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Research Article

The impact of the use of mathematical problem solving on the development of creative thinking skills for prep school students in Arab schools in Israel

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Abstract

In underdeveloped countries, research in mathematics education has been mostly focused on students' geometry abilities based on levels, learning approaches, and textbooks. But, thinking process and level are a problem relevant to the low quality of student achievement. The process and level of students' thinking are due to the conceptual system in operating. In this study, a geometry question at the analysis level was designed to investigate conceptual systems. Students represent and compare two figures by their techniques. Data obtained from the survey and narrative study. Data were analyzed based on three components of activity: input, internal processing, and output. Students represent by copying, revising symbols, rummaging objects, and reconstructing properties. They analyze property geometry on the building block or spatial representation. Students compare through one of the two process models of think, namely: object extraction techniques to structure-property connection and inter-object connection to property extraction. The systematic paths of the two models are different. One produces a creative conceptual formulation before extracting geometry properties. Its creativity is involved in comparisons so there is a leap to a more objective point of view. Therefore, conceptual systems and construction for the conceptual formulation are two ideas for learning situations or solving problems.

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Introduction

The goals of teaching mathematics have gone through many different stages. In the past the goal of teaching mathematics was to focus on the accuracy and speed in performing calculations. But the technological developments have reduced the importance of this goal since a small calculator can perform all these operations more accurately and quickly. Therefore, the goals of teaching mathematics have changed to focus on understanding and perceiving the meaning, and this requires a focus on understanding mathematics as an independent, interconnected subject with its own fundamentals, problems and self-pleasures. While this goal may be sufficient to create a plenty of theoretical mathematicians, it may not be an excuse to overburden pupils with many subjects of mathematics, the primary goal of education as a whole is to prepare the individual to become a useful member for himself/ herself and his/ her community. The important question here is: How does mathematics contribute to this goal? The continuing and growing problems facing humanity require rapid and growing development in the methods of solving them. Therefore, mathematics helps to prepare a useful individual by developing his/her ability to solve the problems of the renewed life with all their types and times. The importance of solving mathematical problems in school comes from the fact that it is the primary objective of teaching and learning process.

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Knowledge, skills, concepts, mathematical generalizations and even all other school subjects are not only goals by themselves, but also they are means and tools that help the individual to solve his/her real problems. In addition, problem solving is the natural way to practice thinking in general, there is no mathematics without thinking and there is no thinking without problems. So the objectives of teaching mathematics focus on developing understanding, meaning and skills alongside with the basic processes, which contribute to rapid scientific development which result endless problems in one's life. It is considered the main goal of problem solving in mathematics is to train students on some ways and methods that help them to solve problems in general.

The school mathematics Study Group in the USA (SMSG) has developed the following set of problem solving goals:

- Provide the student with different types of strategies that help him/her to solve problems.
- Develop the flexibility of student in the way of processing and initiating problem solving.
- Develop some methods to apply geometric representations in the production of new information about the problem.
- Develop some skills in scheduling and organizing the given information and derivatives to take advantage in the solution.
- Deepen the student's understanding of the problem by accustoming him/her to the work of numerical estimates based as in the light of the problem posed

From the above, the importance of problem-solving methods is marked in the following points:

- Help student to discover new concepts.
- Teach student how to develop the concept to use it in solving a new problem.
- Accustom student to critical scientific thinking.
- Help to join and match mathematical concepts.
- Develop some of the student's mental abilities such as visualization, abstraction, analysis, and synthesis.
- Stimulate student's curiosity and discovery.
- Develop student's ability to analyze situations and make decisions.

Mathematics also is one of the study materials that aims to develop creativity and creative thinking. Creativity is not achieved out of the blue; it must be preceded by a problem that challenges the mind, so mathematics can be taken as a field for the development of creativity and creative thinking. Its structural nature allows more than one logical conclusion to the same given introductions, and its connotation structure gives some flexibility in the organization of content. Moreover, mathematics is rich in problem situations for which students can find for each situation multiple and diverse solutions, and its study teaches the student to employ objective criticism of the situation. This will provide the student with some of the basic capabilities for the creative process. Creative thinking in mathematics can be learned as a skill, and then developed with more training, since each learner possesses some degree of thinking (Mufti, 1995).

Mathematics represents an important field of education since it reveals the capabilities of creative thinking and its development for learners in all grades. Mathematics is not just a collection of facts and information, but mainly a way and style of thinking to face mental problems; therefore, successful teaching of mathematics affords learners the capabilities and methods of creative thinking (Aladdin & Abdel, 2003).

Mathematical creative thinking as a science is different from creative thinking as a study subject. Creative thinking as a science seems obvious when the learner solves the mathematical problem in an independent way, not previously known to him, but creative thinking in mathematics as a study subject appears if the learner knows that many mathematical questions can be solved in more than one way, and this itself is the essence of creative thinking (Roshka, 1989).

Creative Thinking Skills

A review of most common tests of creative thinking, that Torrance (1966) and Guilford (1967) tests indicate the most important skills and abilities of creative thinking that researchers have tried to measure, that are:

Fluency

Fluency refers to the ability to produce as many ideas and solutions to a problem as possible. Fluency in mathematics means the learners' ability to give several different solutions to a particular topic or issue. It means accustoming pupils to give several different solutions to a particular topic, issue or obstacle, so that they have the ability to recall the largest number of ideas when exposed to a particular mathematical or geometric problem, and then choose the solution or idea that the pupil finds most convincing. Fluency is divided into sub-aspects: verbal fluency, fluency of thoughts and fluency of expression.

Verbal Fluency

The verbal fluency of mathematics may not be as important as in languages, for instance. We mean the speed of the individual to think and to provide words, or mathematical synonyms, or their imperfections, and generate them in a certain format, or the ability of an individual to produce as many mathematical vocabularies as possible within a given specification in a certain period of time (Austin, 1988).

Write as many attempts as possible to solve the following question:

$$y = -x^2 + 4x - 7$$

A man who is now (71) years old and his son is (33) years, how many years ago was the father three times the age of his son?

The answer……………….... years.

1) 14 2) 11 3) 15 4) 21

List the largest number of objects around us whose size can be calculated?

Fluency of Ideas

Fluency of ideas is the individual's ability to give as many mathematical ideas as possible associated with a certain perceived situation, for example (Davis, 1981):

- Mention all the consequences of doubling the population of Israel.
- Write down as many results as possible for doubling the length of the day to 48 hours.
- Every line has to be the same sum, find the value of y (see figure 1),

17			y
13	**15**		
	10	**11**	**17**
	y		**14**

Figure 1.

Find the Value of y

c. Fluency of Shapes (expression):

Fluency of shapes refers to the ability to change shapes with simple additions, and the ability to quickly draw a number of examples and preferences or adjustments in response to a particular visual stimulus. Examples (Brousseau, 1991):

Example: What can be shaped from the following forms (see figure 2)?

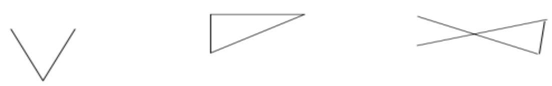

Figure 2.

What can be shaped from the following forms? Question Figure

In the previous example, the student should draw whatever s/he wants to draw, geometric or non-geometric shapes, and the more the answer or the shape s/he draws is meaningful and unique, the more it indicates his/her innovative abilities.

We want to turn the triangle into various drawings that express objects or things. What additions can you add for that (see figure 3)?

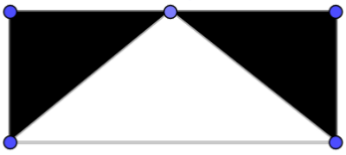

Figure 3.
The Triangle

This is an example of one of the students' solutions (see figure 4):

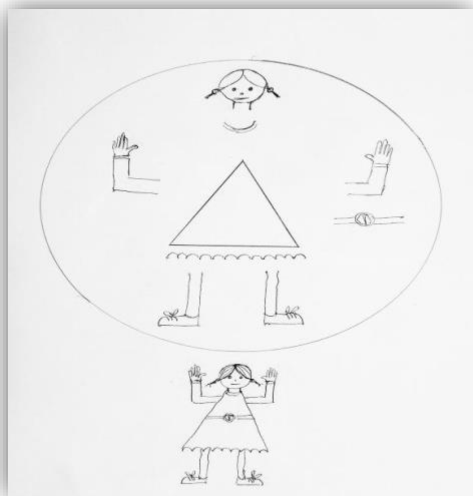

Figure 4.
This is an Example of One of the Student Answer

Flexibility
Flexibility is the ability to vary the mathematical answers and solutions. Flexibility in mathematics means asking learners to mention as many as possible properties of a drawn geometric figure. Here we can notice the development of the student's thinking, and the flexibility that s/he shows in the production of the greatest number of ideas to achieve the presented mathematical situation. One kind of flexibility is that the student can change the way of research, and stop searching in a particular way or narrate preconceived ideas. Another a kind of flexibility is that the student must be aware of the things s/he is looking for, maybe more important than the things s/he needs. Forms of Flexibility:

Automatic Flexibility: It is the speed of the individual to produce as many diverse ideas as possible associated with a problem, and according to this ability the individual tends to automatic initiative in these situations, and does not just respond (Al-Khalili, 2005).

Adaptive Flexibility: It means finding a solution to a problem or facing any situation in the light of the feedback that comes from that situation (Al-Taati, 2004).

The researcher believes that flexibility is the individual's ability to give multiple and different inputs and ideas to solve a problem. Examples of Flexibility (RAMA, 2017):

- ❖ Mention the uses of the caliper ruler for (student, tailor, carpenter, blacksmith …….)
- ❖ Think of all the ways you can design to weigh very light objects.
- ❖ If there were six people at a party and everyone wanted to shake hands with the others only once, how many times did they shake hands at this party?

a) 36 b) 18 c) 15 d) 12

Originality

Originality is the ability to produce mathematical ideas unfamiliar to one's colleagues. Originality in mathematics means the ability to produce authentic responses, i.e., responses not commonly repeated among the colleagues of the person who comes up with these responses. It can be measured in mathematics by asking the learner to give several different solutions to the same mathematical situation, such as giving more than one method to solve a given geometric exercise, or solving an algebraic question by more than one method, for example, (Crrouse, 1987):

In this figure (5), AB, CD are two diameters of the circle perpendicular, M, N two points, and MX, NK, NG, MV, are columns on AB, CD as shown.

To prove that $GK = XV$, the traditional solutions are either:

- Application of triangles $\Delta XEG \cong \Delta EGK$
- Application of Pythagorean theory.

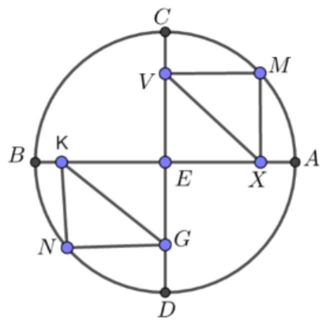

Figure 5.
The circle

Elaboration:

Elaboration means the ability to give many details or make new additions to an idea, problem or mathematical question to develop or enrich it. With this skill, more precise details of the case are discovered or identified and highlighted. For examples (Davis, 1981):

Ahmed bought 10 pens …………….

Add what you want to the question so that it can be solved using a process of:

a) addition b) subtraction c) multiplication d) division

In each item there is a set of triads and there is an adjective to achieve, except one triad has not achieved it. Draw a circle around the irregular triad, and write down what is the adjective: (4, 2, 4), (5, 1, 4), (2, 5, 4), (5, 3, 2), (2, 3, 5), (2, 2, 6).

Sensitivity to Problems

Sensitivity to problems means vigilance to what is in the mind when solving or researching a particular mathematical situation and being attentive to anything new or every change in the path for researching the problem or solving it. Problem sensitivity means awareness of problems, needs, or weaknesses in the educational situation. Sensitivity to problems in mathematics means that some pupils are faster than others in noticing the problem, checking its presence in the situation, and linking the data to their previous experiences. For examples (Brousseau, 1991):

Khalid bought 6 books, 9 notebooks and 7 pens for 15 NIS. If the price of one pen is one NIS. Can you help Khalid to know the price of one book? How?

Notice the outputs of the examples and write down the output of example four?

$$11 \cdot 11 = 4$$
$$22 \cdot 22 = 16$$
$$33 \cdot 33 = 36$$
$$44 \cdot 44 = ?$$

a) 0 b) 16 c) 48 d) 64

The correct answer is b.

Multiple-Choice

Example: Which number, when it is squared, decreases? The answer is ……

a) negative integer
b) positive integer
c) number 1
d) any fraction in which the numerator is less than the denominator

The correct answer is d because:

For example, if we take the number $\frac{1}{2}$ and square it, it becomes $\frac{1}{4}$, which is less. while negative numbers and positive numbers increase when squared, and number 1 remains the same.

Factors Affecting Creative Thinking

We cane summarize the factors that affect the creative thinking in the following chart (figure 6):

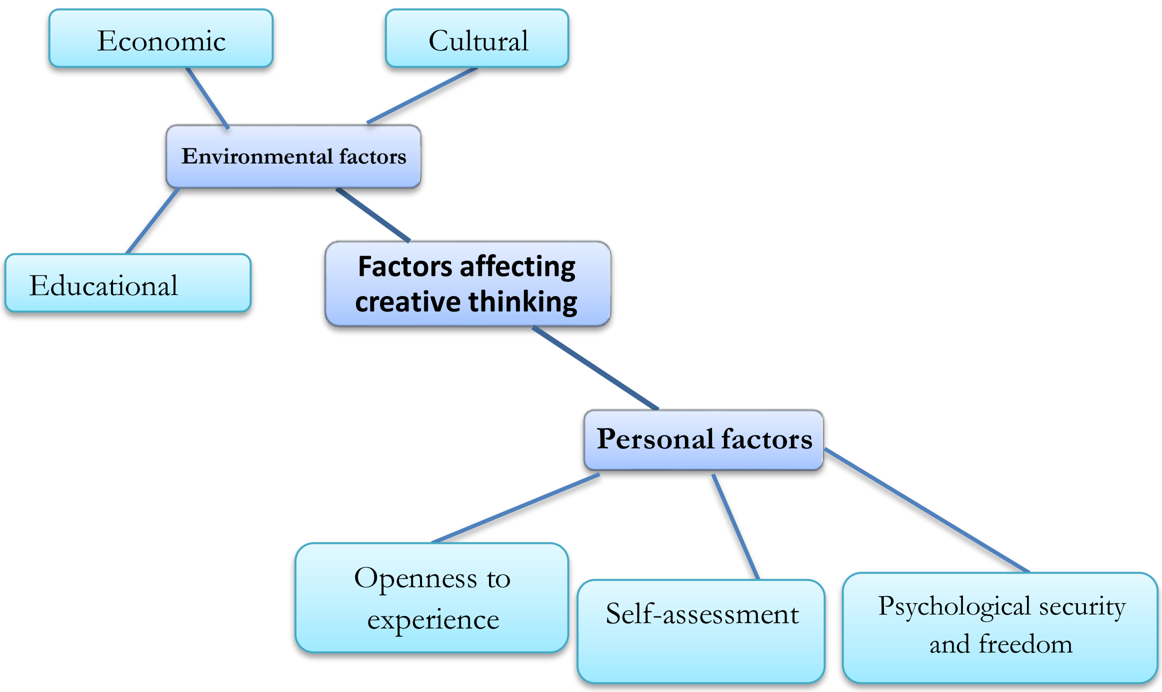

Figure 6.
Factors Affecting Creative Thinking

Personal Factors

There are some factors:

Self-Assessment: Self-assessment means that the individual has a sense of self-confidence, knows well how to evaluate it (high, medium, or low rating), and how an individual sees himself from his personal perspective.

Psychological Security and Freedom: Psychological security is a feeling of reassurance that means lack of fear, absence of anxiety and having self-confidence, while the opposite is called psychological insecurity, which means constant anxiety, lack of confidence and severe phobia from others.

Experience Acquisition (Openness to experience): Experience acquisition relates to many interests and is influenced by imagination, insightful vision, previous knowledge and the acquisition of various experiences.

Environmental Factors

Environmental factors refer to lack of proper place, overcrowding, lack of support from colleagues, lack of material support and socialization in a bossy family.

Educational Patterns: Educational factors refer to the diverse teaching methods and the ways used by an individual in learning, or the teacher in education, as well as the place where the learning process takes place.

Economic Level: A person may not pass through the process of learning, going to university, completing high school, or experience society rejection and criticism for creative ideas, with the lack of alternatives and the lack of appropriate reinforcement for the creators (Al-Ayasr, 2013).

Cultural Level: This refers to the environment surrounding the person, when all people around him are not educated, or do not have the cultural background that appreciates the importance of education. This cultural poverty creates a kind of frustration for the learner. Family is considered the first cell and the basic building block in the building of society, and the place where the individual receives his/her life lessons. It has an impact on the formation of personality and behavior, so the family plays a big role in the development or suppression of the child's creative abilities. For example, a domineering father who imposes his opinions and does not allow to his children to express their opinions, negatively affects the personality of the children and damages their self-confidence, and serves to suppress the creative abilities of the individual (Abu Latifa, 2009).

Personal Stimuli: Personal stimuli are within the person, giving him/her the motivation to reach the goal s/he wants.

Obstacles of Creative Thinking

Several references have indicated that there are many and varied obstacles that stand in the way of the development of creative thinking skills and effective thinking, and perhaps the first step that teachers, coaches and parents should pay their attention to is identifying these obstacles; so that they can be effectively overcome them when applying an educational or training program aimed to develop creative thinking skills (Treffinger & Isaksen 1985). Obstacles of creative thinking were classified into two main groups – personal obstacles and situational obstacles, which we summarize below:

Personal Obstacles

Poor self-esteem: Poor self-esteem is an important factor in creative thinking, because poor self-esteem leads to fear of failure, risk avoidance, and unsafe consequences situations.

Tendency to Conformity: The tendency to comply with the prevailing norms hinders the use of all sensory inputs, and limits the possibilities of imagination and expectation, and thus sets limits on creative thinking.

Excessive Enthusiasm: A strong desire for success and an overzealous enthusiasm for achievements lead to a rush of results before the situation matures, perhaps jumping to a late stage in the creative process without exhausting the prerequisites that may take more time. For example, (Austin, 1988):

Use numbers from 1 to 9; write one number in each circle so that you get a total of 23 in each direction (see figure 7).

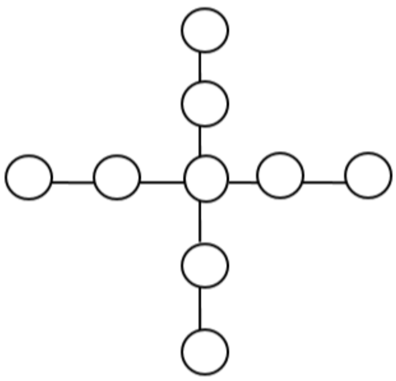

Figure 7.
Example

Saturation

Saturation means reaching a state of overexposure that may result in a loss of awareness of the merits of the status quo, or inaccuracy of views. Saturation is an anti-incubation state or a phased storage of an idea or problem, for example (Austin, 1988):

Use the numbers $\{0, 1, 3, 5, 6\}$ to solve the following code:

		A	*A*	*C*
	×		*A*	*C*
	N	*M*	7	*C*
N	*W*	*W*	*C*	0
N	*N*	7	2	*C*

Figure 8.
"Use numbers from 1 to 9; write one number in each circle so that you get a total of 23 in each direction" Question Figure

Stereotypical Thinking:
Stereotypical thinking is a form of traditional thinking, restricted to habits; Isaksen and Treffinger (1985) considered it the most prominent obstacle to creative thinking. To illustrate the impact of this obstacle, De Bono gives a symbolic example:

A dog used to walk a long way to get the bone his owner put it in the same location behind a fence (see the figure 8). Since the first successful attempt to reach the bone was achieved by taking this long road, the dog held on to it, and it became a habit he did it automatically. If the dog could be guided to this obstacle, he would be able to abandon his habit and find the shortest way to reach his goal.

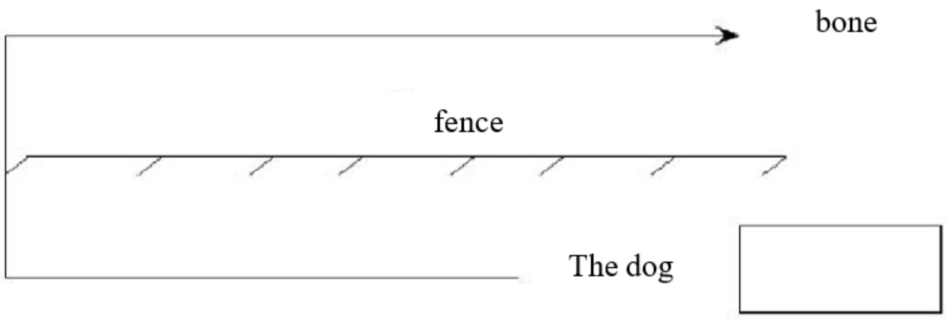

Figure 9.
Example

Another example:
You enter a room with two ropes hanging from the ceiling, and you are asked to tie the two ropes together. There are some tools on a table in the corner of the room, including a hammer, pliers and scissors. You hold the end of one of the ropes and walk towards the other, but you quickly realize that you cannot reach the end of the other rope. You try to extend the range you can reach with the hammer, but it doesn't work. What can you do to solve the problem?

The solution: This problem can be solved using the available tools in an unconventional way. If you attach the hammer to the end of one of the ropes and wave it like a pendulum, you will be able to hold it in the Isaken & Treffinger, (1985) explained this failure by the tendency of individuals to stick to the familiar uses of things, and described this tendency by the terms "inertia" or "functional constancy".

Insensitivity or a Feeling of Helplessness
One of the necessary characteristics of the process of creative thinking is vigilance and a delicate sensitivity to problems. When the sensitivity is weakened by lack of excitement or lack of challenge, the person becomes more inclined to stay in the circle of reactions to what is happening around him, giving up the initiative to explore the dimensions of the problem and to find solutions (Isaken & Treffinger, 1985).

Haste and Improbability of Ambiguity
This trait is related to the desire to find an answer to the problem by taking the first opportunity, without understanding all aspects of the problem, and without working to develop several alternatives or solutions to it, and

then choosing the best one. One of the problems associated with this trait is the unpredictability and evasion of complex or ambiguous situations, and escaping from facing it. Postponement of judgment is also an important characteristic of creative thinking; for example, when brainstorming is practiced, judgment is allowed only after every possible opportunity to generate ideas has been exhausted.

Transfer Habit

When certain mental patterns and structures that have been effective in dealing with new and diverse situations are entrenched, other, more effective strategies are often ignored, and some killer phrases that summarize this obstacle include: "We've always been doing this successfully," or: "We've always been solving the problem this way".

Situational Obstacles

Situational obstacles to creative thinking mean those related to the situation itself, or to prevailing social or cultural aspects. The most important of these obstacles are:

***Resistance to Change*:**There is a general tendency to resist new ideas, and to maintain the status quo by many means for fear of their repercussions on the security and stability of the individual, there are those who believe that modern experience poses a threat to their gains and conditions, and therefore, they respond by using 'deadly' phrases to any new idea (Brousseau, 1991), such as:

- It's not going to work in statement questions.
- This idea of solving the question is very long.

For example, three consecutive natural numbers totaling 300. What are they?
Since the quotient of 300 by 3 is 100. There are three possibilities:

100,101,102

99,100,101

98,99,100

The second possibility is the correct answer.

Imbalance between Competition and Cooperation

There is a need to combine the spirit of competition and the spirit of cooperation for both the individual and the group to achieve new achievements, and excessive consideration of either of them may cause loss of contact with the real problem or progress in solving it; therefore, balance between them is a condition of productive or creative thinking (Coxeter, 1986).

Example for the fifth grade students:
The lengths of the sides of this rectangle are 2 cm and 6 cm (see figure 9):

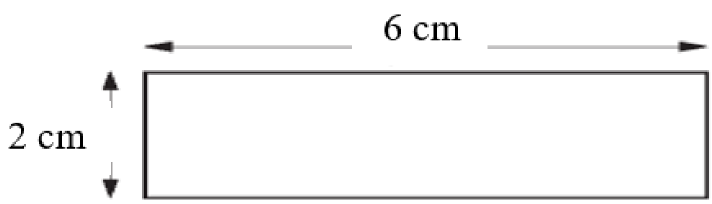

Figure 10.
Example

Indicate the polygon whose perimeter is equal to the perimeter of the rectangle

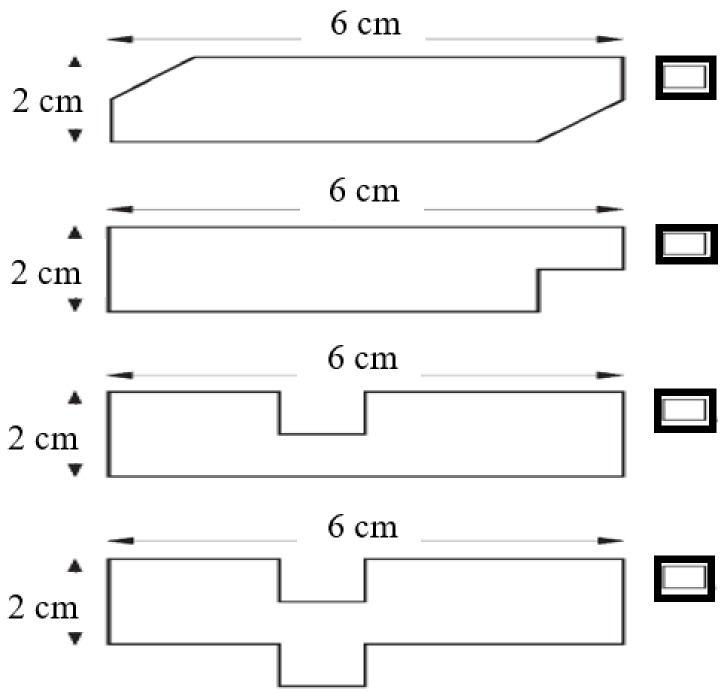

Figure 11.
Example

Comparison between Creative Thinking and Critical Thinking
Perhaps it is not possible to distinguish between creative thinking and critical thinking for the simple reason that any good thinking involves an assessment of quality and the production of what can be described as novelty. It is difficult for the brain to be preoccupied with a complex thinking process without the support of another complex thinking process.
But the outcomes of thinking vary depending on the type of task, and whether it requires creative or critical thinking. Creative thinking must include internal critical thinking.
How critical thinking is involved in creative thinking:

- Self-criticism (for a person): For example, a student criticizes himself for a mistake he made when answering the question.
- External criticism: It comes from other people for a mistake in one's answer/solution.

For example (Austin, 1998): A student solved the following equation: $x^2 - x + 1 = 0$
And from the equation $x^2 - x = -1$ and by analysis a common divisor

$$x^2 = x - 1 \quad (2)$$

$$x \cdot (x - 1) = -1$$

we compensate for the $(x - 1)$ of the equation (2) so

$$x \cdot x^2 = -1$$

$$x^3 = -1$$

$$x = -1$$

When verifying by compensation by the original equation, the solution does not verify, where is the error in which the student falls in?
The correct solution: The mistake lies in the step $x^2 = x - 1$ where this step becomes a requirement to be $(x - 1)$ positive quantity which means x is more than 1.

Trends of Creative Thinking

There are many trends and theories that explain creativity, and the most prominent are (Al-Ayasrah, 2013):

Behavioral Trend: adopted by Skinner, who defined creative thinking as the kind of thinking that is reinforced or substantiated, leading to the possibility of continuity. However, if it does not receive the required reinforcement, this thinking will diminish.

Cognitive Trend: focuses on creative thinking results through the interaction and organization of past and new experiences, while providing a certain maturity, an exciting environment for the individual, and effective training in connectivity and logic.

For example (Edwards, 1979): If a student told to you, I can prove that $5 = 7$ and gave you the following proof:

$1^5 = 1$ as well as $1^7 = 1$ no matter how high the forces = 1

That is $1^5 = 1^7$ because the two results are equal.

And there's a rule in mathematics: if the bases are equal, the exponents are equal, and vice versa. So $5 = 7$

What's wrong with this proof? (The error is that the rule has a complement.)

a) the two quantities are not equal b) 5, 7 are odd numbers

c) the rectangle is less in perimeter d) if the basis is greater than 1

Clairvoyance Trend: This trend in interpretation of creativity was adopted by the German scientist Fertimer, who assumed that creative thinking is clairvoyant and intuitive thinking, so the creative idea is the one in which the problem is formulated and the individual suddenly arrives at the solution by active mental processes, where the situation is treated with a new treatment (Saada, 2015).

Intuition is the formation and verification of mathematical guesses.

For example, (Edwards, 1979):

Find the result of dividing (1122) by (11) without performing the division operation:

a)120 b) 12 c)102 d) 1002

The correct answer is (c).

The Levels of Creative Thinking

Researchers' views on the topic of creative thinking differed. Turns attempted to resolve differences between those views, and proposed five levels of thinking, which are (Tashman, 2010; Al-Ayasrah, 2013; Austin, 1988):

Creative Expressions: This level refers to the development of unique ideas, regardless of their type.

For example, (Edwards, 1979):

Find uncompleted square roots: It is known that the square root of the number 9 is 3, but what about the roots of uncompleted squares? To find their value, it is necessary to estimate their approximate value, for example: Find the root of the number 85?

Productive Creativity: In this level there are strong indications of the availability of certain restrictions that regulate the free performance of individuals.

For example (Edwards, 1979):

How do you prove that the sum of the angles of a triangle equals 180^0? Mention 4 different ways to prove that? What is the simplest of these ways?

Regenerative Creativity: This level represents the individuals' ability to penetrate intellectual principles, and includes substantial improvements through modifications incorporated in new starting point's skills and concepts.

For example (Hoffman, 1988):

Is $(x + y)^2 = x^2 + y^2$ true for every real number x , y?

Innovative Creativity: It refers to the proficiency in using materials.

For example (Hoffman, 1988): Prove that $\frac{n(n+1)}{2}$ is a natural number for every natural number n?

This level includes principles and assumptions, and it is considered the highest degree of creativity

Two numbers x, y square sum of 41 expressed this by symbols
The answer: $x^2 + y^2 = 41$ (Hoffman, 1988).

Modern methods that teachers apply to develop creative thinking in mathematics:

- Encourage the learner to think in a collective way in order to get as many ideas as possible from the discussion with the group.
- Accept the suggested ideas and help the learner to modify and develop them.
- Help the learner assume answers and test them to reach the right solution
- Do not provide ready-made solutions for the problems or ready-made proofs for theorems.
- Give questions that require deep thinking and are open ended.
- Encourage the pupil to produce something new from his/her imagination and invention.

Problem of Research

From all of the above, we can deduce the research question as follows:
What is the impact of the use of mathematical problem-solving on the development of creative thinking skills among prep school students in Arab schools in northern Israel?
The following research hypotheses can be derived:

- There are statistically significant differences at the level of the $\alpha \leq 0.05$ function between the average scores of students in the experimental group and the control group in the post application of the creative thinking test on the fluency skill in favor of the experimental group.
- There are statistically significant differences at the level of the $\alpha \leq 0.05$ function between the average scores of students in the experimental group and the control group in the post application of the creative thinking test on the flexibility skill in favor of the experimental group.
- There are statistically significant differences at the level of the $\alpha \leq 0.05$ function between the average scores of students in the experimental group and the control group in the post application of the creative thinking test on the originality skill in favor of the experimental group.
- There are statistically significant differences at the level of the $\alpha \leq 0.05$ function between the average scores of students in the experimental group and the control group in the post application of the creative thinking test as a whole in favor of the experimental group.

Methods

This chapter discusses the procedures followed in the implementation of the study, the study community and sample, the preparation of the study tool, the statistical methods used, and ensuring its reliability and stability.

Study Procedure

The researcher used the experimental method, which is defined as a method that studies a current phenomenon while inserting changes in one of the factors and monitoring the results of the change. (Al-Aga, Al-Ostath, 2020). The study design was a control group and an experimental group with pre- and post-tests. The independent variable in the study was "solving mathematical problems" and its effect on the dependent variable "creative thinking skills".

Study Community

The study community consists of all preparatory school students in northern Israel, numbering 26,600 students for the 2019-2020 academic year.

Study Sample

The sample was selected from different Arab preparatory schools in northern Israel during the first semester of the 2019-2020 school year. The researcher divided the study sample into two groups: the experimental group (40 students) and the control group (40 students), as presented in Table 1.

Table 1.
The Sample

School	Class	Experimental / Control Group	Number of the students
Arab prep schools from northern Israel	E	Experimental	40
	C	Control	40
Total			80

Teacher's Guide
The teacher's guide is considered a monitor and assistant in the implementation of lessons without problems and flops. It provides directions and guidance that help the teacher to facilitate the educational process and its progress in the right direction. It has been prepared according to the following steps:
Guide Aim: To provide a comprehensive presentation of the role of the teacher in applying the steps of solving mathematical problems in order to achieve the educational goals of the unit. It also helps the teacher to develop creative thinking skills in mathematics in general and in the unit of algebraic fractions in particular for prep school students according to the curriculum of Israeli Ministry of Education.
Guide Content: It consists of the unit of algebraic fractions and solving equations for the preparatory stage according to the Israeli Ministry of Education curriculum in mathematics, as shown in Table 2:

Table 2.
Guide Content

No.	Subject
1	Algebraic fractions
2	Simplifying algebraic fractions
3	Addition of algebraic fractions
4	Subtracting algebraic fractions
5	Multiplying algebraic fractions
6	Dividing algebraic fractions
7	Solving algebraic equations

The guide was written according to the following:
- The objectives of each subject are formulated in a behavioral way, so that the teacher can measure the achievement of class goals and student performance.
- Tools and educational means: the researcher prepared the means that suit the nature of the educational situation according to student's needs.
- Assessment and evaluation: To assess students' understanding of the educational materials.

Data Collection Tool
Creative Thinking Test
The test consisted of 12 questions, to identify and evaluate thinking skills. The objective of the test was to measure the extent to which the prep school students possess creative thinking skills. The researcher prepared the test items according to previous studies, and placed emphasis on examining the following:
Fluency: Refers to the ability to give as many answers as possible to the mathematical problem in a specified period of time.
Flexibility: Means the ability to generate varied thoughts in solving problems.
Originality: Refers to the students' ability to find unique solutions for the group s/he belongs to.

The test was prepared in its initial form with written instructions, and then presented to a competent committee in the field of mathematics including teachers in the field and experts in evaluation and language. The final form of the test was amended according to the recommendations of the expert committee. The test was given to an exploratory sample to determine the level and time required to solve it and the difficulties that it may include. The time required for the test was 120 minutes. The researcher checked the tests, and determined the stability of the test using the alpha Cronbach equation. The result was alpha Cronbach's α=0.63, which is an indicator of the test's validity.

Equivalence of Two Study Groups
The researcher confirms the equivalence of the experimental group and the control group using the following variables:

Mathematical achievement: Through students' achievements in previous exams, as presented in Table 3:

Table 3.
Mathematical Achievement

Group	*Number*	*Mean*	*SD*	*T(79)*
Experimental	40	66.93	16.9	**0.43**
Control	40	65.5	18.48	

Cultural, economic and social level: The experimental and control groups were from the same schools and from a close socio-economic and cultural environment.

Results

The current study aims to reveal the impact of the use of mathematical problem-solving on the development of creative thinking skills for prep school students in Arab schools in northern Israel. To achieve this goal, the researcher's applied his creative thinking test to the study students. And after the application was completed, data was collected to examine the validity of study hypotheses.

First Hypothesis: The first hypothesis states that there are statistically significant differences at $\alpha \leq 0.05$ between the average scores of the students in the experimental group and the control group in the post application of the creative thinking test in the skill of fluency in favor of the experimental group.

In order to test this hypothesis, a t-test was used to calculate the significance of differences between two independent groups to identify the impact of the use of problem-solving in the development of creative thinking skills for prep school students in Arab schools in northern Israel.
Table 4 shows the results:

Table 4.
First Hypothesis Results

Group	**Number of Students**	**Arithmetic Mean**	**Standard Deviation**	**T-test**	η^2
Experimental	40	37.86	13.55	11.52	**0.62**
Control	40	12.98	1.8		

This confirms the obvious impact of the use of mathematical problem-solving on the development of fluency; and also confirms the first hypothesis.

Second Hypothesis: The second hypothesis states that there are statistically significant differences at $\alpha \leq 0.05$ between the average scores of the students in the experimental group and the control group in the post application of the creative thinking test in the flexibility skill in favor of the experimental group.
In order to test this hypothesis, a t–test was used to calculate the significance of differences between two independent groups to identify the impact of the use of problem-solving in the development of creative thinking skills (flexibility) among prep school students in Arab schools in northern Israel.
Table 5 shows the results:

Table 5.
Second Hypothesis' Results

Group	**Number of Students**	**Arithmetic Mean**	**Standard Deviation**	**T-test**	η^2
Experimental	40	27.16	8.33	11.21	**0.6**
Control	40	12.38	0.54		

This confirms the obvious impact of the use of mathematical problem-solving on the development of flexibility, and thus corroborates the second hypothesis too.

Third Hypothesis: the third hypothesis states that there are statistically significant differences at $\alpha \leq 0.05$ between the average scores of the students in the experimental group and the control group in the post application of the creative thinking test in originality skill in favor of the experimental group.

In order to test this hypothesis, a t-test was used to calculate the significance of differences between two independent groups to identify the impact of the use of problem-solving in the development of creative thinking skills (originality) among prep school students in Arab schools in northern Israel. Table 6 shows the results:

Table 6.

Third Hypothesis Results

Group	Number of Students	Arithmetic Mean	Standard Deviation	T-test	η^2
Experimental	40	65	12.88	8.53	**0.47**
Control	40	29.5	2.89		

This confirms the obvious impact of the use of mathematical problem-solving on the development of originality, and consequently confirms the third hypothesis.

Fourth Hypothesis' Results: The fourth hypothesis states that there are statistically significant differences at $\alpha \leq 0.05$ between the average scores of students in the experimental group and the control group in the post application of the creative thinking test as a whole in favor of the experimental group.

To test this hypothesis, a t-test was used to calculate the significance of the differences between two independent groups to identify the impact of the use of problem-solving in the development of creative thinking skills in general among prep school students in Arab schools in northern Israel. Table 7 depicts the results:

Table 7.

Fourth Hypothesis Results

Group	Number of Students	Arithmetic Mean	Standard Deviation	T-test	η^2
Experimental	40	78	10.18	9.43	**0.52**
Control	40	56.6	6.47		

This confirms the obvious impact of the use of mathematical problem-solving on the creative thinking test as a whole, and substantiates the fourth hypothesis.

Discussion and Conclusion

The results of the first hypothesis, whose results found an effect for the use of mathematical problem-solving between the average results of students in the experimental group and the average of the students of control group in favor of the experimental group, coincide with the results of Abu Athrah (2010) and Mustafa (2009), regarding the use of some strategies that work on the development of creative thinking skills, including fluency in the field of mathematics, and this can be explained by the following reasons:

- The use of mathematical problem-solving gives the students an opportunity to solve life problems related to the reality of their lives, which improve the students' fluency of thinking.
- Mathematical problem-solving helps students to apply knowledge in new life situations and increase their connection to this knowledge, leading them to improve their performance.
- The use of mathematical problem-solving improves students' handling of life problems compared to the control group students.
- Mathematical problem-solving increases students' awareness of the importance of studying mathematics in solving daily life problems, prompting students to develop their creative thinking.

The results of the second hypothesis, which found an effect for the use of mathematical problem-solving between the average results of the students of the experimental group and the average of the students of the control group in favor of the experimental group, are consistent with the results of Abu thrah (2010), Giordano (2003), Mann (2005) and Rosa (2000), in the use of some strategies that develop creative thinking skills, including flexibility, in the field of mathematics. This can be explained by the following reasons:

- Solving mathematical problems allows students to take organized steps that greatly contribute to the development of their ability to express and participate effectively, resulting in the development of students' flexibility in producing ideas.
- The use of solving mathematical problems gives the issue a vital character by accustoming students to formulate the issue in their own language, to draw an appropriate diagram of it, and to explain it with a model or sensory means. The diversity in the means of presentation of objects and life situations, and the use of symbols, increase students' creativity.
- Solving mathematical problems is concerned with students' understanding of the topics and problems posed, which develops a spirit of creativity, makes them abler and skilled in dealing with life and its problems, and expands their life experiences and links them to what they had learned at school.
- The use of mathematical problem-solving made students able to solve unfamiliar and more complex problems with more confidence and mental flexibility compared to the control group students.

The results of the third hypothesis, which found an effect for the use of mathematical problem-solving between the average results of the students of the experimental group and the average of the students of the control group in favor of the experimental group, are consistent with the results of the studies of Abu thrah (2010), Giordano (2003), Mann (2005) and Rosa (2000), in the use of some strategies that develop creative thinking skills including originality, in the field of mathematics. This can be explained by the following reasons:

- The use of mathematical problem-solving helps to develop the aesthetic sense of mathematics, to appreciate its importance in real life, and develops a positive inclination towards it, which led to an originality in students' thinking when solving life problems.
- Mathematical problem-solving is used to ask meaningful questions, and gives enough time to think about the answer and clarify the meaning of each question, so that students learn how to address any problem they face by themselves.
- The use of mathematical problem-solving allows students to appreciate reasonable answers, and to use them inversely towards the data, which contributed to the formation of their own issues or problems, which increased their authenticity.

The results of the forth hypothesis, which found an effect in the use of mathematical problem-solving between the average results of the students of the experimental group and the average of the students of the control group in favor of the experimental group, are consistent with the results of the studies of Abu thrah (2010), Giordano (2003), Mann (2005) and Rosa (2000), in the use of some strategies that develop creative thinking skills as a whole in the field of mathematics. This can be explained through the following reasons:

- The diversity of activities and life problems leads to a rise in creativity among students.
- Solving mathematical problems with their multiple, progressive and interrelated steps broadens students' perception, and their inclusion in the steps has helped them to develop their creative thinking.
- Solving mathematical problems increases students' understanding of the problems posed, which developed their fluency of solutions, flexibility of entrances, and originality of solving them, which developed their creative thinking.

We can conclude that the diversity of activities led better understanding of the material, which in turn led to conceptual learning among students and to more creative thinking among students. All of the above can be summarized in the Abd Algani, Hibi and Abo Al-Haija' Instructional Model Offer (see figure 10):

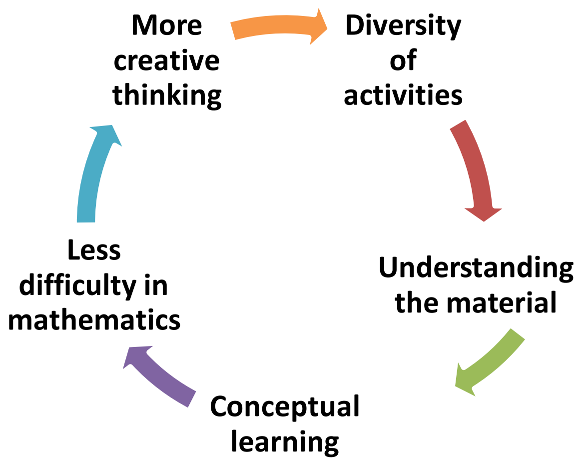

Figure 12.
Abd Algani, Hibi and Abo Al-Haija' Mathematics Instructional Model Offer

Recommendations

In light of the results of the study the researcher recommends:

- To use mathematical modeling in the mathematics curriculum to show the role of mathematical knowledge in solving real-life problems.
- To train students in the faculties of education and teacher training colleges on how to use modeling and mathematical problem-solving in solving life problems.
- Teachers should work to discover students' abilities and inclinations, develop their curiosity, and work to develop these abilities in the right direction.
- The authors of the math curriculum should boost teacher's attention to the importance of mathematical problem-solving, to increase the student's motivation to study mathematics.
- There should be a specialized team to select problems and activities that develop creativity, and to include them in the math curriculum in an appropriate manner that takes into account students' individual differences.
- Apply gradualness in posing problems in the curriculum, so that there are problems solved mentally; some need paper and pen and some need calculators in order to develop creativity in students.
- To focus on organizing the content of the mathematics curriculum in the preparatory stage according to mathematical problem-solving.
- To prepare guides for teachers to teach the mathematics curriculum at the preparatory stage using mathematical problem-solving.

For Further Research

In the light of the objectives of the current study and its results, we can propose the following future studies and research:

- Study the impact of the use of mathematical modeling in teaching other subjects at other educational stages.
- Study the impact of the use of mathematical modeling on the development of visual thinking skills among primary school students.
- Study the effectiveness of training programs for teachers to use mathematical modeling in teaching different school subjects.
- Study and identify the awareness of workers in the educational field concerning the importance of mathematical modeling.

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Yousef Methkal Abd Algani born in Nahif, Israel, June 2, 1981. He graduated from the Department of Software Engineering, Technion in 2002, and graduated B.Sc. on Mathematics and Computer Science in 2008, Haifa Uni', Israel. Furthermore, graduated M.Sc. on mathematics and Computer Science with a thesis in the field of Algebraic Topology, Haifa Uni', Israel and finish my Ph.D. in mathematics Education in 2012 he completes his Teaching Certificate, and in addition to a Certificate on Measurement and Evaluation in Education in Oranim Collage. Worked as a lecturer in Sakhnin College for Teacher Education, in the Department of Mathematics, Sakhnin, also a lecturer in The Arab Academic College for Education in Israel, in the Department of Mathematics, Haifa. He participated in several international conferences, and he published a lot of articles in the field of Mathematics and Mathematics Education.

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