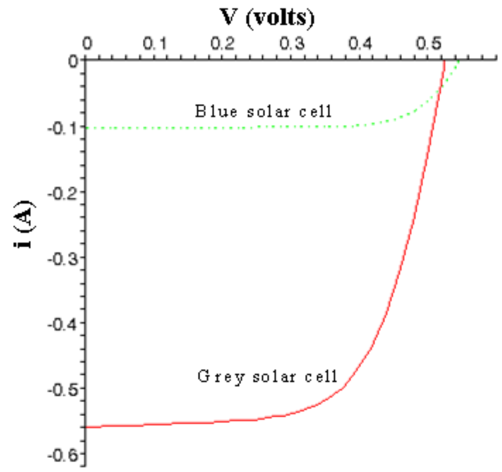
Plastic solar cells have become an intensive field of research because of their advantageous solution processing capability and formation of low-cost, flexible, and large area electronic devices [11]. However, one major limitation to the possible large-scale application of plastic solar cells has been its low efficiency compared to inorganic solar cells. In general, plastic solar cells have large series resistance and small shunt resistance, which tend to reduce the efficiency and hence represent an extreme case where the effects of parasitic resistances are remarkably significant.

Using the experimental data published for a developmental plastic solar cell [12], its different characteristic parameters are calculated using STFT method and are compared with the results obtained by other techniques [Table 3].

4. Results and Discussion

STFT technique is exploited for calculating the parameters of solar cells. The different parameters were calculated using the explicit analytical expression for current in terms of *V* and the model parameters, and its derivative with respect to *V*. The method was applied to three different types of experimental solar cells. Table 1 and Table 2 presents the values of the parameters obtained by STFT method for two experimental solar cells (Grey solar cell and Blue solar cell). Table 3 shows that the evaluated parameters of a plastic solar cell using STFT approach are largely comparable to those obtained by other techniques [6, 12]. Comparison was also made with the existing analytical methods to establish its validity and is apparent in different tables.

To further visualize the analytical expressions obtained by the application of STFT, various characteristic curves are drawn. Figure 1 (a,b) shows the current-voltage characteristics of different solar cells using the analytical solution of STFT. I-V curves for various R_s and R_{sh} are also plotted in Fig. 2 (a,b). In Fig. 3 (a,b), variation of FF of a typical solar cell (in this case blue solar cell) is drawn against R_s and R_{sh} respectively which are in agreement with the theory.



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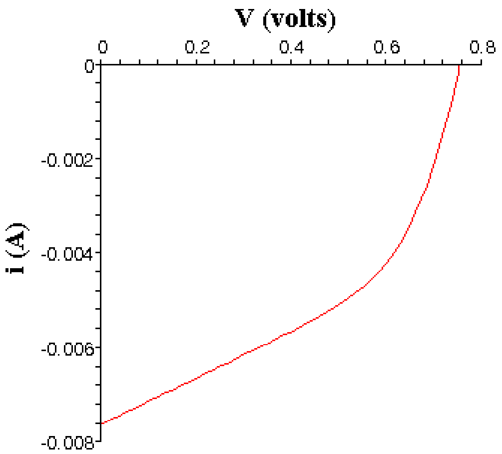

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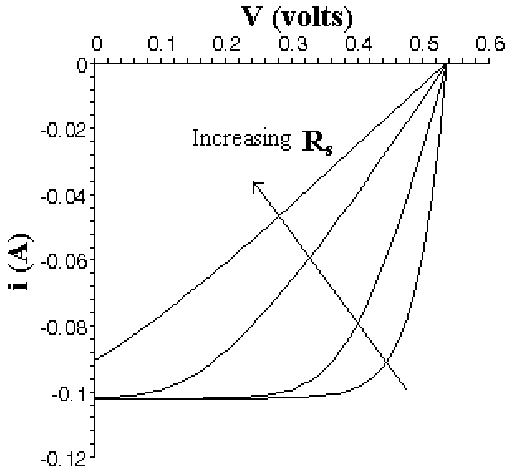
Fig. 1.(a). I-V characteristics of a Grey and Blue solar cell, (b). I-V characteristics of a Plastic solar cell



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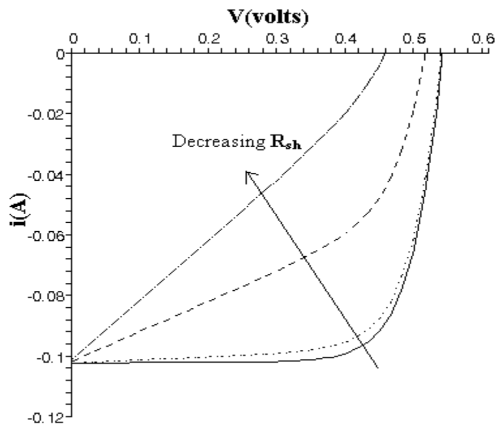
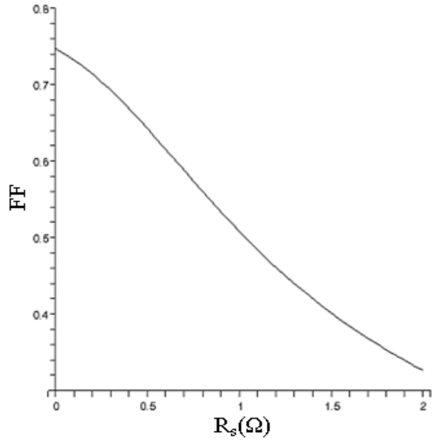


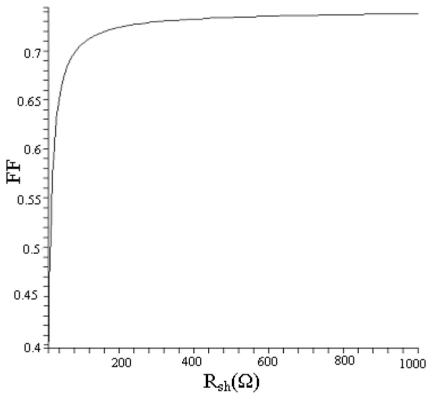
(b)

Fig. 2.(a). Effect of series resistance R_s on the I-V characteristics of a typical solar cell, (b). Effect of shunt resistance R_{sh} on the I-V characteristics of a typical solar cell

The effects of various physical parameters on the output power (P_{out}) are depicted in Figures 4 (a-e) and are in accordance with [13]. P_{out} of an experimental cell (in this case blue solar cell) is plotted against the output voltage (V_{out}) for different values of R_s: 0.07, 0.5, 1.0 and 2.0 Ω [Fig. 4(a)]. In Fig. 4(b), the variation of P_{out} as a function of the V_{out} is depicted for different values of R_{sh}: 20, 50, 100 and 1000 Ω. From the plot, it is clear that the shunt resistance improves the performance of the solar cell. An increase in R_{sh} enhances V_{oc} and FF. Fig. 4(c) shows the change in P_{out} of a solar cell as a function of V_{out} for different values of saturation current I_0: 0.1036, 0.5036, 0.9036 and 1.3036 μA. A decrease in I_0 corresponds to an increase in V_{oc} and FF. The variation of solar cell P_{out} against V_{out} for different values of photocurrent I_{ph}: 0.03023, 0.07023, 0.09023 and 0.1023 A is presented in Fig. 4(d). It is seen that on enhancing the I_{ph}, V_{oc} and the maximum power P_{max} also increases. In Fig. 4(e), it is noticed that V_{oc} and P_{max} enhances as the diode ideality factor increases where we have plotted the variation of the solar cell P_{out} as a function of V_{out} for different values of diode ideality factor n: 1.0, 1.5, 1.7 and 2.0.

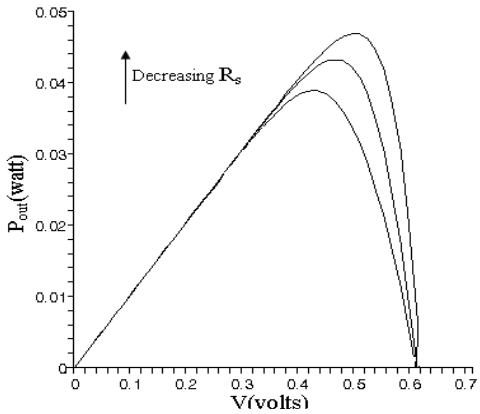


(a)



(b)

Fig. 3. (a). Effect of series resistance R_s on fill factor FF, (b).Effect of shunt resistance R_{sh} on fill factor FF



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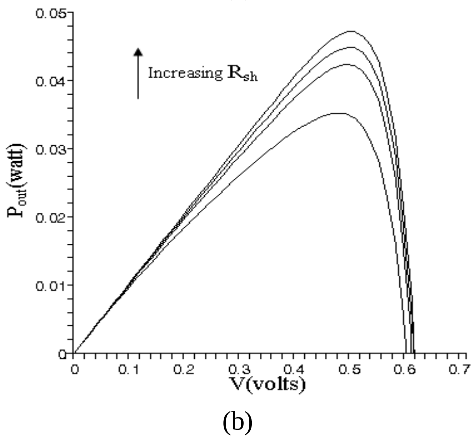

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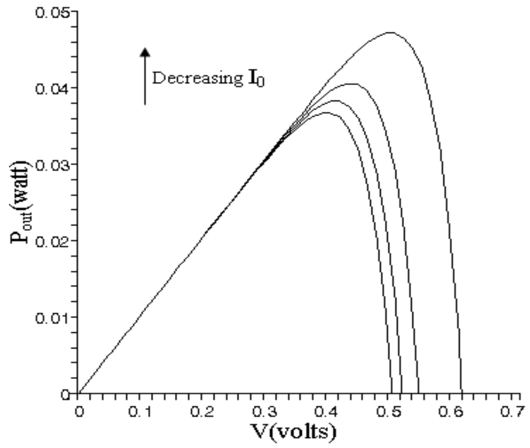

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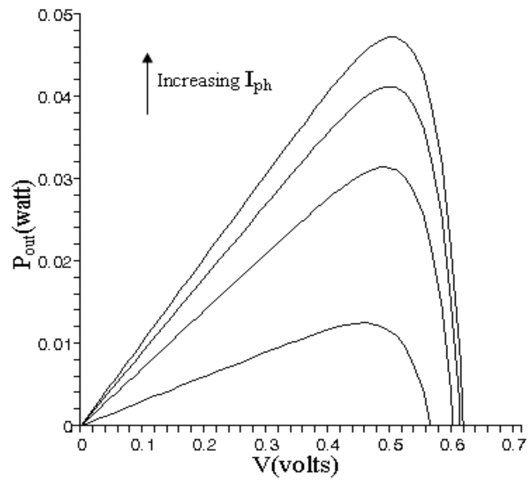

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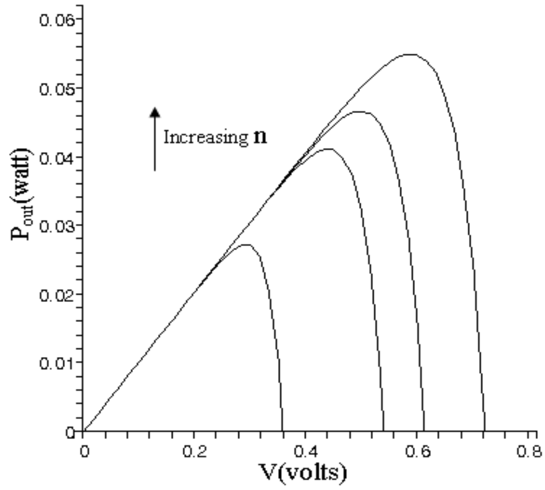

(e)

Fig. 4. (a). Effect of the series resistance R_s on the output power, (b). Effect of the shunt resistance R_{sh} on the output power, (c). Effect of the reverse saturation current I_0 on the output power, (d). Effect of the photocurrent I_{ph} on the output power, (e). Effect of the junction ideality factor n on the output power

5. Conclusion

We presented a new technique of determining the parameters of real solar cells. This method is based on the use of STFT for solving transcendental equations. The calculated values of different parameters presented in the tables and the various characteristic curves obtained through the analytical expressions using STFT validate the proposed method. The effects of different parameters on the power output of a typical real solar cell is deliberated and analyzed.

The proposed method enables the use of the whole I-V characteristics and produces results without any approximations. The results obtained by this analytical method are very accurate which are in good agreement with other established methods. Moreover, it is found that the method of STFT has a lesser calculation time and gives a more accurate result in comparison to the Lambert W-function with the same software and the same system. The main advantage of this method, together with the analytical closed-form structure of the transcendental solution is that the calculation is independent of the response time, and the transcendental calculation scheme for current is independent of the time domain. We may conclude that STFT type solutions could be an effective and powerful analytical approach to extract and study solar cell parameters towards the optimization of its efficiencies.

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