

## Geometric Soft Sets

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### ABSTRACT

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Soft sets are efficient tools to determine uncertainty in systems. In this study, we introduce a new concept called geometric soft sets to present and analyze the geometrical, topological, and structural properties of complex networks.

#### Keywords:

Soft set theory; Computational geometry; Soft computation; Network analysis

#### Article History:

Received: 2017/04/03

Accepted: 2017/06/08

Online: 2017/12/28

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## INTRODUCTION

Recently, the geometric structure of real network datasets has started to get a lot of attention amongst the researchers from various fields. The geometric characterization of such complex networks finds applications related to internet routing, community detection and data mining [3,5,7,24,28]. In real networks, geometrical, topological, and structural properties of data sets emerge from the spontaneous relation of each data element. In the literature, these relations are presented by the graph theoretical concepts [9,31,32]. Moreover, the structural analyses such as domination numbers of several types are presented in [15–18]. Besides, the theory of soft sets is also useful to express such relations [2,6,10,18,29,33].

The soft set theory introduced by Molodtsov in [27] is a mathematical tool dealing with the uncertainty of real-world problems which usually contain uncertain data. The soft set theory depends on the adequacy of the parametrization and differs from the similar theories such as fuzzy set theory, vague set theory, and rough set theory, vague sets theory, and rough sets theory by the adequacy of the parametrization. Mathematically, one may conclude that a soft set over an initial universe is a parametrized family of subsets of the universe. It can be also seen that a soft set is not a set but set systems. By the introduction of the theory in [27], its algebraic [1,19] and topological properties [20,33], interlacing with other theories [14] and the application in other fields [2,6,11,21,30] have been studied intensively.

In this study, we present the emergent geomet-

ric structure of the systems expressed with the soft sets. In Section 2 we give the basic definitions and propositions on soft sets to construct geometric soft sets. We refer [26] to the interested readers for soft set theoretical analogues of the basic set operations. In Section 3, we present the concept of geometric soft sets. Basically we present the soft sets with the parameter mapping of hyperplane inclusions of the points in general positions. Right after the introduction of the geometric realization and structural concepts we study extremal properties of geometric soft sets in this section. Finally, in Section 4, we restrict our parameter mapping to similarity of the distances of points embedded in arbitrary metric spaces. We also construct two families of geometric soft sets which emerge from  $\mathbb{R}^2$  and a stock market network, then analyze them.

## PRELIMINARIES

### Definition 1

[26] A soft set over the universe set  $U$  is defined as a pair  $(F,E)$  such that  $E$  is a set of parameters and  $F : E \rightarrow P(X)$  is a mapping. Mathematically, the soft set  $(F,E)$  is a parameterized family of subsets of the set  $U$  which can be stated as a set of ordered pairs

$$(F, E) = \{(e, F(e)) : e \in E \text{ and } F(e) \subset U\}.$$

### Definition 2

Union of two soft sets of  $(F,A)$  and  $(G,B)$  over the common universe  $U$  is the soft set  $(H,C)$ , where  $C = A \cup B$  and  $\forall e \in C$ ,

$$H(e) = \begin{cases} F(e), & \text{if } e \in A \setminus B \\ G(e), & \text{if } e \in B \setminus A \\ F(e) \cup G(e), & \text{if } e \in A \cap B \end{cases}$$

The union of two soft sets is denoted by  $(F, A) \cup (G, B)$

**Definition 3**

[26] Intersection of two soft sets  $(F, A)$  and  $(G, B)$  over a common universe  $U$  is the soft set  $(H, C)$ , where  $C = A \cap B$  and  $\forall e \in C, H(e) = F(e) \cap G(e)$ , (as both are same set).

The intersection of two soft sets is denoted by  $(F, A) \cap (G, B)$

**Definition 4**

[13] The soft set  $(F, E)$  is called a convex soft set if

$$F(ax + (1-a)y) \supseteq F(x) \cap F(y)$$

for every  $x, y \in E$  and  $a \in [0, 1]$ .

**Definition 5**

[25] The convex hull of a soft set  $(F, E)$ , denoted by  $\widetilde{conv}(F, E)$ , is the smallest convex soft set over the universe  $U$  containing  $(F, E)$ .

**Proposition 1**

[25] The convex hull of a soft set  $(F, E)$  is given by

$$\widetilde{conv}(F, E) = \bigcup_{n \in \mathbb{N}} \bigcup_{B \in C(e, n)} \bigcap \{F(e') : e' \in B\}$$

where

$$C(e, n) = \left\{ \left\{ e_1, \dots, e_n \right\} \subset E : \exists \lambda_i \in [0, 1] \text{ with } \sum_{i=1}^n \lambda_i = 1, e = \sum_{i=1}^n e_i \lambda_i = 1 \right\}$$

**Proposition 2**

[25] The intersection of an arbitrary collection of convex soft sets is a convex soft set.

**GEOMETRIC SOFT SETS**

A set in  $\mathbb{R}^d$  is said to be in general position if no  $d+1$  points lie on a hyperplane with co-dimension 1. Throughout this study we denote  $P(A, k)$  as the set of subsets of  $A$  with  $k$  elements and  $2^A$  as the subsets of  $A$ .

**Definition 6**

Let  $U \subset \mathbb{R}^d$  be the finite set of points in general position and  $A \subseteq U$ . For  $F_A : E \rightarrow 2^A \setminus \{\emptyset\}$  incidence mapping,  $(F_A, E)$  is called a geometric soft set if

- i. for  $A = \{a_1, \dots, a_k\}$ , the tuple  $(e_0, P(A, 1)) \in (F_A, E)$  ;

- ii. for all  $i = 1, \dots, k-2$ ; if  $(e_{k-1}, P(A, k)) \in (F_A, E)$  and  $\exists B \subset A$ , then  $(e_{i-1}, P(B, i)) \in (F_A, E)$  .

Parameter mapping plays important role to define a soft set. Hence, in the view of geometric soft sets we define parameter mapping as incidence relation. Formally, if the tuple  $(e_{k-1}, P(\{a_1, \dots, a_k\}, k)) \in (F_A, E)$ , then there is a hyperplane in  $\mathbb{R}^k$  that contains  $\{a_1, \dots, a_k\}$ .

To clarify the idea, we give the following example. Let us consider two sub-universes as  $A = \{a, b, c\}$  and  $B = \{d, e, f, g\}$ . The soft sets

$$(F_A, E) = \left\{ (e_0, \{\{a\}, \{b\}, \{c\}\}), (e_1, \{\{a, b\}, \{a, c\}\}), (e_2, \{\{a, b, c\}\}) \right\}$$

and

$$(F_B, E) = \left\{ (e_0, \{\{d\}, \{e\}, \{f\}, \{g\}\}), (e_1, \{\{d, e\}, \{d, f\}, \{d, g\}, \{e, f\}, \{e, g\}, \{f, g\}\}), (e_2, \{\{d, e, f\}, \{e, f, g\}, \{d, f, g\}, \{d, e, g\}\}) \right\}$$

The geometric realizations of the soft sets  $(F_A, E)$  and  $(F_B, E)$  are presented in Figure 1. with all triangular areas are included.

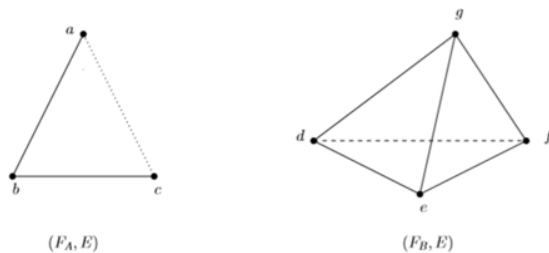


Figure 1. Lower dimensional examples of geometric soft sets.

**Theorem 1**

A geometric soft set with universe  $\mathbb{R}^d$  has a geometric realization in  $\mathbb{R}^{2d+1}$ .

**Proof**

Let  $(F_{\alpha_1}, E)$  be a geometric soft set with  $F_{\alpha_1} : E \rightarrow 2^A \setminus \{\emptyset\}$  and let  $f : A \rightarrow \mathbb{R}^{2d+1}$  be an injection whose image is a set of points in general position. It is well known that any  $2d + 1$  or fewer points in general position are affine independent. Now let  $(F_{\alpha_1}, E)$  and  $(F_{\alpha_2}, E)$  are geometric soft subsets of  $(F_A, E)$  with  $|F_{\alpha_1}(E)| = k_1$  and  $|F_{\alpha_2}(E)| = k_2$ . The soft union of these soft subsets has the cardinality

$$\begin{aligned} |(F_{\alpha_1}, E) \cup (F_{\alpha_2}, E)| &= |F_{\alpha_1}, E| + |F_{\alpha_2}, E| - |(F_{\alpha_1}, E) \cap (F_{\alpha_2}, E)| \\ &= k_1 + k_2 + 1 \leq 2d + 1 \end{aligned}$$

Henceforth, the points in  $\alpha_1$  and  $\alpha_2$  are affine independent.

dent. This also implies that every convex combination  $x$  of points in  $\alpha_1 \cup \alpha_2$  is unique. Besides,  $\widetilde{\text{conv}}(F_{\alpha_1}, E) = \widetilde{\text{conv}}(F_{\alpha_2}, E)$  if and only if  $x$  is a convex combination of  $\alpha_1 \cap \alpha_2$ . Therefore, this implies that  $F_{\alpha_1}(E) \cap F_{\alpha_2}(E)$  is either empty subset of the convex hull  $\text{conv}(\alpha_1 \cap \alpha_2)$  as it is required.

One of the mathematical concepts which may remind the geometric soft sets is the abstract simplicial complexes. Even though the abstract simplicial complexes are the collections of subsets of some vertex set, the main difference emerge from the hereditary structure. For each edge, abstract simplicial complexes contain all of its subsets; however geometric soft sets do not have to. For instance, in the above example, the geometric soft set  $(F_A, E)$  involves the triangle  $\{a, b, c\}$  along with the edges  $\{a, b\}$  and  $\{a, c\}$  but not the  $\{a, c\}$ . There is also one to one correspondence between the soft set  $(F_B, E)$  and the abstract simplicial complex with the vertex set  $\{d, e, f, g\}$ . It is possible to consider simplicial complexes as the hereditary geometric soft sets.

The dimension of a geometric soft set  $(F_A, E)$  is the maximum number  $k$  of  $(e_{k-1}, P(\{a_1, \dots, a_k\}, k))$ , i.e.  $\dim(F_A, E) = \max k$ .

**Definition 7**

Let  $(F_A, E)$  be a geometric soft set. A soft subset  $(F_B, E)$  of  $(F_A, E)$  is a geometric soft subset if and only if  $(F_B, E)$  is also a geometric soft set. For any geometric soft set  $(F_A, E)$  with dimension  $d$  and for any  $0 \leq i \leq d$  the geometric soft subset

$$(F_A, E)^{(i)} = \left\{ (F_{B_i}, E) \subseteq (F_A, E) : \dim(F_{B_i}, E) \leq i \right\}$$

for all  $B_i \subset A$  is called the  $i$ -th soft skeleton of  $(F_A, E)$ .

**Remark 1**

It is straightforward that a geometric soft set is connected if and only if its 1st soft skeleton is connected.

**Definition 8**

Let  $(\mathcal{F}, \mathcal{E}) = \bigcup_{i=1}^n (F_{A_i}, E)$  be a family of geometric soft sets with dimension  $d$ .  $(\mathcal{F}, \mathcal{E})$  is homogeneous if and only if for all  $(F_{A_i}, E)$ , there exists  $j \neq i$  and  $j=1, \dots, n$  such that  $F_{A_i}(e_k) \cap F_{A_j}(e_k) \neq \emptyset$  for any  $k = 1, \dots, d$ . Moreover,  $(\mathcal{F}, \mathcal{E})$  is connected if and only if geometric realization of  $(\mathcal{F}, \mathcal{E})$  is connected.

The sense behind the notion of a homogeneous family of geometric soft sets is the result of gluing geometric soft sets all having the same dimension. For instance, let us define three geometric subsets

$$(F_A, E) = \left\{ (e_0, \{\{a\}, \{b\}\}), (e_1, \{\{a, b\}\}) \right\}$$

$$(F_B, E) = \left\{ (e_0, \{\{b\}, \{c\}, \{d\}\}), (e_1, \{\{b, c\}, \{c, d\}\}), (e_2, \{\{b, c, d\}\}) \right\}$$

$$(F_C, E) = \left\{ (e_0, \{\{c\}, \{d\}, \{e\}, \{f\}\}), (e_1, \{\{c, d\}, \{c, f\}, \{d, f\}, \{e, f\}\}), (e_2, \{\{c, d, f\}\}) \right\}$$

and  $(\mathcal{F}, \mathcal{E}) = (F_A, E) \cup (F_B, E) \cup (F_C, E)$

The geometric realization of  $(\mathcal{F}, \mathcal{E})$  is presented in Figure 2.

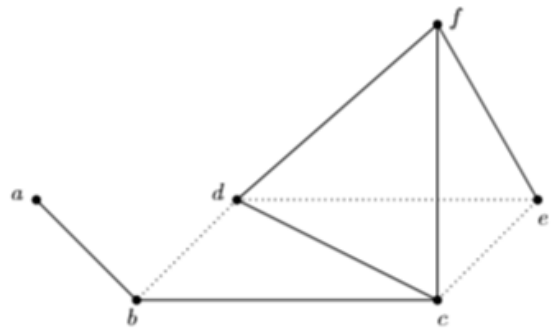


Figure 2. The geometric realization of a family of geometric soft sets

**Definition 9**

Let  $k \geq 2$  and  $(\mathcal{F}, \mathcal{E})$  be a family of geometric soft sets.  $(\mathcal{F}, \mathcal{E})$  is  $k$ -uniform if all of its elements have cardinality  $k$ , i.e.  $|F_A(E)| = k$  for all  $(F_A, E) \in (\mathcal{F}, \mathcal{E})$ .

**Definition 10**

Let  $(\mathfrak{F}, \mathfrak{E})$  be a family of  $k$ -uniform families of geometric soft sets.  $(\mathfrak{F}, \mathfrak{E})$  is called  $k$ -partite if  $A$  of  $(\mathfrak{F}, \mathfrak{E})$  can be partitioned into  $k$  pairwise disjoint soft sets  $A = A_1 \cup A_2 \cup \dots \cup A_k$  with  $A_i \cap A_j = \emptyset$  for  $i \neq j$ , such that  $|S \cap A_i| \leq 1$  for all  $i = 1, \dots, k$  and every  $S \in F_A(E)$ .

**Definition 11**

Let  $(\mathfrak{F}, \mathfrak{E})$  be a family of  $k$ -uniform families of geometric soft sets. If  $(\mathfrak{F}, \mathfrak{E})$  can be partitioned into pairwise disjoint soft sets with  $|A_i| = \ell_i$  such that

$$F_A(E) = \{ S \subset F_A(e) : |S \cap A_i| = 1 \}$$

for all  $i=1, \dots, k$ , then  $(\mathfrak{F}, \mathfrak{E})$  is called complete  $k$ -partite

and denoted as  $(\mathfrak{F}, \mathfrak{E})_k^{(k)}$ .

One of the extremal problem arise in the context of k-uniform geometric soft sets is the maximum number of parameters assigned to sub-universe with fixed  $|A|=n$  and k values without containing certain soft subsets.

**Definition 12**

Let  $k \geq 2$  and let  $(\mathfrak{F}, \mathfrak{E})$  be a family of k-uniform families of geometric soft sets. A k-uniform family that contains no distinct copy of any  $(\mathcal{F}, \mathcal{E}) \in (\mathfrak{F}, \mathfrak{E})$  as a soft subset is called  $(\mathfrak{F}, \mathfrak{E})$ -free. The maximal number of  $(\mathfrak{F}, \mathfrak{E})$ -free k-uniform family on  $|A|=n$ ,  $\max(n, (\mathfrak{F}, \mathfrak{E}))$ , can be given as the maximum number of  $|F_A(E)|$ , i.e.  $\max(n, (\mathfrak{F}, \mathfrak{E})) = \max\{|F_A(E)|\}$ .

**Theorem 2**

Let  $(\mathfrak{F}, \mathfrak{E})$  be a family of k-uniform families of geometric soft sets and  $|A|=n$ . If  $(\mathfrak{F}, \mathfrak{E})$  has no k-partite member, then there is a  $c > 0$  such that  $\max(n, (\mathfrak{F}, \mathfrak{E})) \geq cn^k$ .

**Proof**

Let  $|A|=n$  and  $(\mathfrak{F}, \mathfrak{E})$  be complete k-partite family with

$$\ell_i = \frac{n+i-1}{k}$$

Since all soft subsets of  $(\mathfrak{F}, \mathfrak{E})$  are k-partite they are  $(\mathfrak{F}, \mathfrak{E})$ -free. Hence  $(\mathfrak{F}, \mathfrak{E})$  involves

$$\prod_{1 \leq i \leq k} \frac{n+i-1}{k} \geq \left(\frac{n-l}{k}\right)^k$$

many families for some  $0 \leq l < k$ . This completes the proof.

**GEOMETRIC SOFT SETS IN METRIC SPACES**

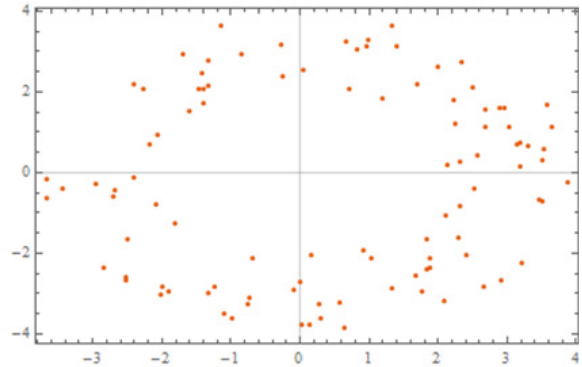
Different restrictions on the parameter map of a geometric soft set yield the soft analogues of well-known computational complexes. In this section, we let the universe of a geometric soft set be any metric space  $\mathcal{P}$ . Then, the geometric soft set  $(F_A^\delta, E)$  can be defined with the parameter mapping

$$F_A^\delta : A \subset \mathcal{P} \rightarrow E$$

as if the tuple  $(e_{k-1}^\delta, P(\{a_1, \dots, a_k\}, k)) \in (F_A, E)$ , then the points  $\{a_1, \dots, a_k\}$  are with the maximum pairwise distance  $\delta$ .

**An Example in  $\mathbb{R}^2$**

Now let us consider the randomly sampled 100 points in a sub-region of  $\mathbb{R}^2$ . The sub-region we choose is  $4 \leq x^2 + y^2 \leq 16$  and the generated points are presented in Figure 3.



**Figure 3.** Randomly sampled points in  $\mathbb{R}^2$

The results emerge from the families of  $(F_A^\delta, E)$  with usual Euclidean metric are presented in Table 1 for different  $\delta$  values.

**Table 1.** Computational results for the family of geometric soft sets

$\delta$ Value	Number of Soft Sets	Maximum Dimension	Connected
5	1	100	Yes
4.694	2	99	Yes
3	28	44	Yes
2	33	28	Yes
1	54	15	Yes
0.853	60	13	Yes
0.852	60	12	No
0.5	68	5	No
0.4	76	5	No
0.3	86	4	No
0.2	94	3	No
0.109	100	1	No

From the computational results presented in Table 1. it can be straightforwardly deduced that the connectivity and dimension of the family of geometric soft sets represented as the  $\delta$  on of randomly sampled points in  $\mathbb{R}^2$  is decreasing  $|(F_A, E)|$  value of  $(F_A^\delta, E)$  decreases. Similarly, the number increases as  $\delta$  descends. The threshold value of  $\delta$  for the connectivity is 0.853. This yields that, for the  $\delta=0.853$  the system represented by the geometric soft sets involves the information with dimension 13 and there exists the continuous geometric flow of the information through the system.

**Geometric Soft Sets Emerge From BIST**

Since global stock markets are continuously in complex interaction they form a complex system and have long been studied by several researchers [4,12,23].

The data used in this section consists of daily data from

the period January 2013 to January 2015. 93 companies operating in Borsa Istanbul (BIST) 100 Index (XU100) are considered. Sessional return  $Cl_i$  for the  $i$ -th company is calculated as the logarithmic return in the value of index compared to previous session's closing value as the logarithmic difference.

The metric used for comparing time series of logarithmic returns is the correlation distance

$$d_{Corr}(i, j) = \sqrt{2(1 - \rho_{ij})},$$

where

$$\rho_{ij} = \frac{Cl_i, Cl_j - Cl_i Cl_j}{\sqrt{(Cl_i^2 - Cl_i^2)(Cl_j^2 - Cl_j^2)}}$$

with  $\langle \dots \rangle$  is the temporal average performed on all the trading days. The data and the correlation distance matrix is presented in Figures 4 and 5, respectively. In Figure 5, it can be concluded that the diagonal elements are zero and the rows and columns which has the darker color are the stocks with long distances. Here, the long distance directly corresponds the smaller correlation amongst the stock markets.

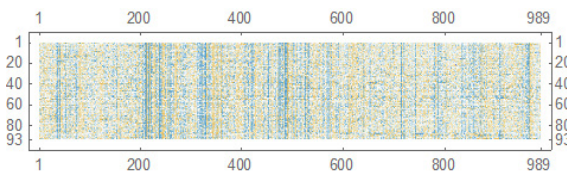


Figure 4. Logarithmic return of the 93 companies operating in 989 days

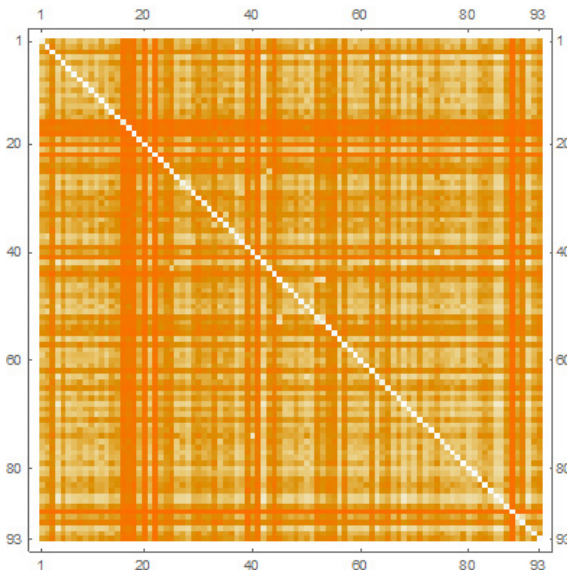


Figure 5. Correlation distances of logarithmic return 93 companies. Light and dark colors indicate strong and weak correlations, respectively

The results emerge from the families of  $(F_A^\delta, E)$  with correlation distance are presented in Table 2 for different  $\delta$  values.

Table 2. Computational results for the family of geometric soft sets

$\delta$ Value	Number of Soft Sets	Maximum Dimension	Connected
2	1	100	Yes
1	1	93	Yes
0.899	1	92	Yes
0.898	2	92	No
0.7	12	82	No
0.6	31	64	No
0.5	56	37	No
0.4	81	10	No
0.3	87	7	No
0.2	86	6	No
0.15	100	1	No

From the computational results presented in Table 2 it can be seen that the connectivity and dimension of the family of geometric soft sets representation of BIST is decreasing as the  $\delta$  value of  $(F_A^\delta, E)$  decreases as same as the previous example. Similarly, the number  $|(F, E)|$  increases as  $\delta$  decreases. The threshold value of  $\delta$  for the connectivity is 0.899 which is also similar to the previous example. This yields that, for the  $\delta=0.898$  the system represented by the geometric soft sets involves the information with dimension 92 and there exists the continuous geometric flow of the information through the system.

## CONCLUSION

In this paper, we have presented a new concept called geometric soft set which emerge from the incidence parameter over the points in general position. After the introduction of the geometric realization of the families of geometric soft sets, we study extremal properties for such families. For fixed number of the sub-universe, we present an upper bound for the crossing of respected hyperplanes.

Several type of complex systems emerge a geometric structure. To obtain this, we have present a restriction on the parameter meter and study different type of geometric soft sets in different metric spaces. First one is the usual metric space of  $\mathbb{R}^2$ , and the second one is the metric space endowed with the correlation distance of time series. We have showed that the geometric soft sets emerge from the BIST has higher dimensions than the ones from usual metric space.

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