Turkish Journal of Science & Technology
16(1), 113-117, 2021

The Helix and Slant Helices Generated by non-Degenerate Curves in $M^3(\delta_0)\subset E_2^4$

Fatma ALMAZ [1], Mihriban ALYAMAÇ KÜLAHCI[2]
[1,2]Department of Mathematics, Science Faculty, Firat University, Elazig, Turkey

[1] fb_fat_almaz@hotmail.com, [2] malyamac@firat.edu.tr

(Geliş/Received: 29/01/2021; Kabul/Accepted: 14/02/2021)

Abstract: In this paper, we investigate helix and slant helices using non-degenerate curves in term of Sabban frame in de Sitter 3-space or Anti de Sitter 3-space $M^3(\delta_0)$. Furthermore, in $M^3(\delta_0)$ 3-space the necessary and sufficient conditions for the non-degenerate curves to be slant helix are given.

MSC[2000]: Primary 53A35; Secondary 53C25
Key words: Sabban frame, helix, slant helix, $M^3(\delta_0)\subset E_2^4$ 3-space.

$M^3(\delta_0)\subset E_2^4$ 3-Uzayında non-Dejenere Eğriler Tarafından Elde Edilen Helis ve Slant Helisler

Öz: Bu çalışmada, de Sitter veya Anti de Sitter $M^3(\delta_0)\subset E_2^4$ 3-uzayında dejenere olmayan eğrilerin helis ve slant helislerini Sabban çatısına göre inceledik. Ayrıca, $M^3(\delta_0)$ 3-uzayında non-dejenere eğrilerin slant helis olması için gerekli ve yeterli şartlar verildi.

Anahtar kelimeler: Sabban çatı, helis, slant helis, $M^3(\delta_0)\subset E_2^4$ 3-uzayı

1. Introduction

In the recent years, there has been remarkable interest in the slant helix and helices among or curve of geometrically constant slope, or so-called general helices are well-known curves in the classical differential geometry of space curves. Helices are characterized by the feature that the tangent makes a constant angle with a fixed straight line (the axis of the general helix), it is known that a curve x is called a slant helix if the principal normal lines of x make a constant angle with a fixed direction.

The concept of slant helix defined by Izumiya and Takeuchi [8, 9]. The geometry of helix and slant helices have been represented in a different ambient spaces by many mathematicians. In [1], the authors studied timelike B_2-slant helices in Minkowski 4-space E_1^4. Ahmad studied the position vectors of a spacelike general helix with reference to the standart frame in E_1^3, [2]. In [3], the notion of a k-type slant helix in Minkowski 4-space E_1^4 was introduced by the authors. In [4], they gave necessary and sufficient conditions to be a slant helix in the Euclidean n-space and they expressed some integral characterizations of such curves in reference to curvature functions. In [14], the authors expressed helices and slant helices in lightlike cone. Camcı and et al. examined some characterizations for a non degenerate curve α to be a generalized helix by using its harmonic curvatures, [6]. Ferrandez and et al. obtained a Lancret-type theorem for null generalized helices in Lorentz-Minkowski space L^n, [7]. Ilarslan and et al. studied the position vectors of a timelike and a null helix in Minkowski 3-space E_1^3, [10]. In [11], the authors examined spherical images the tangent indicatrix and binormal indicatrix of slant helix. In [15], they gave some characterizations for spacelike helices in Minkowski space-time E_1^4. In [17], Turgut and et al. they examined the concept of a slant helix in Minkowski spacetime.

In this paper, we study helix and slant helices according to Sabban frame in de Sitter or Anti de Sitter 3-space.

* Corresponding author: fb_fat_almaz@hotmail.com. ORCID Number of authors: [1]0000-0002-1060-7813, [2]0000-0002-8621-5779

2. Preliminaries

Let E_2^4 be the 4 −dimensional semi Euclidean space with index two with the metric

$$G(a,b)=\langle a,b\rangle=-a_1b_1-a_2b_2+a_3b_3+a_4b_4$$

for all $a=(a_1,a_2,a_3,a_4), b=(b_1,b_2,b_3,b_4)\in\mathbb{R}^4$. Let $\{e_1,e_2,e_3,e_4\}$ be pseudo-orthonormal basis for E_2^4. Then δ_{ij} is Kronecker-delta function such that $\langle e_i,e_j\rangle=\delta_{ij}\varepsilon_i$ for $\varepsilon_1=\varepsilon_2=-1,\varepsilon_3=\varepsilon_4=1$. A vector $x\in E_2^4$ is said to be as spacelike, timelike and lightlike(null) if $\langle x,x\rangle>0$ (or $x=0$), $\langle x,x\rangle<0$ and $\langle x,x\rangle=0$ ($x\neq 0$), respectively. The norm of a vector $x\in E_2^4$ is given as $\|x\|=\sqrt{|\langle x,x\rangle|}$. The signature of a vector x is 1(or 0, −1) if x is spacelike(or null, timelike). Also, de Sitter 3-space with index 2 and Anti de Sitter 3-space in E_2^4 are given as

$$S_2^3=\{x\in E_2^4: g(x,x)=1\}; H_1^3=\{x\in E_2^4: g(x,x)=-1\}, \tag{2.1}$$

respectively. The pseudo vector product of vectors a, b and c is given by

$$a\wedge b\wedge c=\begin{vmatrix}-e_1 & -e_2 & e_3 & e_4\\ a_1 & a_2 & a_3 & a_4\\ b_1 & b_2 & b_3 & b_4\\ c_1 & c_2 & c_3 & c_4\end{vmatrix},$$

where $\{e_1,e_2,e_3,e_4\}$ is the canonical basis of E_2^4 and $a=(a_1,a_2,a_3,a_4)$, $b=(b_1,b_2,b_3,b_4)$, $c=(c_1,c_2,c_3,c_4)\in\mathbb{R}_2^4$. Also, it is clear that $\langle w,a\wedge b\wedge c\rangle=\det(w,a,b,c)$ for any $w\in\mathbb{R}_2^4$. Hence, $a\wedge b\wedge c$ is pseudo-orthogonal to each of the vectors a, b and c. Unless otherwise stated for the sake of brevity, we will use the notation $M^3(\delta_0)$ instead of the symbols S_2^3 or H_1^3. If $\delta_0=1$ or $\delta_0=-1$, then $M^3(1)=S_2^3$ or $M^3(-1)=H_1^3$, respectively.

Let $x: I\subset\mathbb{R}\to M^3(\delta_0)$ be regular curve, where $\mathrm{sign}(x(t))=\delta_0$ for $\forall$ $t\in I$. For non-degenerate curves in $M^3(\delta_0)$. The regular curve x is called as spacelike or timelike if x' is a spacelike or timelike vector where $x'(t)=dx/dt$ at any $t\in I$. The such curves are said to be non-degenerate curve. Let x be a non-degenerate curve, with arc length parametrization $s=s(t)$, that is the curve $x(s)$ is a unit speed curve. Also, the unit tangent vector of x is defined as $t(s)=x'(s)$. Also,

$$\langle x(s),x(s)\rangle=\delta_0, \langle x(s),t'(s)\rangle=-\delta_1; \delta_1=\mathrm{sign}(t(s)) \tag{2.2}$$

The vector $t'(s)+\delta_0\delta_1x(s)$ is pseudo-orthogonal to both $x(s)$ and $t(s)$. Moreover, since $\langle x''(s),x''(s)\rangle\neq\delta_0$ and $t'(s)+\delta_0\delta_1x(s)\neq 0$, the principle normal vector and the binormal vector of x is written as $n(s)=\frac{t'(s)+\delta_0\delta_1x(s)}{\|t'(s)+\delta_0\delta_1x(s)\|}$ and $b(s)=x(s)\wedge t(s)\wedge n(s)$, respectively. In addition, the geodesic curvature of x are given by $\kappa_g(s)=\|t'(s)+\delta_0\delta_1x(s)\|$. Therefore, pseudo-orthonormal frame field $\{x(s),t(s),n(s),b(s)\}$ of $\mathbb{R}_2^4$ along x is said to be the Sabban frame of non-degenerate curve x on $M^3(\delta_0)$, then

$$t(s)\wedge n(s)\wedge b(s)=-\delta_0\delta_3x(s); n(s)\wedge b(s)\wedge x(s)=\delta_1\delta_3t(s);$$
$$b(s)\wedge x(s)\wedge t(s)=-\delta_2\delta_3n(s); x(s)\wedge t(s)\wedge n(s)=b(s),$$

where

$$\langle t(s),t(s)\rangle=\mathrm{sign}(t(s))=\delta_1; \langle n(s),n(s)\rangle=\mathrm{sign}(n(s))=\delta_2$$
$$\langle b(s),b(s)\rangle=\mathrm{sign}(b(s))=\delta_3, \det(x,t,n,b)=-\delta_3. \tag{2.3}$$

Also, let $x: I\to M^3(\delta_0)$ be an immersed spacelike (timelike) curve with k_g, τ_g with respect to the Sabban frame $\{x,t,n,b\}$, such that,

- If $\tau_g=0$, x is said to be as planar curve in $M^3(\delta_0)$.
- If $k_g=1$ and $\tau_g=0$, x is called a horocycle in $M^3(\delta_0)$.
- If both $k_g\neq 0$ and $\tau_g\neq 0$ are constant, it is said to be as helix in $M^3(\delta_0)$.

Now, let's assume that $\langle x''(s),x''(s)\rangle\neq\delta_0$. Then, Frenet-Serret formulas of x is given as

$$\begin{aligned}x'(s)&=t(s)\\ t'(s)&=-\delta_0\delta_1x(s)+k_gn(s)\\ n'(s)&=-\delta_2\delta_1k_gt(s)-\delta_1\delta_3\tau_gb(s)\\ b'(s)&=\delta_1\delta_2\tau_gn(s),\end{aligned} \tag{2.4}$$

where the geodesic torsion of x is given by $\tau_g=\frac{\delta_1\det(x,x',x'',x''')}{k_g^2}$, [18].

Definition 1. Let x be according to the frenet frame $\{t,n,b\}$ in M^3. If there exists a constant vector field $W\neq 0$ in M^3 such that

$$\langle t(s), W_i(s)\rangle = \text{constant},$$

for $\forall s \in I$, then it is called a helix in M^3 and $W(s)$ is said to be an axis of $x(s)$, [9].

Remark 1 The condition $\langle x''(s), x''(s)\rangle \neq \delta_0$ is equivalent to $k_g(s) \neq 0$. Moreover, it may express that $k_g(s) = 0$ and $t'(s) + \delta_0\delta_1 x(s) = 0$ if and only if the non-degenerate curve x is a geodesic in $M^3(\delta_0)$, [5].

3. Helix and Slant Helices in $M^3(\delta_0)$

In this section, the helices and slant helices in $M^3(\delta_0)$ space are expressed using sabban frame.

Definition 3. Let x be a non-degenerate curve with arc length according to the Sabban frame $\{x, t, n, b\}$ in $M^3(\delta_0)$. If there exists a constant vector field $W_i \neq 0$ in $M^3(\delta_0)$ such that

$$\langle t(s), W_i(s)\rangle = \text{constant},$$

for $\forall s \in I$, then x is called a helix in $M^3(\delta_0)$ and $W_i(s)$ is called as axis of $x(s)$.

Theorem 1. Let x be a helix in $M^3(\delta_0)$. Then the axis of the helices x are given as

$$W_i(s) = (\delta_0 As + c_1)x(s) + \frac{A}{\delta_1}t(s) + \left(\frac{A(\delta_0 s + c_2)}{\delta_0\delta_1 k_g}\right)n(s) + \left(\frac{A\delta_3}{\delta_2}\int (\delta_0 s + c_2)\tau_g(s)ds\right)b(s),$$

where $A, c_i \in \mathbb{R}_0^+$.

Proof. Let $W(s)$ be an axis of helix x with the Sabban frame $\{x, t, n, b\}$. Then, $W(s)$ can be written as

$$W_i = m_0^j x + m_1^j t + m_2^j n + m_3^j b, \tag{3.2}$$

where $m_i^j, i \in \{0,1,2,3\}$ are differentiable functions and from definition of the helix, we have $\langle t(s), W_i(s)\rangle = A, A \in \mathbb{R}_0^+$. Furthermore, from (2.2), (2.3) and (3.2), we can write

$$\delta_0 m_0^j = \langle x, W_i\rangle, m_1^j\delta_1 = \langle t, W_i\rangle = A, m_3^j\delta_3 = \langle b, W_i\rangle = A, m_2^j\delta_2 = \langle n, W_i\rangle, \forall s \in I. \tag{3.3}$$

By differentiating on both sides of (3.2) and using (2.4), we get

$$m_0^{j\prime} - m_1^j\delta_0\delta_1 = 0, \tag{3.4a}$$

$$m_0^j + m_1^{j\prime} - m_2^j\delta_2\delta_1 k_g = 0, \tag{3.4b}$$

$$m_1^j k_g + m_2^{j\prime} + m_3^j\delta_2\delta_1\tau_g = 0, \tag{3.4c}$$

$$-m_2^j\delta_3\delta_1\tau_g + m_3^{j\prime} = 0. \tag{3.4d}$$

Substituting $m_1^j\delta_1 = A$ into (3.4a), we have

$$m_0^j = \delta_0 As + c_1. \tag{3.5}$$

After the necessary calculations from (3.4b) and (3.4d) the following equations (3.6) and (3.7) are obtained respectively.

$$m_2^j = \frac{A(\delta_0 s + c_2)}{\delta_0\delta_1 k_g}. \tag{3.6}$$

$$m_3^j = \frac{A\delta_3}{\delta_2}\int (\delta_0 s + c_2)\tau_g(s)ds; c_i, A \in \mathbb{R}_0^+. \tag{3.7}$$

Hence, using (3.3) and m_i^j, $i \in \mathbb{R}$, we have

$$W_i = (\delta_0 As + c_1)x + \frac{A}{\delta_1}t + \left(\frac{A(\delta_0 s + c_2)}{\delta_0\delta_1 k_g}\right)n + \left(\frac{A\delta_3}{\delta_2}\int (\delta_0 s + c_2)\tau_g(s)ds\right)b. \tag{3.8}$$

Theorem 2. Let x be a non-degenerate curve with arc length in $M^3(\delta_0)$. Then x is a helix if and only if

$$k_g + \frac{d}{ds}\left(\frac{\delta_0 s + c_2}{k_g}\right)\frac{1}{\delta_2} + \delta_3\tau_g\int (\delta_0 s + c_2)\tau_g ds = 0. \tag{3.9}$$

Proof. Using the equation (3.4c) and the equations m_i^j, $i \in \{0,1,2,3\}$, j=n,b. Hence, we get (3.9). Conversely, assume that (3.9) holds, we obtain define a vector field W_i . Therefore, from definition of helix, we write $W_i{}' = 0, < t, W_i > = A, A \in \mathbb{R}_0^+$.

Corollary 1. Let $x: I \to M^3(\delta_0)$ is an helix curve with k_g and τ_g relative to the Sabban frame $\{x, t, n, b\}$

1) If the helix x is a horocycle in $M^3(\delta_0)$, x is non-degenerate curve in H_1^3,

2) If the helix x is a planar curve in $M^3(\delta_0)$, since $\tau_g = 0$ the following equation holds

$$\frac{k_g}{\sqrt{k_g^2+\delta_0}}=\frac{c_4}{\delta_0 s+c_2}.$$

3.1 n-type Slant Helix

Definition 4. Let $x(s)$ be a non-degenerate curve with the Sabban frame $\{x,t,n,b\}$ in $M^3(\delta_0)$. If there exists a constant vector field W_n in $M^3(\delta_0)$ such that

$$<n, W_n>= D, D\in \mathbb{R}_0^+,$$

for $\forall s\in I$. Then x is called a helix and W_n is called the $n-$axis of x.

Theorem 3 Let x be n-type slant helix in $M^3(\delta_0)$. Then the n-axes of x are

$$W_n=-D\delta_0\delta_1\delta_3\left(\int\frac{\tau_g}{k_g}\int\tau_g dsds\right)x-D\delta_3\frac{\tau_g}{k_g}\left(\int\tau_g ds\right)t+\frac{D}{\delta_2}n+\frac{D\delta_3\delta_1}{\delta_2}\left(\int\tau_g ds\right)b, \tag{3.10}$$

where $D\in\mathbb{R}_0^+$.

Proof. Let W_n be an axis of n-type helix x with the Sabban frame $\{x,t,n,b\}$. From definition of the n-type helix, we have $\langle n(s), W_n(s)\rangle=D, D\in\mathbb{R}_0^+$. Thus, from equations (3.4), we get

$$m_0^n=-D\delta_0\delta_1\delta_3\int\frac{\tau_g}{k_g}\int\tau_g dsds;\ m_1^n=-D\delta_3\frac{\tau_g}{k_g}\int\tau_g ds;$$

$$m_2^n=\frac{D}{\delta_2};\ m_3^n=\frac{D\delta_3\delta_1}{\delta_2}\int\tau_g ds, \tag{3.11}$$

Considering (3.11) we obtain (3.10).

Theorem 4. For the non-degenerate curve x to be n-slant helix in $M^3(\delta_0)$ the necessary and sufficient condition the following equation is provided

$$\delta_0\delta_1\delta_3\int\frac{\tau_g}{k_g}\int\tau_g dsds+\delta_3\left\{\frac{d}{ds}\left(\frac{\tau_g}{k_g}\right)\int\tau_g ds+\frac{\tau_g^2}{k_g}\right\}+\delta_1 k_g=0. \tag{3.12}$$

Proof. Using the equation (3.4b), we get (4.12). Conversely, assume that (4.12) holds, we can define a vector field W_n as (3.10). Hence, from definition of n-slant helix, we can write $W_n'=0, <n, W_n>$=constant.

Corollary 2. Let $x: I\to M^3(\delta_0)$ is $n-$slant helix curve with k_g and τ_g as regards the Sabban frame $\{x,t,n,b\}$. If the n-slant helix x is a planar curve in $M^3(\delta_0)$, since $k_g=0$ the curve x is also geodesic non-degenerate curve.

3.2 b −type Slant Helix

Definition 5. Let x be non-degenerate curve with the Sabban frame $\{x,\alpha,\beta,y\}$ in $M^3(\delta_0)$. If there is a $W_b\neq 0$ constant vector field in $M^3(\delta_0)$ such that

$$<b, W_b>= B, B\in\mathbb{R}_0^+.$$

Then x is called b-type slant helix and W_b is called the b-axis of x.

Theorem 5. Let x be b-type slant helix in $M^3(\delta_0)$. Then the b-axes of x are given as follows

$$W_b(s)=\left(-B\frac{\delta_0\delta_2}{\delta_3}\int\frac{\tau_g}{k_g}ds+c_3\right)x(s)-B\frac{\delta_1\delta_2}{\delta_3}\frac{\tau_g}{k_g}t(s)+\frac{B}{\delta_3}b(s), \tag{3.13}$$

where $c_3, B\in\mathbb{R}_0^+$.

Proof. From definition of the b-type slant helix and (3.4), we get

$$m_0^b=-B\frac{\delta_0\delta_2}{\delta_3}\int\frac{\tau_g}{k_g}ds+c_3;\ m_1^b=-B\frac{\delta_1\delta_2}{\delta_3}\frac{\tau_g}{k_g};\ m_2^b=0;\ m_3^b=\frac{B}{\delta_3}, \tag{3.14}$$

where $c_3, B\in\mathbb{R}_0^+$. Considering (3.14), we have (3.13).

Theorem 6. For the non-degenerate curve x to be b-slant helix in $M^3(\delta_0)$ the necessary and sufficient condition the following equation is provided

$$\delta_0\int\frac{\tau_g}{k_g}ds+\delta_1\frac{d}{ds}\left(\frac{\tau_g}{k_g}\right)+c_4=0, c_4\in\mathbb{R}_0^+. \tag{3.15}$$

Proof. From definition of b-type slant helix and using (3.4b), we get (3.15). Conversely, assume that (3.15) holds, we can write a vector field W_b as (3.13). Since $W_b'=0$, we obtain $<b, W_b>= B$. Hence, the theorem is provided.

Example 1. Let us consider a non-degenerate curve in $M^3(\delta_0)$ given by

$$x(s) = \left(0, \frac{1-\sqrt{5}}{2}\sinh s, \pm\sqrt{\frac{-1+\sqrt{5}}{2}}, \frac{-1+\sqrt{5}}{2}\cosh s\right),$$

the Sabban frame of x is given as

$$t(s) = \left(0, \frac{1-\sqrt{5}}{2}\cosh s, 0, \frac{-1+\sqrt{5}}{2}\sinh s\right); Q = \sqrt{\frac{-1+\sqrt{5}}{2}},$$

$$n(s) = \left(0, \frac{1-\sqrt{5}}{2Q}(\cosh s + \delta_0\delta_1 \sinh s), \pm 1, \frac{-1+\sqrt{5}}{2Q}(\sinh s + \delta_0\delta_1\cosh s)\right); b(s) = \left((3-\sqrt{5})\delta_0\delta_1, 0,0,0\right),$$

according to Theorem 1, Theorem 3 and Definition 3, Definition 4, the curve x holds helix and n-type helices whose axes are satisfied the following equations

$$\int (\delta_0 s + c_2)\tau_g(s)ds = \frac{c\delta_3}{sinhs\delta_2} - \frac{\coth s\delta_2}{\delta_1\delta_3},$$

$$\int \tau_g(s)ds\left(\frac{\tau_g}{k_g}(\cosh s + \delta_0\delta_1\sinh s) + \frac{\delta_1}{\delta_2}(\sinh s + \delta_0\delta_1\cosh s)\right) = c + \frac{d}{\delta_3\delta_2}; c, d \in IR$$

3. Conclusion

In this study, we examine helix and n-type (and b-type) slant helices due to the Sabban frame given in de Sitter 3-space or Anti de Sitter 3-space $M^3(\delta_0)$. We show that some results of the helix and slant helix curves generated by the Sabban frame in $M^3(\delta_0)$ are given above. We find parameter equation of axis of W of n-type (and b-type) slant helices in terms of the Sabban frame's vector fields.

References

[1] Ali AT, Lopez R. Timelike B_2-Slant Helices in Minkowski Space E_1^4. Archıvum Mathematicum(BRNO) Tomus, 2010; 46: 39-46.
[2] Ali AT. Position vectors of spacelike general helices in Minkowski 3-Space. Nonlinear Analysis 2010; 73: 1118-1126.
[3] Ali AT, Lopez R, Turgut M. k-type partially null and pseudo null slant helices in Minkowski 4-Space. Math. Commun. 2012; 17: 93-103.
[4] Ali AT, Turgut M. Some Characterizations of slant helices in the Euclidean E^n. arXiv: 0904.1187v1 2009; 8 pp.
[5] Barros M, Ferrandes A, Lukas P, Mero.no MA. General helices in the three dimensional Lorentzian space forms. Rocky Mountain J. Math 2001; 31(2): 373-388.
[6] Camci C, Ilarslan K, Kula L, Hacisalihoglu HH. Harmonic curvatures and generalized helices in E^n. Chaos, Solitons and Fractals 2009; 40: 2590-2596.
[7] Ferrandez A, Gimenez A, Lucas P. Null generalized helices in Lorentz-Minkowski Spaces. Journal of Physics A: Math. and General 2002; 35(39): 8243-8251.
[8] Gluck H. Higher curvature of curves in Euclidean space. Amer. Math. Monthly 1996; 73: 699-704.
[9] Izumiya S, Takeuchi N. New special curves and devalopable surface. Turk J. Math. 2004; 28: 153-163.
[10] Ilarslan K, Boyacioglu O. Position vectors of a timelike and a null helix in Minkowski 3-Space. Chaos Solitons and Fractals 2008; 38: 1383-1389.
[11] Kula L, Yayli Y, On slant helix and its spherical indicatrix. App. Math. and Computation 2005; 169: 600-607.
[12] Kulahci M, Bektaş M, Ergüt M. Curves of AW(k)-type in 3-dimensional null cone. Physics Letters A371 2007; 275-277.
[13] Kulahci M, Almaz F. Some characterizations of osculating curves in the lightlike cone. Bol. Soc. Paran. Math. 2017; 35(2): 39-48, 2017.
[14] Kulahci MA, Almaz F, Bektaş M. On helices and slant helices in the lightlike cone. Honam Mathematical J. 2018; 40(2): 305-314.
[15] Liu H. Curves in the lightlike cone. Contributions to Algebra and Geometry Volume 2004; 45(1): 291-303.
[16] Millman RS, Parker GD. Elements of differential geometry, Prentice-Hall Inc. Englewood Cliffs, N. J., 1977.
[17] Onder M, Kocayigit H. Kazaz M. Spacelike helices in Minkowski 4-Space E_1^4 Ann Univ. Ferrara 2010; 56: 335-343.
[18] O'Neill B. Semi-Riemannian Geometry, Academic Press, New York, 1983.
[19] Turgut M, Yilmaz S. Some Characterizations of type-3 slant helices in Minkowski Space-time. Involve 2009; 2(1): 115-121.