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A DIFFERENT APPROACH TO THE 2D FOURIER TRANSFORM OF THE RESPONSE OF A BURIED IMPULSIVE SOURCE

Gömülü Nokta Bir Kaynağın Cevabının 2B Fouirer Dönüşümüne Farklı Bir Yaklaşım

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ABSTRACT

This paper is focused on the 2D Fourier Transform of the response of receivers to a buried impulsive source. Such a transformation was analytically studied by Bolondi, Rocca and Savelli (1978). However, analytical solutions are not always found easily. Therefore, an alternative approach may be useful in handling these sort of problems. In this study, the 2D stationary phase approximation has been used as an approach to the same problem.

ÖZET

Bu çalışmada gömülü bir nokta kaynağın yatay bir kayıt düzleminde oluşturduğu dalga alanının 2 boyutlu Fourier dönüşümü incelenmiştir. Söz konusu dönüşüm Bolondi, Rocca ve Savelli (1978) tarafından analitik olarak bulunmuştur. Oysa analitik çözümleri her zaman bulmak mümkün değildir. O nedenle bu çeşit problemlerin çözümünde farklı bir yaklaşım faydalı olabilir. Bu makalede durağan faz yaklaşımı kullanılarak söz konusu iki boyutlu Fourier dönüşümü elde edilmiştir.

INTRODUCTION

In a study made by Bolondi, Rocca and Savelli (1978), the Fourier transform of the response of the receivers to a buried impulsive source was analytically calculated. It will be shown later that the same result can be obtained in terms of the phase spectrum of the response function, using the stationary phase approximation. First of all, it would be a nice idea to review what the authors did. The response of the receivers to a buried impulsive source is given by the following relationship.

$$R(x,t,z_0) = 0 \qquad t < r/v$$

$$R(x,t,z_0) = \frac{v}{2\pi\sqrt{v^2t^2 - x^2 - z_0^2}} \qquad t > r/v$$

Where, $r^2 = x^2 + z^2$ the distance between the source and receiver, and x , y , z , v , t are the coordinate of the receivers, the depth of the source, the constant velocity of the medium, the travel time, respectively.

Noticing that $R(x,t,0) = \dfrac{v}{2\pi\sqrt{v^2t^2 - x^2}} \qquad t > x/v$

Its 2D Fourier transform follows:

$$R(k_x, w, 0) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{v}{2\pi\sqrt{v^2t^2 - x^2}}\, e^{-i(k_x x + wt)}\, dx\,dt$$

letting

$$q = vt - x$$

$$p = vt + x$$

The Jacobian matrix will be

$$|J| = \begin{vmatrix} \frac{\partial x}{\partial q} & \frac{\partial x}{\partial p} \\ \frac{\partial t}{\partial q} & \frac{\partial t}{\partial p} \end{vmatrix} = \begin{vmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2}v & \frac{1}{2}v \end{vmatrix}$$

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$$|J| = \frac{1}{2v}$$

$$R(k_x, w, 0) = \frac{1}{4\pi}\int_0^{\infty}\frac{1}{\sqrt{q}}\, e^{-iq/2(w/v-k_x)}dq \int_0^{\infty}\frac{1}{\sqrt{p}}\, e^{-ip/2(w/v+k_x)}dp$$

$$R(k_x, w, 0) = \frac{i}{\sqrt{w^2/v^2 - k_x^2}}$$

applying the backwards operator

$$M(k_x, w, 0) = e^{-iz(w^2/v^2 - k_x^2)^{1/2}}$$

$$R(k_x, w, z) = \frac{i}{\sqrt{w^2/v^2 - k_x^2}}\, e^{-iz(w^2/v^2 - k_x^2)^{1/2}} \qquad (1)$$

Equation (1) is the desired 2D Fourier transform.

THEORY

As a different approach, the 2D Fourier transform of the response of the receivers to a buried impulsive source may be obtained using the sationary phase approximation.

To start with,

$$R(x, t, z) = \frac{v}{2\pi\sqrt{v^2 t^2 - x^2 - z_0^2}}$$

if the 2D Fourier transform is taken,

$$R(k_x, w, z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{v}{2\pi\sqrt{v^2 t^2 - x^2 - z_0^2}}\, e^{-iwt + ik_x x}\, dxdt \qquad (2)$$

On the other hand, the Bessel function $J_0(t)$ of the first kind and zero order can be written Papoulis (1962) as follows:

$$J_0(t) = \int_{-\infty}^{\infty}\frac{e^{ik_x x}}{\sqrt{v^2 t^2 - x^2 - z_0^2}}\, dx = \int_{-\pi}^{\pi} e^{i(v^2 t^2 - z_0^2)^{1/2} k_x \sin(B)}\, dB$$

Keeping in mind the 2D stationary phase algorithm Papoulis (1962) and then substituting the value of $J_0(t)$ into equation (2) and letting G denote the total phase, one may take the derivates of G with respect to X and B to obtain the extremum values of the function. If these values are inserted again into the phase function, the following result is obtained.

$$G = -\frac{w^2 z_0}{(w^2 v^2 - v^4 k_x^2)^{1/2}} + \left(\frac{-v^2 z_0^2 w^2}{(w^2 v^2 - v^4 k_x^2)} - z^2\right) k_x + \pi/2$$

$$G = \frac{z_0(w^2 - v^2 k_x^2)}{(w^2 v^2 - v^2 k_x^2)} + \pi/2$$

$$G = -z_0(w^2/v^2 - k_x^2)^{1/2} + \pi/2 \qquad (3)$$

The phase given by equation (3) is identical to the phase in equation (1).

CONCLUSIONS

The 2D Fourier transform of the response of receivers to a buried impulsive source has been obtained using the 2D stationary phase approximation in terms of the phase spectrum. The result is identical to the conclusion arrived by Bolondi, Rocca and Savelli (1978). Therefore this is an alternate approach to the same purpose.

REFERENCES

Bolondi, G., Rocca, F. and Savelli, S. 1978, A frequency domain approach to two-dimensional migration, Geophysical prospecting 26,

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