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PROBLEM SOLVING STRATEGIES USED BY GIFTED SECONDARY SCHOOL STUDENTS TO SOLVE MATH PROBLEMS*

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ABSTRACT

The general purpose of this study was to determine the strategies used by gifted secondary school students in solving mathematics problems. To collect data about the problem solving strategies of the gifted students, a case study was used. The research was performed at the Science and Arts Center of Ankara Province in Turkey. The study was conducted with five 7th grade secondary school students. In the sample chosen, first, the necessary permissions were obtained from the Ministry of National Education in Turkey, and then, the random sampling method was used. A problem solving test that was developed by the researcher was used as the measurement instrument. There were 5mathematics problems in the test, and each problem was checked whether specific problem solving strategies were used or not. Additionally, problems were developed by following the objectivesof the secondary mathematics curriculum. The responsesof the students were evaluated through semi-structured interviews for each problem according to the strategies used, and field notes were taken during the semi-structured interviews by the researcher. The data were analyzed using descriptive analyses and voice thinking techniques. The results of the study revealed that the students used 8 different strategies for solving the mathematics problems. They most frequently used logical reasoning and finding pattern problem solving strategies during the solution of these problems, however the working backward and considering extreme cases strategies were never used by the gifted students.

Keywords: Gifted students, problem solving strategies, mathematics education

ÖZEL YETENEKLİ ORTAOKUL ÖĞRENCİLERİNİN PROBLEM ÇÖZMEDE KULLANDIKLARI PROBLEM ÇÖZME STRATEJİLERİ

ÖZ

Bu çalışmanın genel amacı özel yetenekli ortaokul öğrencilerinin matematik problemlerini çözerken kullandıkları problem çözme stratejilerini belirlemektir. Çalışmada özel yeteneklilerin problem çözme stratejileri hakkında veri toplamak için durum çalışması yöntemi kullanılmıştır. Araştırma, Millî Eğitim Bakanlığından gerekli izinler alındıktan sonra Ankara ilinde yer alan bir Bilim Sanat Merkezinde öğrenim görmekte olan özel yetenekli beş öğrenci ile yürütülmüştür. Çalışmanın örneklemi, kurumda öğrenim görmekte olan yedinci sınıf öğrencileri arasından çalışmaya katılmaya gönüllü olan öğrencilerden oluşmaktadır. Çalışmanın verileri araştırmacı tarafından geliştirilen "Problem Çözme Testi" kullanılarak öğrencilerle yapılan yarı yapılandırılmış görüşmelerle elde edilmiştir. Test, her maddesi ilköğretim matematik öğretim programındayer alan hedefler doğrultusunda geliştirilen ve birden fazla problem çözme stratejisi kullanılmaya uygun olan beş matematik probleminden oluşmaktadır. Elde edilen veriler betimsel analiz ve sesli düşünme tekniği yöntemleri kullanılarak analiz edilmiştir. Araştırmanın sonucunda özel yetenekli öğrencilerin problemleri çözerken sekiz farklı problem çözme stratejisi kullandıkları ortaya çıkmıştır. Öğrenciler en fazla mantıksal akıl yürütme ve ilişki arama stratejilerini kullanmış, geriye doğru çalışma ve uç durumları düşünme stratejilerini hiç kullanmamıştır.

Anahtar Kelimeler: Özel yetenekli öğrenciler, problem çözme stratejileri, matematik eğitimi

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1. INTRODUCTION

This study aimed to investigate which strategies gifted students who studied at the Science and Arts Center (BİLSEM) used when they solved mathematics problems. BİLSEMs are institutions that operate to meet the educational needs of gifted students in Turkey. In the institution, there are several different individual education programs (adaptation, support education, realizing individual talents, developing special talents, and project production) used to develop the abilities of gifted students. A 'completion certificate' is given to the students who complete the programs. Individual and group intelligence tests are used to select the gifted students who are accepted into the institution.

In recent years, the studies on the education of gifted and talented students have increased. Studies have been conducted to diagnose and educate gifted and talented students all over the world. For this reason, studies on the subject in universities have been developing and are ongoing. Countries use different methods in the training of individuals who have advanced intelligence. There are several teaching strategies for gifted learners. For example, the Renzulli's Enrichment Triad (Renzulli, 1986, p.24) or Thinking Actively in a Social Context (Wallace & Adams, 1993) models. Despite these different education programs or models, problem solving is at the forefront in all training programs or models developed for gifted people. According to Montgomery (1996, pp.97-101), one of the most valuable teaching strategies to use when teaching gifted students is problem solving. In order to be able to solve problems, learners need to develop their thinking skills. Developing thinking skills provides gifted individuals with broad opportunities in problem solving. For this reason, the measures for developing problem solving and setting skills are included in programs for gifted students.

In the literature, the components and definitions of giftedness vary. For example, Renzulli (2002a) defined giftedness as a combination of above-average talent, creativity, and commitment in a three-ring theory. According to Renzulli, giftedness is a combination of all three components. A high level of creativity is associated with the originality of thought. Furthermore, task commitment is associated with aspecial interest that a person has in a subject (Renzulli, 2002b, 2003). In addition to this, there have also been studies stating that the interaction of gifted students with their environment is importantfor their development (Sternberg et al., 2011). For example, Mönks (1992) and Tannanbaum (2004) described giftedness with individual or environmental factors (school, peer, or parental factors). The common point of these definitions isthat giftedness is explained through special skills.

1.1. Mathematically giftedness

Mathematics is considered as one of the main subjects for gifted students who want to be creative professionals in mathematics, engineering, space sciences, or technology in the future. Therefore, mathematics requires much more attention and importance to uncover the potential of these students. Although much research and manyapproaches have been developed to identify mathematical giftedness (Sousa, 2003), there is no common and clear definition in the literature about it (Singer et al., 2016). Miller (1990) defined mathematical giftedness as an extraordinarily high ability to understand mathematical ideas and reason mathematically. On the other hand, researchers such as Krutetskii (1976), Sriraman (2003), and Lee (2005), who have observed the intellectual characteristics of mathematical giftedness, stated that these students were very different from ordinary students in terms of speed and depth because of their insightful reasoning.

Research has shown that mathematically gifted students learn differently than their peers in the same age group. It has been suggested that high-ability gifted students should have a stronger curriculum that is qualitatively better (Davis & Rimm, 1998; VanTassel-Baska, 1994). They require a specially differentiated curriculum to meet their specific learning styles (Johnson, 2006). On the other hand, mathematically talented students vary greatly in their degree of talent and motivation. For this reason, no single approach is best for all of these students. The design of the instructional program for each studentin mathematics should be based on an analysis of their individual abilities and needs (Miller, 1990).

The basic goal of mathematics teaching and learning is to develop the ability of the studentto solve a broad range of complex problems (Stanic & Kilpatrick, 1988; NCTM, 1989). In recent years, it has been important to give opportunities to students in mathematics education where they can generate their problems. This helps the student to develop their math skills and is applied to gifted students as well. Miller (1990) explained that gifted students need mathematical programs that develop independent exploratory behavior, problem solving, looking for patterns, and organizing data to find relationships. Additionally, mathematically gifted students also need a variety of different topics and educational diversity opportunities for solving more complex problems (Rotigel & Fello, 2004). Heinze (2005), stated that some forms of mathematical giftedness may be an ability to give a precise analysis of problems and reasons for solutions. Therefore, gifted students should be provided with environments that allow for a higher level of thinking and encourage problem solving to develop their skills. These situations in the literature have shown that mathematics is important in problem solving for gifted

students. For this reason, it was believed that determining the problem solving strategies used by gifted students in problem solving would be useful in developing their skills.

1.2. Purpose of the study

This study was aimed to investigate theproblem solving strategies of the 7th-grade secondary school gifted studentswhen they solved a mathematics problem solving test.

2. METHODS

2.1. Research design

In this study, the case study design was used. The case study is differentiated from other research strategies because the focus of the research is a bounded system or case (Brown, 2008). The case study is a research design by which one or more cases, events, individuals, or programs are examined in-depth and analyzed. Case studies are primarily a means of investigating what has happened in a real environment, collecting and analyzing data in an orderly manner, and determining the results that emerge. The emergent results clearly explain why the event occurred in that way and the matters that future studies need to focus on in more detail (Creswell, 2013; Merriam, 1998).

2.2. Study group

The study was carried out at a Science and Arts Center (BİLSEM) of Ankara province in Turkey during the 2018-2019 educational years. The participants of the study consisted of five gifted students. Before applying the research, necessary permissions were obtained from the Ministry of Education. In the sample chosen, the random sampling method was used. The students participated in the study in their own free time. All of the students had been educated for one or more years in mathematics within the BİLSEM system, and they also took mathematics lessons in formal education schools. Their formal schools varied greatly; however, all of the students were located in communities near Ankara. For this study, 12 high-ability gifted students were chosen to be interviewed. However, this study was conducted with five of these students who had been randomly selected, as it was part of a doctoral dissertation. The participants in this study reflected participation in the doctoral dissertation. The study group was fairly small for a qualitative study but sufficient to make generalizations about the findings obtained from the study (Robinson, 2014). The participants all showed high levels of mathematics ability in their schools and they voluntarily participated in the study.

2.3. Instrument

A Problem Solving Test was developed by the researcher in order to determine the solution strategies used by the 7th-grade secondary school gifted students. The problem solving test consisted of 14 open-ended problems; however, only 5 problems were used in the study, which were developed by following the objectivesof the secondary mathematics curriculum (MEB, 2013). The questions in the test were developed for students to use for more than one problem solving strategy.

The interviews with the students, who were willing to participate, were held between October 2017 and May 2018. Between these dates, all of the interview data were gathered. The interviews lasted from 40 min to1hour. The interviews were open-ended and limited by the permission from the Ministry of National Education in Turkey. Moreover, in the interviews, the think aloud techniques were used to deepen the problem solving processes of the gifted students in detail.

2.4. Data collection and analysis

In this study, semi-structured interviews with gifted students and a problem solving test were the tools used to determine the problem solving strategies used by the gifted student when they solved mathematics problems. Interview questions were prepared to inquire about the problem solving processes of the gifted students, to determine the strategies that they used. For this reason, qualitative techniques were used in the analysis of the data that were generated by the semi-structured interviews. Descriptive analysis and think aloud techniques were used in the analysis of the qualitative data as well.

The data was obtained as below, in Figure 1.

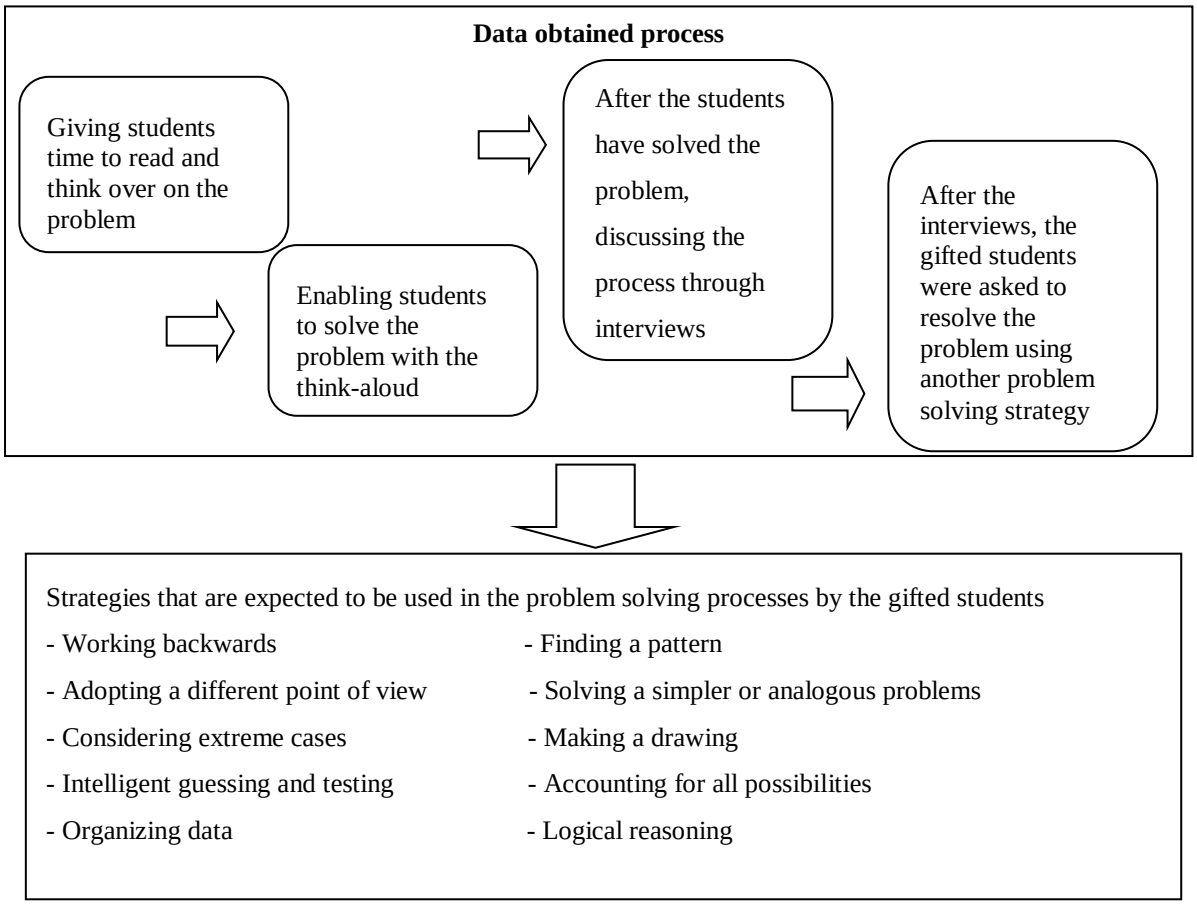

Figure 1. Data collection process

First of all, the data obtained from the interviews were transcribed in two stages. In the first stage, a researcher transcribed the interviews in Microsoft Word. In the second stage, a researcher read through the written documents of the interviews and determined the problem solving strategies used by the gifted students. Problem solving strategies (Posamentier & Krulik, 1998) include working backwards, finding a pattern, adopting a different point of view, solving a simpler or analogous problem, considering extreme cases, making a drawing, intelligent guessing and testing, accounting for all possibilities, organizing data, and logical reasoning. On the other hand, to ensure accuracy, another researcher studied the data at different times and determined the problem solving strategies. Then, the two investigations by the researchers were compared using the Huberman and Miles (1994) interoperator reliability formula, and the percentage of agreement between the encoders was calculated. Thus, it was determined that the percentage of agreement between the coders was 93%. After the coding process, comparison analyses were performed using the continuous comparison analysis technique for each item. The results obtained were taken directly from the test answer sheets of the students. Yin (1994) stated that the data obtained from qualitative studies directly reflect the opinions of the individuals.

2.5. Ethical consent of the researcher

In this study, all rules stated to be followed within the scope of "Higher Education Institutions Scientific Research and Publication Ethics Directive" were followed. None of the actions stated under the title "Actions Against Scientific Research and Publication Ethics", which is the second part of the directive, have not been carried out.

Ethics committee permission information

Name of the board conducting the ethical evaluation: Adıyaman University Social and Humanities Ethics Committee

Date of the ethical assessment decision: 11/01/2021

Ethics evaluation certificate issue number:42

3. FINDINGS

Analysis of the responses of the students showed eight distinct solution strategies: finding a pattern (a), adopting a different point of view (b), solving a simpler or analogous problem (c), organizing data (d), intelligent guessing, and testing (e), accounting for all possibilities (f), logical reasoning (g), and making a drawing (h). Table 1 given below shows the distribution of the strategies used for each problem.

Table 1.
Distribution of Strategies Used by Gifted Students in Solving Problems

	Strategies used				
Problems	**Gifted student 1**	**Gifted student 2**	**Gifted student 3**	**Gifted student 4**	**Gifted student 5**
Problem 1	a,e,g	a,e,g	a,e,g	a,b,e,g	a,e,g
Problem 2	a,d,g	a,d,f,g	d,f	a,f,g	a,d,g
Problem 3	a,c,e,g	a,c,e,g	a,c,d,e,g	a,g	a,d,g
Problem 4	a,e,g	a,e,g	a,g	a,e,g	a,e,f,g
Problem 5	d,g,h	d,e,g	e,g	d,e,g	d,g

Problem 1 was searching the relationship between variables x and y in the table in Figure 2. In the problem, the gifted student was asked to find the equation that showed the relationship between these values (Fig. 2).

x	*y*
1	0
2	3
3	8
4	15

Figure 2: Image of the first question

The gifted students used a total of four different problem solving strategies in this question (Table 1). The strategies used by students in solving this problem were: finding a pattern, adopting a different point of view, intelligent guessing and testing, and logical reasoning. In Table 1, it can be seen that the gifted students mostly used similar problem solving strategies in the solution of Problem 1. Only the fourth gifted student used the adopting a different point of view problem solving strategy. The student explained the solution of the problem as follows:

"*hımm… The first thing I see is the differences between the x and y values. The x values always increase, but the y values increase more and more, and by different amounts. The difference here is 3 (the difference between 0 and 3 in the y column), but the difference in the other is 5 (the difference between 8 and 3 in the y column) and the other is 7. So, all are odd numbers starting from 3. I can say that the difference after here will be 9. The next y column value should be 15 plus 9. That is 24. The next x column value should be 5.*"

In his solution thus far, the student primarily used the relationships between the numbers (finding a pattern strategy). Then, he used the logical reasoning strategy, by making inferences based on the relationships that he obtained. The student continued to solve the problem as follows:

"*The increase in the y values starts from 3 and is by two. I have to find something from here. Hmm… if I say the differences n… no no no… I can't say n because it doesn't show odd numbers. It should be like 2n-1. hımm… then (writes)… y = x + (2x-1)… hımm… but I don't think this is going anywhere…* "

After thinking for a while ...

"*hımm ok… I square the x values since the squares of the x values go as 1,4,9,16. I think I found it (laughs)… then, the y-values are a minus of squares of the x-values. Then, the equation is* $y = x^2-1$."

Thus, the student obtained a relationship between the x and y values. From this solution, it was seen that the student first used the intelligent guessing and testing strategy, and using this strategy, he reached the conclusion based on his prediction for the solution. At the same time, he used the adopting a different point of view strategy and he looked at the given in the question from a different perspective.

Problem 2 was a non-routine verbal question. The question was as follows: "*Selda played a video game 43 times last year. She has never lost more than 2 games in a row. How many games could Selda have lost at most?*"

The gifted students used a total of four different problem solving strategies in this question (Table 1). The strategies used by the students in solving this problem were: finding a pattern, organizing data, accounting for all possibilities, and logical reasoning. Gifted student 2 used all of these strategies while solving this problem.

"*She lost at most two games back-to-back..hımmm…so… to find the maximum number of games she lost, I have to put 2 lost games back-to-back always, then, I have to put a winning game. Such as LLWLLWLLW and so on.*"

He wrote all of the games like this, until game 43. He searched relationships between the data by organizing them.

"*Then I need to count all of the lost games. (he is counting). It is 29. Because she lost the last game. It must be true. Because if I make the list like WLLWLLW … she is going to win the last game, so then the maximum number of lost games would be 28. So, the first version that I put here is the correct one for this question.*"

Hence, the student examined the data in a logical context and accounted for all of the possible situations for the solution. The student was asked to solve the problem in a different way. Then, he checked the solution by doing it another way, as follows:

"*Let me do it another way and check the solution..hmm.. since she is losing every two games out of 3, I have to find out how many 3's arein 43. To do that, I need to divide 43 by 3 (making the division)…I find 14, but 1 is a reminder. That means she has lost 2 times 14.. 28 games and she has also lost the reminder one. So, that makes it 29. The same way she won 14 games.*"

Problem 3 was a question about operations with rational numbers. In this problem, the gifted students were asked to find a general expression to solve the problem (Fig. 3).

$$\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\frac{1}{4\times 5}+\frac{1}{5\times 6}+\cdots+\frac{1}{9\times 10}$$

Figure 3: Image of the third question.

The gifted students used a total of five different problem solving strategies in this question (Table 1). The strategies used by students in solving this problem were: finding a pattern, solving a simpler or analogous problem, intelligent guessing and testing, organizing data, and logical reasoning. Gifted student 3 used all of these strategies while solving this problem.

"*hmm…general expression…it seems a little hard too… I can say that it is 1 over n times n plus 1(writing down). But that is the last one…I have to find a general expression for all the addition. I can calculate until 10 but it does not ask me the result. It is asking for a general expression. Hmmm… I am really stuck here… let me try to get n(n+1) out of the parentheses. So, it should be (writing down) n(n+1) …no-no… it is not like that… I have to find a common denominator.*"

It was understood that the student understood the question from his explanations and tried to find a generalization. At this stage, he made several attempts to find the solution. After thinking for a while, he decided to find the sum of the fractions.

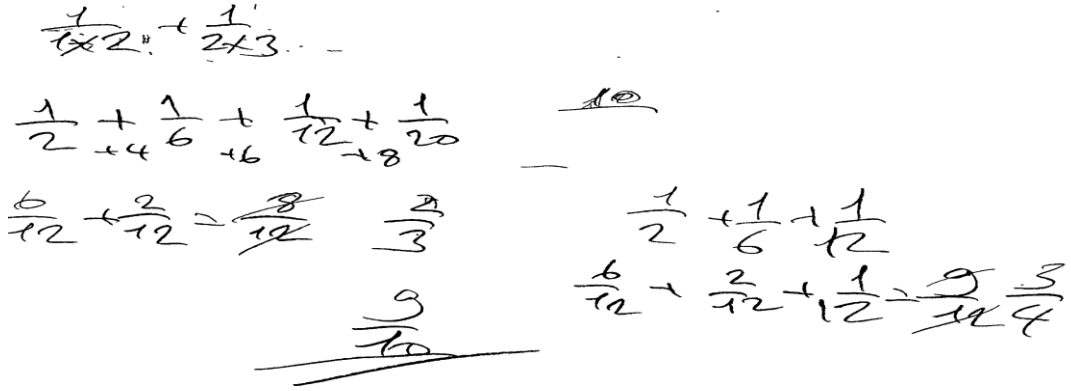

Figure 4: Visual that gifted student 3 used in solving the third problem.

After getting the sums, he got the results $\frac{2}{3}, \frac{3}{4}$ (Fig. 4). Hence, he realized the solution to the problem. It can be said that discovering the relationships between the data and logical reasoning was beneficial in this solution.

Problem 4 was finding the unknown number in the number pattern "4-7-15-29 -? – 117". The gifted students used a total of four different problem solving strategies in this question (Table 1). The strategies used by students in solving this problem were: finding a pattern, intelligent guessing and testing, accounting for all possibilities, and logical reasoning. Gifted student 5 used all of these strategies while solving this problem.

"*So, there are 4, 7, 15, 29, a number, and 117. 4 and 7increases by 3, but 15 minus 7 is 8, not 3. The differences are 3, 8, 14. From 3 to 8, the difference is 5. From 8 to 14, the difference is 6. Maybe the next difference is 7. Let me try... 14 plus 7 is 21… 29 plus 21 is 50. So, the 5th number is 50, then the difference should be 21 plus 8 is 29, and 29 plus 50 is 79. It should be 117. So, it is not working. Hmmm…. from 4 to 7….hmm…2 times 4 is 8 and minus 1 gives us 7. What about the next one…2 times 7 is 14 minus 1 is 13. Hmmm… it should be 15. I am confused. Let me try the next one… 2 times 15 is 30 minus 1 is 29. This one works, but the second one does not work. The second one is 2 times 7 is 14, so I need to add 1. Maybe it goes like 1minus 1plus…I am not sure.. but I will try... 4 times 2 minus 1 is 7. Then, 7 times 2 plus 1is 15. Then, 15 times 2 minus 1 is 29. Then, 29 times 2 plus 1 is …hmm...59. Then,59 times 2 minus 1 is…yes 117. So, the missing number should be 59.*"

First, the student searched for a relationship between the data. Thus, the student examined all of the possible situations that may occur in the solution and constantly tested the accuracy of the situations obtained. Then, the student solved the problem as a result of her experiments.

Problem 5 was a non-routine verbal problem. The problem was as follows: "*The king called his vizier. He gave 10 pouches and 1000 gold. Place these gold in pouches in such a way that no matter what number of gold pieces I ask from you, you can give me the number of gold pieces that I want by simply collecting the pouches, without opening the pouches (minimizing or increasing gold)?*"

The gifted students used a total of four different problem solving strategies in this question (Table 1). The strategies used by students in solving this problem were: organizing data, intelligent guessing and testing, accounting for all possibilities, logical reasoning, and making a drawing. In Table 1, it can be seen that the students mostly used similar problem solving strategies in the solution to Problem 5. Only gifted student 1 used the making a drawing problem solving strategy. The student explained the solution to the problem as follows.

"*hmm.. This is so hard. Maybe I can find something. First of all, the king can ask for 980 from us, or 1. Then, there must be 1 in the first pouch.*"

He drew 10 pouches on paper (Fig. 5).

"*hmm.. 999 gold pieces left. Let's say he wanted 2 from me. Then, I put 2 in the second pouch, because there is 1 in the first pouch already. If he wants 3 gold pieces, I can add the first and second pouches. He can ask for 4 gold pieces. So, let me put 4 gold pieces in the third pouch. If he wants 5,6, or 7 gold pieces, I can add the pouches.*"

Thus, he discovered that the next pouches should have 8 and 16 gold pieces.

"*16 is twice that of 8, and the others go on like this. But, will these gold pieces add up to 1000? hmm.. We try.*"

Hence, he wrote 512 in the last pouch. Then, he collected them all and he found that the result was 1023.

"*23 gold pieces extra now… Then, I have to subtract 23 from the tenth pouch. Since in the first 9pouches, there can be no excess gold. Then, there must be 489 gold pieces in the last pouch.*"

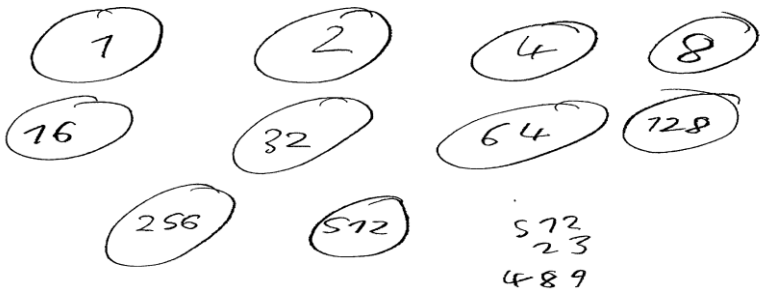

Figure 5: Visual that Gifted student 1 used in solving the Problem 5.

Thus, the student found the solution to the problem. From this solution, it was seen that the student first used the logical reasoning strategy. Using this strategy, he created a logical layout for the solution. Then, he used the strategy of organizing the data strategy by sorting the data according to this logical order. At the same time, he made the data visible using the making a drawing strategy.

4. DISCUSSION AND RESULTS

The research findings revealed that the 7th-grade mathematically gifted students most frequently preferred to use logical reasoning and finding pattern problem solving strategies. The present data were in agreement with the results of the studies of Durmaz and Altun (2014) and Intaros et al. (2014). In the study of Durmaz and Altun (2014), they found that gifted students mostly used the strategy of finding a pattern when they solved math problems. On the other hand, in the study of Intaros et al. (2014), investigating the strategies used by 7th-grade students in problem solving, determined that the students used the logical reasoning strategy effectively. In this respect, the results obtained from the current study were similar to those of the studies conducted in the literature.

Adopting a different point of view, solving a simpler or analogous problem, and accounting for all possibilities strategies were used very little by the gifted students. In the literature, it has been stated that the strategy of solving simpler or analogous problems was less used and it was a difficult strategy for students to use (Intaros et al., 2014; Verschaffel et al., 1999; Yazgan, 2007; Yıldız et al., 2012). However, the importance of this problem solving strategy has been expressed in many studies (Vershaffel et al., 1999; Yazgan & Bintaş, 2005; Yazgan, 2007). Similarly, it has been stated that in the literature that adopting a different point of view strategy was a more difficult strategy to learn and apply than the other strategies (Vershaffel et al., 1999). This can explain why these strategies were used very little by the gifted students. On the other hand, working backwards and considering extreme cases strategies were never used by the gifted students in this study.

During the process of obtaining the data, the students were asked to solve the questions again using a different strategy after solving the questions. However, only one of the students who participated in the study solved the question again with a different strategy. This situation was consistent with the results of Karaca (2012). As a result of the study, the researcher stated that the students mostly used at least one strategy when they solved problems. This was related to the knowledge and experience of the students. Gifted students can show different approaches to problem solving based on their experiences and knowledge. Swings and Peterson (1988) stated that understanding mathematical knowledge and establishing the relationship between this information occurs during the problem solving process. Therefore, as Rotigel and Fello (2004) pointed out, mathematically gifted students should be left alone with problems that strain their mental capacity in mathematics lessons, since they are very different from ordinary students in terms of speed and depth because of their insightful reasoning (Krutetskii, 1976; Lee, 2005; Sriraman, 2003). As a matter of fact, it was revealed in the study that the mathematically gifted students tended to use more strategies when they tried to solve challenging problems.

In general, all of the gifted students solved the problems using more than one strategy (Table 1). This was consistent with the results of the studies of Tertemiz et al. (2017) and Billstein et al. (2013). Tertemiz et al. (2017) stated in their study that gifted students can use at least two different problem solving strategies to solve non-routine problems. Similarly, Billstein et al. (2013) stated that more than one strategy can be used in solving problems. In this respect, it can be stated that mathematically gifted students can use their mathematical knowledge and skills flexibly in the problem solving process. This shows that mathematically gifted students should be more encouraged to use problem solving strategies.

In addition to this, it was determined in the semi-structured interviews with the gifted students that they had not received any training about strategies. Nevertheless, they developed their strategies while solving the problems. As stated by Miller (1990), gifted students need mathematical programs that develop independent exploratory behavior, problem solving, looking for patterns, and organizing data to find relationships. Therefore, suitable education programs for the needs of mathematically gifted students should be developed and students should be supported.

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GENİŞ ÖZET

1. GİRİŞ

Araştırmanın genel amacı, yedinci sınıf özel yetenekli öğrencilerin matematik problemlerini çözerken kullandıkları problem çözme stratejilerinin incelenmesidir.

2. YÖNTEM

Araştırmada nitel araştırma yöntemlerinden durum çalışması metodu kullanılmıştır. Araştırmanın katılımcıları Ankara ilinde yer alan Yasemin Karakaya Bilim Sanat Merkezine devam etmekte olan yedinci sınıf özel yetenekli öğrencilerden oluşmaktadır. Katılımcılar Millî Eğitim Bakanlığından gerekli izinler alındıktan sonra kurumda öğrenimine devam etmekte olan gönüllü öğrenciler arasından rastgele seçilmiştir. Çalışmaya katılan tüm öğrenciler bir yıldan fazla merkezde öğrenim görmektedir ve öğrencilerin tümü kurumda matematik derslerine katılmıştır. Böylece belirlenen 5 adet yedinci sınıf özel yetenekli öğrenci ile yarı yapılandırılmış görüşmeler yardımıyla çalışma yürütülmüştür. Yarı yapılandırılmış görüşmeler araştırmacı tarafından hazırlanan "Problem Çözme Testi (PÇT)" kullanılarak 40 ile 60 dakika arasında değişen süreler boyunca gerçekleştirilmiştir. PÇT beş sorudan oluşmaktadır. PÇT'de yer alan tüm sorular ilköğretim matematik öğretim programında yer alan hedeflere uygun ve öğrencilerin soruların çözümünde birden fazla strateji kullanmalarına olanak tanıyacak şekilde geliştirilmiştir. Görüşmelerde ayrıca sesli düşünme tekniği kullanılarak öğrencilerin çözümlerinin derinlemesine incelenmesi sağlanmıştır. Elde edilen veriler yazılı hâle getirilerek iki araştırmacı tarafından farklı zamanlarda değerlendirilmiştir. Öğrencilerin soruların çözümünde kullandıkları problem çözme stratejilerinin kodlamaları Posamentier ve Krulik (1998) tarafından geliştirilen problem çözme stratejileri esas alınarak belirlenmiştir. Alan yazında birçok problem çözme stratejisi yer almaktadır fakat bunlar temelde aynıdır. Bu nedenle çalışmada tüm stratejileri kapsamasından dolayı Posamentier ve Krulik (1998) tarafından geliştirilen stratejiler kodlamalar için esas alınmıştır. Böylece çalışmada kullanılan stratejiler; geriye doğru çalışma, ilişki arama, farklı bakış açısı ile düşünme, daha basit benzer problemlerle çözme, verileri organize etme, tahmin etme test etme, çizim yapma, uç durumları düşünme, tüm olası durumları düşünme ve mantıksal akıl yürütme stratejileri olarak belirlenmiştir. İki araştırmacı tarafından yapılan analiz sonuçları Huberman ve Miles (1994) tarafından geliştirilen kodlayıcılar arası uyuşma yüzdesi formülü kullanılarak %93 bulunmuştur. Karşılaştırma sonucunda farklı olan durumlar için araştırmacılar bir araya gelmiş, farklılıklar değerlendirilerek ortak kodlamalar gerçekleştirilmiştir.

3. SONUÇ VE TARTIŞMA

Araştırmanın sonucunda tüm öğrencilerin soruların çözümünde en az bir tane problem çözme stratejisini kullandıkları belirlenmiştir. Bununla birlikte öğrencilerin tamamı problemleri çözerken birden fazla problem çözme stratejisinden yararlanmayı tercih etmiştir. Alan yazında yapılan farklı çalışmalarda bu bulguya benzer sonuçlar elde edilmiştir. Öğrenciler problemler üzerinde düşünürken yeni stratejiler oluşturmayı ve aynı zamanda oluşturdukları stratejileri farklı problemleri çözerken kullanmayı öğrenir. Bu nedenle bir problemin çözümü için birden fazla farklı strateji kullanabilir (Billstein vd., 2013). Tertemiz vd. (2017) yaptıkları çalışmalarında özel yetenekli öğrencilerin rutin olmayan problemleri çözmede en az iki farklı problem çözme stratejisini kullanabildiklerini söylemektedir. Bu bakımdan özel yetenekli öğrencilerin matematik bilgi ve becerilerini esnek bir şekilde soru çözümlerinde kullanabildikleri, bu nedenle soruları çözerken farklı stratejilerden aynı anda yararlanabildikleri söylenebilir.

Öğrencilerle yapılan görüşmelerde öğrencilerin problem çözme stratejilerinin neler olduğuna yönelik bilgilerinin olmadığı ve bu yönde bir eğitim almadıkları sonucu ortaya çıkmıştır. Bu durumda özel yetenekli öğrencilerin problem çözme stratejileriyle ilgili bilgileri olmasa dahi problemlere kendiliğinden informal olarak çözüm stratejileri geliştirdikleri söylenebilir. Bu sonuç Yazgan ve Bintaş'ın (2005) yaptıkları çalışmalarında, öğrencilerin problem çözme stratejilerine yönelik eğitim almasalar dabazı stratejilerin informal olarak kendiliğinden gelişebildiği sonucuyla örtüşmektedir. Benzer olarak Azak (2015) da yaptığı çalışmasında öğrencilerin bazı problem çözme stratejilerini eğitim almadan kullanabildiklerini belirtmektedir.

Çalışmaya katılan tüm öğrenciler problemlerin çözümlerinde sekiz farklı problem çözme stratejisi kullanmıştır. Bu stratejiler; ilişki arama, farklı bakış açısı ile düşünme, daha basit benzer problemle çözme, verileri organize etme, tahmin etme test etme, tüm olası durumları düşünme, çizim yapma ve mantıksal akıl yürütmedir. Bunun yanı sıra öğrencilerin hiçbiri uç durumları düşünme ve geriye doğru çalışma stratejilerini kullanmamıştır.

Çalışmaya katılan özel yetenekli öğrenciler çoğunlukla mantıksal akıl yürütme ve ilişki arama stratejilerini kullanarak soruları yanıtlamıştır. Durmaz ve Altun (2014) yürüttükleri çalışmalarında öğrencilerin matematik problemlerini çözerken çoğunlukla ilişki arama stratejisini kullandıklarını tespit etmiştir. Diğer yandan Intaros

vd. (2014) çalışmalarında yedinci sınıf öğrencilerinin mantıksal akıl yürütme stratejisini sıklıkla kullandıklarını belirmektedir. Elde edilen bu sonuçlar Durmaz ve Altun (2014) ile Intaros vd. (2014) tarafından yürütülen çalışmalardan elde edilen sonuçlarla benzerlik göstermektedir.

Farklı bakış açısı ile düşünme, daha basit benzer problemle çözme ve tüm olası durumları düşünme stratejileri özel yetenekli öğrenciler tarafından soruların çözümünde az kullanılmıştır. Nitekim alan yazında yer alan birçok çalışmada bu stratejilerden daha basit benzer problemlerle çözme stratejisinin öğrenciler tarafından kullanılmakta zorlanılan bir strateji olduğu ve bu nedenle bu stratejinin az kullanıldığı ifade edilmektedir (Intaros vd., 2014; Verschaffel vd., 1999; Yazgan, 2007; Yıldız vd., 2012). Diğer yandan bu stratejinin önemine dair çalışmalar da alan yazında yer almaktadır (Vershaffel vd., 1999; Yazgan & Bintaş, 2005; Yazgan, 2007). Ayrıca Lee vd. (2014) çalışmalarında öğrencilerin çoğu için bu stratejinin kullanımının zor olduğunu belirtmekte ve zorluğundan dolayı öğrencilerin farklı stratejilerle problemleri çözdüklerini ifade etmektedir. Araştırmacılara göre bu stratejiler tüm olası durumları düşünme ve çizim yapma stratejileridir. Çizim yapma stratejisi öğrencilerin hem problemi anlamasında hem de çözmesinde etkili bir stratejidir. Buna rağmen bu çalışmada öğrenciler tarafından az kullanılmıştır.

Verileri organize etme ile tahmin etme test etme stratejileri de öğrencilerin problem çözümlerinde sıklıkla kullandıkları stratejilerdir. Altun (2005) ve Yazgan (2007) rutin olmayan problemlerin çözümlerinde öğrencilerin işlem becerilerinin yeterli olmadığı durumlarda verileri organize etme stratejisine yöneldiklerini belirtmektedir. Akkan vd. (2012) ortaokul öğrencilerinin aritmetikten cebire geçiş süreçlerini problem çözme bağlamında inceledikleri çalışmalarında, öğrencilerin problem çözerken tahmin etme test etme stratejisini sıklıkla kullanarak problemlerdeki bilinmeyenleri elde etmeye çalıştıklarını söylemektedir. Benzer olarak Pilten (2008) de tahmin etme test etme stratejisinin öğrenciler tarafından sık kullanıldığını ifade etmektedir. Bu çalışmada da özel yetenekli öğrencilerin soruları çözerken bu stratejileri diğer stratejiler yardımıyla kullandıkları tespit edilmiştir.

Öğrencilerle yapılan yarı yapılandırılmış görüşmelerde çözdükleri soruları farklı stratejiler kullanarak tekrar çözmeleri istenmiştir. Fakat sadece bir öğrencinin soru çözümünde farklı strateji kullanarak soruyu cevapladığı görülmüştür. Bu durum Karaca (2012) tarafından yapılan çalışmanın sonucuyla örtüşmektedir. Öğrenciler çoğunlukla bir stratejiyle çözüme ulaştıklarında farklı stratejileri kullanmayı tercih etmemektedir. Öğrencilerin belirli stratejilere yönelmeleri ve farklı çözüm yollarını denememeleri problem çözme stratejileri hakkında yeterli bilgiye sahip olmamalarından da kaynaklanabilir. Özmen vd. (2011) yaptıkları çalışmalarında benzer bir durumla karşılaşmış ve öğretmen adaylarının problemleri çözerken strateji çeşitliliğinden çok belirli stratejilere yöneldiklerini belirlemiştir.

Yapılan çalışma sonucunda özel yetenekli öğrencilerin matematik derslerinde problem çözme stratejilerinin neler olduğuna yönelik bilgilendirilmedikleri, buna rağmen yetenekleri sayesinde problem çözme süreçlerindeki düşünceleri ve kullandıkları stratejiler ile ilgili matematiksel anlayışlarındaki yeterliliklerinin yeterli düzeyde olduğu sonucuna ulaşılmıştır. Alan yazında yapılan çalışmalar incelendiğinde problem çözme sürecinde farklı stratejiler kullanımının öneminin vurgulandığı görülmektedir. Bu bakımdan özel yetenekli öğrencilerin problem çözmede farklı stratejiler geliştirebilmelerine yönelik kazanımlarının arttırılması sağlanarak potansiyellerinin desteklenmesi önem arz etmektedir.

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